Grades 2–8

Vmath®

Research Foundation

VOYAGER SOPRIS LEARNING®

Vmath Research Foundation

Table of Contents

Introduction

Performance on the National Assessment of Educational Progress (NAEP) provides information on student performance at the national and state levels (National Center for Education Statistics, 2013). NAEP results for 2013 indicate only 42% of fourth-grade students, 35% of eighth-grade students, and 26% of 12th-grade students are performing at a proficient level in mathematics. Research (e.g., Cross & Hynes, 1994; Forbringer & Fuchs, 2014; Gersten, Beckmann, et al., 2009) indicates many students do not learn effectively from the instruction provided in their regular math classes and may benefit from early interventions. They struggle continually with the math concepts, skills, and strategies necessary for success. These students often fall even further behind as their core program moves on to more difficult material. They score poorly on important tests, have to attend summer school, and may even be denied advancement to the next grade level. However, examples in the literature (Campbell, 1995; Griffin, Case, & Siegler, 1994; Forbringer & Fuchs, 2014; Silver & Stein, 1996) demonstrate that all children, including those who have been traditionally underserved, can learn mathematics when they have access to high-quality instructional programs that support their learning.

This paper introduces the program *Vmath*®. After describing the consequences of poor mathematical abilities, the guiding principles for assisting students who struggle with mathematics will be discussed. Following this discussion, the aims and components of the *Vmath* curriculum will be identified. The aim of this paper is not to provide a guide for using the *Vmath* program, but rather to describe the research and rationale behind the program.

Consequences of Poor Mathematical Abilities

On an individual basis, students who do not acquire basic competencies in mathematics before leaving high school will be disadvantaged in the workforce and in their ability to function in routine day-to-day activities (Geary, Hoard, Nugent, & Bailey, 2012). Routine activities could include grocery shopping, paying taxes, and meeting college prerequisites (Thompson, n.d.). Christie (2013) reports math-challenged homeowners are five times more likely to default on their loans. Workman (2014) reports the lack of basic math skills can result in severe money problems, noting arithmetic is essential for budgeting and understanding percentages and interest rates. Mielach (2012) reports shoppers with poor basic math skills, unable to determine the best bargain, are often selecting the more expensive options when faced with converting percentages.

Weak knowledge of fractions also can have large, long-term, consequences (Bailey, Hoard, Nugent, & Geary, 2012; Jordan et al., 2013; Siegler & Pyke, 2013). Fractions are essential for learning algebra and more advanced mathematics. They are also important for tasks such as managing personal finances, doing home repairs, and understanding rate of change. Siegler et al. (2012) found fifth-grade students' knowledge of fractions uniquely predicted students' knowledge of algebra and overall mathematics achievement in high school, even after controlling for other types of mathematical knowledge, verbal and nonverbal IQ, reading comprehension, working memory, and family income and education. It follows then that students who do not succeed in algebra are less likely to graduate from college than higher-achieving students and will have few career opportunities in science, technology, engineering, and mathematics (STEM) disciplines (Jordan et al., 2013).

According to the *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (National Mathematics Advisory Panel [NMAP], 2008), "sound education in mathematics across the population is a national interest" (p. xii). Having command of mathematical skills has brought societies advantages in "medicine and health, in technology and commerce, in navigation and exploration, in defense and finance, and in the ability to understand past failures and to forecast future developments" (NMAP, 2008, p. xi). The future workforce will undoubtedly have to handle increasingly complicated quantitative concepts in all areas of employment. The World Bank's *STEP Skills Measurement—Snapshot* 2014 (Valerio, Sanchez Puerta, Pierre, Rajadel, & Taborda, 2014) points out that "technology has shifted the focus from the qualifications needed for doing a 'life-time' job to the skills needed to do specific tasks in jobs that are constantly changing" (p. 2). The World Bank report concludes that "more than ever a person has to be equipped with a solid foundation of multiple generic skills that enable further acquisition of job-specific (technical) skills and adaptation to repeated employment changes over the work life" (Valerio et al., 2014, p. 2).

Guiding Principles for Assisting Students Who Struggle with Mathematics

Different types of sources have been influential in the development of *Vmath*. Standards had a direct effect on what skills and concepts were included in *Vmath*. Standards considered in the *Vmath* development included state standards from Texas, Florida, California, and Virginia, and the *Common Core State Standards Mathematics* (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Additionally, various reports and practice guides were used because they reviewed the available research and made recommendations for effective practices for mathematics instruction. Both of these sources are described below.

Larson (2012) describes the use of standards as an effort to reform mathematics instruction since the 1950s. The standards effort began with "new math" in the 1950s and 1960s, followed by the "back-to-basics" movement in the 1970s. For the next 30 years, the National Council of Teachers of Mathematics (NCTM) led the way with a call for a greater emphasis on problem solving and higher standards for all students, beginning with the groundbreaking report *An Agenda for Action* (1980) through the seminal report *Principles and Standards for School Mathematics* (2000).

