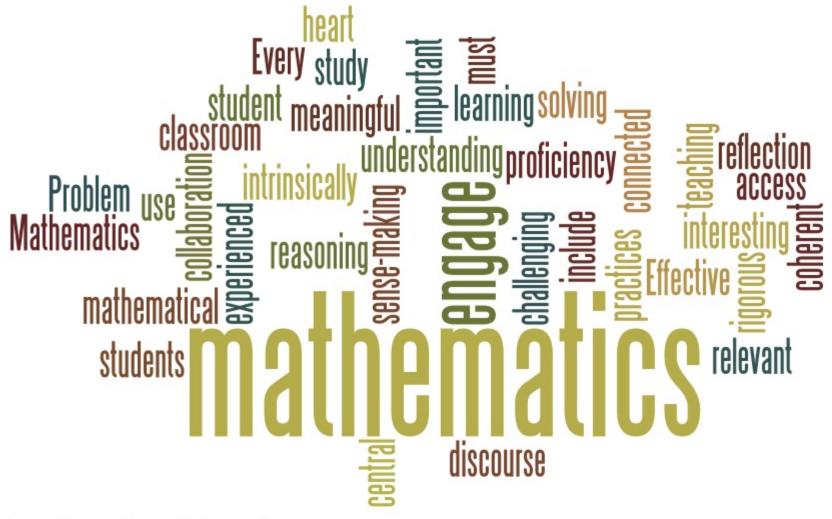


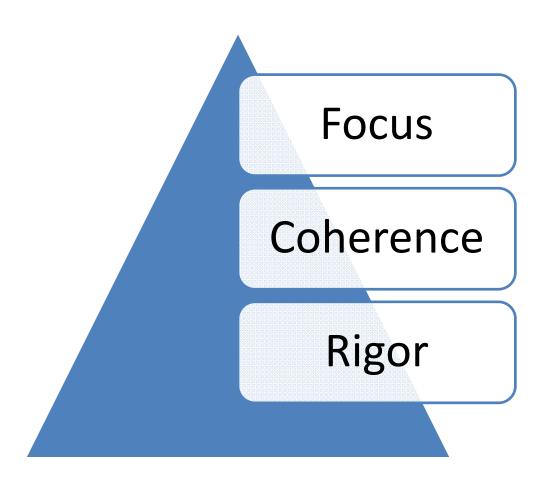


Mathematics



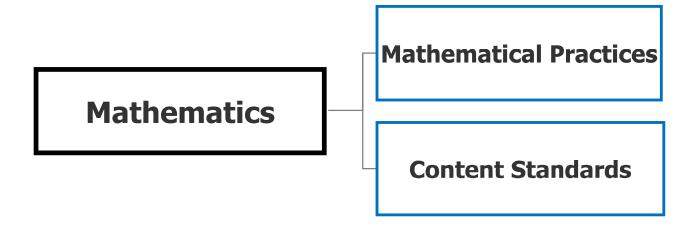


Key Shifts in Mathematics













Make sense of problems and persevere in solving them

> Reason abstractly and quantitatively

Construct viable arguments and critique the reasoning of others

Model with mathematics

Use appropriate tools strategically

Use appropriate tools strategically

Look for and make use of structure

Look for and express regularity in repeated reasoning

Mathematical Practices

Mathematics

Content Standards





1. Make sense of problems and persevere in solving them.

- Explain the meaning of the problem
- Identify appropriate strategies to solve the problem
- Check for reasonableness of solution

2. Reason abstractly and quantitatively.

- Recognize relationships between numbers and quantities
- Translate given information to create a mathematical representation

Mathematical Practice



3. Construct viable arguments and critique the reasoning of others.

- Use observation and prior knowledge to make conjectures and construct arguments
- Justify the approach used to solve a problem
- Compare and contrast logical arguments

4. Model with mathematics.

- Use a variety of models to represent and solve real-world problems.
- Simplify a complicated problem

Mathematical Practice



5. Use appropriate tools strategically.

- Select and use appropriate tools to best solve problems.
- Use estimation to predict reasonable solutions
- Compare and contrast logical arguments

6. Attend to precision.

- Understand symbols and use them
- Calculate answers efficiently and accurately
- Communicate using clear mathematical definitions, vocabulary, and symbols

