Teacher Edition

I

Vmath®

Third Edition

VOYAGER SOPRIS LEARNING™

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These state standards are addressed on the following pages.

MAFS.8.EE.1.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$

Module 2: 62

MAFS.8.EE.1.2
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Module 2: 84; Module 7: 302

MAFS.8.EE.1.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

Module 2: 78

MAFS.8.EE.1.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Module 2: 78

MAFS.8.EE.2.5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Module 5: 224

MAFS.8.EE.2.6
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Module 4: 186; Module 5: 219, 232

MAFS.8.EE.3.7
Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Module 3: 108

MAFS.8.EE.3.8
Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Module 5: 234

MAFS.8.F.1.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Module 4: 166; Module 5: 212

MAFS.8.F.1.2
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Module 4: 182

MAFS.8.F.1.3
Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Module 5: 214, 232

MAFS.8.F.2.4
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Module 4: 163, 166; Module 5: 212

MAFS.8.F.2.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Module 4: 163, 166; Module 5: 212

MAFS.8.G.1.1
Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.

Module 6: 280, 268, 252

MAFS.8.G.1.2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Module 5: 268, 288

MAFS.8.G.1.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Module 6: 276

MAFS.8.G.1.4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Module 6: 252, 284

MAFS.8.G.1.5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Module 6: 248, 260

MAFS.8.G.2.6
Explain a proof of the Pythagorean Theorem and its converse.

Module 7: 334

MAFS.8.G.2.7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Module 7: 334

MAFS.8.G.2.8
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Module 7: 334

MAFS.8.G.3.9
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Module 7: 326

MAFS.8.NS.1.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Module 2: 92

MAFS.8.NS.1.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Module 2: 92

MAFS.8.SP.1.1
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Module 4: 158

MAFS.8.SP.1.2
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Module 4: 158

MAFS.8.SP.1.3
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

Module 4: 158

MAFS.8.SP.1.4
Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Module 4: 158

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What Is *Vmath*?

Vmath® is a targeted math intervention program for struggling students in grades 2–8 that provides additional opportunities to master critical grade-level standards. *Vmath* is specifically designed to reinforce grade-level expectations. With a blended print and technology solution, or a digital only option, *Vmath* delivers essential content using strategies proven to accelerate and motivate at-risk students. Through a balanced, systematic approach, *Vmath* creates successful learning experiences for students and develops confident, independent learners of mathematics.

LEVEL C

- Addition
- Subtraction
- Measurement
- Money and Geometry
- Time, Graphing, and Data
- Fractions and Concepts of Multiplying and Dividing

LEVEL D

- Whole Numbers
- Whole Number Addition and Subtraction
- Whole Number Multiplication
- Whole Number Division
- Fractions and Money
- Data, Measurement, and Geometry

LEVEL E

- Operations with Whole Numbers: Addition and Subtraction
- Operations with Whole Numbers: Multiplication and Division
- Understanding Fractions and Equivalence
- Operations on Fractions and Relationship to Decimals
- Geometry
- Measurement and Data

LEVEL F

- Whole Numbers and Decimals
- Operations with Whole Numbers and Decimals
- Fractions
- Algebraic Reasoning
- Data Analysis
- Geometry and Measurement

LEVEL G

- Rational Numbers Part A
- Rational Numbers Part B
- Expressions, Equations, and Inequalities
- Proportional Thinking
- Geometry
- Data

LEVEL H

- Rational Number Operations Part A
- Rational Number Operations Part B
- Expressions and Equations
- Proportionality
- Geometry
- Data, Probability, and Statistics

LEVEL I

- Real Numbers
- Equations
- Functions Part A
- Functions Part B
- Transforming Geometry
- Geometry

Vmath Key Features

- 7 levels (C–I, appropriate for grades 2–8)
- 7 modules per level
- Wraps around any core math program and provides additional support in the skills expected by new, more rigorous standards
- Provides an easy delivery model to address the needs of students requiring more instruction
- Focuses on the standards of each grade level, reinforcing the skills taught in the classroom and the skills students struggle with most, while providing foundational lessons to help scaffold the instruction
- Builds on the coherence and progressions of the skills taught in the standards
- Incorporates rigor into the intervention solution by providing conceptual development, procedural skill and fluency, and application activities such as writing, manipulatives, and the new Math Flash lessons
- Provides a built-in assessment model to inform teachers, support data-driven decision making, and ensure that instruction is always differentiated to meet each student at his or her level
- Delivers focused, standards-based instruction, while also providing foundational skills necessary for grade-level success
- Levels D–I of *Vmath* include four different lesson types:
    - ***Vmath* Lessons**—four-step scaffolded instruction specific to concepts and skills related to grade-level expectations for both problem-solving and concept development
    - **Hands-On Lessons**—activities specific to the use and application of manipulatives to help students develop deeper conceptual understanding
    - **Gizmos™ Lessons**—instruction using online digital manipulatives to help reinforce conceptual learning
    - **Math Flash Lessons**—short lessons that reinforce the concepts and skills most frequently tested



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What's New to *Vmath* Third Edition?

With the updated grade-level expectations, customers have asked for intervention solutions that are more closely aligned, yet still provide explicit instruction, print and technology components, a strong assessment system, error analysis, detailed lesson support, and a data management system. In addition to maintaining these components, *Vmath* Third Edition also has maintained the Hands-On and Gizmos lessons, Algebraic Thinking, Write Math, and Critical Thinking open-ended response items, as well as the reteach tools and differentiation suggestions.

New features include:

- **Foundational Skills**—Each *Vmath* module is built on the progression of the standards. These foundational skills, critical for the standards in the current grade level, provide students with the opportunity to receive the instruction they may not have mastered from previous grade levels or to provide students with a review of these skills due to summer loss.
- **New Lessons**—
    - Math Flash lessons (Levels D–I) are quick 20-minute lessons designed to optimize the time to bridge the content gap. Math Flash lessons focus on concepts, test-taking, or open-ended response.
    - New *Vmath* lessons have been added to the program to ensure a tighter alignment to the standards.
    - *Vmath* lessons have been thoroughly revised so students receive the skills needed for success in the core.
- **Concept Development**—Building on the representational and concrete conceptual development of the Hands-On and Gizmos lessons, *Vmath* Third Edition expands the use of abstract conceptual development by providing more explicit language for the teacher during the Build the Concept component of the lesson.
- **Problem Solving**—Students who struggle with math often have difficulties making the connection between the algorithm and the skills needed to solve problems. *Vmath* Third Edition increases the explicit language used to strategically teach students how to formulate a plan and explore the options of creating plans as they apply their knowledge of content.
- **eBooks**—The eBook is a rich, interactive, and enhanced multimedia experience that retains the fidelity of the *Vmath* materials, while providing users with electronic content. eBooks can be accessed from iPads, Android platforms, PCs, and Macs. The *Vmath* eBook allows users to click on pictures and icons to access multimedia content and provides communication options between the teacher and the student.

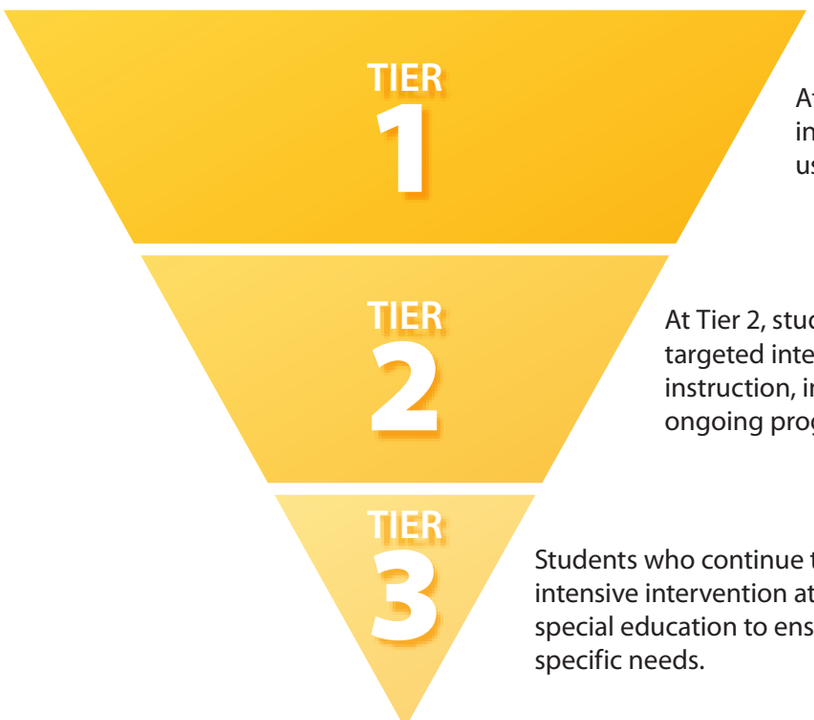
Who Is *Vmath* For?

Response to Intervention (RTI) is the practice of providing evidence-based, high-quality instruction targeted to students' needs. Meeting the RTI requirements of Tier II instruction, *Vmath* supports students who need additional practice and reinforcement of the grade-level skills. *Vmath* provides evidence-based instruction, uses data to guide instructional decisions, and evaluates instructional effectiveness.

Vmath is designed to complement and strengthen core math instruction by providing additional daily instruction for small groups of students.

The *Vmath* lesson content and format are specially designed to meet the needs of students who struggle with grade-level math concepts. *Vmath* delivers the essential content using explicit instruction and a variety of lessons to motivate students.

The instructional strategies used in the *Vmath* lessons are effective for at-risk students and provide the necessary teacher support to assure that research-based best practices are consistently delivered to every student. This support allows teachers of all experience levels to deliver high-quality instruction using the suggested dialogue and questioning strategies provided for each lesson.



At Tier 1, students achieve proficiency in the general education classroom, using the core curriculum.

At Tier 2, students who are struggling receive targeted intervention through differentiated instruction, informed by assessments and ongoing progress monitoring.

Students who continue to struggle receive additional, intensive intervention at Tier 3 and may be referred to special education to ensure instruction meets their specific needs.

What Makes *Vmath* Work?

Vmath offers several resources to help target the skills students need for success:

- The Foundations module at the beginning of each level provides instruction in the skills of the previous grade level(s). This instruction refreshes the earlier skills that are needed for success in the students' current grade level.
- In addition, Preskill Lessons (PL) are taught at the beginning of Modules 2–7.
- Additional preskill components are embedded in many of the lessons.
- Extra Practice is included in every module.
- Reteach lessons are included in every module.
- The Check Up, completed at the end of each lesson, provides the teacher with information regarding students' understanding of the content as well as tools to help students overcome any misconceptions regarding the content or skills.
- Teachers receive data to help guide instructional decisions.

Foundations for Success (National Mathematics Advisory Panel [NMAP], 2008), available at www.ed.gov/MathPanel, highlights evidence-based practice necessary for student success. *Vmath* responds to each of these recommendations with critical components that support the struggling math student through intervention.

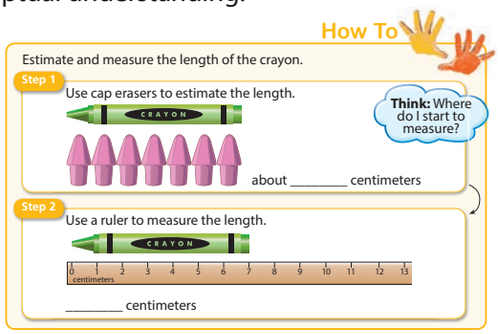
- **Emphasize Critical Concepts**—*Vmath* provides students with additional instruction in the concepts outlined by state and national standards. In addition, *Vmath* provides foundational lessons to help scaffold students to meet the grade-level expectations.
- **Teach Critical Foundations to Mastery**—*Vmath* provides teachers with assessment data to help guide when differentiation, reteach, or extra practice is necessary.
- **Balance Conceptual Understanding, Fluency, and Problem Solving**—*Vmath* provides explicit instruction in both conceptual understanding and problem solving. In addition, VmathLive® supports students through mental math, computational fluency skills, and procedural understanding and fluency.
- **Use a Combination of Teacher-Centered and Student-Centered Approaches**—While providing explicit instruction to ensure the best instructional delivery, *Vmath* also provides a variety of lessons and lesson components within each module that support hands-on experience, discussion, writing, and problem solving.
- **Provide a Variety of Conceptual-Learning Lessons**—*Vmath* provides concrete learning in Level C Center Activities and in Levels D–I Hands-On lessons. In addition, students in Levels D–I have access to Gizmos, online simulations. *Vmath* lessons also include Build the Concept models to support students in their mathematical development.

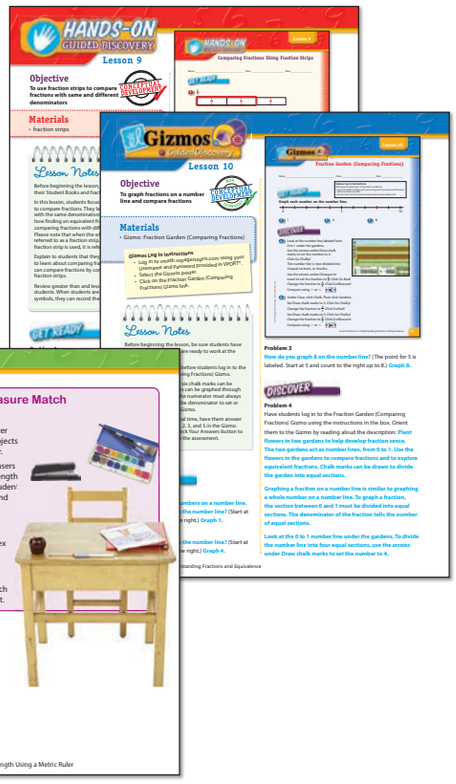
Explicit Instruction

Vmath employs the explicit instruction model supported by the National Mathematics Advisory Panel. *Vmath* lessons use a four-step lesson format consistent with the tenets of direct instruction. Each lesson involves teacher modeling, student and teacher interaction, independent work, and error analysis. *Vmath* gives detailed, explicit instructions for each lesson, allowing teachers to deliver the lesson comfortably and quickly and students to hear the consistency of the math language.

Conceptual Understanding

Vmath Hands-On and Gizmos lessons in Levels D–I, integrated at point of use, help teachers present important math concepts using either common manipulatives or Gizmos, online virtual manipulatives developed by Explore Learning. Detailed support and design ensure high-quality instruction. Additionally, in the daily *Vmath* lessons, the Build the Concept box uses visual models and pictorial representations to help students develop conceptual understanding.





Vmath Level C includes Center Activities to reinforce skills taught in each daily lesson.

Vocabulary

Vmath lessons reinforce the recursive academic vocabulary critical for student understanding. Teachers introduce the words at the start of each lesson and reinforce the use of the specific language as they follow the explicit language in the lesson. Students in turn are provided with multiple exposures and the opportunity to acquire the language as part of their skill development.

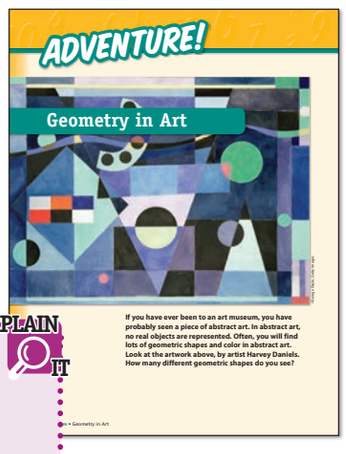
Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **Commutative Property of Addition** states that the order of the addends can be changed without affecting the sum
- **Associative Property of Addition** states that the grouping of the addends can be changed without affecting the sum

Adventures

Adventures pose relevant project-like questions in which students read, use data, answer open-ended questions, or write short paragraphs. All Adventures are included in a separate eBook. Teachers have access to Adventures for all levels, C–I.



Application of Skills

Several components of *Vmath* are geared to help students apply their learning. Each daily lesson provides a Critical Thinking, Explain It, Write Math, or Algebraic Thinking component.

Each of these features is designed to promote reasoning and decision making in math.

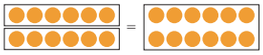
EXPLAIN IT

Find 3 × 5. Then find 5 × 3. What do the products show about the order of the factors in a multiplication problem? Explain.

_____
_____
_____

ALGEBRAIC THINKING

Use the model to complete the statement:



2 × _____ = 12

WRITE MATH

Which answer choices in problem 2 are the least reasonable? Explain. _____
_____
_____

CRITICAL THINKING

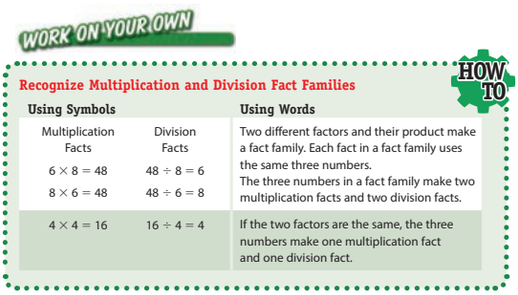
Without finding the actual product, tell whether 80 × 70 is greater than or less than 500. Explain. _____
_____
_____

In addition, throughout the Teacher Edition, teachers see an icon that reminds them to stop and have students discuss and/or share their thinking. Students should discuss their answers in pairs or small groups. If students' answers differ, students should work together to solve the problem.



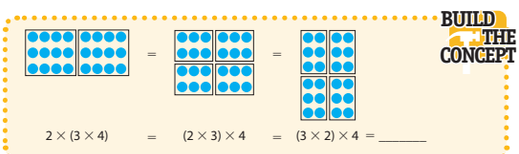
Procedural Understanding

Students learn a strategy and are given tools to help them remember the procedures of a strategy while using those tools to build conceptual understanding. Struggling math students need reinforcement to remember these procedures as they learn. The How To box provides students with detailed steps so they can repeat the procedure on their own.



Build the Concept boxes help students analyze what they are learning and how the procedure applies to the concept.

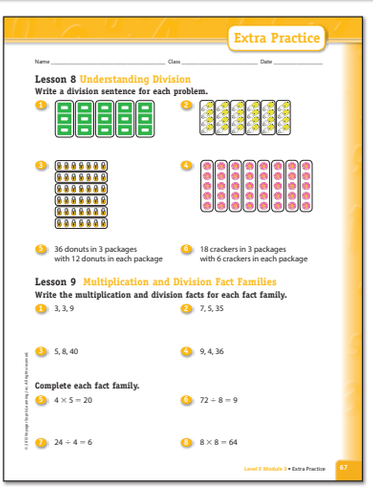
As students develop these reasoning skills, the connection to what is taught in core instruction will become stronger.



VmathLive, the technology component of *Vmath*, provides students with the opportunity to hone their computational fluency skills as well as their strategy and fluency with mental models. VmathLive aligns with daily lessons, supports the standards, and extends the school day, as it is available anywhere and, anytime students have access to the Internet.



In addition, students have Extra Practice pages for each of the *Vmath* four-step lessons that reinforce automaticity of the concepts and skills they are learning in *Vmath*. Hands-On, Gizmos, and Math Flash lessons do not include Extra Practice pages.



Problem Solving

Students who struggle with math often have difficulties making the connection between the algorithm and the skills needed to solve problems. As the student begins to make connections between the concepts of math and the skills needed for success, problem solving is introduced strategically throughout each *Vmath* module. These problem-solving components help students begin to learn how to formulate a plan.

