


Preparing Struggling Students (Teachers and Administrators) for the Common Core in Mathematics

*A Framework for Thinking about the Issue*

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Who is Ed Reform?

And When Will He Go Away?

1997 – 2013 PISA and TIMSS again and again

2008 National Math Panel Report

2007 NCTM Curriculum Focal Points

2001 Adding it Up

2000 PISA

2000 NCTM Standards

Late 1990s Riley Endorsements/ Mathematicians' Backlash

Mid to Late 1990s High Stakes State Testing


1995 TIMSS (Before Third became Trends)

Mid - 1990s NSF Curricula

America 2000 and Goals 2000 in the 1990s

1989 NCTM Standards

Math Policy and Practice



May 7, 2014



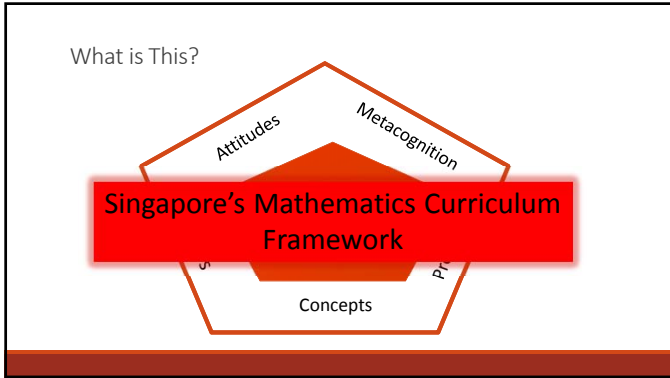
The Virtues of the Common Core

- Shared Expectations
  - Relationship between state tests and the NAEP: .23 (IES, 2005)
- Efficiency
  - Cut down on 50 state standards, curriculum, & assessments
- Quality of Assessments
  - \$330 m. USDE funding:
    - SMARTER Balanced Assessment Coalition
    - Partnership for Assessment of Readiness for College and Careers (PARCC)

*Common Core Standards: The New US Intended Curriculum* Porter, McMaken, Hwang, & Yang (April 2011)

Virtues of the Common Core

- A *Leaner* Set of Standards for all *Learners*
  - A response to the behavioral objectives/ laundry list of so many state standards
  - International benchmarking and previous research
    - "Fewer Topics, Greater Depth" *Facing the Consequences*, Schmidt et al. (1999)



### 2009 PISA Results: "OUR Sputnik Moment"

1. Shanghai	11. Belgium
2. Korea	12. Norway
3. Finland	13. Estonia
4. Hong Kong	14. Switzerland
5. Singapore	15. Poland
6. Canada	16. Iceland
7. New Zealand	17. United States
8. Japan	18. Lichtenstein
9. Australia	19. Sweden
10. Netherlands	20. Germany

### TIMSS 2011: Mathematics at Grade 8

"The 11 education systems with average mathematics scores above the U.S. score were:"

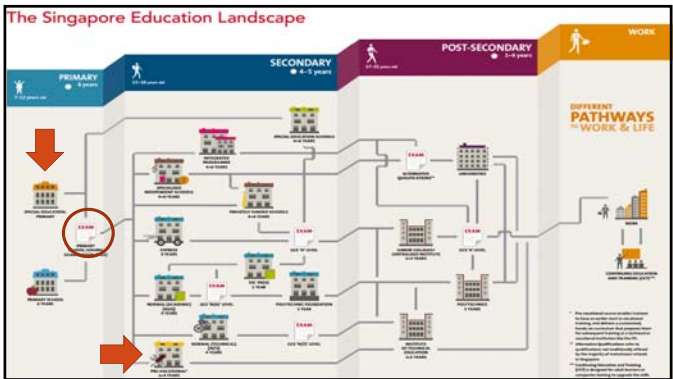
Korea	Massachusetts-USA
Singapore	Minnesota-USA
Chinese Taipei	Russia
Hong Kong	North Carolina-USA
Japan	Quebec
	Indiana-USA