While most states used the principles and standards outlined in *Principles and Standards for School Mathematics* (NCTM, 2000) for teaching mathematics and developing state standards (Powell, Fuchs, & Fuchs, 2013), many are now embracing the CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Porter, McMaken, Hwang, and Yang (2011) studied the differences and commonalities between state standards and CCSSM. They found that the CCSSM emphasized higher-level thinking and conceptual understanding over memorization and procedures. While the CCSSM outlines highly specific standards by grade level, the NCTM standards do not enumerate as many skills, and skills are described in grade bands. Both sets of standards cover the same larger categories of skills: numbers, operations, algebra, measurement, geometry, and data analysis. Powell et al. (2013) draw the conclusion that both of these sets of standards have had and are having an influence on mathematics instruction in the United States.

Three reports were most influential in the development of *Vmath*. They are *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (NMAP, 2008); *Mathematics Instruction for Students with Learning Disabilities or Difficulty Learning Mathematics: A Guide for Teachers* (Jayanthi, Gersten, & Baker, 2008); and *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (Gersten, Beckmann, et al., 2009). Forbringer and Fuchs (2014) combined information from these reports to produce the following guidelines for effective mathematics instruction.

Core instruction recommendations:

1. Emphasize critical concepts: For K–8, streamline to emphasize critical topics. Schmidt, Wang, and McKnight (2005) found international comparison of concepts covered in core curricula showed high-performing nations covered five or six concepts compared with more than 20 for U.S. students.
2. Teach critical foundations to mastery: NMAP (2008) indicated some skills need to be mastered rather than being revisited year over year without mastery. Some skills and concepts are prerequisites for more advanced mathematics and therefore need to be mastered.
3. Balance conceptual understanding, fluency, and problem solving: These are mutually supportive and important components.
4. Use a combination of teacher-centered and student-centered approaches: Inquiry methods, which are more student centered, are generally taught by general education teachers; and direct or explicit models of instruction, which are more teacher centered, are generally taught by special education teachers. A balance of the two approaches is most effective.
5. Follow the CRA sequence: The progression is known as concrete-representational-abstract (CRA) sequence (Peterson, Mercer, & O'Shea, 1988; Sousa, 2007; Witzel, 2005). Younger and older students benefit from progressing from concrete manipulations to representational models to abstract symbols when understanding occurs.

Intervention recommendations to support students who struggle in mathematics:

1. Teach critical content during interventions:
    a. Focus on whole numbers and rational numbers: Gersten, Beckmann, et al. (2009) recommend focusing intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4–8. When older students struggle with whole numbers and operations, they would benefit from in-depth coverage as well. As students in grades 4–8 focus on rational numbers, they should also focus on advanced topics in whole number arithmetic, such as long division.

   b. Basic facts: Studies (Geary, 1993, 2003; Goldman, Pellegrino, & Mertz, 1988; Hasselbring, Goin, & Bransford, 1988; Jordan, Hanich, & Kaplan, 2003) have found students who struggle in mathematics often lack automaticity with basic facts, leading to a lack of cognitive energy to focus on and understand new concepts.
   c. Problem solving: Solving mathematical word problems is often very difficult for students who struggle with mathematics (Geary, 2003; Hanich, Jordan, Kaplan, & Dick, 2001). Students who struggle with mathematics often do not understand the first step of Polya's (1957, 2004) four-step process, Understanding the Problem. Also, students lack the strategic knowledge necessary for the second step, Devising a Plan, to solve the problem. Teaching students the underlying structures allows them to learn and then recognize problem patterns, increasing problem solving performance.
2. Effective instructional methods for struggling students:
   a. Explicit instruction: Studies (Gersten, Chard, et al., 2009; Jayanthi et al., 2008; NMAP, 2008) show learning increases for students who struggle with mathematics when systematic, explicit instruction is used by teachers. It is still suggested that a balanced approach between inquiry and explicit instruction be used for all students, however the emphasis should be on explicit instruction for students who struggle.
   b. Manipulative and visual representation: Hecht, Vogi, and Torgesen (2007) found students who struggle with mathematics also have difficulty understanding abstract symbols. For these students to gain conceptual understanding, instruction that follows the CRA continuum is beneficial. Students first need to experience the concepts by dramatizing problems or using manipulatives. After mastering a skill using manipulatives, students are ready to use two-dimensional representations before moving to abstract numbers and symbols to solve problems.
   c. Motivation: Students who struggle with mathematics have generally experienced failure or frustration when they try problems and therefore approach mathematics with trepidation. Using motivational strategies can improve learning outcomes (see, for example, Epstein, Atkins, Cullinan, Kutash, & Weaver, 2008; Fuchs et al., 2005; Marzano, Pickering, & Pollock, 2001). Epstein et al. (2008) suggest offering "a variety of activities and materials at a pace and level of difficulty appropriate to the range of student abilities in the class" (p. 23). Sternberg (2005) contends without motivation, a student will never try. Relating mathematical content to students' previous experiences and personal interests will make the information more meaningful and relevant (NCTM, 2000).