Some of the *Vmath* Problem-Solving strategies include solving simpler problems, finding patterns, and working backward.

PROBLEM-SOLVING

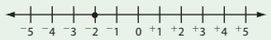
Using a 4-Step Plan

The high temperature yesterday was ⁻2°F. The forecast for today says the high temperature will be the opposite of the high temperature yesterday. What is the forecast for the high temperature today?

a. **Find:** the forecast for the high temperature today

b. **How?** Use a 4-step plan.

c. **Solve.** Find the opposite of ⁻2°F.



high temperature yesterday: _____

opposite: _____

d. **Is the answer reasonable? Explain.** _____

_____

Differentiation Opportunities

English Language Learners

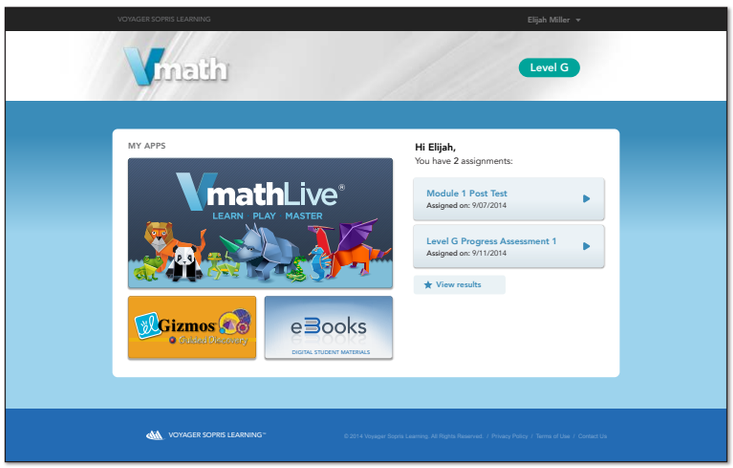
Use the VmathLive Animated Math Dictionary to review the terms *less than symbol* and *greater than symbol*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

To distinguish between the less than symbol, <, and the greater than symbol, >, show students that the less than symbol looks like a tilted 4 and the greater than symbol looks like a tilted 7 and that 4 is less than 7.

When working comparison problems, have students say the math sentences aloud, reinforcing the names for the symbols.

Students with Special Needs

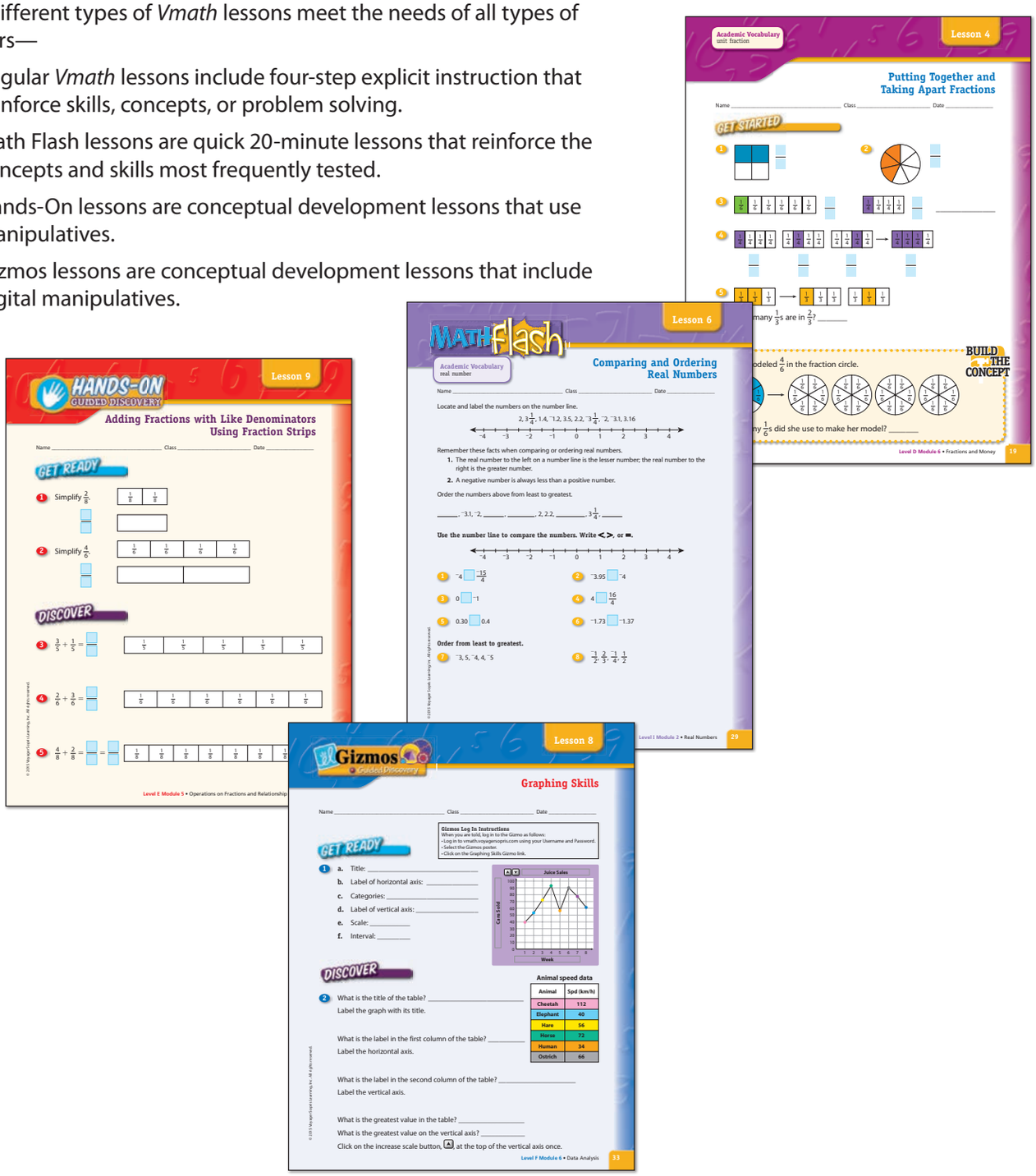
Have students draw a number line for reference that shows ⁻10 to 10, labeling the left arrow with the words *Lesser, Less than,* and *Least,* all of which begin with the letter *L*. This will be a visual cue for students to remember that numbers farther to the left on a number line are less than numbers toward the right.



Four Types of Lessons

Four different types of *Vmath* lessons meet the needs of all types of learners—

1. Regular *Vmath* lessons include four-step explicit instruction that reinforce skills, concepts, or problem solving.
2. Math Flash lessons are quick 20-minute lessons that reinforce the concepts and skills most frequently tested.
3. Hands-On lessons are conceptual development lessons that use manipulatives.
4. Gizmos lessons are conceptual development lessons that include digital manipulatives.



Instructional Principles Supported by Research

Vmath uses widely accepted principles of effective intervention instruction for struggling students. *Vmath* provides a balance of conceptual understanding, fluency, and problem solving.

***Vmath* Instructional Principles**	**Supporting Research**
Explicit Instruction	The direct, systematic instruction in *Vmath* provides carefully sequenced, specific, and detailed dialogue for every lesson. This form of highly structured lesson delivery is based on the guiding principles of effective instructional design outlined in the book *Designing Effective Mathematics Instruction: A Direct Instruction Approach* by Marcy Stein, Diane Kinder, Jerry Silbert, and Douglas Carnine (2006).
Visual Models	NMAP (2008) endorses the use of models and visual images as important ways to promote conceptual understanding. Well-chosen visual models, in conjunction with conceptual explanations, can help students understand and remember key math concepts.
Conceptual Understanding	Concept development is taught through concrete, representational, and abstract instruction (Butler et al., 2003; Gersten & Clark, 2010; Witzel, Mercer, & Miller, 2003).
Problem Solving	The problem-solving plan in *Vmath* incorporates the four phases of the problem-solving plan devised by George Polya in his book *How to Solve It* (1957).
Procedural Skill and Fluency	*Vmath* supports students in the acquisition of procedures and knowing when to use procedures appropriately (Bos & Vaughn, 1998; Rhymer, Dittmer, & Skinner, 2000).
Error Analysis	The teacher uses the error analysis procedures to correct the student's misconception or misapplication. Error analysis is another of the common attributes that have been identified as positively affecting student learning (Smith & Geller, 2004).
Use of Assessments	"The use of formative assessment improves students' learning, especially if teachers have additional guidance on using the assessment to design and individualize instruction." —National Mathematics Advisory Panel, 2008

What Are the *Vmath* Components?

Teacher Materials

Teacher Edition—includes all modules

Student Math Pack—includes one copy of each **Student Edition Module**

Teacher Edition eBook—includes all modules

Student Math Pack eBook—7 modules

Reteach eBook

Adventure eBook

Assessment Guide eBook

***Vmath* Teacher Center**



Student Materials

Student Math Pack—7 modules

Student Math Pack eBook—7 modules

Reteach eBook

Adventure eBook

Assessment eBook

***Vmath* Student Center**—includes

- **VmathLive**
- **Gizmos**
- ***Vmath* Testing Center**



Scope and Sequence

WHOLE NUMBERS	C	D	E	F	G	H	I
Recognize odd and even numbers	●	●					
Know place value	●	●	●	●			
Use number lines	●	●	●	●	●	●	
Distinguish prime and composite numbers				●			
Write numeric expressions				●	●	●	●
Discover numeric patterns		▲	▲	●			
Solve for unknowns	●	●	●	●	●	●	●
Find greatest common factor					●	▲	▲
Find least common multiple					●		
Understand absolute value					●		
Describe relationships between sets of real numbers					●	●	●
Compare and order real numbers			●	●	●	●	●

ADDITION	C	D	E	F	G	H	I
Know basic addition facts	●	▲					
Perform multi-digit addition with and without regrouping	●	▲	●	▲			
Solve one- and multi-step addition word problems	●	●	●	▲			
Use addition of equal grouping when equal shares are joined	●	▲	▲	▲			
Describe relationships of addition	●	▲					
Use mental math to add 10 or 100 to a given number	●	▲					
Add using strategies based on place value	●	▲	●	▲			
Add and subtract within 1,000		●					
Represent one- and two-step problems involving addition and subtraction to whole numbers to 1,000 using models		●		▲			
Round to the nearest 10, 100, 1,000, or 10,000 to estimate solutions in addition problems		●	●				
Add rational numbers				●	●	●	▲

KEY ● = Taught ▲ = Introduced or Reinforced

SUBTRACTION	C	D	E	F	G	H	I
Know basic subtraction facts	●						
Perform multi-digit subtraction with and without regrouping	●	▲	●	▲			
Solve one- and multi-step subtraction word problems	●	●	●	▲			
Use subtraction of equal groups when equal shares are separated	●	▲	▲	▲			
Describe relationships of subtraction	●	▲					
Use mental math to subtract 10 or 100 from a given a number	●	▲	●				
Add and subtract within 1,000		●					
Represent one- and two-step problems involving addition and subtraction to whole numbers to 1,000 using models		●					
Round to nearest 10, 100, 1,000, or 10,000 to estimate solutions in subtraction problems		●	●	●			
Subtract rational numbers				●	●	●	▲

PLACE VALUE	C	D	E	F	G	H	I
Round whole numbers to nearest 10, 100, 1,000, or 10,000	●	●	▲				
Compose and decompose numbers up to 100,000 as a sum of ten thousands, thousands, hundreds, tens, and ones	●	●	▲	▲			
Compare and order whole numbers	●	●	▲	▲			

MULTIPLICATION	C	D	E	F	G	H	I
Interpret products of whole numbers		●					
Use multiplication within 100 to solve word problems involving equal groups		●					
Fluently multiply within 100		●					
Understand multiplication properties		●	▲	▲			
Multiply two-digit by one-digit numbers		●	▲	▲			
Determine unknown whole numbers in multiplication problems		●	▲				
Understand multiplication patterns of operations		●	●	▲			
Multiply four-digit by one-digit numbers and two-digit by two-digit numbers			●	▲			
Solve one- and multi-step problems			●	▲			
Multiply rational numbers				●	●	●	▲

KEY ● = Taught ▲ = Introduced or Reinforced

DIVISION	C	D	E	F	G	H	I
Interpret quotients of whole numbers		●		▲			
Use division within 100 to solve word problems involving equal groups		●		▲			
Fluently divide within 100		●					
Understand division properties		●	▲	●			
Divide two-digit by one-digit numbers		●	▲				
Determine unknown whole numbers in division		●	▲				
Understand division patterns of operations		●	●	●			
Find quotients up to four-digit divided by one-digit			●				
Solve one- and multi-step problems			●	▲			
Divide rational numbers				●	●	●	▲

MEASUREMENT	C	D	E	F	G	H	I
Measure lengths of objects	●	●					
Estimate lengths of objects	●	▲	●				
Compare lengths of objects	●	▲					
Represent numbers on a number line as length from 0	●	▲					
Solve problems involving length, volume, mass, and money	●		●	●			
Collect data of repeated lengths	●						
Organize length data in picture and bar graphs	●	▲					
Draw conclusions about length when organized in tables	●						
Use measurements of liquid (capacity) and weight		●					
Identify relative sizes of measurement units (customary and metric)			●				
Use area and perimeter formulas for rectangles			●				
Generate number or shape patterns following given rules (i.e., input-output tables)			●				
Convert units of measurement			●	●	●	●	●
Know and use the formula for volume				●			
Use unit cubes and cubic units to measure volume				●			

KEY ● = Taught ▲ = Introduced or Reinforced

DATA	C	D	E	F	G	H	I
Draw and organize data into graphs, line plots, stem-and-leaf plots, scatterplots, and frequency tables	●	●	●	●			
Write and solve one-step and multi-step problems using data	●	●	●	●			
Read graphs to make conclusions about a collection of data	●	●	▲	▲	●	●	
Make predictions using data in graphs	●	▲				●	
Summarize numerical data including mean, median, mode, range, and interquartile range					●	●	
Display numerical data graphically					●	●	
Use graphical representations of numeric data to describe the center, spread, and shape of the data distribution					●	●	
Compare measures of central tendency					●		
Compare numeric data						●	
Understand probability						●	
Use data from random samples and information from populations						●	

TIME	C	D	E	F	G	H	I
Read and write time to the nearest minute	●	▲	▲				
Time intervals		●	●				

MONEY	C	D	E	F	G	H	I
Determine the value of coins	●	●					
Solve word problems using money	●	▲					
Add, subtract, and make change			●				
Solve for products of decimals				●			

KEY ● = Taught ▲ = Introduced or Reinforced

GEOMETRY							
	C	D	E	F	G	H	I
Recognize shapes	●	▲	▲	●			
Know the attributes of shapes and classify shapes	●	●	●	●			
Draw shapes	●						
Decompose shapes	●	●	▲			●	▲
Use concrete models to find areas of shapes	●						
Classify and sort two- and three-dimensional figures		●	●	●			
Solve problems involving perimeter		●		●			
Recognize unit squares as one square unit of area		●		●	▲		
Find area		●	●				
Understand the concept of angles			●			●	●
Classify figures based on the presence of parallel lines, perpendicular lines, or angles			●				
Determine the measure of an unknown angle			●				
Illustrate the measure of an angle			●		●	●	●
Identify and draw lines of symmetry			●				
Describe key attributes of the coordinate plane				●			
Graph ordered pairs				●	▲		
Model area formulas by decomposing and rearranging parts of the shapes					●	●	●
Draw polygons in the coordinate plane given coordinates for the vertices					●		
Find volume				●	●	●	●
Solve problems involving scale drawing of geometric figures						●	
Describe π as the ratio of the circumference to the diameter						●	
Use models to determine the formulas for circumference and area of a circle						●	
Determine the area of composite figures						●	
Solve problems for total surface area						●	●
Apply the Pythagorean Theorem							●
Compare shapes using rotations, dilations, translations, and reflections							●

KEY ● = Taught ▲ = Introduced or Reinforced

INTEGERS	C	D	E	F	G	H	I
Identify integers					●		
Interpret absolute value for positive or negative quantities					●		
Represent integers as points on a number line					●	●	
Extend integers to points on a coordinate axes					●		
Add, subtract, multiply, and divide integers fluently					●		
Understand that integers describe quantity					●	●	
Add and subtract positive and negative integers					●	●	
Multiply and divide integers						●	
Know and apply the properties of integer exponents							●

FRACTIONS	C	D	E	F	G	H	I
Partition shapes into equal shares	●						
Understand halves, fourths, and eighths	●	●					
Count fractional parts	●	▲					
Join objects to show equivalent sets	●	▲	▲				
Divide objects to create equivalent sets	▲	●	●				
Represent fractions greater than 0 and less than or equal to 1		●	▲				
Understand that the unit fraction $\frac{1}{b}$ represents the quantity formed by one part of a whole		●	▲				
Represent equivalent fractions		●	▲				
Compare fractions		●	●	●			
Determine fraction equivalence			●	●			
Represent the value of digits in whole numbers and real numbers			●				
Add and subtract fractions				●	●	●	
Multiply fractions				●	●	●	
Divide fractions				●	●		
Understand $\frac{1}{b} \times q$ as a partition of q into b equal parts				▲			
Determine quantities as increased or decreased when multiplied by a fraction					▲		
Extend representations for division to include fraction notations					▲		
Add and subtract positive and negative fractions						●	
Multiply and divide positive and negative fractions						●	

KEY ● = Taught ▲ = Introduced or Reinforced

DECIMALS	C	D	E	F	G	H	I
Represent fractions as decimals			▲	▲			
Represent decimals using concrete models			●				
Use decimal notations			●	●			
Compare and order decimals			●	●			
Add and subtract decimals			●	●	●	●	
Round decimals				●			
Multiply and divide decimals				●	●	●	
Multiply and divide positive and negative decimals						●	
Convert rational numbers to decimals						●	

EXPRESSIONS AND EQUATIONS	C	D	E	F	G	H	I
Write and evaluate expressions	●	●	●	●	●	●	●
Distinguish between expressions and equations					●		
Use variables			●	●	●	▲	▲
Identify parts of expressions					●		
Evaluate expressions at specific values for their variables					●		
Generate equivalent expressions					●	●	●
Identify when two expressions are equivalent					▲	▲	▲
Find solutions for one-variable, one-step equations					●	▲	▲
Write equations with dependent and independent variables					●	▲	
Analyze the relationship between variables using graphs and tables					●		
Write one-variable, one-step equations and inequalities to represent constraints within problems					●	▲	
Represent solutions for one-variable, one-step equations and inequalities on number lines					●		
Solve one-variable, one-step equations and inequalities that represent problems					●		
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients						●	
Write one-variable, two-step equations and inequalities to represent constraints						●	

KEY ● = Taught ▲ = Introduced or Reinforced

EXPRESSIONS AND EQUATIONS (cont.)	C	D	E	F	G	H	I
Represent solutions for one-variable, two-step equations and inequalities						●	●
Solve one-variable, two-step equations and inequalities that present problems						●	●
Represent linear relationships using multiple representations						●	●
Perform operations with numbers expressed in scientific notation							●
Solve linear equations with and without rational number coefficients							●

PROPORTIONALITY	C	D	E	F	G	H	I
Understand the concept of a ratio					●	▲	
Differentiate between additive and multiplicative relationships that represent mathematical problems					●		
Understand unit rate as associated with a ratio					●	●	
Represent ratio and rates using scale factors, tables, graphs, proportions, fractions, and decimals					●	●	
Find a percent of a quantity as a rate per 100					●		
Solve problems finding the whole, given a part and the percent					●	●	
Use ratio reasoning to convert measurement units					▲	●	
Identify proportional relationships and constant rates of change						●	
Represent proportional relationships						●	●
Determine mean absolute deviation							●
Construct and interpret bivariate data							●
Graph proportional relationships							●
Use data to determine slope							●

FUNCTIONS	C	D	E	F	G	H	I
Describe the functional relationship between two quantities							●
Understand function rules							●
Model a linear relationship between quantities							●
Solve problems involving direct variation							●

KEY ● = Taught ▲ = Introduced or Reinforced

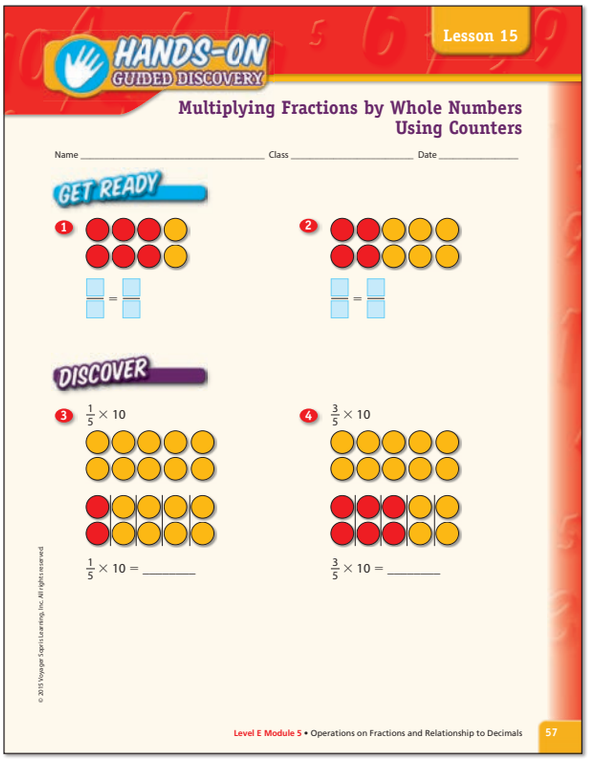
Vmath Builds Conceptual Understanding

Manipulatives, both concrete and representative models, are beneficial tools for teaching students important mathematical concepts (NMAP, 2008). Conceptual development in math is especially important to struggling students who may not have grasped math concepts as presented in their core program. *Vmath* uses three types of conceptual development instruction: concrete, representational, and abstract.