Highlights from TIMSS 2011 US Department of Education

- ### Singapore's Long Road to Reform
- Survival Driven Phase: 1959 – 1978**
    - Organize schools
    - Move from low skill to high skill labor
    - Education was disorganized, quality of education was low
  - Efficiency Driven Phase: 1979 – 1996**
    - Focused effort to improve the quality of labor
    - Shift from one size fits all to multiple pathways
    - The Curriculum Institute created high quality textbooks
  - Ability-based, Aspiration Driven Phase: 1997 to Present**
    - Refine career paths for students and teachers
    - Refine teacher preparation
    - "Teach less, Learn more"

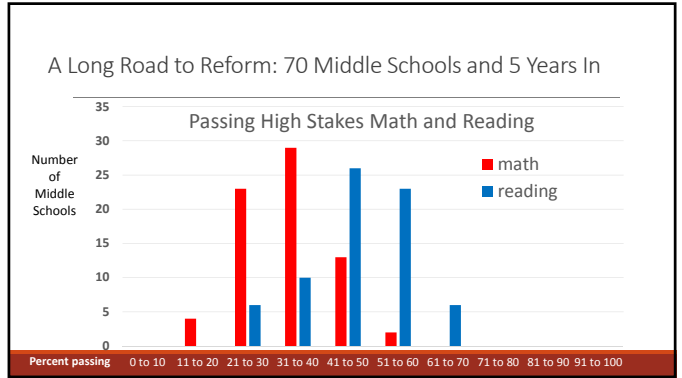
### Singapore: After 35 Years of Reform

*Thinking Schools, Learning Nation* encompassed a wide range of initiatives over a number of years that were designed to **tailor education to the abilities and interests of students**, to provide more **flexibility and choice** for students and to transform the structures of education.



### Math Policy and Practice

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### The Common Core and Diverse Student Performance

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The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in **abilities, needs, learning rates, and achievement levels** of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

### Intentional Design Limitations

**What the Standards do NOT define:**


- How teachers should teach
- All that can or should be taught
- The nature of advanced work beyond the core
- The interventions needed for students well below grade level**
- The full range of support for English language learners and students with special needs
- Everything needed to be college and career ready

COUNCIL OF CHIEF STATE SCHOOL OFFICERS (CCSSO), NATIONAL GOVERNORS ASSOCIATION CENTER FOR BEST PRACTICES (NGA CENTER), JUNE 2010

### You Can't Do This Until You Do This

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$$\frac{8}{1} \cdot \frac{1}{8} (3x + 4) = 5 \cdot \frac{8}{1}$$




A Robust Understanding of Algebraic Equations

### You Can't Do This Until You Do This

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$$\frac{8}{1} \cdot \frac{1}{8} (3x + 4) = 5 \cdot \frac{8}{1}$$



A Robust Understanding of Fractions

You Can't Do This Until You Do This

$$3x + 4 + -4 = 40 + -4$$



A Robust Understanding of Integers

You Can't Do This Until You Do This

$$\frac{3}{3}x = \frac{36}{3}$$



A Robust Understanding of Multiplication/Division

You Can't Do This Until You Do This

Concepts



Procedures

One Builds on the Other

And You JUST Can't Do This

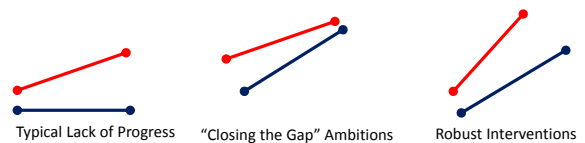


Fill Holes Independently

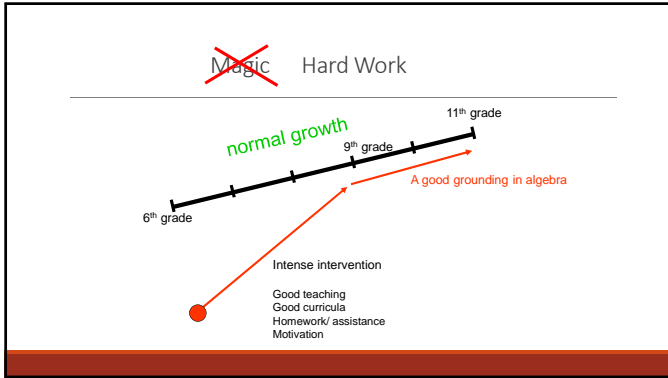
Translations

- Sequences are called **learning progressions**
- The problem of *different learning rates* is called **diverse learners**
- Catching everyone up by putting students *at grade level* is called **magic**

What Happens with Robust Interventions?