Aims for *Vmath*

Vmath is a targeted math intervention program for struggling students in grades 2–8, reinforcing grade-level expectations and standards. Meeting the Response to Intervention (RtI) requirements of Tier II instruction (Forbringer & Fuchs, 2014), *Vmath* supports students who are receiving core instruction but need additional practice and reinforcement of the grade-level skills. The third edition of *Vmath* is specifically designed to reinforce the performance expectations of rigorous new standards, with special attention given to whole numbers in grades 2–5 and rational numbers in grades 4–8. The second edition of *Vmath* is more appropriate for students who are several years behind grade level and lack many foundational skills. While this paper focuses mostly on the third edition of *Vmath*, many of the topics discussed are also relevant to the second edition.

The aims for *Vmath* include providing an easy delivery model for teachers with detailed lesson support to address the needs of students requiring more instruction to meet new, more rigorous standards. *Vmath* builds on the progressions of skills, incorporating conceptual development, procedural understanding and practice, corrective feedback, fluency practice, and use of concrete and virtual manipulatives. *Vmath* continues to provide foundational lessons to help scaffold the instruction.

Vmath Components and Design

Vmath is designed using widely accepted principles of effective instruction for students struggling with mathematics. This would include students who have been determined to have a mathematical learning disability and those who have persistently low mathematical achievement (Geary, 2011). The direct, systematic instruction in *Vmath* provides carefully sequenced, specific, and detailed dialogue for every lesson, allowing teachers to clearly model solutions to specific problems. The discussion that follows features the topics of most importance to an intervention curriculum.

Explicit Instruction

Rosenshine (1983), in a review of research on teacher effectiveness, concluded that highly interactive, briskly paced, clearly presented instruction was related to high rates of student success, and he referred to this type of teaching as direct instruction (explicit, teacher-directive practices). The term *direct instruction* is generally used to refer to the instructional theory work of Engelmann and Carnine (1991). Research provides consistent support for using an explicit approach to teaching mathematics. Adams and Engelmann (1996) analyzed 34 intervention studies and found this approach to be more successful in 32 of the 34 studies they reviewed. Bottge (2001) stated teachers should continue to foster competence in basic skills by providing students explicit instruction, and Kroesbergen and Van Luit (2003) reported in a study of 58 research reports that direct instruction was found to be more effective than mediated instruction.

Explicit systematic instruction is defined by NMAP (2008) as "teachers explaining and demonstrating specific strategies and allowing students many opportunities to ask and answer questions and to think aloud about the decisions they make while solving problems" (p. 48). Additionally, explicit systematic instruction involves the sequencing of problems to highlight critical features. Doabler et al. (2012) indicates regular use of teacher modeling and demonstrations, visual representations of math ideas, frequent opportunities for student practice, and instructional scaffolding are part of explicit and systematic mathematics instruction.

Vmath based its form of highly structured lesson delivery on the guiding principles of effective instructional design described by Stein and colleagues (Stein, Kinder, Silbert, & Carnine, 2006; Stein, Silbert, & Carnine, 1997) and recommended by NMAP (2008). *Vmath* lessons use a four-step format: Get Started allows for teacher modeling (I do); Try It Together with student and teacher interaction during practice (We do); Work On Your Own that includes independent student work (You do); and Check Up with error analysis during each lesson. The detailed explicit instructions in every lesson allow teachers to deliver the lesson comfortably and quickly and allow students to hear the consistency of the math language being used.

Conceptual Understanding

Conceptual knowledge is the linking or defining of relationships between pieces of information. This implies understanding the meaning and underlying principles of mathematical concepts (Jones, 2012). Conceptual understanding is also associated with deep, flexible knowledge that is associated with operational or procedural step-by-step understanding (Jones, Inglis, Gilmore, & Hodgen, 2013). Conceptual understanding, computational fluency, and problem-solving skills are all essential and mutually reinforcing for students to be successful in mathematics (NMAP, 2008).

NCTM (2000) recommended that students be able to use manipulatives in order to help develop understanding of mathematical concepts. Carbonneau, Marley, and Selig (2013) summarized four theoretical explanations of how manipulatives facilitate learning: (a) by supporting the development of abstract reasoning; (b) by stimulating the students' real-world knowledge; (c) by providing the learner with an opportunity to enact the concept for improved encoding; and (d) by providing opportunities for students to discover mathematical concepts through exploration. Fuchs, Powell, Hamlett, and Fuchs (2008) found some combination of manipulatives and visual representations may promote conceptual understanding of mathematics.

Vmath uses multiple types of instruction promoting conceptual development. The Hands-On lessons make use of concrete and representational manipulatives to help students develop a deeper understanding of the targeted mathematical concept. Concrete manipulatives include base-10 pieces, fraction strips, counters, rulers, unit cubes, and algebra tiles. Additionally, copy masters of nets, decimal grid paper, dot paper, and grid paper are provided for these lessons.

