Lesson 1

Fractions on a Number Line

Problem Solving:

Partitioning the Number Line

Fractions on a Number Line

Vocabulary

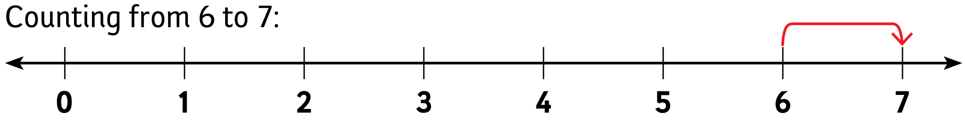
whole numbers
consecutive
predictable
fractions
infinite
denominator
numerator

Where are fractions on a number line?

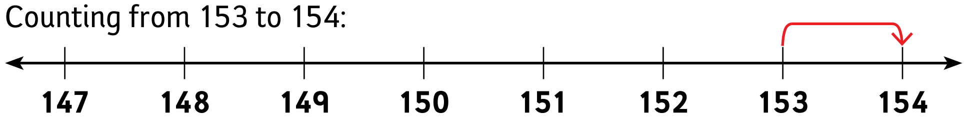
All the numbers on the first two number lines shown are **whole numbers**. When we count with **consecutive** whole numbers, we can count forward or backward in a **predictable** way. We can tell what number will come next by adding 1 to the current number. We can tell what number came before by subtracting 1 from the current number. Counting with whole numbers can continue forever.

Whole Numbers on a Number Line

Counting from 6 to 7:

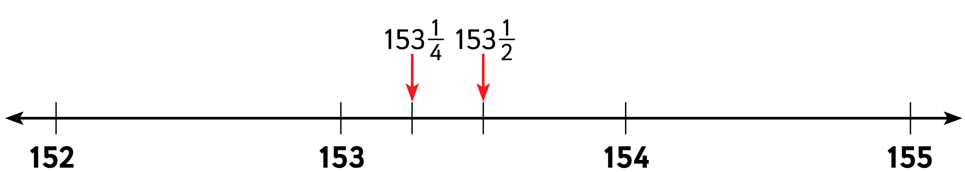


Counting from 153 to 154:



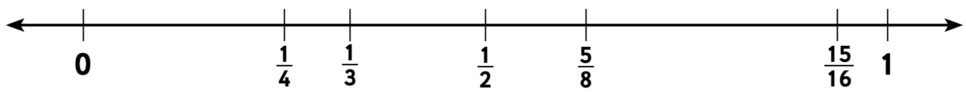
The Numbers between Whole Numbers on a Number Line

Are there any numbers *between* the whole numbers on a number line? Yes! These numbers are **fractions**, or numbers that have a fractional part, and they can be found between every pair of consecutive whole numbers on a number line.



Here are some interesting concepts. First, there are an **infinite** number of fractions between any two consecutive whole numbers on a number line.

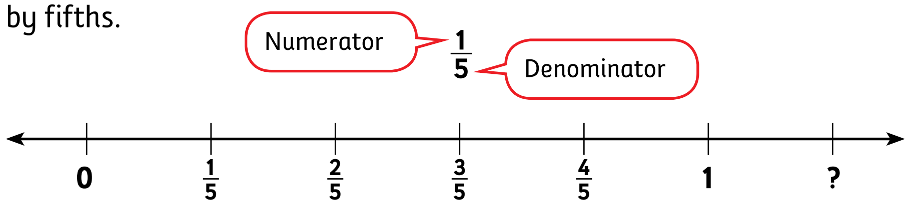
Second, look at the fractions below. Do they appear to have a pattern? Can the fraction that comes next be predicted?



POWER CONCEPT

There are an infinite number of fractions between any two consecutive whole numbers.

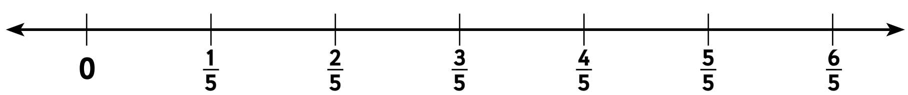
The simplest way to count with fractions in a predictable manner is to count by using fractions with the same **denominator**. Let's count by fifths.



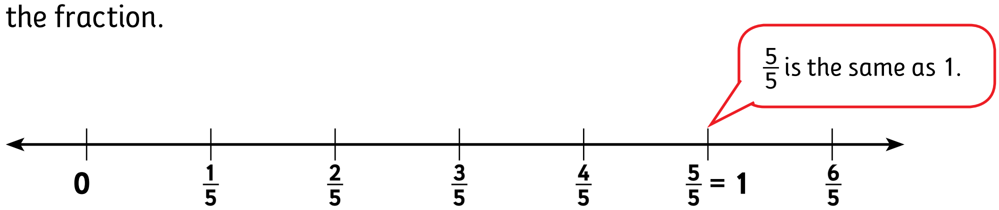
POWER CONCEPT

We can easily predict what fraction comes next when the denominators are the same: $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \ldots$

We can continue to count beyond $\frac{4}{5}$ by adding 1 to the **numerator**. So $\frac{5}{5}$ follows $\frac{4}{5}$, $\frac{6}{5}$ follows $\frac{5}{5}$, and so on.



Notice that $\frac{5}{5}$ is in the same location as 1. When the numerator and denominator of a fraction are the same number, the fraction is *equal* to 1. To make it easier to remember this fact, we can write 1 beside the fraction.



Apply Skills
Turn to ***Interactive Text***, page 2.

Reinforce Understanding
Use the ***Unit 1 Lesson 1 Teacher Talk Tutorial*** to review lesson concepts.