Hands-On Lessons

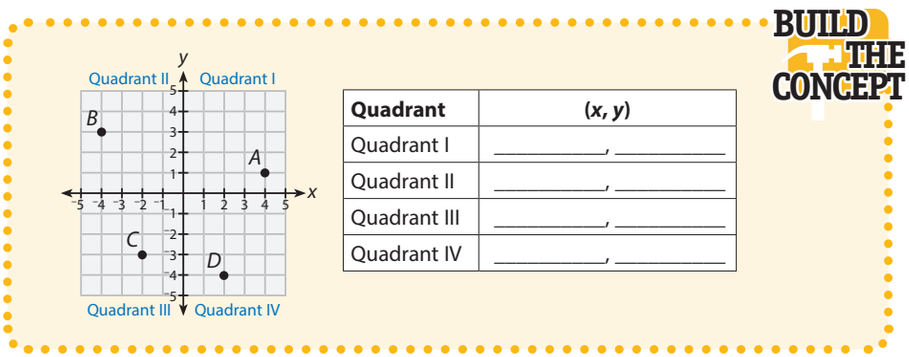
The following concrete manipulatives are used in the *Vmath* Hands-On lessons for Levels D–I. These manipulatives are commonly found in the mathematics classroom. Additionally, copy masters of nets, decimal grid paper, dot paper, and grid paper are provided in the Teacher Editions of each level wherever these materials are required for the lesson.

- Base-10 pieces
- Fraction strips
- Counters
- Rulers
- Unit cubes
- Algebra tiles



Build the Concept

The Build the Concept box uses visual models to help students develop a deeper understanding of the targeted mathematical concept. Along with the model, the teacher provides explicit language to help students connect the visual representations to the standard symbolic representations used in mathematics.



Quadrant	(x, y)
Quadrant I	_____, _____
Quadrant II	_____, _____
Quadrant III	_____, _____
Quadrant IV	_____, _____

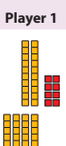
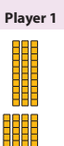
Level C Center Activities

Level C Center Activities provide students with an opportunity to work with partners or groups to further reinforce the skills taught in the lesson.

Center 1: Play Teamwork Addition

1. This game needs two players.
2. Player 1 uses base-10 pieces to make an addition problem. Each number should be made of both tens rods and ones blocks.
3. Player 2 writes the addition problem on paper.
4. Player 1 uses base-10 pieces to solve the problem.
5. Player 2 solves the problem on paper.
6. Check to see if both answers match. Write down 1 point for a match.
7. Change roles. Repeat and see how many points you can get.



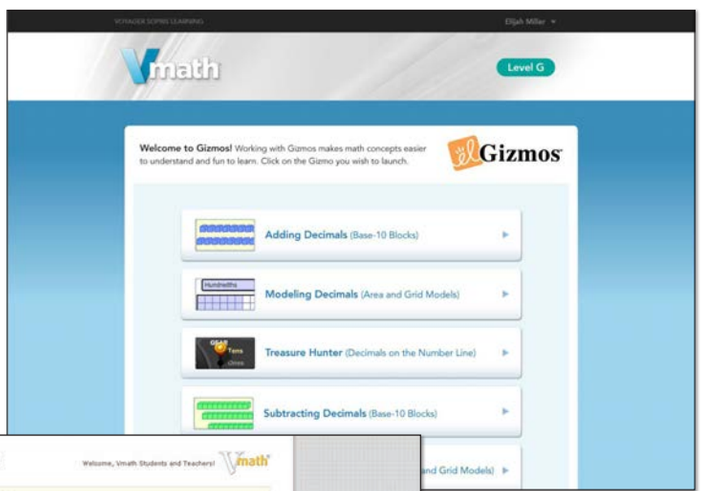
Player 1	Player 2	Player 1	Player 2
	28 +45		1 28 +45 73

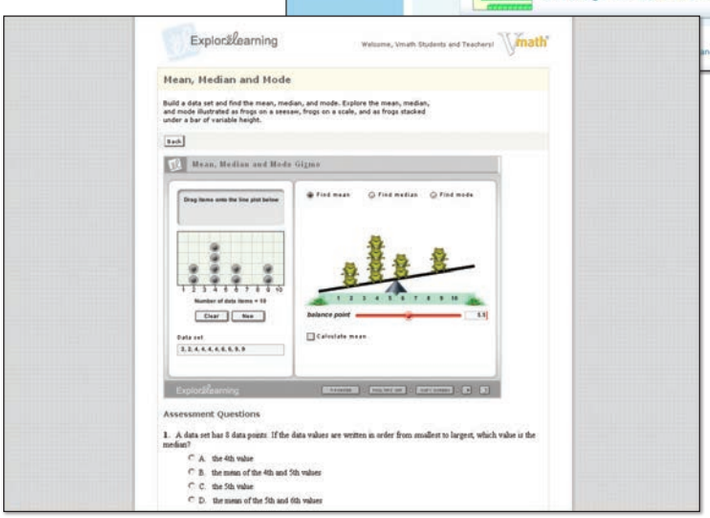
Gizmos Lessons

ExploreLearning offers a catalog of modular, interactive simulations in math and science for teachers and students in grades 3–12 called Gizmos. These simulations are fun, easy to use, and flexible enough to support many different teaching styles and contexts. A subset of Gizmos is available for *Vmath* students. Students can access the Gizmo available with *Vmath* using these steps:

1. Access VmathLive at www.vmath.voyagersopris.com.
2. Sign in to VmathLive with a Username and Password.
3. Select the Gizmos poster.
4. Click on the Gizmo referenced in the lesson.

To find out more about other Gizmos for math and science not included with *Vmath*, visit ExploreLearning at www.explorelearning.com.





Vmath Improves Problem-Solving Skills

"Solving problems is not only a goal of learning mathematics but also a major means of doing so."
Principles and Standards for School Mathematics, National Council of Teachers of Mathematics (2000)

As students master fundamental math concepts and skills, they must also develop appropriate problem-solving skills and strategies. Problem solving is an essential component of the *Vmath* program in two important ways: problem-solving strategies are explicitly taught in every *Vmath* module; and problem-solving strategies are integrated as a feature in many *Vmath* lessons.

Problem-Solving Lessons

Not all students develop problem-solving strategies naturally. This is more likely the case with students targeted for math intervention. Many students need explicit instruction in mathematical problem-solving skills and strategies to solve problems. For this reason, in each of the seven modules per level, *Vmath* provides specific lessons entirely devoted to teaching problem-solving strategies.

These lessons, which integrate instruction and practice in problem solving, emphasize the four-step process most often used in core math programs: Understand, Plan, Solve, and Look Back. Employing explicit instruction techniques, *Vmath* presents lessons on several high-utility problem-solving strategies, such as finding a pattern, working backward, using a table, solving a simpler problem, and using estimation.

The problem-solving lessons provide the opportunity for students to apply the four-step process. During the Get Started section of the lesson, the teacher guides students through the four-step process by asking questions and modeling the thought process used in the particular problem-solving strategy. Throughout the Try It Together section, the teacher continues to guide students through the problem-solving process using the How To box, which outlines a step-by-step process. An example of the How To box in a Problem-Solving lesson is shown.



HOW TO

Solve a Problem Using Patterns

When released into the air from the ground, a helium-filled balloon rises 9 inches every second. What is the height of the balloon after 5 seconds?

1. **What am I trying to find?** the height of the balloon after 5 seconds
2. **What is the pattern?** Add 9 for every second.
3. **Solve.**

Seconds	1	2	3	4	5
Computation		$9 + 9 = 18$	$18 + 9 = 27$	$27 + 9 = 36$	$36 + 9 = 45$
Height Gain (inches)	9	18	27	36	45

4. **Is the answer reasonable? Explain.** Yes, because $9 + 9 + 9 + 9 + 9 = 45$.

Problem-Solving Strategy Practice

In the Problem-Solving strategy box incorporated into many *Vmath* lessons, students are guided to solve a problem through the use of a problem-solving strategy. Students then apply the strategy themselves to the problem that follows in the Problem-Solving: New and Review section of the lesson. For instance, students are shown an example of the strategy of Using a Table for a lesson on ordering whole numbers. Students then apply what they have learned to solve the problem that follows.

Student Edition

PROBLEM-SOLVING

Using a Table

There are 1,124 students at Meadow School, 1,142 students at River School, and 1,064 students at Valley School. Which school has the least number of students?

a. **Find:** the school with the least number of students

b. **How?** Use a table.

c. **Solve.**

	Thousands	Hundreds	Tens	Ones
Meadow	1	1	2	4
River	1	1	4	2
Valley	1	0	6	4

least to greatest: _____; _____; _____

_____ School has the least number of students.

d. **Is the answer reasonable? Explain.** _____

_____

_____

Problem-Solving: New and Review

Solve each problem.

14 There are 1,102 students at Poe School, 1,021 students at Mill School, and 1,120 students at Cove School. Which school has the greatest number of students?

	Thousands	Hundreds	Tens	Ones
Poe				
Mill				
Cove				

Teacher Edition

PROBLEM-SOLVING This problem illustrates the **Using a Table** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem using a table. The instruction is immediately followed by application of the strategy in problem 14.

The problem states the number of students at each of three schools.

Have students first look at the table with the numbers of students filled in.

Because all of the numbers have the same number of digits, students must find the greatest place value where the digits are different. Have students draw a ring around the digits in the hundreds place. The least of the digits in the hundreds place is 0, so Valley School has the least number of students.

Next, students will look at the digits in the tens place. Two is less than 4, so of the remaining schools, Meadow School has fewer students. Finally, River School has the greatest number of students.

Ask students to explain why Valley School has the least number of students.

Reasonableness

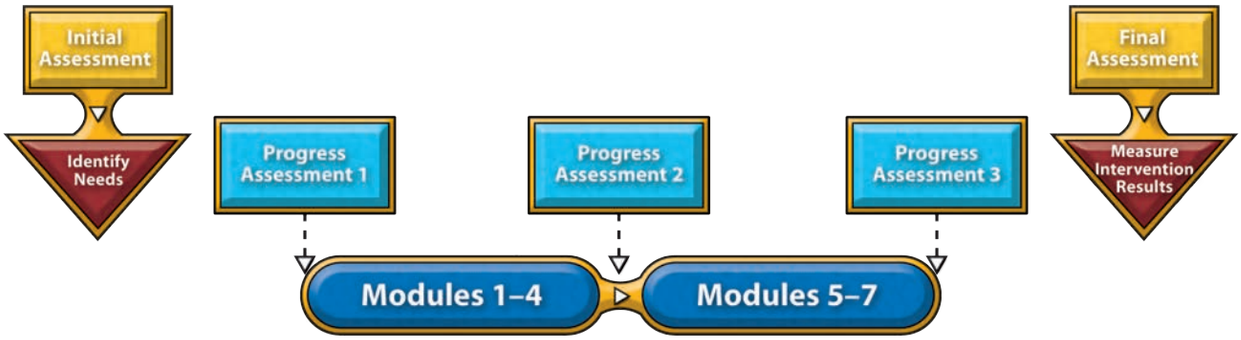
In each lesson, the third problem of each Check Up section generally focuses on applying one or more of the strategies of reasonableness and estimation. These high-utility strategies focus primarily on getting the correct answer to a problem. As students learn and apply these strategies automatically, they become better test takers. These strategies enable them to:

- check every answer to be sure it makes sense.
- eliminate unreasonable answers to increase the probability of selecting the correct answer.
- choose the best answer in a multiple-choice format.
- get correct answers to problems they thought they couldn't solve.

Armed with a toolbox of problem-solving and reasoning strategies, *Vmath* students become confident problem solvers who perform with a high degree of accuracy that is reflected in higher test scores.

Vmath Assessment System

Vmath Assessments can be administered in two ways. Online assessments can be administered through the Student Center (www.vmath.voyagersopris.com) after being scheduled by the teacher through VPORT. Paper-and-pencil assessments can also be administered. Assessments are available to print through the teacher eBook Assessment Guide.



Initial Assessment

The Initial Assessment can be given to the entire class or designated students at the beginning of *Vmath* instruction. This assessment identifies strengths and weaknesses of individual struggling students and determines (along with district criteria) appropriate entry points into instruction. Each Initial Assessment consists of 35 questions that act as indicators of understanding of skills taught in each *Vmath* module. Scores provide indicators of where students begin struggling with the material and show a starting place in the instructional sequence.

Final Assessment

The Final Assessment is given to the entire class or designated students upon completion of the full *Vmath* course of instruction or at the end of the school year. By comparing scores from the Initial and Final Assessments, teachers and coaches can document student growth and mastery throughout the course of intervention.

Progress Assessments Powered by The Quantile Framework® for Mathematics

The Progress Assessments powered by The Quantile Framework for Mathematics are administered three times throughout the school year to monitor student progress. These tests cover all five content strands in the NCTM Standards—number sense and operations, algebra, geometry, measurement, and data analysis and probability. Each 30-item test yields a Quantile measure that indicates to teachers what mathematics the student is ready for next and provides an indication of how a student is likely to perform on his or her state assessment.

Module Pre-Tests and Post-Tests

Pre-Tests and Post-Tests provide information needed to monitor student growth and mastery of the concepts, skills, and strategies taught in each module. Each test has 10 multiple-choice items. Pre-Tests determine students' prior knowledge of module content, and Post-Tests determine students' degree of mastery of module content after instruction.

Informal Assessment and Error Analysis

Ongoing informal assessments help teachers gauge student understanding. *Vmath* lessons facilitate opportunities to see students' reactions to instructions, listen as students respond to the frequent questions, and evaluate daily work. Lessons include specific review and reteaching suggestions and appropriate questioning techniques. The Check Up questions presented in multiple-choice and short-answer formats allow teachers to easily and informally assess students after each lesson. The question distracters are based on common student errors, so the teacher can be informed as to the exact cause of the student's misconception. This type of error analysis is a common attribute identified as positively affecting student learning.

Instructional Design Supports RTI Model

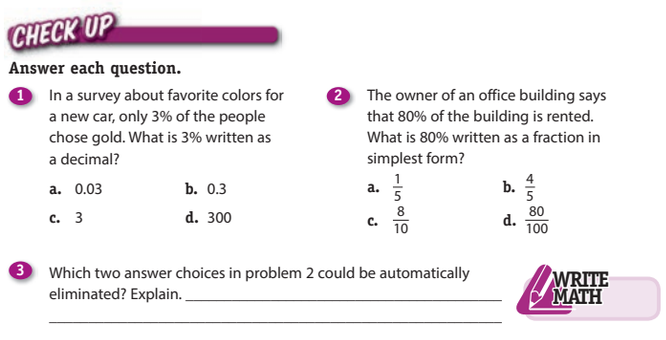
Vmath lessons are designed to support movement between Tier 2 and Tier 3 based on instructional need. Each lesson is characterized by a clear four-step lesson format consistent with the tenets of direct instruction. The first two steps, Get Started and Try It Together, are taught in a small group. The next two steps, Work on Your Own and Check Up, provide the teacher with tools to monitor progress and determine whether instruction needs to be intensified for individual students.

Students complete the Work on Your Own section and check their own answers according to instructions provided.

The Check Up questions presented in multiple-choice and short-answer formats and accompanying error analysis suggestions in the Teacher Editions allow teachers to easily assess mastery. The question distracters are based on common student errors, so the teacher can be informed as to the exact cause of the student's misconception.

Vmath includes several resources designed to enable teachers to easily intensify and reinforce instruction.

- VmathLive activities
- Reteach Copy Masters
- Extra Practice problems





ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b: The student did not place a 0 in the tenths place of the decimal as a placeholder. Use Additional Resources in G.5.7 to reteach how to write a percent as a decimal.

If student answered 1d: The student moved the decimal point two places to the right instead of the left. Remind the student how to write a percent as a decimal.

If student answered 2c or 2d: The student wrote the percent as a fraction, but did not write the fraction in simplest form. Review with the student how to write a fraction in simplest form.



If student answered problem 3 incorrectly: While students work together, review with the student that the problem asks for a fraction in simplest form. Both $\frac{8}{10}$ and $\frac{80}{100}$ are not written in simplest form, so both fractions can be automatically eliminated as the correct answer.

Differentiation Informed by Data and Resources

Vmath offers multiple opportunities to assess, reinforce, and differentiate instruction.