The Build the Concept boxes use visual models to communicate the mathematical concept. Along with the model, the teacher is provided with explicit language to help students connect the visual representations to the standard symbolic representations used in mathematics. These pictorial representations in the Build the Concept boxes reinforce the instruction from the core curriculum and help students use the numbers and symbols in problems.

The *Gizmos*® lessons provide carefully sequenced lessons built into the *Vmath* modules. These online, virtual manipulatives provide interactive simulations that can be accessed anytime, anywhere. The Get Ready section of these lessons reviews the prerequisite skills needed to make the most of the *Gizmos*. In the Discover section, teachers guide students to discover important math concepts. The specific step-by-step instruction takes the guesswork out of using the *Gizmos*.

With the use of the multiple types of instruction, *Vmath* teaches the underlying concepts, or big ideas, needed for conceptual understanding of mathematics. The instruction provides the structure by which students understand the transition from concrete to representational to abstract, recommended by NMAP (2008) and shown to be effective for young and older students (Peterson et al., 1988; Sousa, 2007; Witzel, 2005) and students who struggle to understand abstract symbols (Hecht et al., 2007).

Procedural Knowledge and Fluency

A procedure is defined by Rittle-Johnson and Schneider (in press) as a series of steps, or actions, done to accomplish a goal. They go on to state "procedures can be (1) algorithms—a predetermined sequence of actions that will lead to the correct answer when executed correctly, or (2) possible actions that must be sequenced appropriately to solve a given problem (e.g., equation-solving steps)" (Defining Conceptual and Procedural Knowledge section, para 7). It follows that knowledge of procedures, or procedural knowledge, is the ability to execute action sequences (i.e., procedures) to solve mathematical problems from basic facts to complex equations.

NMAP (2008) states "conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations together support effective and efficient problem solving" (p. 26). Computational facility with whole number operations requires students have automatic recall of addition and related subtraction facts, and of multiplication and related division facts. The ability to use the algorithms efficiently requires the automatic recall of number facts. This ability to recall number facts automatically reinforces the fluent use of algorithms as well.

Gersten, Beckmann, et al. (2009) state quick retrieval of basic arithmetic facts (addition, subtraction, multiplication, and division) is critical for success in mathematics. Research (Geary, 2004, 2011; Geary, Hoard, & Bailey, 2012; Jordan et al., 2003; Price, Mazzocco, & Ansari, 2013) finds students with mathematics difficulties exhibit immature procedural strategies and are not fluent in basic math facts. The inability to retrieve math facts in a timely way is likely to impede student understanding of concepts being taught. The recommendation is to add time or devote part of the intervention session to practicing and becoming proficient in recall of basic math facts.

Procedural knowledge and fluency are taught and reinforced in multiple ways in *Vmath*. During lessons, students learn strategies and are given tools to help them remember the procedures of the strategies. The How To boxes in the lessons provide reinforcement and practice of the procedures and strategies. The Build the Concept boxes help students analyze what they are learning and how the procedure applies to the concept being taught. Each of the four-step *Vmath* lessons has an Extra Practice page that reinforces automaticity of the concepts and skill students are learning. Practice of basic math facts can be best accomplished using *VmathLive*®, the online program supplementing *Vmath*. In the Play section of *VmathLive*, students enter a live, online competition with other students. As they play, students increase their mental math skills, developing and strengthening computational fluency.

Problem Solving

Being able to apply mathematical knowledge to solve real-life problems is the main reason for learning mathematics (Forbringer & Fuchs, 2014). NCTM (2000) defines problem solving as "engaging in a task for which the solution method is not known in advance" (p. 52). Research (i.e., Geary, 2003; Hanich et al., 2001) has shown students who struggle with mathematics have difficulty solving word problems. Polya's (1957, 2004) four-step process to solving problems is often used as a model for mathematics instruction. Students who struggle with mathematics have difficulty in the first step, Understanding the Problem, and in the ability to address the second step, Devising a Plan, due to a lack of strategic knowledge (Carnine, Silbert, Kame'enui, & Tarver, 2009; Forbringer & Fuchs, 2014).

The metacognitive competencies required for problem solving prove to be difficult for students who struggle with mathematics (Forbringer & Fuchs, 2014). These metacognitive competencies include:

> (1) processing the language of the problem and understanding what is being asked, 2) identifying and organizing relevant information, 3) selecting a problem-solving strategy, 4) remembering and executing the strategy steps in the proper sequence, 5) performing necessary computations and accurately recording solutions, and finally 6) checking to make sure the computation was executed successfully and that the answer makes sense (Forbringer & Fuchs, 2014, p. 216).

Gersten, Beckmann, et al. (2009) recommend systematic explicit instruction on solving word problems with a focus on the problems' underlying structure, specifically problem types with similar mathematical structures. Additionally, Gersten, Beckmann, et al. recommend systematic explicit instruction on the structural connections between familiar and unfamiliar problems so students will know when to apply the solution methods they have learned. With a superficial format change, key vocabulary changes, or the inclusion of irrelevant information, students might believe they are looking at a new and unfamiliar problem when in fact they are able to apply a solution that is already part of their repertoire.