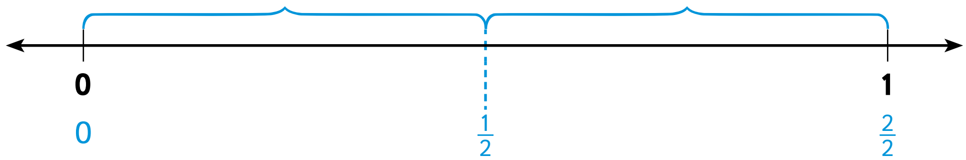
▶Problem Solving: Partitioning the Number Line

Vocabulary

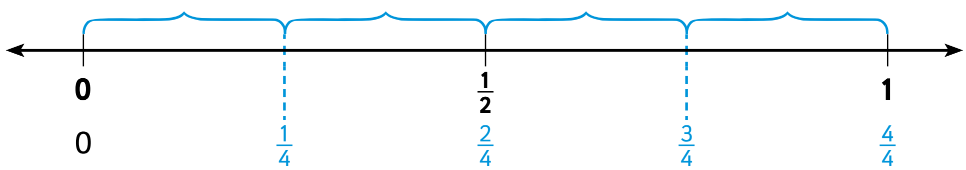
partition
length model

How do we partition the number line?

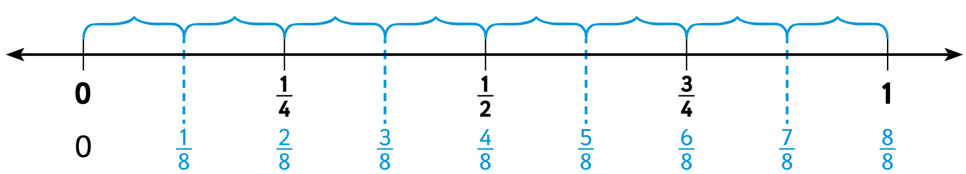
When we divide the number line into equal parts, we are *partitioning* the number line. One way to **partition** the number line is to repeatedly find the number that is halfway between two numbers already on the number line. When we partition from 0 to 1 into two equal parts, $\frac{1}{2}$ is halfway between 0 and 1. In other words, the distance from 0 to $\frac{1}{2}$ is the same as the distance from $\frac{1}{2}$ to 1.



Now partition each of the two parts into two equal parts. So $\frac{1}{4}$ is halfway between 0 and $\frac{1}{2}$ and $\frac{3}{4}$ is halfway between $\frac{1}{2}$ and 1.



Again, partition each part into two equal parts. Look at the fraction halfway between 0 and $\frac{1}{4}$. It is $\frac{1}{8}$. Notice that when we partition by repeatedly finding the number that is halfway between two numbers, the denominator doubles.



It is important to understand that when we divide distances on the number line in half, the denominators double even though the fractions are getting smaller. For example, $\frac{1}{8}$ is smaller than $\frac{1}{4}$, which is smaller than $\frac{1}{2}$. This is easy to see on a number line where we are comparing fractions by comparing their distances from 0. We call this model a **length model**.

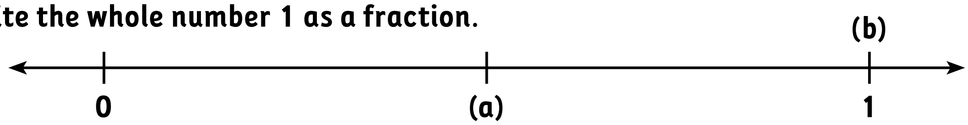
Problem-Solving Activity
Turn to ***Interactive Text***, page 4.

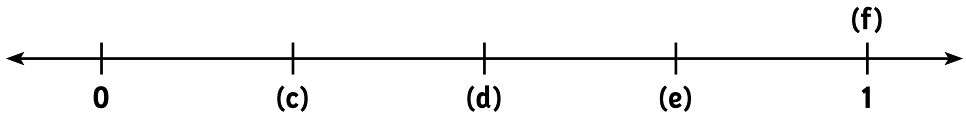
Reinforce Understanding
Use the ***Unit 1 Lesson 1 Problem Solving Teacher Talk Tutorial*** to review lesson concepts.

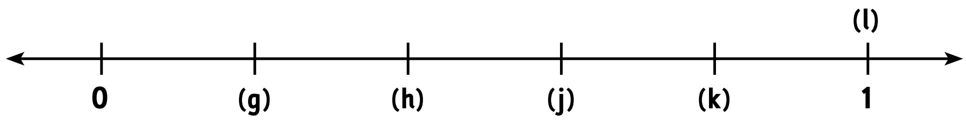
Homework

Activity 1

Find the fractions for the letters on the number line. Remember that we can write the whole number 1 as a fraction.

1. 

2. 

3. 

Activity 2

Write the correct multiple in each empty box in the list.

Model

0	2	4	6	8	10	12	14	16	18

1.

0	10	20		40			70		90

2.

0	5				25			40	

3.

0	4			16			28		

4.

0	6	12			30		42		54

Activity 3 • Distributed Practice

Solve.

1. $354 + 489$
2. $203 - 177$
3. 112×32
4. $1{,}045 + 992$
5. $431 - 27$
6. $4\overline{)248}$

Lesson 2 | ▶Connecting Fractions and Fair Shares to Geometry

Problem Solving:

▶Noncongruent Fair Shares

▶Connecting Fractions and Fair Shares to Geometry

Vocabulary

fair share
congruent

How can fractions on a number line be related to fair shares?

Think about how we partitioned a number line in Lesson 1. Each segment between fractions on the number line was the same length. Look at this number line. It is divided into fifths. Each segment is a **fair share** because each segment is the same length.

Each $\frac{1}{5}$ segment is the same length.

One-dimensional objects like number lines are divided into fair shares that look the same. The fair shares are **congruent** line segments. Fair shares are not always congruent when we use two-dimensional shapes like rectangles or squares. Look at the rectangles in Example 1. Each has been divided into fair shares called fourths.