Data-Driven Instruction

Initial Assessment—The Initial Assessment, given at the start of the *Vmath* implementation, provides teachers with valuable information regarding students' mathematical strengths and weaknesses. Along with district criteria, it can also be used to determine entry points into *Vmath* instruction.

Students who score less than 60 percent on the Initial Assessment will benefit from the Foundations Module at the beginning of each level.

Name or ID	Jane B.		Amit K.		Dan C.		Ravi L.		José P.		Maria G.		
	Number Correct		Number Correct		Number Correct		Number Correct		Number Correct		Number Correct		
Questions	I	F	I	F	I	F	I	F	I	F	I	F	Level G Module
1–5	5		3		5		2		3		2		1
6–10	5		3		5		2		3		2		2
11–15	4		2		5		1		3		2		3
16–20	4		1		4		1		3		1		4
21–25	5		1		4		0		3		1		5
26–30	2		0		2		0		2		1		6
31–35	1		0		1		0		1		0		7
Total	26		10		26		6		18		9		

Module Pre-Tests and Post-Tests—These tests provide information that is used to monitor student growth and mastery of the concepts, skills, and strategies taught in each module.

Using data from module Pre-Tests, the teacher can determine students' prior knowledge of module content. If students score above 70 percent on the Pre-Test, students are ready for the lessons in the module. Scores of less than 70 percent on the Pre-Test indicate that students would benefit from additional skill acquisition from the PLs (Preskill Lessons) found at the beginning of each module.

Pre-Test	Informs Instruction
Student scores less than 70%	Teach PLs at the start of each module and main lessons
Student scores above 70%	Teach main lessons

Post-Tests determine students' degree of mastery of module content after instruction. The data from the Post-Test can be used to determine additional instruction needed to help reinforce new learning.

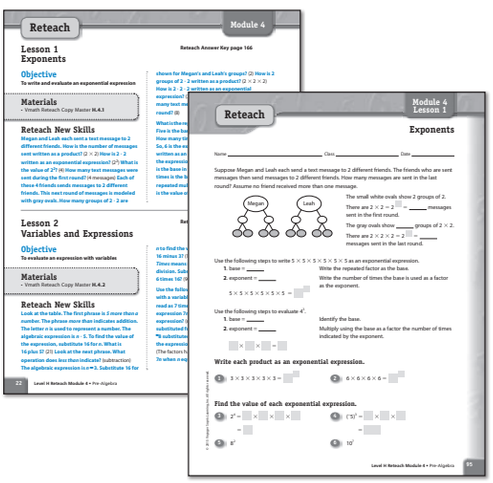
Post-Test	Informs Instruction
Student scores less than 70% or makes no gain from Pre-Test	Analyze results and use Reteach and/or VmathLive assignments
Student scores above 70%	No additional instruction is necessary

Progress Assessments powered by The Quantile Framework® for Mathematics—administered three times per year—monitor student progress in math by providing a Quantile® measure that indicates to teachers what mathematics students are ready for. Each Assessment Guide provides a Quantile lesson map that can also be used to guide instruction. The Quantile score of the lessons when compared with the Quantile score of the student(s) can help determine pacing for a lesson.

Lesson	Lesson Name	Quantile Measure
1	Using a Problem-Solving Plan	No measured Quantile number
2	Modeling Decimals	470Q
3	Comparing and Ordering Decimals	600Q
4	Adding Decimals	580Q
5	Subtracting Decimals	580Q
6	Multiplying Decimals	700Q
7	Dividing Decimals by Whole Numbers	700Q
8	Dividing Decimals by Decimals	700Q
9	Rounding and Estimating Decimals	600Q
10	Order of Operations	500Q

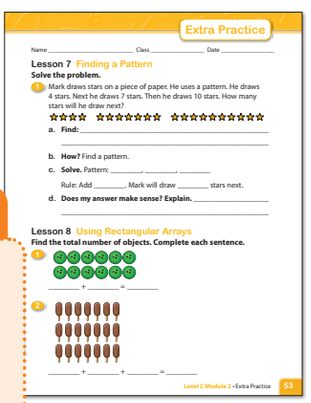
Resource Tools

Reteach Lessons—Some students may have difficulty grasping a particular math concept presented in a *Vmath* lesson the first time the concept is taught. When this happens, teachers will need to use different approaches to reteach these struggling learners. The *Vmath* program includes Reteach Copy Masters designed to enable teachers to easily reteach each *Vmath* lesson using these different approaches. Hands-On, Gizmos, and Math Flash lessons do not have Reteach Lessons.



Each student page of the Reteach Copy Masters includes a sample problem that in most cases uses an alternative strategy to present the lesson concept or skill. The example is followed by practice problems that incorporate the alternative strategy. Just as with the main *Vmath* lessons, explicit teacher dialogue is included for each Reteach student page to help the teacher effectively present the alternative method.

Extra Practice Pages—Each *Vmath* module includes extra practice lessons that can be used to reinforce critical skills and concepts taught in the *Vmath* four-step lessons.



Integrated Support for Students with Special Needs—To enhance instruction for students with special needs, lesson-specific teaching strategies are included in the Teacher Editions. The teaching strategies for students with special needs provide teachers with adaptations to meet the learning challenges of these students.

Students with Special Needs

Have students work together to find the perimeter of the classroom. Gather several yardsticks. Discuss a strategy for measuring the room. (Students measure each wall of the classroom.) Have a few students measure the room with yardsticks while the rest of the students record the information. As a group, add the measurements together to find the perimeter of the classroom.

In the example at the right from the Teacher Edition lesson in a Geometry module, students can use yardsticks to measure the perimeter of the classroom. While the lesson teaches perimeter, the Integrated Support give a hands-on understanding of the concept. Students are encouraged to discuss the strategy before measuring.

Integrated ELL Support—To enhance instruction for English Language Learners, lesson-specific teaching strategies are included in the Teacher Editions. ELL strategies suggest detailed activities that focus on increasing student understanding of the language of mathematics. In the example at the right from the Teacher Edition of a lesson on perimeter, students create index cards with polygons with side lengths. Students are often encouraged to write definitions for terms in their own words as well as collaborate with other students to write their own definitions.

Students then calculate the perimeters of the polygons on the index cards.

English Language Learners

Use the VmathLive Animated Math Dictionary to review the term *perimeter*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Create a set of note cards with drawings of polygons. Indicate the length of each side of the polygon on a card. Match students with partners. Have students work together to write their own definitions for the term *perimeter*. Distribute note cards to students. Have students find the perimeter of the figure on their card. For extra practice, distribute rulers and string, and have students find the perimeter of a classroom object, such as a desk.

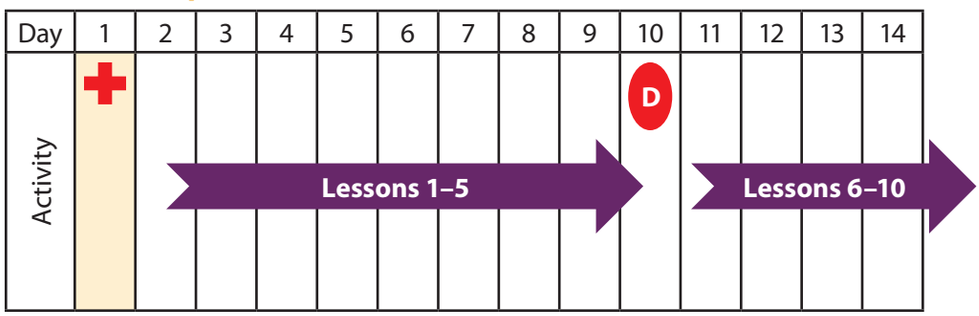
Pacing Guide at the Module Level

Each *Vmath* level includes seven modules. The first module is a Foundations Module, which is used if the majority of students score below 70 percent on the *Vmath* Initial Assessment. The remaining modules provide additional instruction and practice in the grade-level expectations taught in grade-level core classrooms.

Each module contains 10 or 15 lessons. In addition, two Preskill Lessons are used in Modules 2–7 if students score below 70 percent on the module Pre-Test.

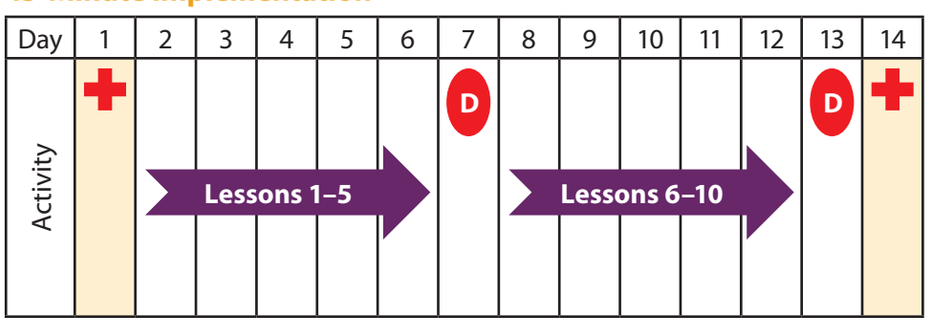
Lessons can be used in several implementation models: 20- to 30-minute sessions or 45- to 50-minute sessions.

20-Minute Implementation

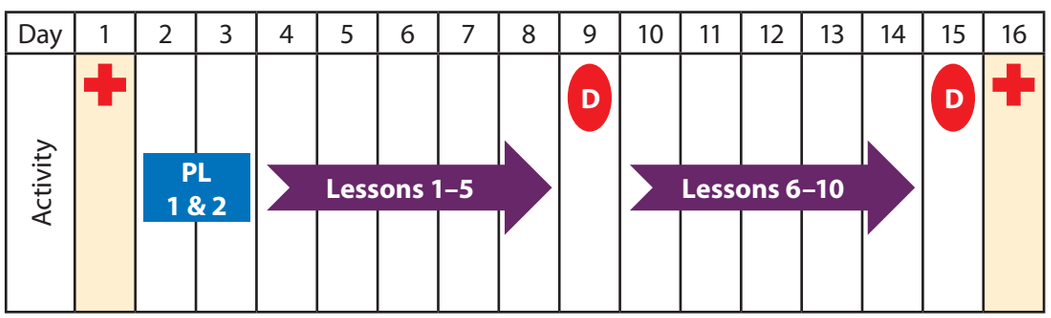


Example based on students scoring above 70 percent on Module Pre-Test

45-Minute Implementation



Example based on students scoring above 70 percent on Module Pre-Test



Example based on students scoring below 70 percent on Module Pre-Test

KEY ✚ = Pre- and Post-Test D = Differentiation

Pacing Guide at the Lesson Level

Each of the different lessons included within *Vmath* has a predictable lesson structure and can be adjusted to a 20- to 30-minute implementation or a 45- to 50-minute implementation. They can also be adjusted to fit multiple scheduling options. The following charts outline some pacing suggestions at the lesson level based on various implementation models.

20- to 30-Minute Implementation

***Vmath* Lesson—2-day Lesson Cycle**

Day One		
Lesson Structure		
Get Started	Teacher introduces vocabulary and models the concept or skill to be used.	6–8 minutes
Try It Together	Teacher scaffolds the modeling to have the students respond. This section includes the Build the Concept.	6–12 minutes
Work On Your Own	Students begin Work On Your Own.	8–10 minutes
Day Two		
Lesson Structure		
Get Started	Teacher reviews the Get Started.	2–3 minutes
Work On Your Own	Students complete Work On Your Own from Day 1 and check their results.	6–10 minutes
Check Up	Students complete all components of Check Up. Teacher uses the Error Analysis to identify misconceptions or reteaching needs.	12–17 minutes

Math Flash Lessons (Levels D–I)

Taught entirely in one 20- to 30-minute block

Hands-On and Gizmos Lessons (Levels D–I)

Day One		
Lesson Structure		
Get Ready	Teacher reviews prerequisite skills before modeling new concepts as students respond orally and follow along.	5–10 minutes
Discover	As students use manipulatives to discover key math concepts, teacher guides students' learning through questioning.	15–20 minutes
Day Two		
Lesson Structure		
Get Ready and Discover Box	Teacher reviews the skills and students review the use of the manipulatives or Gizmo.	3–4 minutes
Discover Box	Students explain what they have observed.	6–12 minutes
Explore More	Students engage in further exploration of new concepts as they apply their learning.	11–14 minutes

45-Minute Implementation

***Vmath* Lessons**

Lesson Structure		
Get Started	Teacher introduces vocabulary and models the concept or skill to be used.	5 minutes
Try It Together	Teacher scaffolds the modeling to have the students respond. This section includes the Build the Concept model.	10 minutes
Work On Your Own	Students complete Work On Your Own.	15 minutes
Check Up	Students complete all components of Check Up. Teacher uses the Error Analysis to identify misconceptions or reteaching needs.	15 minutes

Lesson Structure (Levels D–I)		
Math Flash	Students complete the lesson.	20–30 minutes
VmathLive or Reteach	Teacher assesses differentiated instruction opportunities.	15–20 minutes

Hands-On and Gizmos Lessons (Levels D–I)

Lesson Structure		
Get Ready	Teacher reviews prerequisite skills before modeling new concepts as students respond orally and follow along.	5 minutes
Discover	As students use manipulatives to discover key math concepts, teacher guides students' learning through questioning.	15 minutes
Discover Box	Students explain what they have observed.	10 minutes
Explore More	Students engage in further exploration of new concepts as they apply their learning.	15 minutes

Daily Support for Teachers Through Technology

The *Vmath* Teacher Center provides teachers access to *Vmath*—anytime, anywhere.



The *Vmath* eBook provides online resources that support teachers in the successful daily implementation of this intervention curriculum.

- Eliminates the need for multiple books
- Provides access to the complete online Teacher Edition as well as all the ancillary books such as the Assessment Guide, Reteach Book, and Alignments
- Provides a link to the student components
- Supports instruction through Online Tools
- Allows teachers to review and plan lessons anytime, anywhere
- Includes enhanced components in the Teacher Edition with just-in-time professional development and support
- Provides teacher/student interactivity and allows for immediate feedback

VPORT

The key to effective instruction is real-time data that tracks students' progress throughout the year. The *Vmath* Assessment System includes VPORT, an online data management system that lets educators collect and report student results. Using the data gathered in VPORT, teachers and district leaders are able to identify students' needs, adjust instruction, monitor student progress in meeting grade-level goals, and evaluate the effectiveness of instruction.

VPORT gives immediate feedback on all assessment results. Using VPORT, teachers, schools, and districts can . . .

- input scores from all assessments.
- view each student's scores.
- print a variety of reports for multiple uses.

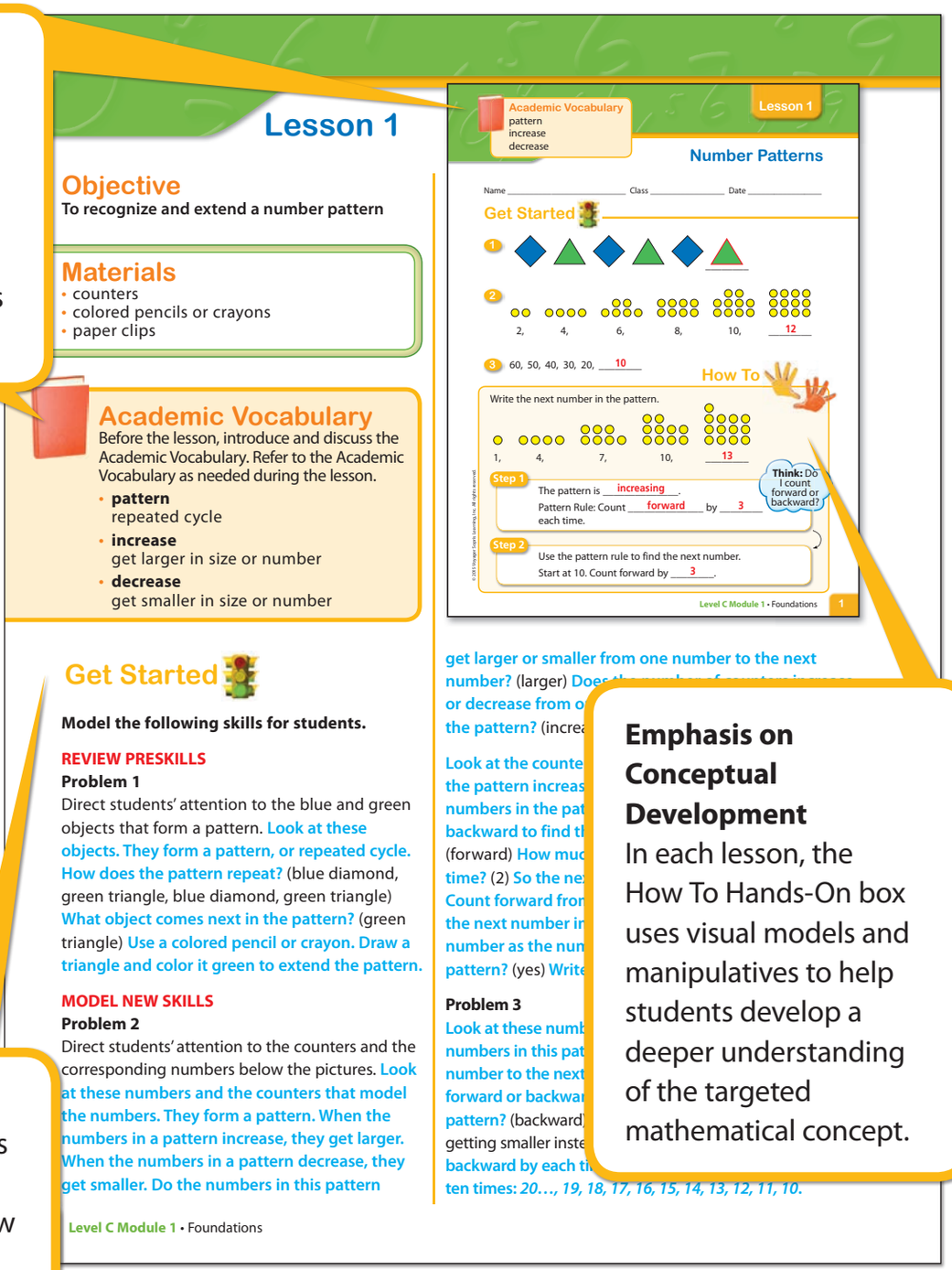
Daily Support for Teachers Through Print and eBooks

Vmath Lesson Level C Walk-Through

Vocabulary Development
Emphasis on learning the language of math enables ELL and struggling students to succeed.

Get Started
The teacher reviews prerequisite skills and models the new skills of the lesson as students work in their Student Books.

Emphasis on Conceptual Development
In each lesson, the How To Hands-On box uses visual models and manipulatives to help students develop a deeper understanding of the targeted mathematical concept.



Lesson 1

Objective
To recognize and extend a number pattern

Materials
- counters
- colored pencils or crayons
- paper clips

Academic Vocabulary
Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.
- **pattern** repeated cycle
- **increase** get larger in size or number
- **decrease** get smaller in size or number

Get Started

Model the following skills for students.