Mathematical Practice



7. Look for and make use of structure.

- Use patterns to make sense of mathematics and connect to prior knowledge
- Analyze a complex problem by breaking it down into smaller parts

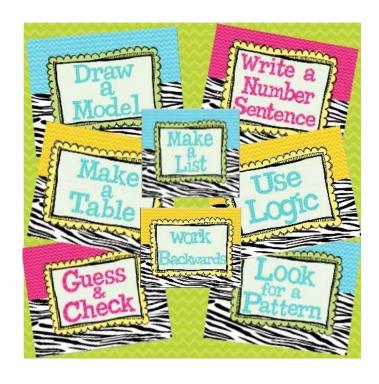
8. Look for and express regularity in repeated reasoning.

- Recognize similarities and patterns in repeated trials
- Generalize the process to create shortcuts which may lead to creating a formula.
- Communicate using clear mathematical definitions, vocabulary, and symbols

Mathematical Practices

Problem-Solving Strategies

- Look for a pattern
- Construct a table
- Make an organized list
- Act it out
- Draw a picture
- Use objects
- Work backwards
- Write an equation
- Solve a simpler problem
- Make a model

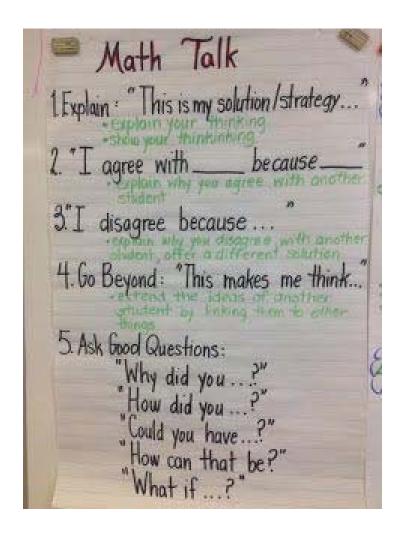


Marilyn Burns, 1992





Assessing Reasoning



Concepts and **Procedures**

Problem Solving Modeling and Data **Analysis**

Communicating Reasoning

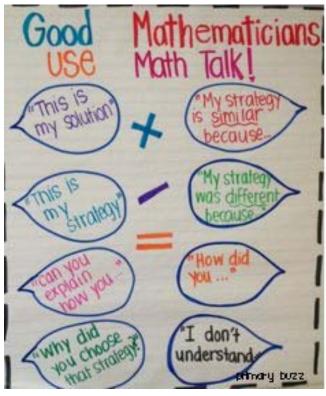
Student-centered instruction

 Have your students explain their reasoning in all instances

Encourage your students to talk with one another about their reasoning

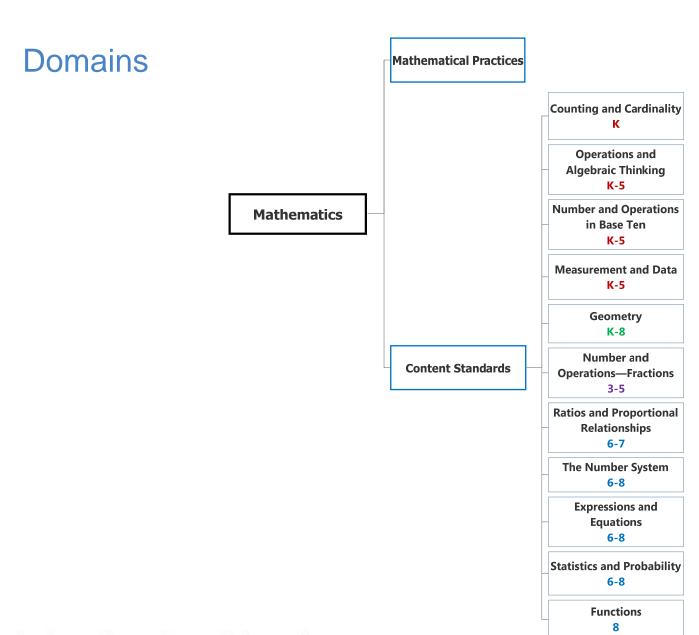
 Make writing an integral part of math learning

- Embed math activities in other context
- Use manipulatives whenever possible
- Make calculators available











Mathematical Content Standards

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

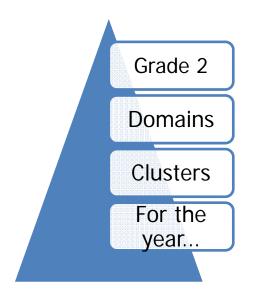
Standard

- Use place value understanding to round whole numbers to the nearest 10 or 100.
- 2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Cluster



Clusters



Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and Interpret data.