Example 1

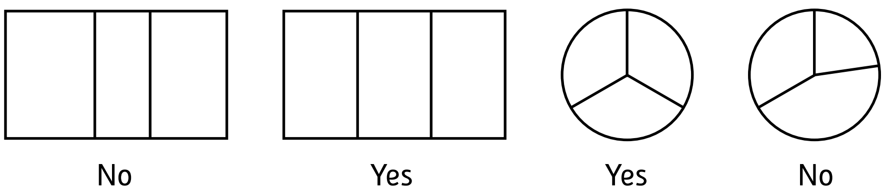
Partition each rectangle into fair shares called fourths.

In each rectangle, the fair shares are *congruent*. The area and shape of each fair share is the same. Here is another way to think about these fair shares. In each rectangle, the fair shares can be stacked on top of each other and they would look exactly alike.

It is easy to make the mistake that we can divide up shapes any way we want and call it a fractional part such as fourths or thirds. We cannot. The parts must be fair shares. Example 2 compares fair shares with non-fair shares.

Example 2

Are these fair shares for thirds?



Fractional parts must be fair shares. This means they must have the same **area**.

Apply Skills
Turn to ***Interactive Text***, page 6.

Reinforce Understanding
Use the ***Unit 1 Lesson 2 Teacher Talk Tutorial*** to review lesson concepts.

▶**Problem Solving:** Noncongruent Fair Shares

Vocabulary

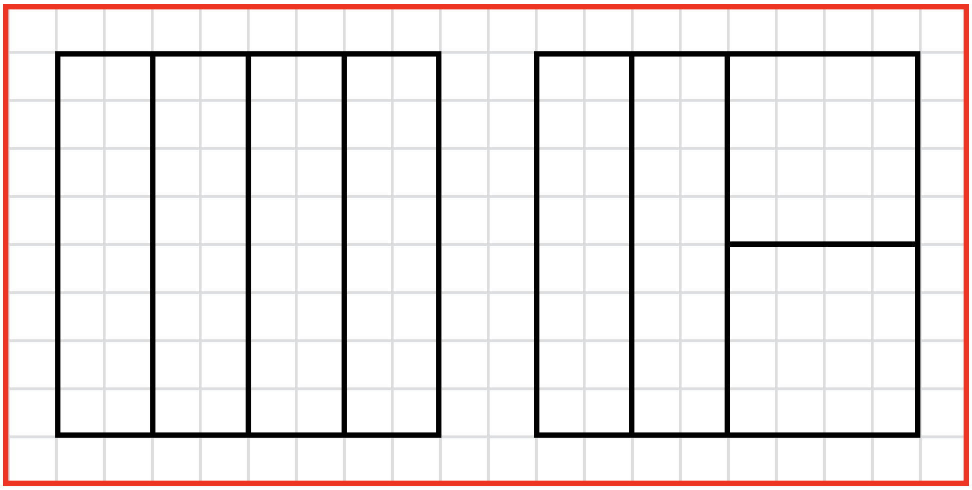
area
area models
noncongruent

Do fair shares always have to be congruent?

So far we have only looked at fair shares that are congruent. Fair shares do not have to be congruent. They only must have the same **area**. Look at the shapes on the grid below. Both shapes have been divided into fair shares.

Shape A: Congruent fair shares

Shape B: Noncongruent fair shares



Shape A shows a square that has been divided into fourths. Each fair share is 16 square units. On a grid, simply count the small squares to find the area. That is why we call these models **area models**. These fair shares also happen to be congruent.

Shape B is also divided into fourths. Each fair share is 16 square units, and we can count the small squares to confirm this. But these fair shares are **noncongruent**.

Most of the time we work with congruent fair shares. But it is important to remember that fair shares for two-dimensional shapes only need to have the same area. Putting shapes on a grid helps us confirm that the areas of noncongruent fair shares are the same.

POWER CONCEPT

Fair shares for two-dimensional shapes must have the same area. They do not have to be congruent.

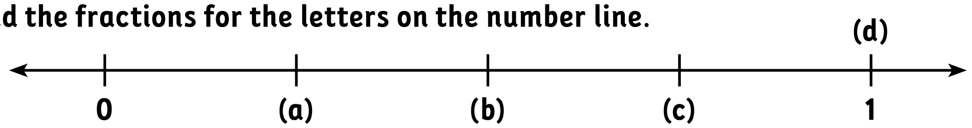
Problem-Solving Activity
Turn to ***Interactive Text***, page 8.

Reinforce Understanding
Use the ***Unit 1 Lesson 2 Problem Solving Teacher Talk Tutorial*** to review lesson concepts.

Homework

Activity 1

Find the fractions for the letters on the number line.

1.


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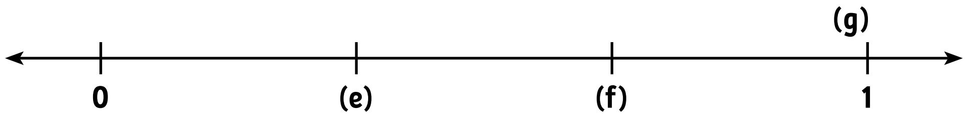

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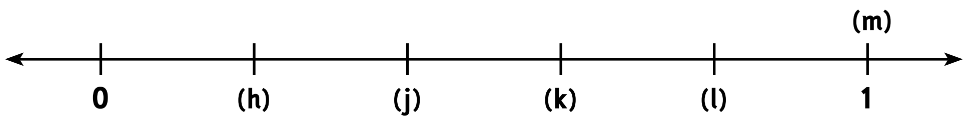


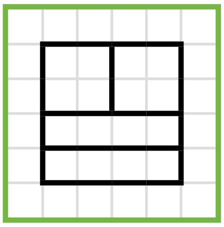
Activity 2

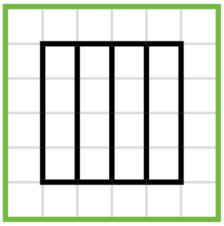
Divide the rectangles into the fair shares indicated.