Vmath makes problem solving an essential component in two ways. First, in every module and level, there are lessons that are entirely devoted to teaching problem-solving strategies. Second, problem-solving strategy practice is incorporated into many of the *Vmath* lessons, including the teacher-explicit language needed to guide the students toward solving the problem using the appropriate strategy.

Vmath makes use of Polya's (1957, 2004) four-step problem-solving process: Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking Back. Explicit instruction is coupled with each step, ensuring, in particular, students are able to recognize the underlying structure in the first step, Understanding the Problem, and apply the appropriate solution method in the second step, Devising a Plan. In the Looking Back step, students practice strategies enabling them to check every answer to be sure it makes sense; eliminate unreasonable answers to increase the probability of selecting the correct answer; and get correct answers to problems they thought they could not solve. Along with presenting high-utility problem-solving strategies during the lessons, *Vmath* also makes extensive use of visual representations during problem-solving instruction to strengthen student understanding of the relationships between mathematical ideas and abstract symbols (Hecht et al., 2007).

Use of Assessments

NMAP (2008) recommends the use of formative assessment, defined as "the ongoing monitoring of student learning to inform instruction" (p. 46), as a requirement for effective instruction. Additionally, the report states that when teachers are provided with specific suggestions on how to use the assessment data to provide differentiated instruction, the effect is significant. Further, the report indicates the use of formative assessment can lead to increased precision in how the instructional time is used in a classroom or intervention period. Information derived from formative assessments can assist teachers in identifying specific student needs.

Gersten, Beckmann, et al. (2009) recommended monitoring student progress for those receiving supplemental instruction and for other students who may be at risk. The level of evidence for this recommendation was low due to the lack of studies meeting the standards for inclusion, but the panel's expert opinion was exercised, so the recommendation was included. Based on an RtI model, Gersten, Beckmann, et al. recommend monitoring progress of students receiving instruction in Tier 2 and 3 and borderline Tier 1 at least once a month. Additionally, the use of curriculum-embedded assessments in interventions can be used as often as every day or as infrequently as once every other week.

Vmath provides multiple opportunities to assess students, leading to reinforcement, differentiated instruction, and grouping recommendations. The Progress Assessment of Mathematics, powered by the Quantile Framework® for mathematics, are administered three times a year to monitor student progress. These tests cover all five content strands in the NCTM (2000) standards—number sense and operations, algebra, geometry, measurement, and data analysis and probability. Each test yields a Quantile measure that can be used to inform teachers about the mathematical skills and content students are ready for next. Nearly every lesson has a Quantile measure that can be compared to the students' Quantile scores to determine appropriate material and to help determine pacing for a lesson.

The next assessment opportunity is the Initial Assessment that can be given to an entire class or a small group of students at the beginning of *Vmath* instruction. This assessment identifies strengths and weaknesses of individual struggling students and, combined with other district information and criteria, determines appropriate entry points into instruction. The Initial Assessment acts as an indicator of the skills taught in each *Vmath* module, showing where students begin to struggle with the material. The Final Assessment is administered at the completion of the *Vmath* course of instruction or at the end of the school year. Comparison of the scores from the Initial Assessment and the Final Assessment are an indication of student growth and mastery.

The last formative assessments are the Module Pre-Tests and Post-Tests. These assessments monitor student growth and mastery of concepts, skills, and strategies taught in each module. Using data from the Module Pre-Test, the teacher can determine students' prior knowledge of the module content. If students or a group of students score less than 70% correct on the Module Pre-Test, this is an indication additional skill acquisition from the Preskill Lessons is warranted. The Module Post-Tests determine students' degree of mastery of module content after instruction. Based on data from the Module Post-Test, teachers can determine if Reteach lessons are needed or if reinforcement of new material is needed using the *VmathLive* lessons.

Vmath also includes informal assessment and error analysis that completes the assessment system. Ongoing informal assessments help teachers gauge student understanding of the material being taught. Every *Vmath* lesson facilitates opportunities to see students' reactions to instruction, listen as students respond to the frequent questions, and evaluate daily work. Lessons include specific review, reteaching suggestions, and appropriate questioning techniques. The Work On Your Own section of the lesson allows students to apply newly and previously learned concepts on their own, answering skill-building and problem-solving questions. The Check Up questions are presented in multiple-choice and short-answer formats with accompanying error analysis suggestions. The question distractors are based on common student errors. The Error Analysis provides targeted information about the exact cause of the student's misconception along with specific correction procedures leading to materials to intensify and reinforce instruction.

Technology Component for Practice and Reinforcement

NCTM (2000) states "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). NCTM cautions that while they feel technology should be available to all students, it should not be the only instruction. Raines and Clark (2011) state computer-based instruction has a positive impact on and assists students in learning mathematics concepts. They conclude that technology alone will not improve instruction, but that there are positives to using computer and web-based tools.