The Rhetoric and Reality of Gap Closing, Ceci & Papierno (2005), *American Psychologist*.



How Do We Begin to Think about The Needs of Low Struggling Students and Those in Special Education?

Which Perspective is "Right"?

One Mathematician's Perspective

Teach Division of Fractions "Conceptually"

$n \div m = k$  therefore  $n = mk$

by analogy

$\frac{a}{b} \div \frac{c}{d} = \frac{x}{y}$  therefore  $\frac{a}{b} = \frac{x}{y} \cdot \frac{c}{d}$

$\frac{a}{b} \cdot \frac{d}{c} = \frac{x}{y}$

$n \div m = \frac{n}{m}$

$\frac{m}{1} \cdot \frac{n}{m} = \frac{m}{1} \cdot k$

$\frac{m}{1} \cdot \frac{n}{m} = mk$

The Common Core – Grade 6

**The Number System** **6.NS**

**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**

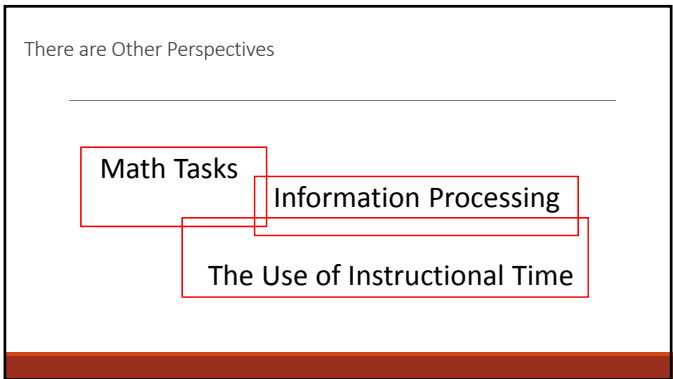
1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi?

The Common Core – Grade 6


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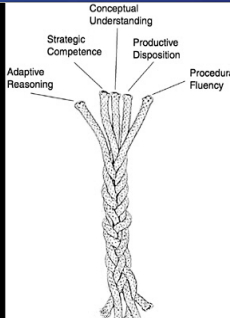
## Math Policy and Practice



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## Adding It Up (2001)

- 1) **Conceptual understanding** refers to the “integrated and functional grasp of mathematical ideas”, which “enables them [students] to learn new ideas by connecting those ideas to what they already know.” A few of the benefits of building conceptual understanding are that it supports retention, and prevents common errors.
- 2) **Procedural fluency** is defined as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Mental gymnastics- Flexibility with numbers.
- 3) **Strategic competence** is the ability to formulate, represent, and solve mathematical problems.
- 4) **Adaptive reasoning** is the capacity for logical thought, reflection, explanation, and justification.
- 5) **Productive disposition** is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

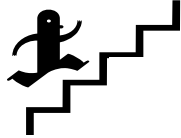


### Information Processing and Learning

- Schematic understanding and background knowledge
  - *How People Learn (2001)*
  - *How Students Learn (2005)*
- The role of metacognition in learning
- Types of memory: text/symbolic and visual
- The role of automaticity and proficiency in learning (*Adding It Up* = fluency)
  - Massed and distributed practice on reasonable skills
  - The role of timed practice

### Mary Kay Stein and Math Tasks

- Low Level Demands
  - Memorization
  - Procedures without connections
- High Level Demands
  - Procedures with connections
  - Doing mathematics



### What Is the Average of These Numbers?

8   9   10   11   12


Can you find 5 other numbers whose average is 10 using these two numbers?