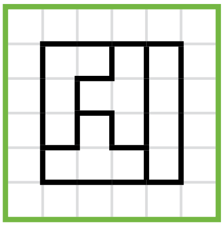
1. Fourths
2. Thirds
3. Fifths
4. Sixths
5. Eighths

Activity 3

The following shapes have been divided into fourths. Tell whether the fourths are congruent fair shares. Answer Y for yes or N for no. If you answer N, tell why they are not congruent.

1. 

2. 

3. 

Activity 4 • Distributed Practice

Solve.

1. $\begin{array}{r} 1{,}059 \\ +\ 368 \\ \hline \end{array}$

2. $\begin{array}{r} 1{,}002 \\ -\ 209 \\ \hline \end{array}$

3. $5\overline{)305}$

4. $\begin{array}{r} 9{,}213 \\ +\ 8{,}827 \\ \hline \end{array}$

5. $\begin{array}{r} 89 \\ \times\ 66 \\ \hline \end{array}$

6. $6\overline{)186}$

Lesson 3 | Part-to-Whole Relationships

Problem Solving: Representing Fractions with Cuisenaire Rods

Part-to-Whole Relationships

Vocabulary

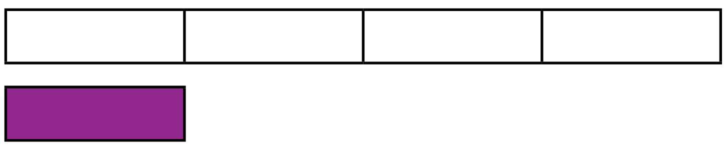
unit fraction
Cuisenaire rods

What is a part-to-whole relationship?

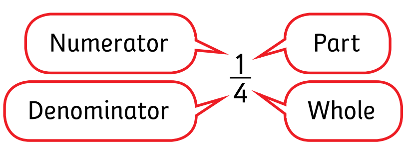
One of the most important ideas about fractions is the part-to-whole relationship that a fraction describes. To describe this relationship, first identify the "whole" and then compare the part or parts to the whole. The top rectangle shows the whole and the shaded rectangle underneath shows one of the parts.

The whole

One part



Notice that the whole is made of four congruent parts. We can use the fraction $\frac{1}{4}$ to name the shaded rectangle because it shows one of the four equal parts. The fraction $\frac{1}{4}$ is called a **unit fraction** because it names one part of the whole.



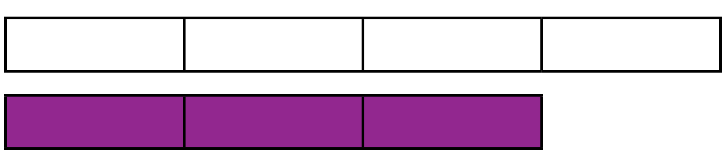
POWER CONCEPT

A unit fraction is a fraction that names one part of the whole.

The shaded rectangles below show a different part-to-whole relationship. The top rectangle shows that the whole is made of four equal parts. There are three parts to the whole. We can think of the three parts as three unit fractions.

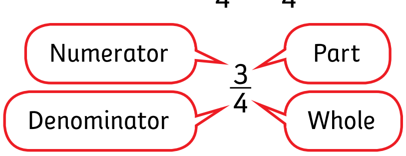
The whole

Three parts



The fraction shown by this picture can be written as

$$3 \times \frac{1}{4}, \text{ or } \frac{3}{4}.$$



Understanding part-to-whole relationships gets trickier when there are no lines on the rectangles. Today, we will use a math tool for modeling fractions called **Cuisenaire rods**.

Example 1

Compare the two rods to see the part-to-whole relationship.
Look at the Cuisenaire rods below. How does the part compare to the whole? Is the shorter rod $\frac{1}{2}$ the size of the longer rod? Is it $\frac{1}{3}$ or $\frac{1}{4}$ the size?

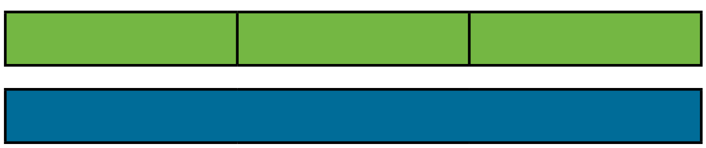
One part

The whole

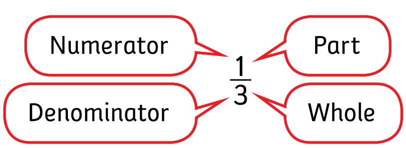
The only way to tell for sure is to get more of the parts to make a whole.

Three parts

The whole



Because it takes three parts to make the whole, the unit fraction is $\frac{1}{3}$.



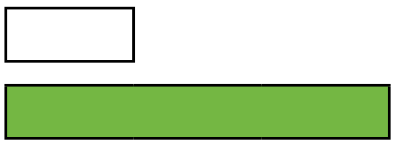
Rods with different lengths can represent the same part-to-whole relationship. It's all about the relationship of the part to its whole. Example 2 shows this.

Example 2

What is the part-to-whole relationship shown with the two rods below?

One part

The whole

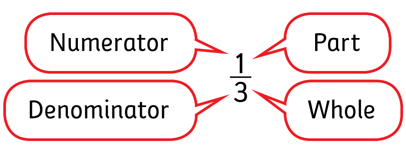


Again, three parts are needed to make the whole.

Three parts

The whole

So the unit fraction is $\frac{1}{3}$.



Even though the part and whole rods in Example 2 are shorter than the corresponding rods in Example 1, the part-to-whole relationship is still $\frac{1}{3}$.