The NMAP (2008) recommends two actions with regard to technology use. The first recommendation advises "high-quality computer-assisted instruction (CAI) drill and practice, implemented with fidelity, be considered as a useful tool in developing students' automaticity (i.e., fast, accurate, and effortless performance on computation)" (NMAP, 2008, p. 51). The second recommendation is related to the introduction and teaching of new subject-matter content. The recommendation advises "high-quality computer-assisted instruction (CAI) tutorials, implemented with fidelity, be considered as a potentially useful tool in introducing and teaching specific subject-matter content to specific populations" (NMAP, 2008, p. 51).

In Recommendation 6, Gersten, Beckmann, et al. (2009) discuss the inclusion of time within the intervention period to build fluent retrieval of basic arithmetic facts. The panel states "cumulative review is critical if students are to maintain fluency and proficiency with mathematics facts" (Gersten, Beckmann, et al., 2009, p. 38). While the recommendation involves the use of intervention time for practicing mathematics facts, the panel also believes students should "spend time after instruction with extensive practice on quick retrieval of facts through the use of materials such as flash cards or technology" (Gersten, Beckmann, et al., 2009, p.40). Research (Woodward, 2006) indicates integrating strategy instruction with timed practice drills helps students achieve automaticity in multiplication facts. Shin, Sutherland, Norris & Soloway (2012) found evidence that game technology positively impacted elementary students' learning of arithmetic and actually influenced students' arithmetic learning.

Vmath is available to students anytime, anywhere through three avenues: eBooks, *VmathLive*, and the *Vmath* Testing Center. The eBook is a rich, interactive, and enhanced multimedia experience that retains the fidelity of the *Vmath* materials while providing users with electronic content. The *Vmath* Testing Center may be used by teachers to administer assessments to each student or a whole class. For this discussion, *VmathLive* will be the focus since the other two, eBooks and the *Vmath* Testing Center, facilitate the instruction and assessment referred to previously.

VmathLive is an online program supplementing *Vmath* by providing motivating practice on skills and concepts covered in the *Vmath* modules. The main components of *VmathLive* are the Learn and Play sections, but there is also an animated math glossary that defines common math terms using animated visual models. The Learn section matches the *Vmath* modules and can be used as alternative reteaching opportunities or as computer-assisted practice that reinforces the lessons taught by the teacher. If students need support in the Learn section, they can unfold hints on the screen and access a video, in English or Spanish, to guide them through the problem-solving steps. The Play section has timed live competition with other students, leveled to match the students' needs. As students play, they increase their mental math skills, increasing computational fluency. The combination of the Learn and Play sections in *VmathLive* helps students gain confidence and continuously improve their results. King (2011) concluded that receiving an interactive game intervention (*VmathLive*) combined with remedial instruction was superior to remediation alone or no intervention at all for 128 seventh grade students in an 18-week quasi-experimental study.

Motivation

Students who struggle with mathematics often have experienced failure or frustration when they attempted mathematical tasks in the past, causing them to approach mathematics with trepidation (Forbringer & Fuchs, 2014). One suggestion (Forbringer & Fuchs, 2014; Gersten, Beckmann, et al., 2009; Epstein et al., 2008) is to use additional positive motivational strategies with students who struggle with mathematics. Gersten, Beckmann, et al. (2009) states "praising students for their effort and for being engaged as they work through mathematics problems is a powerful motivational tool that can be effective in increasing students' academic achievement" (p. 44). It is suggested that interventions should include components that promote student effort, persistence, and achievement. These components could include praise and rewards and may be useful for improving mathematical achievement.

Verbal praise should be given for students' effort, for listening carefully, and for following the lesson in a systematic fashion (Fuchs et al., 2005). Gersten, Beckmann, et al. (2009) provide three recommendations for carrying out motivational strategies. Praise or reinforcement should be immediate and specifically related to the students' effort and engagement. Generic praise ("good job") is ineffective. Rewarding students' accomplishments is also effective. Accomplishments could include completion of math tasks or accurate work. Rewards could include using praise, applauding, or more tangible means, such as points or tokens that can be used at a later time. The final recommendation is allowing students to chart their progress and set goals for improvement. Goal setting could include beating a previous score or receiving the maximum score. This type of goal setting, according to Gersten, Beckmann, et al., is believed to help develop self-regulated learning because students take responsibility for setting and achieving the goals.

In *Vmath*, there are ample opportunities for teachers to use positive motivational strategies within the lessons, within a module, based on assessments, and in using *VmathLive*. All of the recommendations described by Gersten, Beckmann, et al. (2009) would work within the *Vmath* program. *Vmath* would fit into just about any point or token system in use in the intervention classroom or as an extension of the core classroom. Goal setting and monitoring progress can be accomplished via the reports available through the VPORT® data management system.