Geometry

Reason with shapes and their attributes.



Specific Standards

Operations and Algebraic Thinking

2.0A

Represent and solve problems involving addition and subtraction.

 Use addition and subtraction within 100 to solve one- and two-step. word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1

Add and subtract within 20.

Fluently add and subtract within 20 using mental strategies.² By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

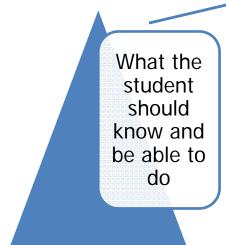
- 3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten

2.NBT

Understand place value.

- Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens called a "hundred."
 - The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- Count within 1000; skip-count by 5s, 10s, and 100s.
- Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
- Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.





Domain	Operations and Algebraic Thinking 1.0A	Operations and Algebraic Thinking 2.0A	Operations and Algebraic Thinking 3.OA
Cluster	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving multiplication and division.
Miles the student sout	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.		Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
Cluster		Work with equal groups of objects to gain foundations for multiplication.	
Standard		Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing	
What the student must know and be able to do		objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	
Standard		Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the	
What the student must know and be able to do		total as a sum of equal addends.	



Examples

	Result Unknown Change Unknown		Start Unknown		
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5		
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ?-2 = 3		
	Total Unknown	Addend Unknown	Both Addends Unknown ¹		
Put Together/ Take Apart ²	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1		
			5 = 2 + 3, 5 = 3 + 2		
	Difference Unknown	Bigger Unknown	Smaller Unknown		
Communi	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?		
Compare ³	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5		



Domain	Operations and Algebraic Thinking 1.0A	Operations and Algebraic Thinking 2.OA	Operations and Algebraic Thinking 3.OA
Cluster	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving multiplication and division.
Standard What the student must know and be able to do	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
Cluster		Work with equal groups of objects to gain foundations for multiplication.	
Standard		Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing	
What the student must know and be able to do		objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	
Standard What the student must know and be able to do		Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	



Domain	Operations and Algebraic Thinking 1.OA	Operations and Algebraic Thinking 2.OA	Operations and Algebraic Thinking 3.0A
Cluster	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving multiplication and division.
Standard What the student must know and be able to do	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
Cluster		Work with equal groups of objects to gain foundations for multiplication.	
Standard What the student must know and be able to do		Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	
Standard What the student must know and be able to do		Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	



Examples

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division) ? × 6 = 18, and 18 ÷ 6 = ?		
	3 × 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?			
	There are 3 bags with 6 plums in each bag. How many plums are there in all?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?	If 18 plums are to be packed 6 to a bag, then how many bags are needed?		
Equal Groups	Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?		
Arrays,4 Area ⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?		
	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?		
Compare	Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?		
General	a x b = ?	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$		



Step 1

Start with the big picture

What are the domains and clusters taught at the grade level?

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and Interpret data.

Geometry

· Reason with shapes and their attributes.



Step 2

What are the standards within the clusters that students must know and be able to do?

Operations and Algebraic Thinking

2.OA

Represent and solve problems involving addition and subtraction.

 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

- Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal
- 4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten

2.NBT

Understand place value.

- 1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens called a "hundred."
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- Count within 1000; skip-count by 5s, 10s, and 100s.
- Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
- Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.



Step 3

Examine what is taught a grade level below and a grade level above for the domain and cluster.