REVIEW PRESKILLS
Problem 1
Direct students' attention to the blue and green objects that form a pattern. Look at these objects. They form a pattern, or repeated cycle. How does the pattern repeat? (blue diamond, green triangle, blue diamond, green triangle) What object comes next in the pattern? (green triangle) Use a colored pencil or crayon. Draw a triangle and color it green to extend the pattern.

MODEL NEW SKILLS
Problem 2
Direct students' attention to the counters and the corresponding numbers below the pictures. Look at these numbers and the counters that model the numbers. They form a pattern. When the numbers in a pattern increase, they get larger. When the numbers in a pattern decrease, they get smaller. Do the numbers in this pattern

Academic Vocabulary
pattern
increase
decrease

Lesson 1

Number Patterns

Name _____ Class _____ Date _____

Get Started

1

2
2, 4, 6, 8, 10, 12

3 60, 50, 40, 30, 20, 10

How To

Write the next number in the pattern.

1, 4, 7, 10, 13

Step 1
The pattern is increasing.
Pattern Rule: Count forward by 3 each time.

Think: Do I count forward or backward?

Step 2
Use the pattern rule to find the next number.
Start at 10. Count forward by 3.

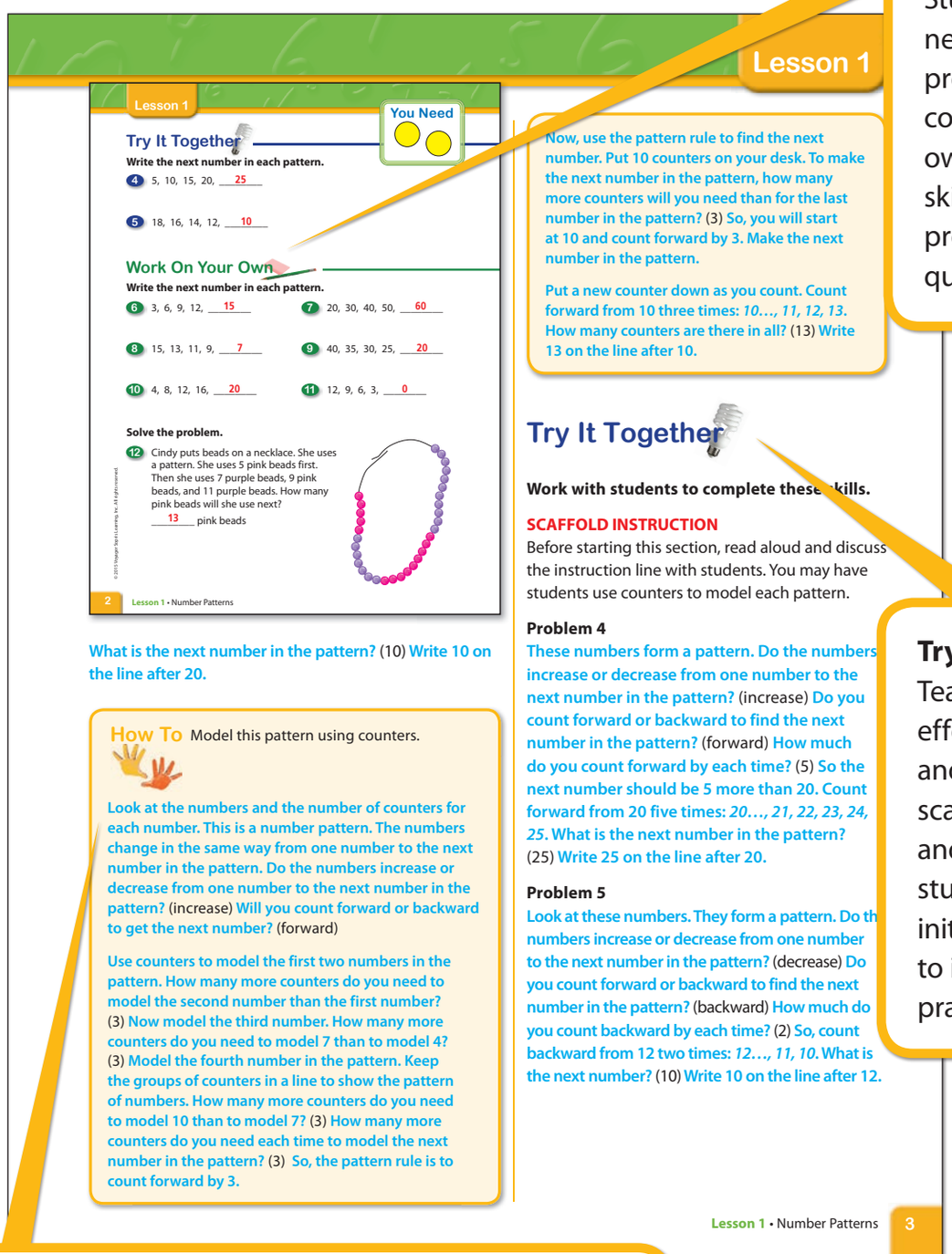
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Level C Module 1 • Foundations 1

get larger or smaller from one number to the next number? (larger) Does th… or decrease from o… the pattern? (increa…

Look at the counte… the pattern increas… numbers in the pat… backward to find t… (forward) How muc… time? (2) So the ne… Count forward fro… the next number i… number as the nu… pattern? (yes) Write…

Problem 3
Look at these numb… numbers in this pat… number to the next… forward or backwa… pattern? (backward… getting smaller inste… backward by each ti… ten times: *20…, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10.*



Work On Your Own
Students apply new learning and previously learned concepts on their own to answer skill-building and problem-solving questions.

Try It Together
Teachers use effective prompts and questions to scaffold instruction and transition students from initial learning to independent practice.

Carefully Crafted Teacher Dialogue
Teachers are provided with effective questioning strategies as they teach new skills. Students orally respond to the teachers' questions and write their answers in their Student Books as the teacher models correct work.

Building Strong Problem-Solvers
Special emphasis is placed on helping struggling students build problem-solving skills as they learn and apply strategies such as drawing a picture, using a table, and finding a pattern.

Lesson 1

Work On Your Own

MONITOR INDEPENDENT WORK

How To Before students begin independent work, review the **HOW TO HANDS-ON** example. As you review each step, emphasize the words of mathematics by having students repeat the actions for each step.

Problem 11 Reminder
Be sure students recognize that the next number in the pattern is 0.

Explain It As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

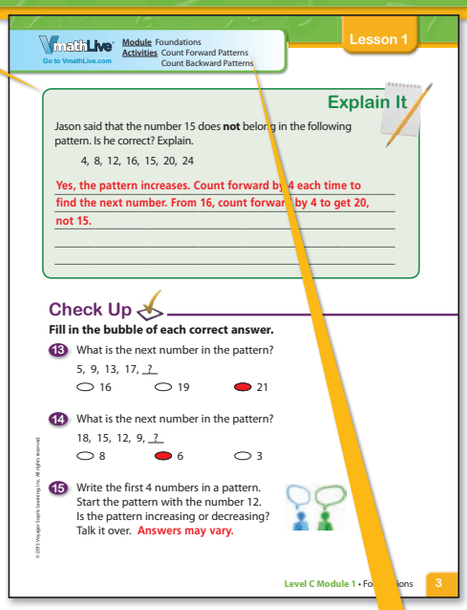
Check Up

ASSESS INFORMALLY

Error Analysis
Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered the first answer in problem 13: The student counted backward instead of counting forward. Use Additional Resources in C.1.1 to reteach how to recognize and extend a pattern.

If student answered the second answer in problem 13 or the first answer in problem 14: The student counted in the correct direction, but by the wrong number. Use Additional Resources in C.1.1 to reteach how to recognize and extend a number pattern.



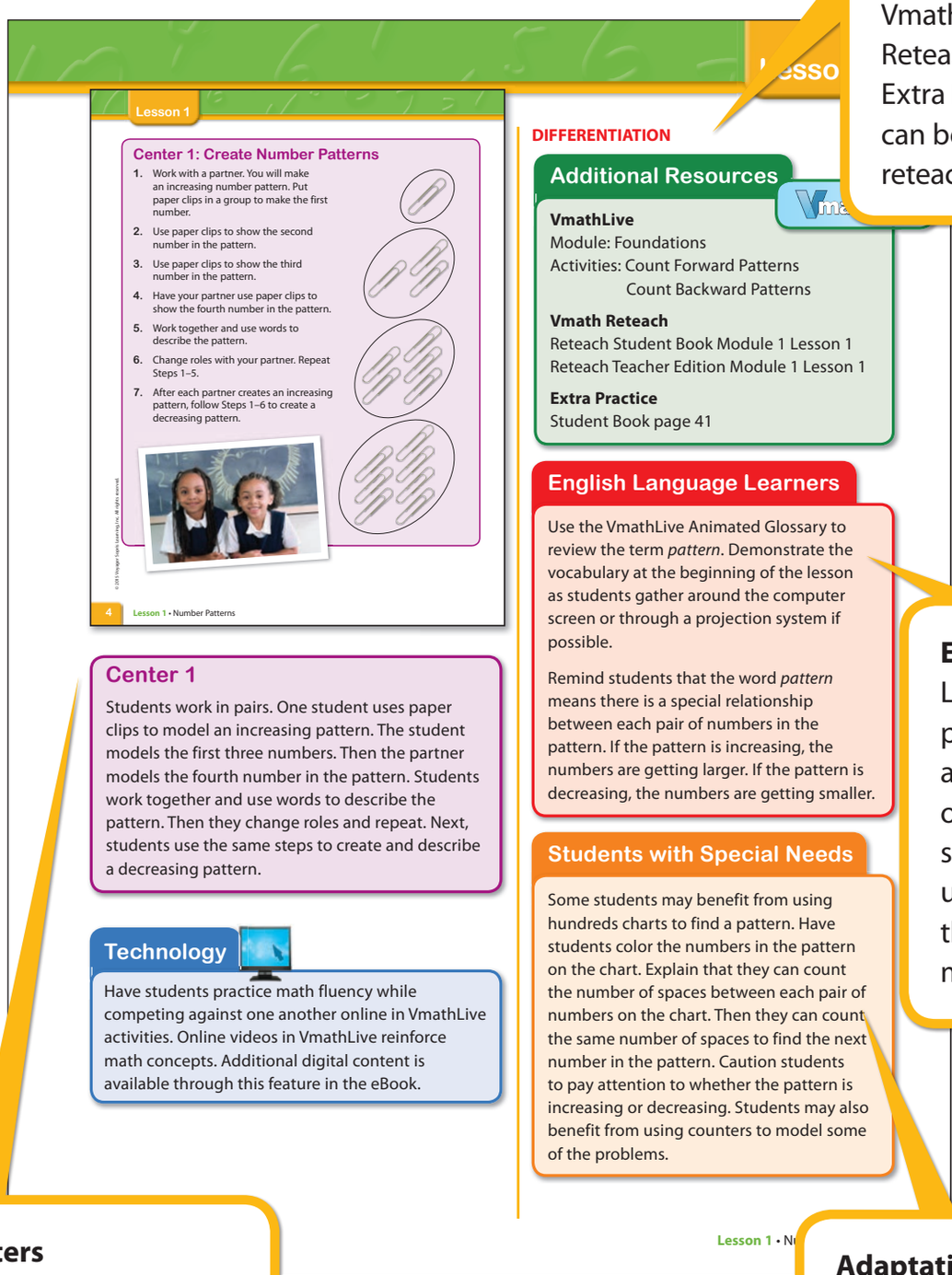
If student answered the third answer in problem 14: The student confused the number to count back[ward] by with the next number in the pattern. Use Additional Resources in C.1.1 to reteach how to recognize and extend a number pattern.

If students answered problem 15 incorrectly: While students work together, review how to create a number pattern. Students should first decide whether the pattern is increasing or decreasing. Then they must count forward (or backward) by the same amount from one number to the next to create the pattern.

Check Up
Daily informal assessments allow teachers to check for understanding and intervene with targeted corrective feedback.

Error Analysis and Corrective Procedures
Error analysis tips and specific correction procedures help teachers review and reteach as needed.

Blended Solution
Vmath integrates teacher-led instruction with engaging computer-assisted practice that reinforces the lesson and provides alternative reteaching opportunities.



Additional Resources
VmathLive activities, Reteach lessons, and Extra Practice problems can be used for practice, reteaching, or review.

ELL Teaching Tips
Lesson tips provide specific activities that focus on increasing students' understanding of the language of math.

Centers
Students are given activities to do with partners or groups to further reinforce the skill taught in the lesson.

Adaptations for Students with Special Needs
Teaching strategies in the lessons provide adaptations that support students with special needs.

Vmath Lesson Levels D–I Walk-Through

Updated Academic Vocabulary
Lessons introduce and reinforce critical vocabulary.

Lesson 7

Objective

To write an equivalent fraction

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **equivalent fractions**
  fractions that represent or name the same number

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

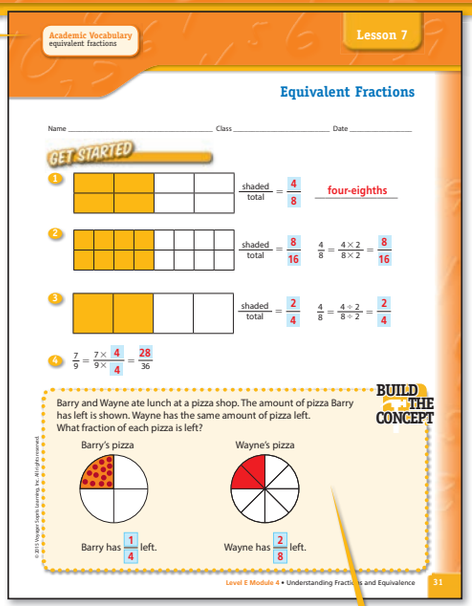
The rectangle in problem 1 is divided into equal parts. How many parts are shaded orange? (4) What is the total number of equal parts? (8) What fractional part of the rectangle is shaded orange? ($\frac{4}{8}$)

Write the fraction using words by writing the numerator in words, writing a hyphen, then writing the denominator as an ordinal number. How is the fraction written using words? (four-eighths)

MODEL NEW SKILLS

Problem 2

Look at the picture in problem 2. What fractional part of the rectangle is shaded orange? ($\frac{8}{16}$) Look at the part of the rectangle in problem 1 that is shaded orange and compare it with the part of the rectangle in problem 2 that is shaded orange. What can be said about the shaded parts of the two rectangles? (They represent the same amount.) Equivalent fractions are fractions that represent or name the same part of a whole. The fractions $\frac{4}{8}$ and $\frac{8}{16}$ are equivalent fractions.



Equivalent fractions can be found by either multiplying or dividing the numerator and denominator of a fraction by the same number. Start with the fraction $\frac{4}{8}$. Multiply both the numerator and the denominator by the number 2. What is 4 times 2? (8) What is 8 times 2? (16) The fractions $\frac{4}{8}$ and $\frac{8}{16}$ are equivalent fractions; this is shown in the pictures in problem 1 and problem 2.

Problem 3

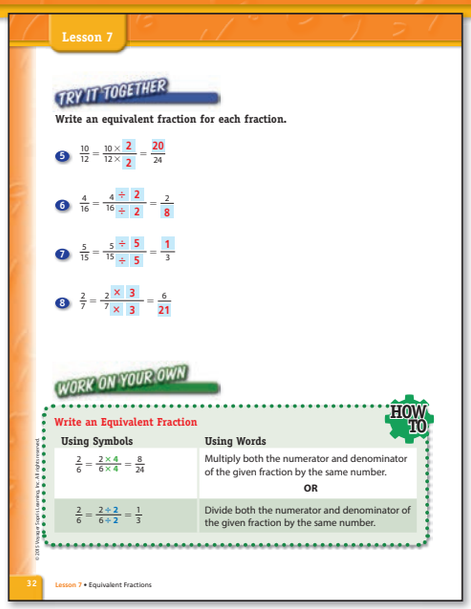
Look at the picture in problem 3. What fractional part of the rectangle is shaded orange? ($\frac{2}{4}$) Look at the part of the rectangles in problems 1 and 2 that are shaded orange and compare them with the part of the rectangle in problem 3 that is shaded orange. What can be said about the fractional parts? (They represent the same amount.) The fractions $\frac{4}{8}$ and $\frac{8}{16}$ and $\frac{2}{4}$ are all equivalent; they name the same number.

Another way to show that $\frac{2}{4}$ is equivalent to $\frac{4}{8}$ is to use division. Start with the fraction $\frac{4}{8}$. Divide both the numerator and denominator by 2. What is 4 divided by 2? (2) What is 8 divided by 2? (4) So $\frac{4}{8}$ is equivalent to $\frac{2}{4}$.

odule 4 • Understanding Fractions and Equivalence

Systematic Instruction
For students who struggle with math, studies show that learning increases when teachers use systematic, explicit instruction.

Pictorial Representations
Build the Concept reinforces the instruction from core and helps students learn to use the numbers and symbols in problems.


Lesson 7

TRY IT TOGETHER

Write an equivalent fraction for each fraction.

5. $\frac{10}{12} = \frac{10 \times 2}{12 \times 2} = \frac{20}{24}$

6. $\frac{4}{16} = \frac{4 \div 2}{16 \div 2} = \frac{2}{8}$

7. $\frac{5}{15} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$

8. $\frac{2}{7} = \frac{2 \times 3}{7 \times 3} = \frac{6}{21}$

WORK ON YOUR OWN

HOW 10

Write an Equivalent Fraction

Using Symbols	Using Words
$\frac{2}{6} = \frac{2 \times 4}{6 \times 4} = \frac{8}{24}$	Multiply both the numerator and denominator of the given fraction by the same number.
	OR
$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$	Divide both the numerator and denominator of the given fraction by the same number.

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32 Lesson 7 • Equivalent Fractions

Problem 4

In this problem, a fraction equivalent to $\frac{7}{9}$ will be found. The denominator of $\frac{7}{9}$ is 9. The denominator in the equivalent fraction is 36. The equivalent fraction has a denominator that is greater than 9. Multiplication was used to find the denominator of the equivalent fraction. Nine times what number equals 36? (4) The same operation must be applied to the numerator of the fraction $\frac{7}{9}$ to get the equivalent fraction. What is 7 times 4? (28) The fraction $\frac{7}{9}$ is equivalent to $\frac{28}{36}$.

BUILD THE CONCEPT

Model how to use a picture to write equivalent fractions.