Summary

Geary (2011) summarized several articles that indicate below-average mathematical competencies at the beginning of schooling were associated with elevated risk of poor mathematical competencies at the end of schooling, regardless of family background or the child's social and emotional functioning or their intelligence and reading ability. Price et al. (2013) state "school-entry math skills are a stronger predictor of later academic achievement than early reading or socio-emotional skills" (p. 156) and "low mathematical competence is associated with lower indices of life success" (p. 156). The indices of life success include lower rates of full-time employment, higher rates of employment in low-paying manual occupations, more frequent periods of unemployment, and a lower ability to take advantage of employer-offered training and thus lower rates of promotion (Geary, 2011).

There is a significant body of research that describes instructional procedures and methods that have been found to be effective for teaching mathematics (Forbringer & Fuchs, 2014). It is also clear that with high-quality instructional programs, students who struggle with mathematics can learn mathematics (Forbringer & Fuchs, 2014; Campbell, 1995; Griffin et al., 1994; Silver & Stein, 1996). This paper has described many of these practices and the research showing they are of value when trying to improve student achievement in mathematics.

Vmath was designed and built using the most relevant standards and the proven instructional procedures and methods. Teachers will find the program easy to use with all instructional material readily available. The carefully crafted teacher dialogue provides effective questioning strategies as teachers teach new skills. *Vmath* provides students the opportunity to master the critical foundations they need for success at their grade level. The critical concepts are taught in the *Vmath* program, while extra practice and reteach lessons are included to help provide students the time they need to acquire the skills. Using a balanced, systematic approach, *Vmath* creates successful learning experiences for students, leading to the development of confident, independent learners of mathematics who are able to master grade-level standards.

References

Adams, G. L., & Engelmann, S. (1996). *Research on direct instruction: 25 years beyond DISTAR.* Seattle, WA: Educational Achievement Systems.

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 113*(3), 447–455.

Bottge, B. A. (2001). Reconceptualizing mathematics problem solving for low-achieving students. *Remedial and Special Education, 22*(2), 102–112.

Campbell, P. (1995). Redefining the girl problem in mathematics. In W. Secada and E. Fennema (Eds.), *New directions for equity in mathematics education,* (pp. 225–241). New York, NY: Cambridge University Press.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380–400.

Carnine, D. W., Silbert, J., Kame'enui, E. J., & Tarver, S. G. (2009). *Direct reading instruction* (5th Ed.). Columbus, OH: Pearson.

Christie, L. (2013, June). Bad at math? You're more likely to lose your home. *CNN Money.* Retrieved from http://money.cnn.com/2013/06/25/real_estate/mortgage-math/

Cross, L., & Hynes, M. C. (1994). Assessing mathematics learning for students with learning differences. *Arithmetic Teacher, 41*(7), 371–377.

Doabler, C. T., Cary, M. S., Jungjohann, K., Clarke, B., Fien, H., Baker, S., … Chard, D. (2012). Enhancing core mathematics instruction for students at risk for mathematics disabilities. *Teaching Exceptional Children, 44*(4), 48–57.

Engelmann, S., & Carnine, D. (1991). *Theory of instruction: Principles and applications.* Eugene, OR: ADI Press.

Epstein, M., Atkins, M., Cullinan, D., Kutash, K., & Weaver, R. (2008). *Reducing behavior problems in the elementary school classroom: A practice guide* (NCEE #2008-012). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides

Forbringer, L. L., & Fuchs, W. W. (2014). *RtI in math: Evidence-based interventions for struggling students.* New York, NY: Routledge.

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology, 97*(3), 493–513.

Fuchs, L. S., Powell, S. R., Hamlett, C. L., & Fuchs, D. (2008). Remediating computational deficits at third grade: A randomized field trial. *Journal of Research on Educational Effectiveness, 1*(1), 2–32.

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*(2), 345–362.

Geary, D. C. (2003). Learning disabilities in arithmetic: Problem-solving differences and cognitive deficits. In H. L. Swanson, K. R. Harris, & S. Graham (Eds.), *Handbook of learning disabilities* (pp. 199–212). New York, NY: Guilford.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4–15.

Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental and Behavioral Pediatrics, 32*(3), 250–263.

Geary, D. C., Hoard, M. K., & Bailey, D. H. (2012). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal of Learning Disabilities, 45*(4), 291–307.

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. *Journal of Educational Psychology, 104*(1), 206–223.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*(3), 1202–1242.

Goldman, S. R., Pellegrino, J. W., & Mertz, D. L. (1988). Extended practice of addition facts: Strategy changes in learning disabled students. *Cognition & Instruction, 5*(3), 223–265.

Griffin, S. A., Case, R., & Siegler, R. S. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 25–49). Cambridge, MA: MIT Press.

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93*(3), 615–626.

Hasselbring, T. S., Goin, L. I., & Bransford, J. D. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20*(6), 1–7.

Hecht, S. A., Vogi, K. J., & Torgesen, J. K. (2007). Fraction skills and proportional reasoning. In D. B. Berch & M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origin of mathematical learning difficulties and disabilities* (pp. 121–132). Baltimore, MD: Paul H. Brooks.