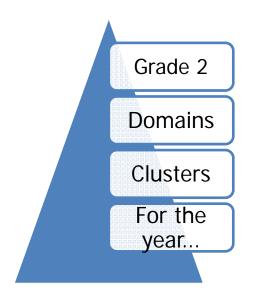
Domain	Operations and Algebraic Thinking 1.0A	Operations and Algebraic Thinking 2.OA	Operations and Algebraic Thinking 3.OA
Cluster	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving multiplication and division.
Standard What the student must know and be able to do	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7 .
Cluster		Work with equal groups of objects to gain foundations for multiplication.	
Standard		Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing	
What the student must know and be able to do		objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	
Standard What the student must know and be able to do		Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	



PLANNING



Clusters



Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and Interpret data.

Geometry

Reason with shapes and their attributes.



Pre-planning: Identifying Your Resources

Mathematics	Title
Core Curriculum	Everyday Math Math Connects Saxon Math Houghton-McGraw Holt Mathematics
Intervention Curriculum	Vmath DoTheMath
District Resources	First In Math (online)
Other	Kahn Academy Videos www.ixl.com



Planning: Mapping out your domains and clusters for the year

Suggested Pacing	Operations & Algebraic Thinking	Number & Operations in Base Ten	Measurement & Data	Geometry
Quarter 1 (45 days)	Add & Subtract Within 20 (1 standard)	Understand Place Value (4 standards)	Measure and estimate lengths in standard units. (4 standards)	
Quarter 2 (45 days)	Represent and solve problems involving addition and subtraction. (1 standard)	Use place value understanding and properties of operations to add and subtract. (5 standards)	Relate addition & subtraction with length (2 standards)	
Quarter 3 (45 days)			Work with time and money (2 standards)	Reason with shapes and their attributes (3 standards)
Quarter 4 (45 days)	Work with equal groups of objects to gain foundations for multiplication. (2 standards)		Represent and interpret data. (2 standards)	

Planning: Unpacking the standard for lesson plans

Unpacking Standards for Instruction

Mathematics 2.OA.2

Domain: Operations and Algebraic Thinking (OA)

Cluster: Add and subtract within 20

Standard: Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.

Prerequisites skills:

 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. (First Grade)

 Fluently add and subtract within 5.

(Kindergarten)

Vocabulary:

Count Combine Sum Take away Put together

How is this knowledge/concept/skill assessed on state/district exam?

Write an addition sentence based on the picture (for example, 2 + 1 = 3).



Resources:

Math Expressions - Addition/Subtraction - pages 23-25

www.ixl.com

VmathLive



Planning: Documenting student mastery

Kindergarten Mathematics						
	Counting a	nd Cardinality				
Evidence of Mastery	Know number names and the count sequence.	Count to 100 b yones and by tens.	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).		Count to tell the number of objects.	Understand the relationship between numbers and quantities; connect counting to cardinality.
Student 1						
Student 2						
Student 3	8					
Student 4						
Student 5						
Student 6						
Student 7						
Student 8	5					
Student 9	8					
Student 10	2			-		
Student 11	8			-		
Student 12	8			1		
Student 13		I	1			



Further Study

The Hunt Institute – YouTube Videos

https://www.youtube.com/user/TheHuntInstitute#g/u

- The Importance of Coherence in Mathematics
- The Mathematics Standards: How They Were Developed and Who Was Involved
- Common Core State Standards: Principles of Development
- The Crucial Role of Higher Education and Business in Developing the Standards
- The Mathematics Standards: Key Changes and Their Evidence



ASSESSMENTS



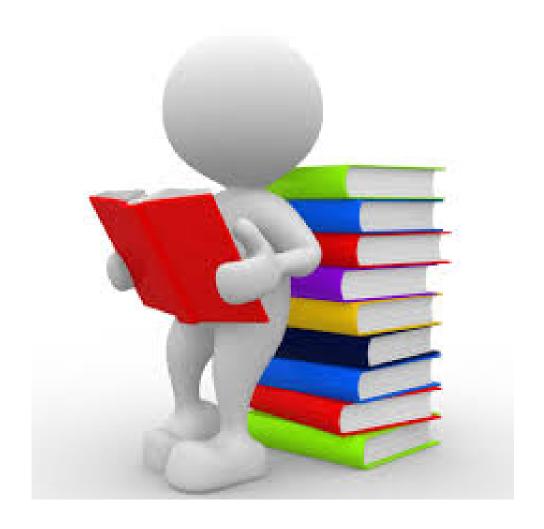


http://www.smarterbalanced.org/



http://www.parcconline.org/







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