2   14        

### Variations in Task Difficulty

- One of the Oldest Principles in Special Education: Insure High Levels of Success
- The Work of Alfred Bandura and Self-Efficacy
  - Consistently “easy” tasks lead to a false sense of self-efficacy
  - Students need variations in tasks so that:
    - They monitor their approach to / strategies for different kinds of tasks
    - Develop persistence

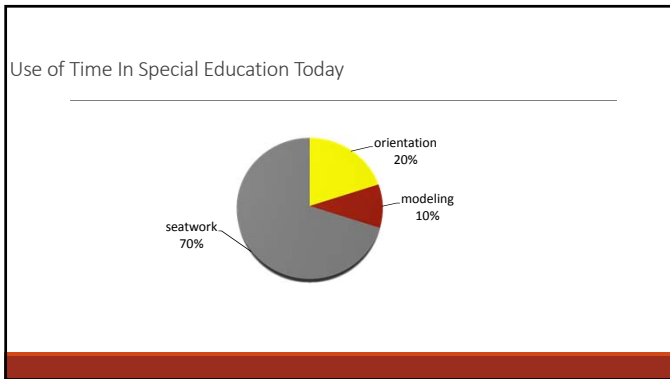
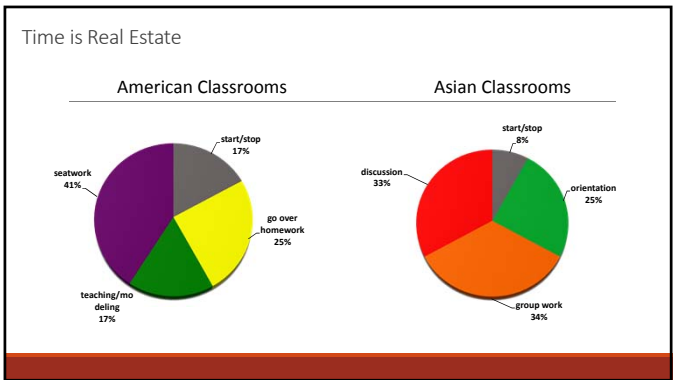
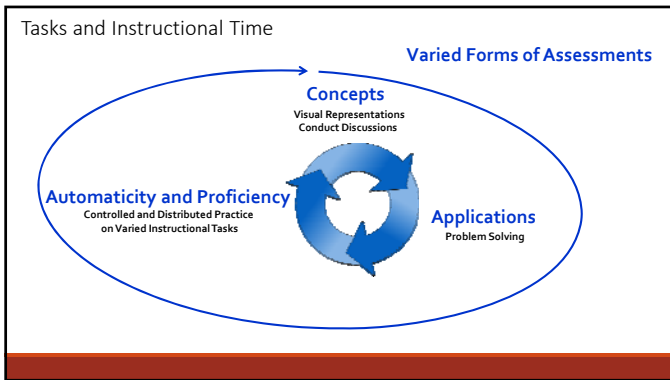
**IES Practitioner's Guide**  
**Math Problem Solving Grades 4-8**



WOODWARD, J., BECKMANN, S., DRISCOLL, M., FRANKE, M., HERZIG, P., JITENDRA, A., KOEDINGER, K. R., & OGBUEHI, P. (2012). *IMPROVING MATHEMATICAL PROBLEM SOLVING IN GRADES 4 THROUGH 8: A PRACTICE GUIDE* (NCEE #). WASHINGTON, DC: NATIONAL CENTER FOR EDUCATION EVALUATION AND REGIONAL ASSISTANCE, INSTITUTE OF EDUCATION SCIENCES, U.S. DEPARTMENT OF EDUCATION. RETRIEVED FROM [HTTP://IES.ED.GOV/NCEE/WWC/PUBLICATIONS/PRACTICEGUIDES](http://ies.ed.gov/NCEE/WWC/PUBLICATIONS/PRACTICEGUIDES)

**Key Recommendations**

1. Prepare problems and use them in whole-class instruction
2. Assist students in monitoring and reflecting on the problem-solving process
3. Teach students how to use visual representations
4. Expose students to multiple problem-solving strategies
5. Help students recognize and articulate mathematical concepts and notation

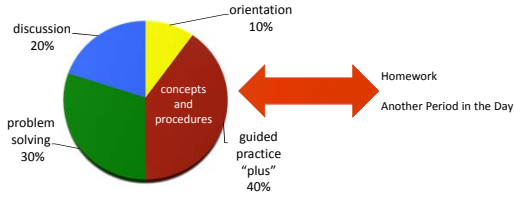


**Special Education Equations**

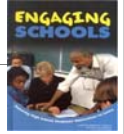
**Rate x Time = Distance**

at which you learn	amount of time on the type of math task(s)	progress through mathematics
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Implication for Automaticity and Proficiency Instruction



Homework and Giving Up



A survey of 13,000 8th graders

- 32% dropped out because they could not keep up with school work.