Jayanthi, M., Gersten, R., & Baker, S. (2008). *Mathematics instruction for students with learning disabilities or difficulty learning mathematics: A guide for teachers.* Portsmouth, NH: RMC Research Corporation, Center on Instruction.

Jones, I., Inglis, M., Gilmore, C., & Hodgen, J. (2013). Measuring conceptual understanding: The case of fractions. In A. M. Lindmeier, & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 113–120), Kiel, Germany: IGPME.

Jones, J. C. (2012). *Visualizing elementary and middle school mathematics methods.* Hoboken, NJ: Wiley.

Jordan, N. C., Hanich, O. B., & Kaplan, D. (2003). A longitudinal study of children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834–850.

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology, 116*(1), 45–58.

King, A. (2011). *Using interactive games to improve math achievement among middle school students in need of remediation* (Doctoral dissertation). Retrieved from ProQuest Dissertations and Theses Database (AAT 3468187).

Kroesbergen, E. H., & Van Luit, J. E. H. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education, 24*(2), 97–114.

Larson, M. R. (2012). Will CCSSM matter in ten years? *Teaching Children Mathematics, 19*(2), 108–115.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). Classroom instruction that works: *Research-based strategies for increasing student achievement.* Alexandria, VA: ASCD.

Mielach, D. (2012, August). Poor math skills costing Americans big bucks. Yahoo! News. Retrieved from http://news.yahoo.com/poor-math-skills-costing-americans-big-bucks-141826709.html

National Center for Education Statistics. (2013). *The nation's report card: A first look: 2013 mathematics and reading* (NCES 2014–451). Washington, DC: Institutes of Education Sciences, U.S. Department of Education.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics for the 1980s.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards mathematics.* Washington, DC: Author.

National Mathematics Advisory Panel [NMAP]. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4*(1), 52–56.

Polya, G. (1957). *How to solve it.* New York, NY: Doubleday.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common Core Standards: The new U.S. intended curriculum. *Educational Researcher, 40*(3), 103–116.

Powell, S. R., Fuchs, L. S., & Fuchs, D. (2013). Reaching the mountaintop: Addressing the Common Core Standards in mathematics for students with mathematics difficulties. *Learning Disabilities Research & Practice, 28*(1), 38–48.

Price, G. R., Mazzocco, M. M. M., & Ansari, D. (2013). Why mental arithmetic counts: Brain activation during single digit arithmetic predicts high school math scores. *The Journal of Neuroscience, 33*(1), 156–163.

Raines, J. M., & Clark, L. M. (2011). A brief overview on using technology to engage students in mathematics. *Current Issues in Education, 14*(2), 1–8.

Rittle-Johnson, B., & Schneider, M. (in press). Developing conceptual and procedural knowledge of mathematics. In R. Cohen Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition.* New York, NY: Oxford University Press.

Rosenshine, B. (1983). Teaching functions in instructional programs. *Elementary School Journal, 83,* 335–351.

Schmidt, W. H., Wang, H. C., & McKnight, C. C. (2005). Curriculum coherence. An examination of U.S. mathematics and science content standards from an international perspective. *Journal of Curriculum Studies, 37*(5), 525–559.

Shin, N., Sutherland, L. M., Norris, C. A., & Soloway, E. (2012). Effects of game technology on elementary student learning in mathematics. *British Journal of Educational Technology, 43*(4), 540–560.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23,* 691–697.

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology, 49*(10), 1994–2004.

Silver, E. A., & Stein, M. K. (1996, January). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*(4), 476–521.

Sousa, D. A. (2007). *How the brain learns mathematics.* Thousand Oaks, CA: Corwin.

Stein, M., Kinder, D., Silbert, J., & Carnine, D. W. (2006). *Designing effective mathematics instruction: A direct instruction approach* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

Stein, M., Silbert, J., & Carnine, D. (1997). *Designing effective mathematics instruction: A direct instruction approach.* Upper Saddle River, NJ: Merrill/Prentice Hall.

Sternberg, R. (2005). Intelligence, competence, and expertise. In A. Elliot & C. Dweck (Eds.), *Handbook of competence and motivation.* New York, NY: Guilford Press.

Thompson, V. (n.d.). Can a lack of math skills hurt you in the future? *Global Post.* Retrieved from http://everydaylife.globalpost.com/can-lack-math-skills-hurt-future-9767.html

Valerio, A., Sanchez Puerta, M. L., Pierre, G., Rajadel, T., & Taborda, S. M. (2014). *STEP skills measurement: Snapshot 2014.* Washington, DC: The World Bank.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal, 3*(2), 49–60.

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly, 29*(4), 269–289.

Workman, D. (2014, September). Poor math skills contribute to Canadian debt problems. *CreditCards.com.* Retrieved from http://canada.creditcards.com/credit-card-news/poor-math-skills-1264.php