POWER CONCEPT

The part-to-whole relationship is a comparison of the part to what we define as the whole.

Apply Skills
Turn to ***Interactive Text***, page 10.

Reinforce Understanding
Use the ***Unit 1 Lesson 3 Teacher Talk Tutorial*** to review lesson concepts.

▶Problem Solving: Representing Fractions with Cuisenaire Rods

How do we select Cuisenaire rods to model a fraction?

We have been shown Cuisenaire rods and asked to describe the part-to-whole relationship. Now we will choose Cuisenaire rods to show a fraction. Let's look at an example.

Example 1

Select Cuisenaire rods to show $\frac{1}{5}$.

Because the 5 in $\frac{1}{5}$ means that there are 5 parts in the whole, first choose something small for one part.

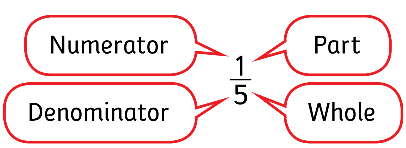
One part 

Now put five of these parts together and find a rod that represents the whole.

Five parts
The whole


These rods show $\frac{1}{5}$.

One part
The whole

What if we are given a fraction that is not a unit fraction? Let's look at an example.

Example 2

Select Cuisenaire rods to show $\frac{2}{5}$.

First, repeat the process in Example 1 to find the unit fraction. These rods show $\frac{1}{5}$.

One part

The whole

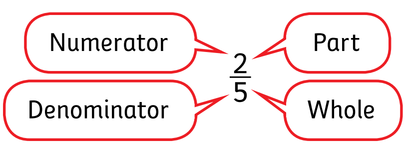
Because the numerator of $\frac{2}{5}$ is 2, we need two unit fractions.

Two parts

The whole

The fraction can be written as

$$2 \times \frac{1}{5}, \text{ or } \frac{2}{5}.$$



Notice how important the unit fraction is when we work with part-to-whole relationships.

Problem-Solving Activity
Turn to ***Interactive Text***, page 12.

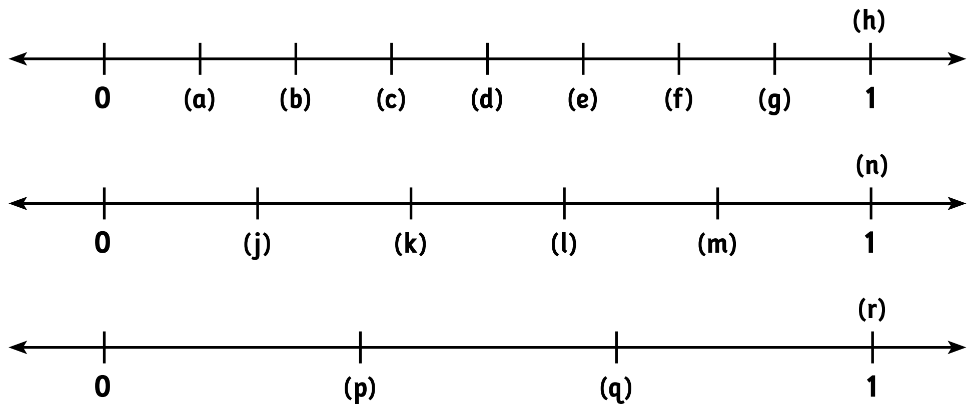
Reinforce Understanding
Use the ***Unit 1 Lesson 3 Problem Solving Teacher Talk Tutorial*** to review lesson concepts.

Homework

Activity 1

Find the fractions for the letters on the number line.

1. 2. 3.



Activity 2

Divide the rectangles into the fair shares indicated.

1. Fifths
2. Tenths
3. Fourths
4. Eighths
5. Halves
6. Thirds

Homework

Activity 3

Name the unit fraction represented by each pair of rods.

1. One part

The whole

2. One part

The whole

3. One part

The whole

4. One part

The whole

Activity 4 • Distributed Practice

Solve.

1. $\begin{array}{r} 277 \\ +\,4{,}234 \\ \hline \end{array}$

2. $\begin{array}{r} 4{,}001 \\ -\,2{,}001 \\ \hline \end{array}$

3. $3\overline{)633}$

4. $\begin{array}{r} 903 \\ +\,1{,}209 \\ \hline \end{array}$

5. $\begin{array}{r} 75 \\ \times\ \ 75 \\ \hline \end{array}$

6. $5\overline{)1{,}520}$

7. $\begin{array}{r} 4{,}001 \\ +\ \ 701 \\ \hline \end{array}$

8. $\begin{array}{r} 808 \\ \times\ \ 25 \\ \hline \end{array}$

9. $8\overline{)280}$

Lesson 4 | Working from the Whole to the Part

Problem Solving:

Finding the Part in Shapes on a Grid

Working from the Whole to the Part

How do we find a part when given the whole?

In the last lesson, we talked about part-to-whole relationships. We used Cuisenaire rods to recognize this relationship. There were two rods. The shorter rod represented the part and the longer rod represented the whole.

Example 1

What fraction represents the relationship between the two rods?

One part

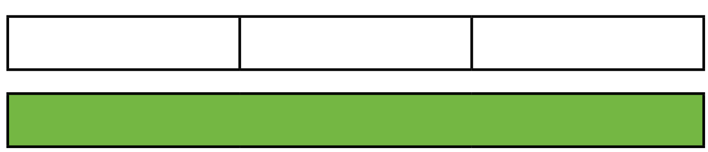
The whole

It would take three of the parts to make the whole. That means the one part is the unit fraction $\frac{1}{3}$.

We can check the answer by using extra parts to make a whole.

Three parts

The whole



It is important to be able to think about what the part looks like when we are just given the whole. Here is an example.