How much of Barry's pizza is left? ($\frac{1}{4}$) How many equal parts is Wayne's pizza cut into? (8) Shade an amount of Wayne's pizza that is equal to the amount of Barry's pizza that is left. How many sections of Wayne's pizza are shaded? (2) How much of Wayne's pizza is left? ($\frac{2}{8}$) The fractions $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

Is the numerator or the denominator of the equivalent fraction given? (denominator) What is the denominator in the equivalent fraction? (24) What is the denominator in $\frac{10}{12}$? (12) What operation was used to find the denominator of the equivalent fraction? (multiplication) Twelve times what number equals 24? (2) What is the next step? (Multiply the numerator of $\frac{10}{12}$ by 2.) What is 10 times 2? (20) What fraction is equivalent to $\frac{10}{12}$? ($\frac{20}{24}$)

Problem 6

Is the numerator or the denominator of the equivalent fraction given? (numerator) What is the numerator in the equivalent fraction? (2) What operation was used to find the numerator of the equivalent fraction? (division) Four divided by what number equals 2? (2) What is the next step? (Divide the denominator of $\frac{4}{16}$ by 2.) What is 16 divided by 2? (8) What fraction is equivalent to $\frac{4}{16}$? ($\frac{2}{8}$)

Problem 7

What operation is needed to find the fraction with a denominator of 3 that is equivalent to $\frac{5}{15}$? (division) What does the numerator of $\frac{5}{15}$ need to be divided by? (5) What fraction with a denominator of 3 is equivalent to $\frac{5}{15}$? ($\frac{1}{3}$)

Problem 8

What operation is needed to find the fraction with a numerator of 6 that is equivalent to $\frac{2}{7}$? (multiplication) What fraction with a numerator of 6 is equivalent to $\frac{2}{7}$? ($\frac{6}{21}$)

Try It Together
Teachers use effective prompts and questions to scaffold instruction and transition students from initial learning to independent practice.

Conceptual Understanding
Build the Concept provides updated conceptual development with explicit language.

Self-Checking
Studies show that students who monitor their own work are more engaged and productive during instruction and are also more aware of their efforts.

VmathLive
The integration of VmathLive promotes procedural and conceptual automaticity.

Tracking Success
Students with math deficits often face math instruction with trepidation. *Vmath* provides opportunities each day for students to track their success as well as quickly receive feedback to avoid any misconceptions.

WORK ON YOUR OWN

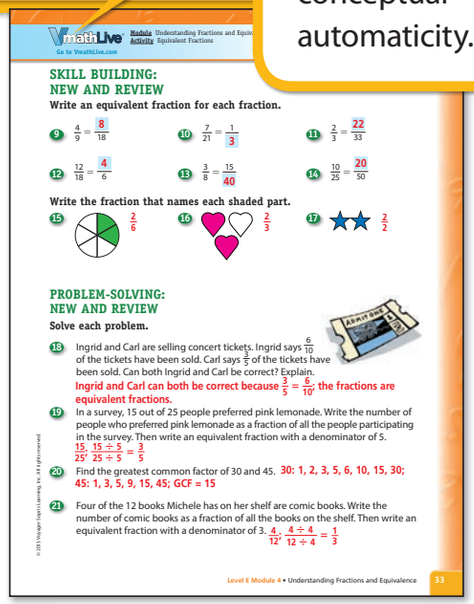
MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 9–21
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 19 **Reminder**
First, write a fraction with the number of people who preferred pink lemonade as the numerator and the total number of people surveyed as the denominator. Then review how to find equivalent fractions by dividing the numerator and denominator by the same number.


VmathLive Module Activity Understanding Fractions and Equivalent Fractions
Go to VmathLive.com

SKILL BUILDING: NEW AND REVIEW
Write an equivalent fraction for each fraction.

9. $\frac{4}{9} = \frac{8}{18}$
10. $\frac{7}{21} = \frac{1}{3}$
11. $\frac{2}{3} = \frac{22}{33}$
12. $\frac{12}{18} = \frac{4}{6}$
13. $\frac{3}{8} = \frac{15}{40}$
14. $\frac{10}{25} = \frac{20}{50}$

Write the fraction that names each shaded part.

15. $\frac{2}{6}$
16. $\frac{2}{3}$
17. $\frac{2}{2}$

PROBLEM-SOLVING: NEW AND REVIEW
Solve each problem.

18. Ingrid and Carl are selling concert tickets. Ingrid says $\frac{6}{10}$ of the tickets have been sold. Carl says $\frac{3}{5}$ of the tickets have been sold. Can both Ingrid and Carl be correct? Explain.
Ingrid and Carl can both be correct because $\frac{3}{5} = \frac{6}{10}$; the fractions are equivalent fractions.
19. In a survey, 15 out of 25 people preferred pink lemonade. Write the number of people who preferred pink lemonade as a fraction of all the people participating in the survey. Then write an equivalent fraction with a denominator of 5.
$\frac{15}{25}$; $\frac{15 \div 5}{25 \div 5} = \frac{3}{5}$
20. Find the greatest common factor of 30 and 45. 30: 1, 2, 3, 5, 6, 10, 15, 30; 45: 1, 3, 5, 9, 15, 45; GCF = 15
21. Four of the 12 books Michele has on her shelf are comic books. Write the number of comic books as a fraction of all the books on the shelf. Then write an equivalent fraction with a denominator of 3. $\frac{4}{12}$; $\frac{4 \div 4}{12 \div 4} = \frac{1}{3}$

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Level E Module 4 • Understanding Fractions and Equivalence 33

CHECK UP

ASSESS INFORMALLY

Error Analysis
Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problems 1 or 2 incorrectly: Remind the student that the numerator and denominator of the original fraction must be either multiplied by the same number or divided by the same number to obtain an equivalent fraction. Use Additional Resources in E.4.7 to reteach how to find equivalent fractions.

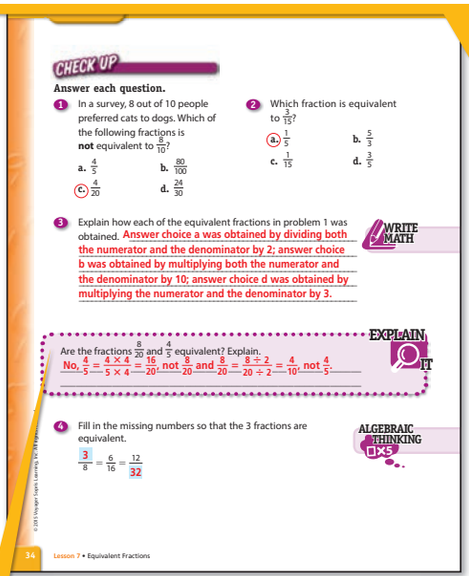

WRITE MATH

If student answered problem 3 incorrectly: While students work together, review the steps for writing equivalent fractions with the student. Then have the student identify how the fractions $\frac{4}{5}$, $\frac{80}{100}$, and $\frac{24}{30}$ were obtained from $\frac{8}{10}$.

Student Interaction
Research shows that cooperative learning and discussion reinforce skills during intervention.

Additional Resources

Additional Resources provide a balance to the teacher-centered explicit instruction by providing inquiry-based lesson components, including writing and discussion.



CHECK UP

Answer each question.

1. In a survey, 8 out of 10 people preferred cats to dogs. Which of the following fractions is **not** equivalent to $\frac{8}{10}$?
   a. $\frac{4}{5}$ b. $\frac{80}{100}$ c. $\frac{4}{20}$ d. $\frac{24}{30}$

2. Which fraction is equivalent to $\frac{3}{15}$?
   a. $\frac{1}{5}$ b. $\frac{5}{3}$ c. $\frac{1}{15}$ d. $\frac{3}{5}$

3. Explain how each of the equivalent fractions in problem 1 was obtained. WRITE MATH
   Answer choice a was obtained by dividing both the numerator and the denominator by 2; answer choice b was obtained by multiplying both the numerator and the denominator by 10; answer choice d was obtained by multiplying the numerator and the denominator by 3.

EXPLAIN IT
Are the fractions $\frac{8}{20}$ and $\frac{4}{5}$ equivalent? Explain.
No, $\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$, not $\frac{8}{20}$ and $\frac{8}{20} = \frac{8 \div 2}{20 \div 2} = \frac{4}{10}$, not $\frac{4}{5}$.

4. Fill in the missing numbers so that the 3 fractions are equivalent. ALGEBRAIC THINKING
   $\frac{3}{8} = \frac{6}{16} = \frac{12}{32}$

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EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

Explain It

Explain It is an excellent way to build students' ability to reason and communicate their mathematical thinking. This portion of the lesson is also an excellent way to assess these same skills. The *Vmath* eBook (student and teacher) provides a link to a rubric that can be used to help score the students' writing in Explain It.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Understanding Fractions and Equivalence
Activity: Equivalent Fractions

Vmath Reteach
Reteach Student Book Module 4 Lesson 7
Reteach Teacher Edition Module 4 Lesson 7

Extra Practice
Student Book page 44

English Language Learners

Use the VmathLive Animated Glossary to review the term *equivalent fractions*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Have students work with a partner to name synonyms for *equivalent*. Possible answers include *equal to, the same as, identical.* Write several fractions on the board. Have pairs work together to find at least one equivalent fraction for each fraction on the board.

ELL Teaching Tips

Lesson tips provide specific activities that focus on increasing students' understanding of the language of math.

Students with Special Needs

When solving word problems involving equivalent fractions, students may find it helpful to draw models of the fractions given in the problem. A common error that occurs when drawing these models is using unequal whole models. Emphasize to students how important it is to draw the models so that they are the same size and the same shape. If they are not, the shaded fractions may show an incorrect answer.

Adaptations for Students with Special Needs

Teaching strategies in the lessons provide adaptations that support students with special needs.

Math Flash Lesson Levels D–I Walk-Through

Lesson 1

Objective

To use a protractor to measure angles and to find missing measures of angles

Model the following skills for students.

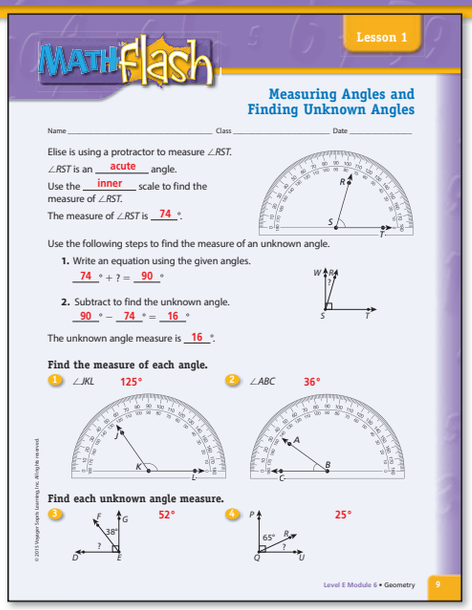
Look at the angle Elise is measuring. Angle *RST* is shown on a protractor. What type of angle is ∠*RST*? (acute angle) **Is the measure of an acute angle greater than or less than the measure of a right angle?** (less than)

The protractor has two scales, an inner scale and an outer scale. Both scales go from 0° to 180° in opposite directions. Look at the ray *SR*. Between what two numbers is it pointing on the inner scale? (70 and 80) **Between what two numbers is it pointing on the outer scale?** (100 and 110) **Because ∠*RST* is an acute angle, use the inner scale. Count the tick marks from 70. How many tick marks is it to where ray *SR* is pointing?** (4) **What is the measure of ∠*RST*?** (74°)

Work with students to complete the following skills.

You can find the unknown measure of an angle using given angle measures. Look at the diagram. You know the measure of ∠*RST* and the measure of a right angle. What is the measure of ∠*RST*? (74°) **What is the measure of a right angle?** (90°)

To find the unknown angle measure, first write an equation. The measures of which two angles equal the measure of a right angle? (∠*RST* and the angle with the unknown measure) **So, to find the unknown angle measure, use the equation 74° + ? = 90°. Solve by using the opposite operation, subtraction. What is the unknown angle measure?** (16°)



Lesson 1

Measuring Angles and Finding Unknown Angles

Name _____ Class _____ Date _____

Elise is using a protractor to measure ∠*RST*.

∠*RST* is an acute angle.

Use the inner scale to find the measure of ∠*RST*.

The measure of ∠*RST* is 74°.

Use the following steps to find the measure of an unknown angle.

1. Write an equation using the given angles.
   74° + ? = 90°
2. Subtract to find the unknown angle.
   90° − 74° = 16°

The unknown angle measure is 16°.

Find the measure of each angle.

1. ∠*JKL* 125°
2. ∠*ABC* 36°

Find each unknown angle measure.

3. 52°
4. 25°

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Level E Module 6 • Geometry 9

Explicit Instruction
Explicit dialogue provides further clarification and instruction regarding frequently tested concepts.

Hands-On Lesson Levels D–I Walk-Through

Background Information for Teachers
Lesson Notes provide manipulative management tips, identify necessary preskills, and provide background information to help teachers effectively teach the lesson.

HANDS-ON GUIDED DISCOVERY

Lesson 8

Objective

To use fraction strips to find the simplest form of a fraction

CONCEPTUAL DEVELOPMENT

Materials

- Copy Master: Fraction Strips
- Scissors
- Envelopes or small plastic bags

Lesson Notes

Before beginning the lesson, be sure each student has a Student Book, a pair of scissors, a copy of Copy Master: Fraction Strips, and an envelope or small plastic bag.

This conceptual lesson is designed to give students a visual method of understanding the meaning of the simplest form of a fraction. When using fraction strips, remind students that the denominator of the fraction tells the total number of parts and the numerator tells how many parts to use.

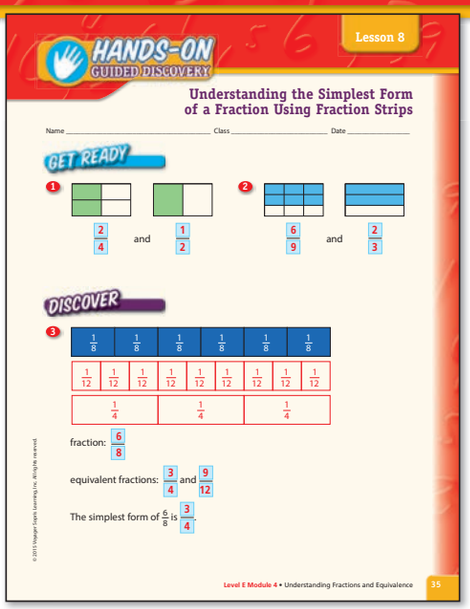
Remind students that when the entire fraction strip is used, it is called a fraction strip. When just one or a few parts of the entire fraction strip are used, each separate piece is called a fraction bar.

Have students use scissors to cut apart the fraction strips on their copy of Copy Master: Fraction Strips. Have students save the fraction strips in an envelope or small plastic bag.

GET READY

Problem 1

The rectangles in problem 1 are the same size. Into how many equal parts is the first rectangle divided? (4) **How many parts are shaded?** (2) **What fractional part of the rectangle is shaded?** ($\frac{2}{4}$) **The second rectangle is divided into 2 equal parts instead of 4 equal parts. What part of the rectangle is shaded?** ($\frac{1}{2}$) **What do you notice about the amount of shading on these?** (It is the same.) **The**



Problem 2

In problem 2, the rectangles are the same size. The first rectangle is divided into how many equal parts? (9) **How many parts are shaded?** (6) **What fractional part of the rectangle is shaded?** ($\frac{6}{9}$) **The second rectangle is divided into 3 equal parts instead of 9 equal parts. What part of the rectangle is shaded?** ($\frac{2}{3}$) **What do you notice about the amount of shading on these?** (It is the same.) **The fractions $\frac{6}{9}$ and $\frac{2}{3}$ are equivalent fractions.**

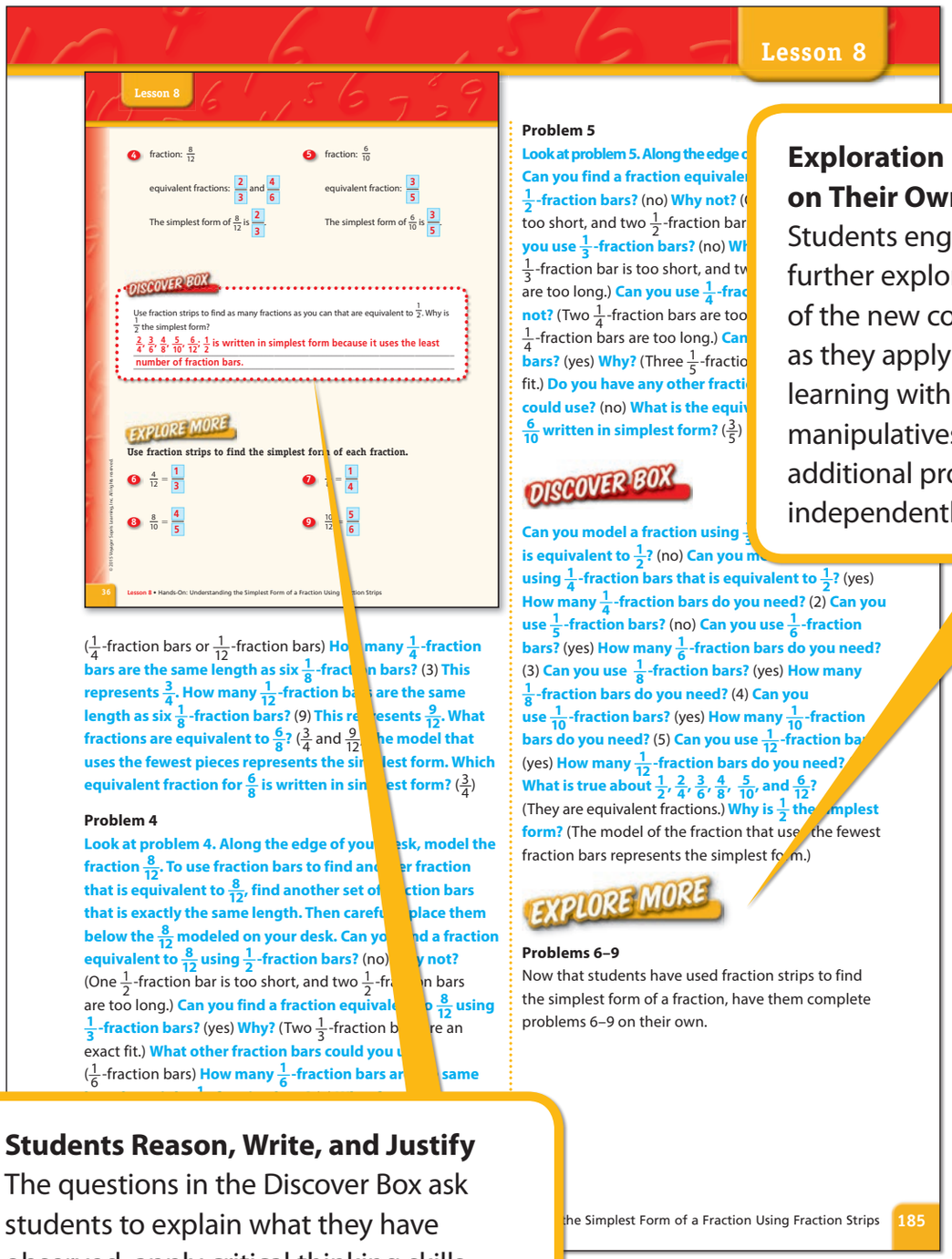
DISCOVER

Problem 3

What fraction is modeled by the fraction bars in problem 3? ($\frac{6}{8}$) **To use fraction bars to find another fraction that is equivalent to $\frac{6}{8}$, place the fraction bars in the space below $\frac{6}{8}$. Be sure to line up the left fraction bars. Can you find a fraction equivalent to $\frac{6}{8}$ using $\frac{1}{2}$ fraction bars?** (no) **Why not?** (One $\frac{1}{2}$-fraction bar is too short and two $\frac{1}{2}$-fraction bars are too long.) **Can you find a fraction equivalent to $\frac{6}{8}$ using $\frac{1}{3}$-fraction bars?** (yes) **Why?** (Three $\frac{1}{3}$-fraction bars are too short, and four $\frac{1}{3}$-fraction bars are too long.) **Use the other bars to see if you can find a fraction equivalent to $\frac{6}{8}$.** (Answers will vary.) **It may take some time to rearrange the**

Getting Students Ready to Learn
In the Get Ready section of the lesson, teachers review prerequisite skills before modeling new concepts as students respond orally and follow along in their Student Books.