A survey of 100,000 7th through 11th grade students

- Students from families with low SES and students of color reported *less understanding* of teachers' lessons and comprehension of the material that they read for school.
- Although *they spent about as much time on homework* as the other students in the same classes, they were much less likely to complete their homework.
- In a compelling ethnographic study of urban high schools, one student put it this way, "Mr. Yana, when he talks I just can't follow what he's saying. *So I just give up.*"

Academic Press

Students do not have to be in the same classes

- Class size matters
- Attending to beliefs matters (self-efficacy)
- School connectedness matters



National Research Council  
Institute of Medicine  
of the National Academies (2003)

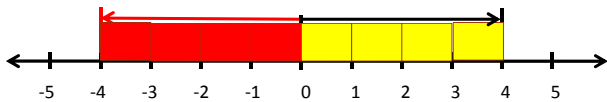
One Math Example: Taking Time to Make Sense

$$3x + 4 + -4 = 40 + -4$$



A Robust Understanding of Integers

When Opposites Don't Attract, They Cancel



Addition and Subtraction of Integers

- Subtraction of Integers: Where the challenge begins:

$$4 + -4 = 0$$

- Adding the Opposite Involves Cancellation



Subtraction of Positive Integers

$$4 + -4 =$$



Subtraction of Positive Integers

$$4 + -4 =$$



Subtraction of Positive Integers

$$4 - 4 = 0$$



Subtraction is Removal from a Set

Subtraction of Negative Integers

$$4 - -4 =$$

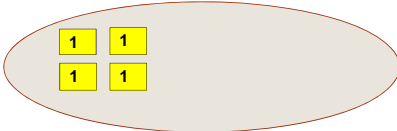
This is where understanding breaks down

Think in terms of subtraction:

You have a set of 4 and you are trying to remove 4 negatives from the set. What is the issue? What do we need to understand?

Subtraction of Negative Integers

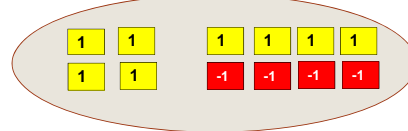
$$4 - (-4) =$$



There are no negatives to remove from the set.

Subtraction of Negative Integers

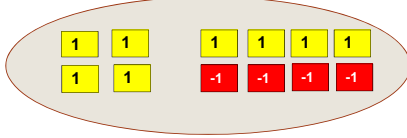
$$4 - (-4) =$$



We add  $4 + -4$  to the set, which equals 0. The set still has a value of 4.

## Subtraction of Negative Integers

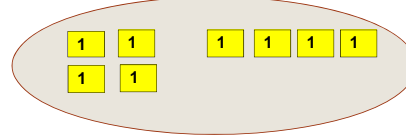
$$4 - (-4) =$$



We now have -4 to remove from the set.

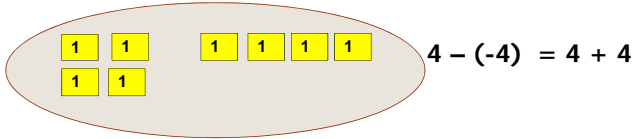
## Subtraction of Negative Integers

$$4 - (-4) = 8$$



## Subtraction of Negative Integers

$$4 - (-4) = 8$$



Notice that by adding the zero pairs and removing the negatives, the problem became an addition problem. You started with +4 and added +4 to this set.

## Massed and Distributed Practice is also Necessary

$$1 + 3 = \quad -1 + -3 = \quad -4 + 3 = \quad -1 - -3 = \quad 10 + -5 =$$

$$\frac{4}{8} + \frac{2}{3} = \quad \frac{7}{8} - \frac{2}{3} = \quad \frac{1}{5} \cdot \frac{2}{3} = \quad \frac{4}{8} \div \frac{2}{3} =$$

## Concluding Remarks

- This is hard work: *It takes time and intensity*
- There are no instant shortcuts to grade level mathematics for those who are significantly behind
- For struggling students, it's much more than symbol manipulations
  - Learning theory
  - Visual representations
  - Varied tasks