Example 2

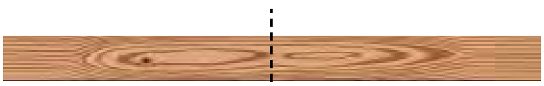
Divide rectangles into fair shares to find the part.

Kari cuts a board into pieces to make a window frame. She needs $\frac{1}{4}$ of the board for the bottom of the window frame. Even though she will measure the exact length of the $\frac{1}{4}$ part, she still needs to understand what $\frac{1}{4}$ of the board would look like before she makes the cut.

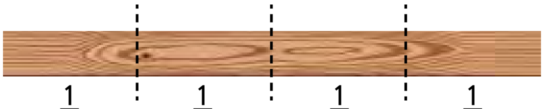
POWER CONCEPT

Visualizing the part from the whole helps us to understand fractions.

One way for Kari to think about $\frac{1}{4}$ is to use the "halving" strategy to make fair shares out of a rectangle. Kari looks at the whole board and decides how to cut it into fair shares that are fourths. She could first cut the board into two fair share pieces. Each piece is $\frac{1}{2}$ of the whole board.



Then she could cut each piece into two fair share pieces. This will make four fair share pieces. Each piece is $\frac{1}{4}$ of the whole board.



When we compare the part to the whole, we can see that it is $\frac{1}{4}$ of the whole.

One part

The whole

Kari has an important skill. She can visualize the unit fraction based on the whole. This is an important way of understanding part-to-whole relationships.

Apply Skills
Turn to ***Interactive Text***, page 14.

Reinforce Understanding
Use the ***Unit 1 Lesson 4 Teacher Talk Tutorial*** to review lesson concepts.

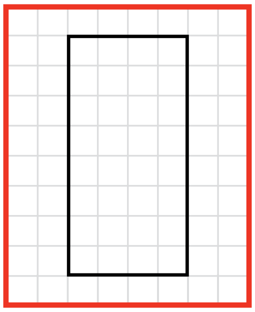
▶Problem Solving: Finding the Part in Shapes on a Grid

How do we find the part when the whole is a shape?

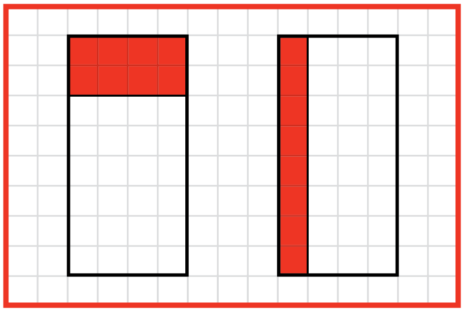
We used Cuisenaire rods to show and name a unit fraction. Now we will look at a shape on a grid and show a unit fraction. In Example 1, we will find two different ways to show the unit fraction.

Example 1

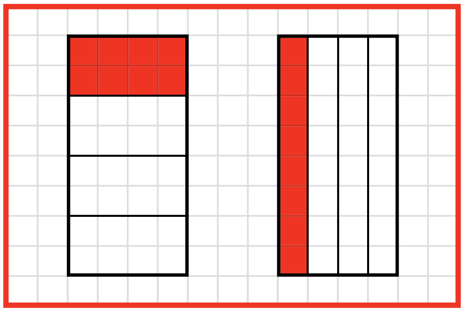
The shape on the grid is the whole. Show $\frac{1}{4}$.



Look at the whole shape and visualize what $\frac{1}{4}$ could look like.



We can check by dividing the rest of the grid into fair share units.



Problem-Solving Activity
Turn to ***Interactive Text***, page 15.

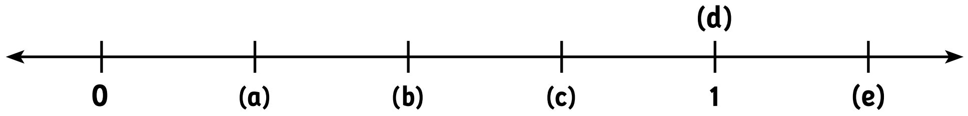
Reinforce Understanding
Use the ***Unit 1 Lesson 4 Problem Solving Teacher Talk Tutorial*** to review lesson concepts.

Homework

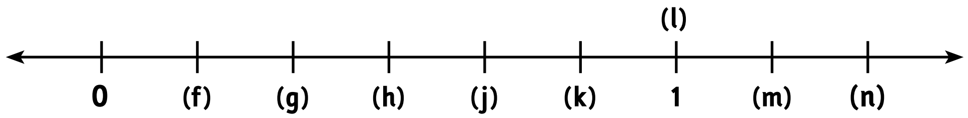
Activity 1

Find the fractions for the letters on the number line. Notice that one or more fractions are greater than 1 on each number line.

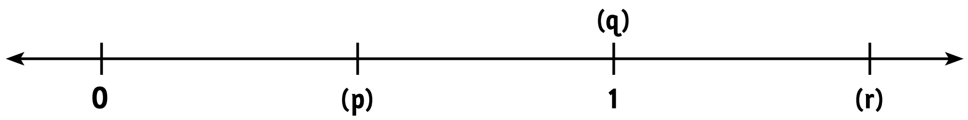
1.



2.



3.



Activity 2

Divide the rectangles into the fair shares indicated.