Students Discover Key Math Concepts
As students use manipulatives to discover key math concepts, teachers guide student learning through the use of effective questioning strategies.



Exploration on Their Own
Students engage in further exploration of the new concepts as they apply their learning with the use of manipulatives to solve additional problems independently.

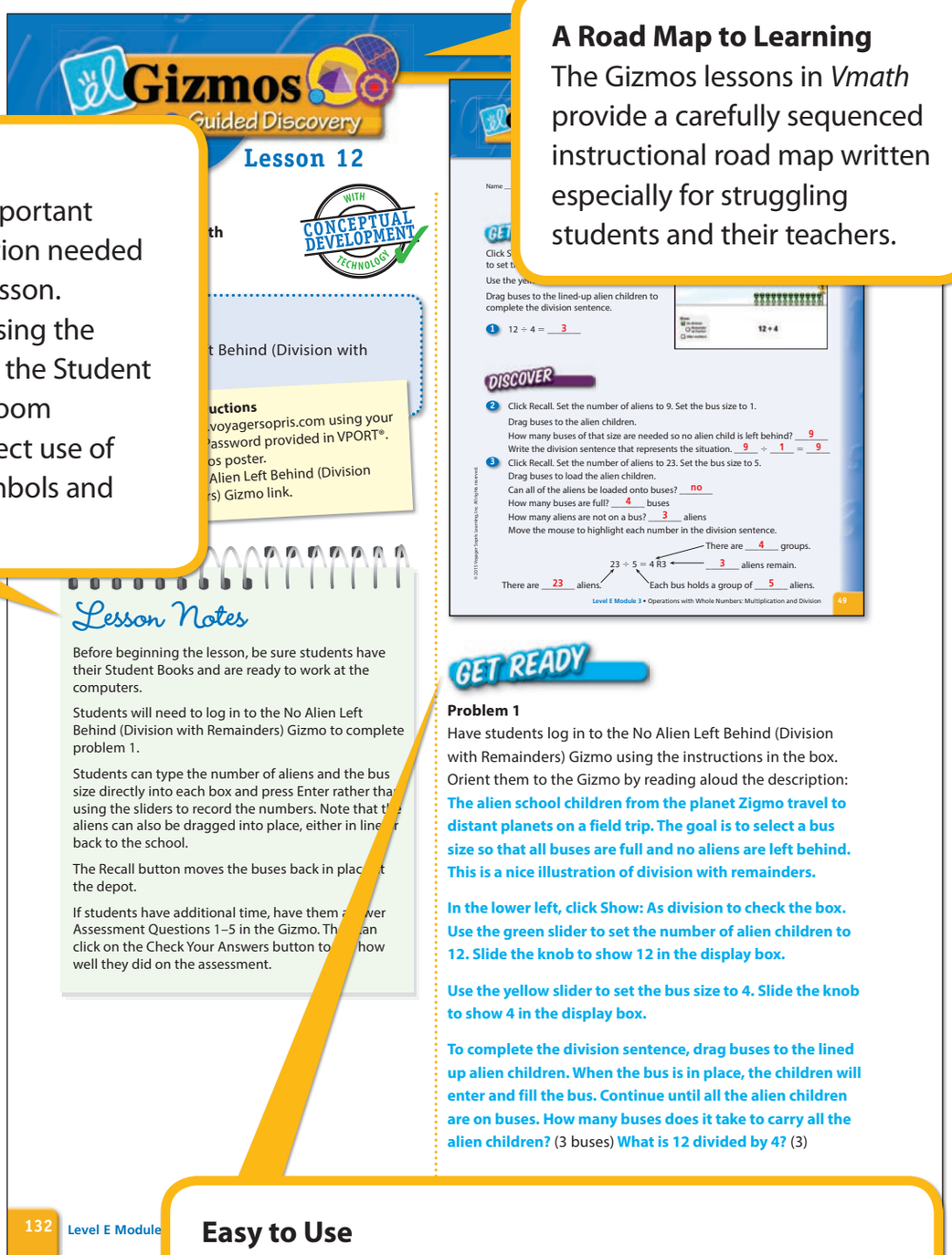
Students Reason, Write, and Justify
The questions in the Discover Box ask students to explain what they have observed, apply critical thinking skills, and use deductive reasoning as they communicate their conclusions with peers and teachers.

Gizmos Lesson Levels D–I Walk-Through

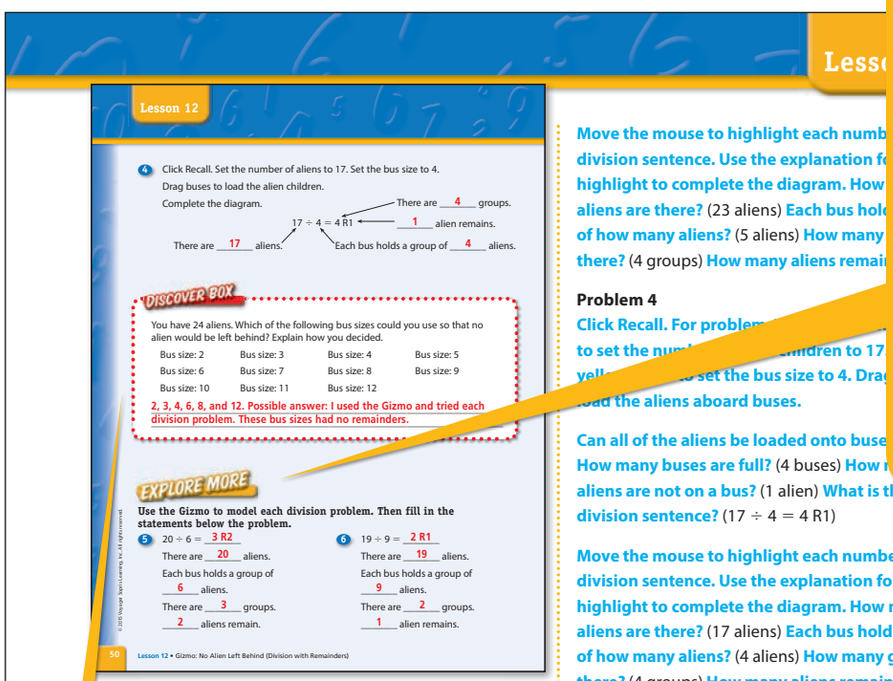
Teacher Guidance
Lesson Notes give important background information needed when teaching the lesson.

- Suggestions for using the Gizmo along with the Student Book in the classroom
- Notes on the correct use of mathematical symbols and vocabulary

A Road Map to Learning
The Gizmos lessons in *Vmath* provide a carefully sequenced instructional road map written especially for struggling students and their teachers.



Easy to Use
These online, virtual manipulatives provide interactive simulations that can be accessed any time, anywhere. In the Get Ready section, teachers and students review prerequisite skills needed in using the Gizmo to discover concepts.



Lesson 12

4 Click Recall. Set the number of aliens to 17. Set the bus size to 4.
Drag buses to load the alien children.
Complete the diagram.

17 ÷ 4 = 4 R1
There are 17 aliens.
There are 4 groups.
1 alien remains.
Each bus holds a group of 4 aliens.

DISCOVER BOX

You have 24 aliens. Which of the following bus sizes could you use so that no alien would be left behind? Explain how you decided.

Bus size: 2 Bus size: 3 Bus size: 4 Bus size: 5
Bus size: 6 Bus size: 7 Bus size: 8 Bus size: 9
Bus size: 10 Bus size: 11 Bus size: 12

2, 3, 4, 6, 8, and 12. Possible answer: I used the Gizmo and tried each division problem. These bus sizes had no remainders.

EXPLORE MORE

Use the Gizmo to model each division problem. Then fill in the statements below the problem.

5 20 ÷ 6 = 3 R2
There are 20 aliens.
Each bus holds a group of 6 aliens.
There are 3 groups.
2 aliens remain.

6 19 ÷ 9 = 2 R1
There are 19 aliens.
Each bus holds a group of 9 aliens.
There are 2 groups.
1 alien remains.

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DISCOVER

Problem 2

Click Recall. Use the green slider to set the number of alien children to 9. Use the yellow slider to set the bus size to 1.

Drag the buses to the alien children. How many buses of that size are needed so no alien child is left behind? (9)
What division sentence represents the situation?
(9 ÷ 1 = 9)

Problem 3

Click Recall. For problem 3, use the green slider to set the number of alien children to 23. Use the yellow slider to set the bus size to 5.

Drag buses to where the alien children are standing in line to load the aliens aboard buses. A bus that will not be completely filled cannot be partially loaded. All buses must be full or they will not load.

Move the mouse to highlight each numb
division sentence. Use the explanation fo
highlight to complete the diagram. How
aliens are there? (23 aliens) Each bus hol
of how many aliens? (5 aliens) How many
there? (4 groups) How many aliens remai

Problem 4

Click Recall. For problem
to set the num ildren to 17
yell set the bus size to 4. Drag
oad the aliens aboard buses.

Can all of the aliens be loaded onto buse
How many buses are full? (4 buses) How
aliens are not on a bus? (1 alien) What is the
division sentence? (17 ÷ 4 = 4 R1)

Move the mouse to highlight each number in the division sentence. Use the explanation for each highlight to complete the diagram. How many aliens are there? (17 aliens) Each bus holds a group of how many aliens? (4 aliens) How many groups are there? (4 groups) How many aliens remain? (1 alien)

DISCOVER BOX

Look at problems 2–4, which you have completed. Think about when there were no aliens left behind.

You have 24 aliens. Use the Gizmo to determine which of the following bus sizes could be used so that alien would be left behind. Which bus sizes work? (2, 3, 12)

Explain how you de ssible
the Gizmo and tried each di
bus sizes had no remainders.)

EXPLORE MORE

Problems 5 and 6

Have students work independently. I
that the dividend, or the first numbe
nce, is the number of aliens, an
d number, or the divisor, is the
t on each bus.

o: No Alien Left Behind (Division

Making Connections

In the Explore More section, students work independently using the Gizmo to solve additional problems by applying concepts they discovered together. Gizmos help students make connections between mathematical concepts and processes.

Supporting Different Learning Styles

Students with different learning styles can visualize and understand essential concepts through the use of Gizmos. The questions in the Discover Box allow the teacher to get a clear picture of whether students have grasped the concepts presented in the lesson.

Supporting Teachers as They Teach

In the Discover section, teachers guide students to discover important math concepts as they use the Gizmos to answer questions together. Specific step-by-step instructions help teachers feel comfortable using Gizmos and take the guesswork out of how Gizmos work.

Daily Support for Students Through Technology

Vmath is accessible to students—anytime, anywhere.

The eBook provides students with . . .

- the complete set of *Vmath* modules.
- access to all pages.
- connectivity to submit pages to the teacher for feedback.

VmathLive provides students with . . .

- activities to reinforce instruction.
- Fluency Practice.
- competition with other students.

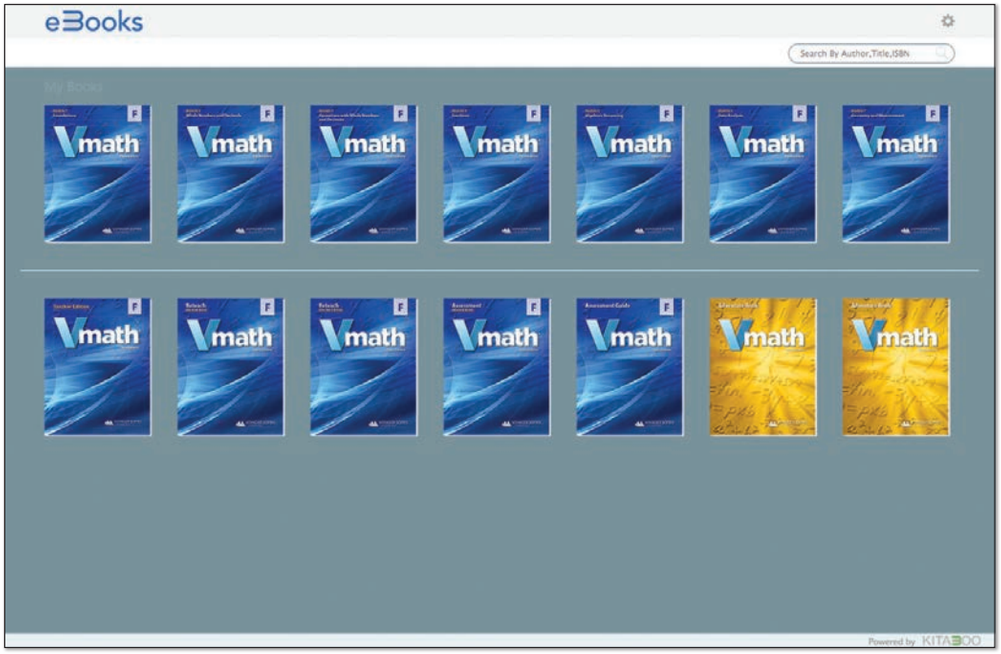
Vmath Testing Center provides students with . . .

- access to online assessments.



Using the eBook to Support Instruction

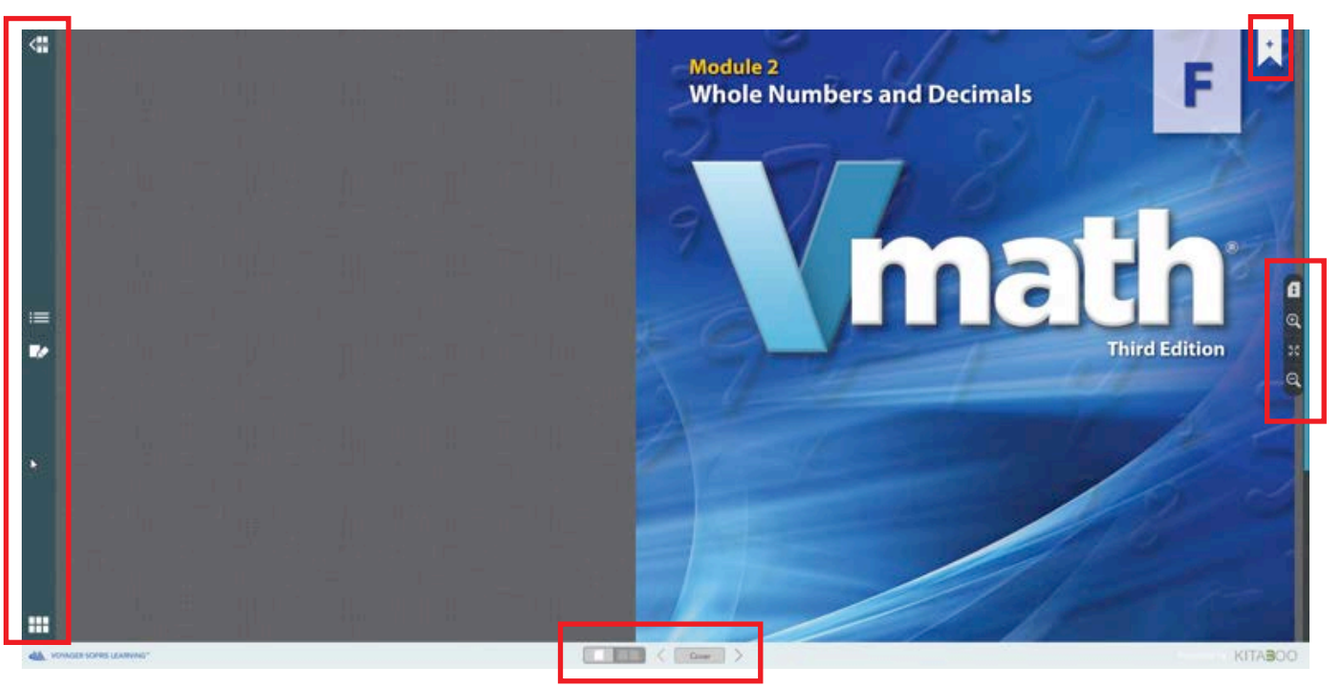
The eBook is a rich, interactive, and enhanced multimedia experience that retains the fidelity of the *Vmath* materials, while providing users with electronic content. eBooks can be accessed from iPads, Android platforms, PCs, and Macs. The *Vmath* eBook allows users to click on pictures and icons to access multimedia content and provides communication options between the teacher and the student.



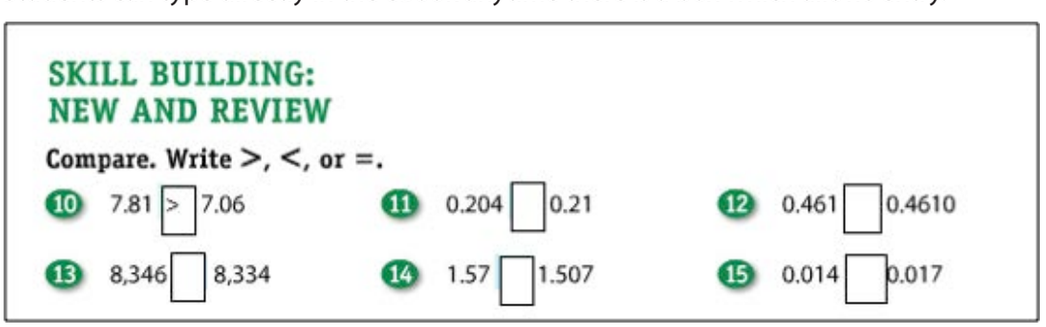
Student Access



1. Log in to the *Vmath* Student Center: Students will access their eBook by logging into the *Vmath* Student Center at www.vmath.voyagersopris.com and using their unique username and password.
2. Choose the eBook poster: When students select the eBook poster from the Student Center page, they will be taken to the eBook platform.
3. Select the appropriate book: Students will select the module book corresponding to the instruction or the Reteach Book, as directed by the teacher.
4. Navigate the book: There are several different tools within the book to help navigate.
    a. The menu bar on the left side of the screen can be used to return to the bookshelf, see the book's content, review the data (work submitted), or view the thumbnail view of each page.
    b. The menu bar on the right side of the screen can be used to increase or decrease the size of the book, center the page, or adjust the height.
    c. The menu at the bottom of the page can be used to turn pages and see page numbers
    d. There is also a bookmark in the top right corner so upon subsequent visits to the book students can quickly access where they stopped before.



Students can type directly in the eBook anytime there is a box which allows entry.