1. Fourths

2. Eighths

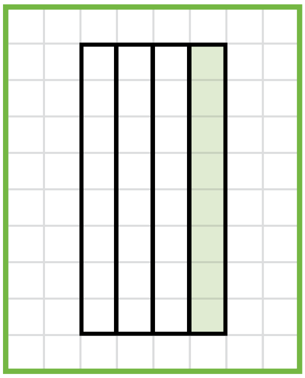
3. Thirds

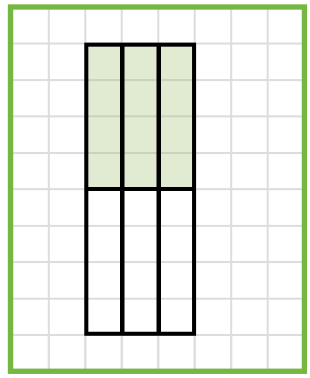
4. Halves

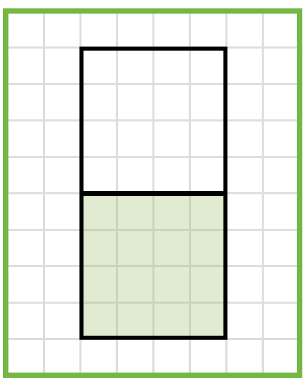
5. Tenths

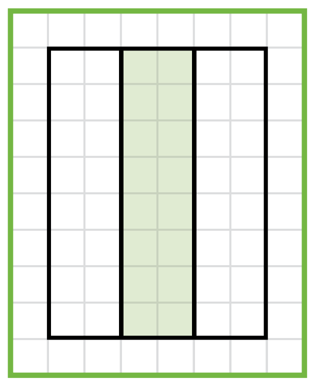
Activity 3

Tell the fraction represented by the shaded part in each of the shapes.

1. 

2. 

3. 

4. 

Activity 4 • Distributed Practice

Solve.

1. $\begin{array}{r} 1{,}799 \\ +\ 808 \\ \hline \end{array}$

2. $\begin{array}{r} 9{,}032 \\ -\ 4{,}501 \\ \hline \end{array}$

3. $5\overline{)550}$

4. $\begin{array}{r} 63 \\ +\ 5{,}607 \\ \hline \end{array}$

5. $\begin{array}{r} 122 \\ \times\ 31 \\ \hline \end{array}$

6. $6\overline{)492}$

7. $\begin{array}{r} 5{,}005 \\ -\ 931 \\ \hline \end{array}$

8. $\begin{array}{r} 45 \\ \times\ 120 \\ \hline \end{array}$

9. $10\overline{)360}$

Lesson 5 ▶Going Beyond the Unit Fraction

Monitoring Progress:

▶Quiz 1

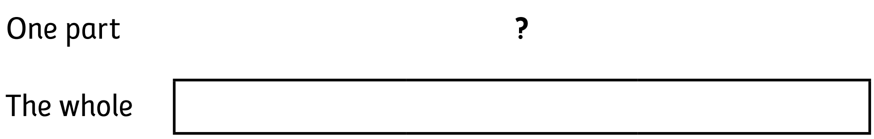
▶Going Beyond the Unit Fraction

How do we model fractions other than unit fractions when given the whole?

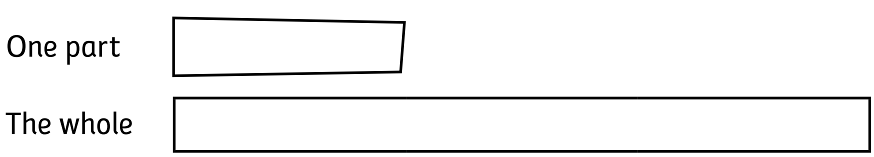
In Lesson 4, we started with "the whole" and found the part by dividing the whole into fair share parts. One part is the unit fraction. Let's review finding a unit fraction. Can we imagine the unit fraction $\frac{1}{3}$ when given the whole?

Example 1

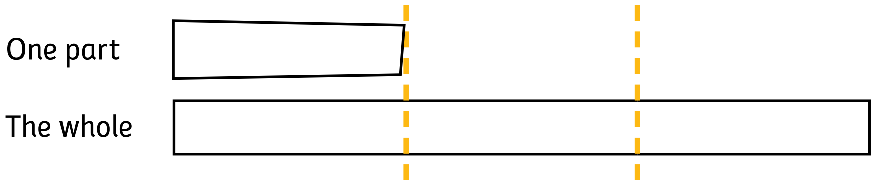
Complete a model for $\frac{1}{3}$ given the whole.



After working with thirds for a while, we begin to develop a good visual for the unit fraction $\frac{1}{3}$. We can even be able to sketch it freehand like this:



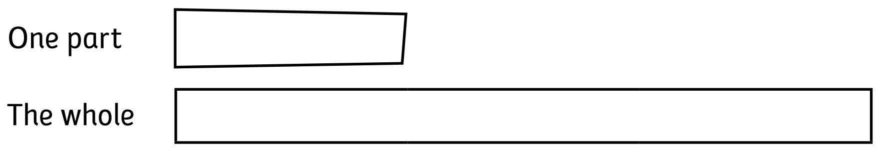
We can always divide the shape into fair shares to check whether the sketch is accurate.



Once the unit fraction is known, it is easier to find other fractions. Example 2 shows how to find another fraction given a unit fraction.

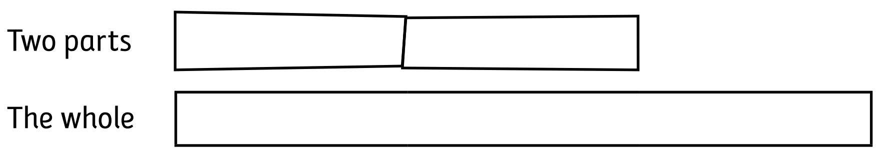
Example 2

Sketch a model of $\frac{2}{3}$.

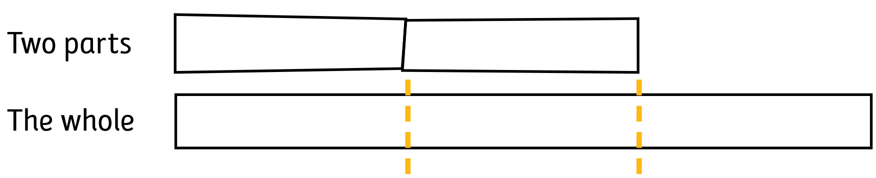


To sketch $\frac{2}{3}$, we need to sketch two unit fractions.

$$2 \times \frac{1}{3} = \frac{2}{3}$$



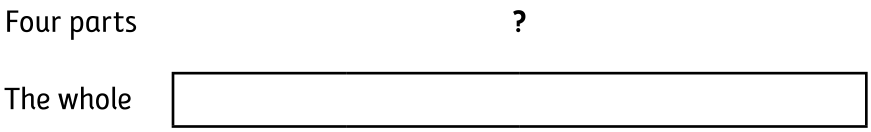
We can always divide the shape into fair shares to check whether the sketch is accurate.



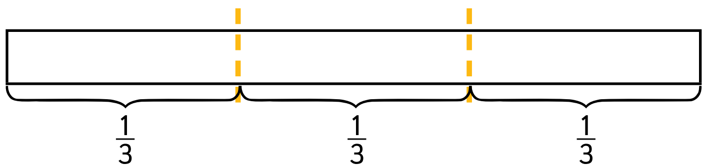
Some fractions are harder to visualize and sketch than others. One fraction that can be harder to sketch is $\frac{4}{6}$. It helps to first divide the whole into thirds, then divide each third fair share in half. Example 3 shows how to sketch $\frac{4}{6}$ using this strategy.

Example 3

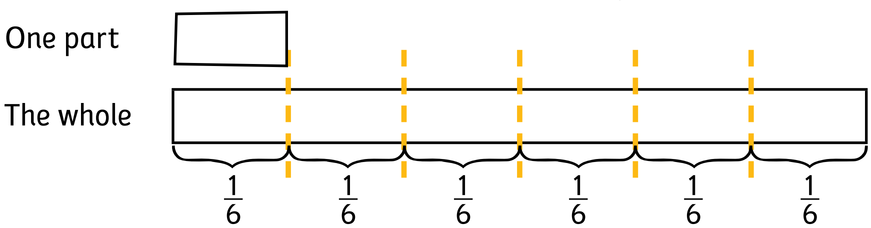
Use the "halving" strategy to sketch a model for $\frac{4}{6}$.



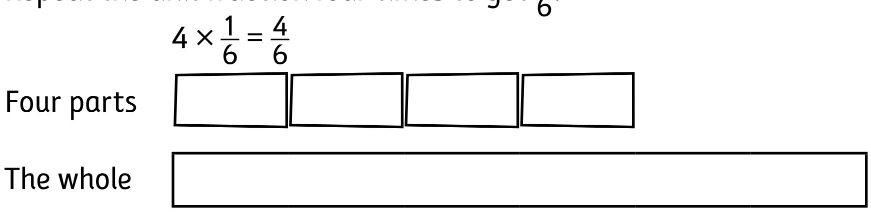
First, divide the whole into thirds. This is what Kari does when she cuts boards.



Next, divide each fair share third in half. The whole now has six equal parts. Use the whole to sketch the unit fraction $\frac{1}{6}$.



Repeat the unit fraction four times to get $\frac{4}{6}$.



Kari uses sixths all the time and is good at visualizing the unit fraction $\frac{1}{6}$ and repeating the part four times to get $\frac{4}{6}$. But if we don't have the same kind of experience as Kari, we can use this method of finding thirds and using the halving strategy to find sixths.

Apply Skills
Turn to ***Interactive Text***, page 19.

Monitoring Progress
Quiz 1

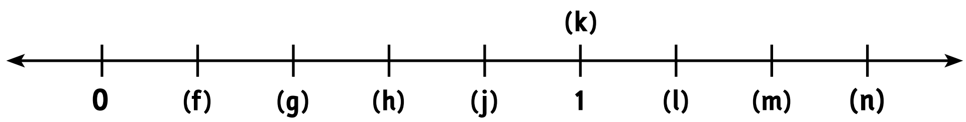
Reinforce Understanding
Use the ***Unit 1 Lesson 5 Teacher Talk Tutorial*** to review lesson concepts.

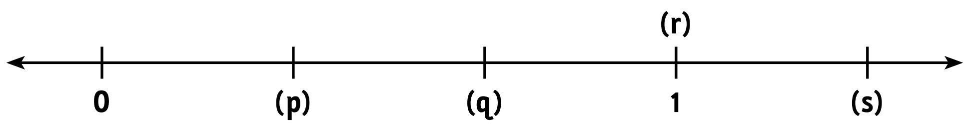
Homework

Activity 1

Write the correct fraction for each letter. Some fractions are greater than 1.

1. 

2. 

3. 

Activity 2

Divide the rectangles into the fair shares indicated.

1. Eighths

2. Fifths

3. Tenths

4. Thirds

5. Sixths

Homework

Activity 3

Draw a rod to represent one whole for each problem. Then sketch the fraction.

1. $\frac{1}{3}$

2. $\frac{1}{4}$

3. $\frac{3}{4}$

4. $\frac{2}{3}$

Activity 4 • Distributed Practice

Solve.

1. $\begin{array}{r} 1{,}879 \\ +\ 925 \\ \hline \end{array}$

2. $\begin{array}{r} 8{,}021 \\ -\ 3{,}591 \\ \hline \end{array}$

3. $4\overline{)552}$

4. $\begin{array}{r} 74 \\ +\ 6{,}719 \\ \hline \end{array}$

5. $\begin{array}{r} 237 \\ \times\ 42 \\ \hline \end{array}$

6. $8\overline{)496}$

7. $\begin{array}{r} 2{,}002 \\ -\ 947 \\ \hline \end{array}$

8. $\begin{array}{r} 63 \\ \times\ 140 \\ \hline \end{array}$

9. $10\overline{)580}$