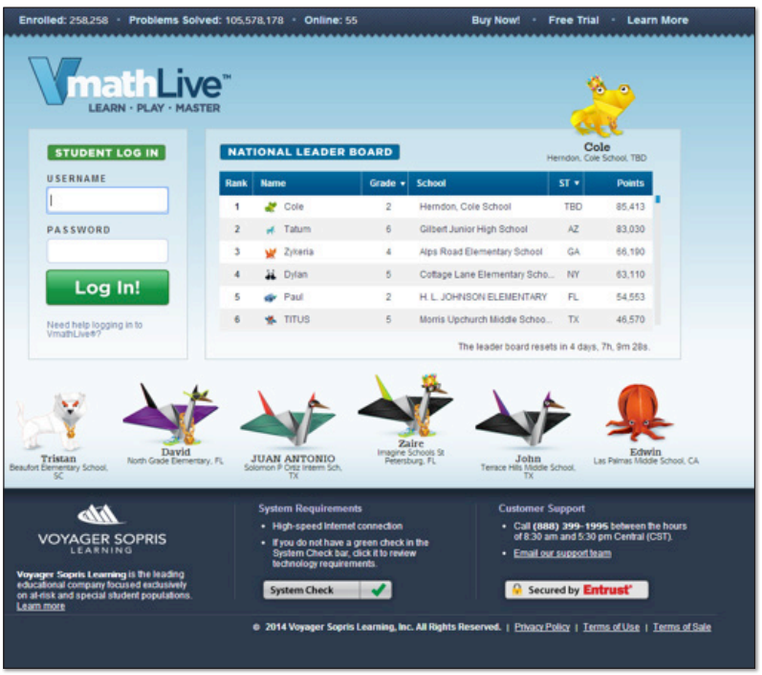
Teacher Access



1. Log in to VPORT: Teachers will access their eBook by logging into VPORT at www.vmath.voyagersopris.com and using their unique username and password.
2. Select the HOME tab and the eBook poster: When teachers select the eBook poster from the Home page, they will be taken to the eBook platform.
3. Select the appropriate book: Teachers will have access to the Teacher Edition, Assessment Guide, Reteach eBook, Adventure eBook, as well as each of the Student Books.
4. Navigate the book: There are several different tools within the book to help navigate.
    a. The menu bar on the left side of the screen can be used to return to the bookshelf, see the book's content, review the data (work submitted), or view the thumbnail view of each page.
    b. The menu bar on the right side of the screen can be used to increase or decrease the size of the book, center the page, or adjust the height.
    c. The menu at the bottom of the page can be used to turn pages and see page numbers.
    d. There is also a bookmark in the top right corner so upon subsequent visits to the book students can quickly access where they left off.
5. View the data: Teachers can view the notes, highlights, and submitted work by clicking on the data icon in the navigation menu.

VmathLive®

Vmath's blended instructional model combines teacher-led instruction with print material and individualized instruction and practice with technology. VmathLive is an online program supplementing *Vmath* by providing motivating practice on the skills and concepts covered in the *Vmath* modules.



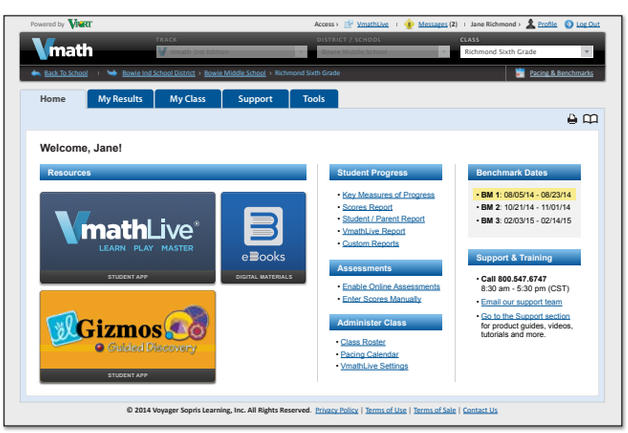
Because VmathLive provides access any time of the day, students can log in at home, at school, or in any after-school setting where they can get online. In this way, VmathLive extends the school day and provides a link between school and home.

Logging In

There are two ways for teachers to log in to VmathLive.

1. Go to www.vmathlive.com and click the Log In button. (This is the only means for students to log in.) Enter your Username and Password in the Student Log In box.
2. Teachers can also log in through VPORT® by clicking on the Access VmathLive icon at the top of the page.





VmathLive Features

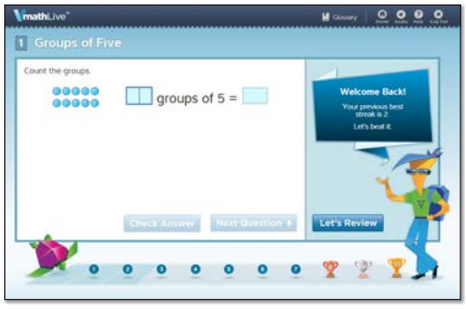
The main components of the VmathLive program are the Learn and Play sections. VmathLive also includes an Animated Math Glossary.



Learn

The VmathLive course matches the *Vmath* modules. Students access interactive activities that engage and motivate them as they practice essential concepts and skills.

Teachers also can identify areas of weakness at a glance by accessing the VmathLive report through the tab in the Teacher Center.



Video

As students require support in the Learn activities, they can unfold hints on the screen and access a video to guide them through the steps necessary to solve problems. These videos are available in both English and Spanish.



Play

In the Play section of VmathLive, students enter a live, online competition with other students. As they play, students increase their mental math skills, developing computational fluency.



Animated Math Glossary

VmathLive includes an animated math glossary that defines common math terms. Students can access the glossary and receive clarity around the terms through animated visual models.



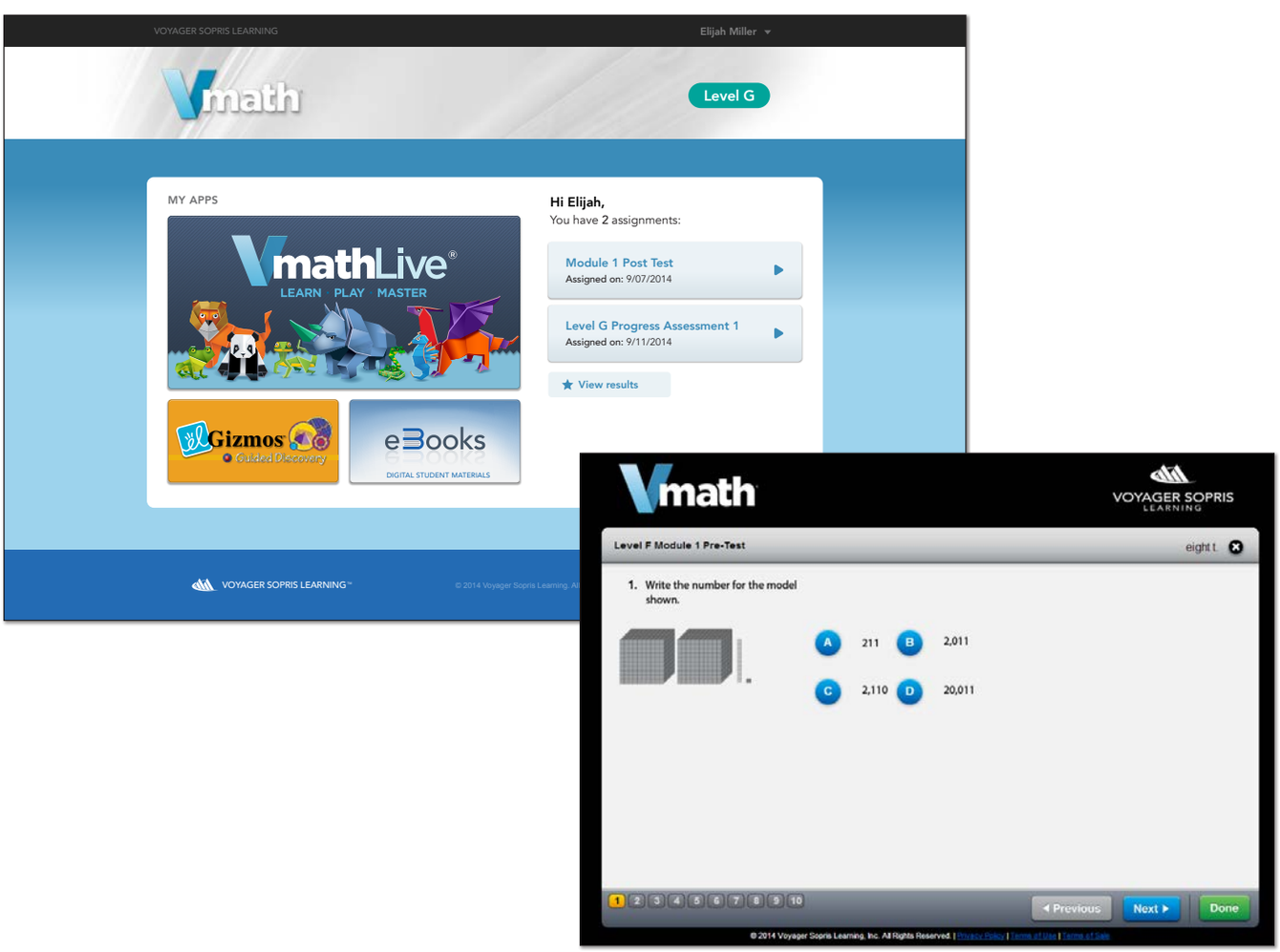
Vmath Testing Center

Vmath students may complete *Vmath* assessments online by logging in to VmathLive. Teachers may use the *Vmath* Testing Center to administer assessments to each student or to the whole class. Paper-and-pencil assessments can also be administered by printing the assessments from the Assessment Guide in the eBook.

Before students can take an assessment online, the teacher must schedule the assessment in VPORT.

After the teacher schedules the assessment, students log in to their student page (www.vmath.voyagersopris.com), where any assessments or assignments will be listed.

Once students have completed the assessment, teachers can view the class results in VPORT in the MY RESULTS tab.



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Foundations

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Lesson 1

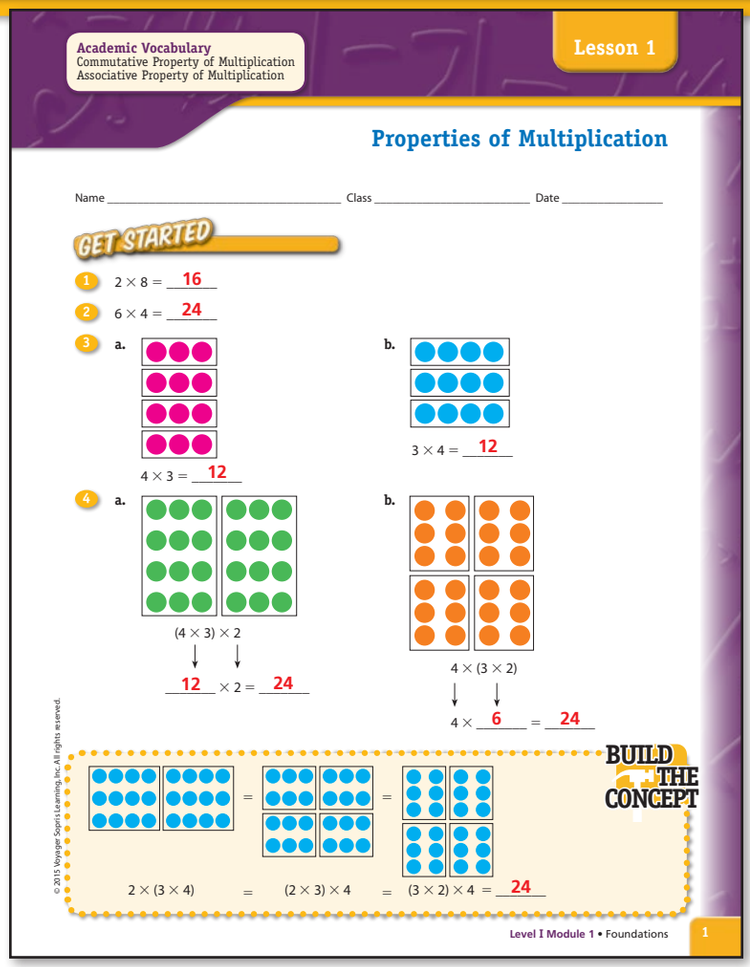
Objective

To find the product of two or more factors using the Commutative and Associative Properties of Multiplication

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **Commutative Property of Multiplication** two factors can be multiplied in any order and the product will be the same
- **Associative Property of Multiplication** three factors can be grouped in different ways and the product will be the same



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

This multiplication problem is read as *2 times 8*. This is a basic multiplication fact. If the multiplication fact is not memorized, count 2 times by 8. What is the product of 2 and 8? (16)

Problem 2

This multiplication problem is read as *6 times 4*. This is a basic multiplication fact. If the multiplication fact is not memorized, count 6 times by 4. What is the product of 6 and 4? (24)

MODEL NEW SKILLS

Problem 3

Look at problem 3a. The problem 4 times 3 means 4 groups of 3. How many total circles are in 4 groups of 3 circles? (12)

Look at problem 3b. The problem 3 times 4 means 3 groups of 4. How many total circles are in 3 groups of 4 circles? (12) **Notice that 3 times 4 has the same factors as 4 times 3, but the factors are written in a different order.**

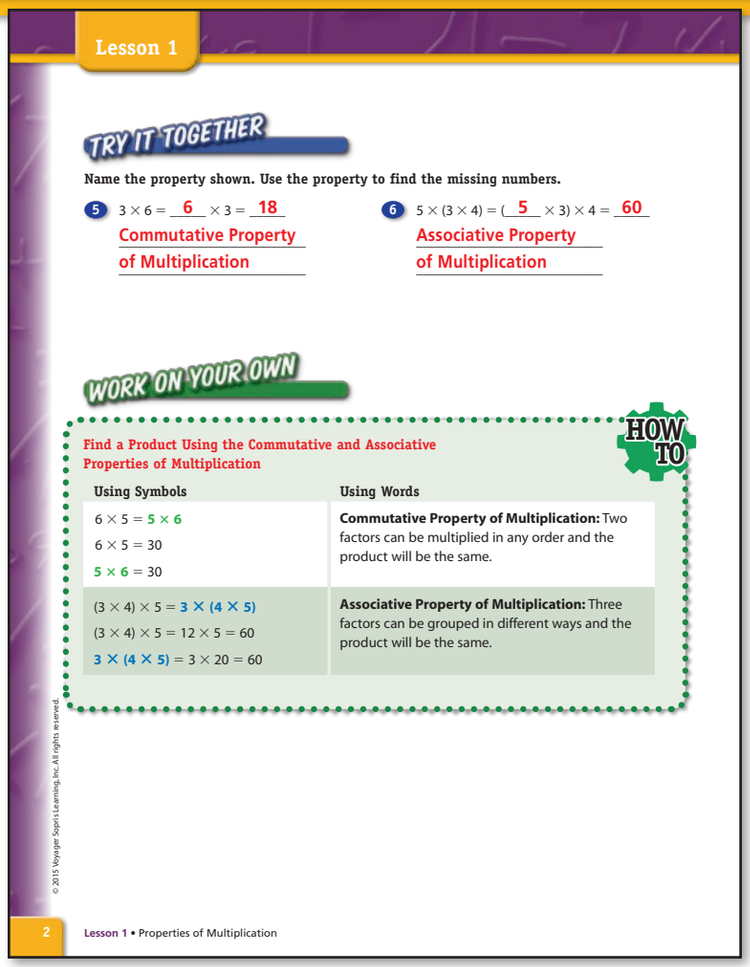
Problems 3a and 3b show the Commutative Property of Multiplication. The Commutative Property of Multiplication states that two factors can be multiplied in any order and the product will be the same. Four times 3 and 3 times 4 are the same because the factors are the same.

Problem 4

Look at problem 4a. This problem is read as the *quantity of 4 times 3 times 2*. Multiply the numbers in parentheses first. The problem 4 times 3 is in parentheses. What is 4 times 3? (12) **Now multiply 12 and 2 to find the final product. What is 12 times 2?** (24)

Look at problem 4b. This problem is read as *4 times the quantity of 3 times 2*. Multiply the numbers in parentheses first. What is the problem in parentheses? (3 times 2) **What is 3 times 2?** (6) **Now multiply 4 by 6 to find the final product. What is 4 times 6?** (24)

Problems 4a and 4b show the Associative Property of Multiplication. The Associative Property of Multiplication states that three factors can be grouped in different ways and the product will be the same. The products in problems 4a and 4b are the same because the factors are the same, only grouped differently.



Lesson 1

TRY IT TOGETHER

Name the property shown. Use the property to find the missing numbers.

5 $3 \times 6 = \underline{6} \times 3 = \underline{18}$
Commutative Property of Multiplication

6 $5 \times (3 \times 4) = (\underline{5} \times 3) \times 4 = \underline{60}$
Associative Property of Multiplication

WORK ON YOUR OWN

HOW TO

Find a Product Using the Commutative and Associative Properties of Multiplication

Using Symbols	Using Words
$6 \times 5 = 5 \times 6$ $6 \times 5 = 30$ $5 \times 6 = 30$	**Commutative Property of Multiplication:** Two factors can be multiplied in any order and the product will be the same.
$(3 \times 4) \times 5 = 3 \times (4 \times 5)$ $(3 \times 4) \times 5 = 12 \times 5 = 60$ $3 \times (4 \times 5) = 3 \times 20 = 60$	**Associative Property of Multiplication:** Three factors can be grouped in different ways and the product will be the same.

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2 Lesson 1 • Properties of Multiplication

BUILD THE CONCEPT

Model how to use the Commutative and Associative Properties of Multiplication.

The models show the Commutative and Associative Properties of Multiplication. Look at the first model. This represents the multiplication problem *2 times the quantity of 3 times 4* because there are 2 groups each of 3 rows of 4 circles.

Look at the next model. How many groups are there? (4) **How many groups of 3 circles are there in each group?** (2) **This represents the multiplication problem *the quantity of 2 times 3 times 4*. Look at the third model. How many groups are there?** (4) **How many groups of 2 circles are there in each group?** (3) **What multiplication problem does this represent?** ((3 × 2) × 4)

Now, how many total circles are there in each model? (24) **What property was used to show the change from the first model to the second model?** (Associative Property of Multiplication) **Why?** (The factors were regrouped by the parentheses.) **What property was used to change from the second model to the third model?** (Commutative Property of Multiplication) **Why?** (The numbers in parentheses were flipped.) **Regardless of the order or grouping, the product of the three factors is 24.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What property can be used to write 3 times 6 another way? (Commutative Property of Multiplication) **What does the Commutative Property of Multiplication state?** (Two factors can be multiplied in any order and the product will be the same.)

What is another way to write 3 times 6? (6 times 3) **What is the product of 3 and 6?** (18) **What is the product of 6 and 3?** (18)

Problem 6

What property can be used to write 5 × (3 × 4) another way? (Associative Property of Multiplication) **What does the Associative Property of Multiplication state?** (Three factors can be grouped in different ways and the product will be the same.) **What is another way to write 5 × (3 × 4)?** ((5 × 3) × 4) **What is 3 times 4?** (12) **What is 5 times 12?** (60) **What is 5 times the product of 3 and 4?** (60) **What is the product of 5 and 3 times 4?** (60)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 7–19

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 17 Reminder

Be sure students understand that the problem asks for ways to group the numbers and also the name of the property. Students should not change the order of the numbers.

CHECK UP

ASSESS INFORMALLY

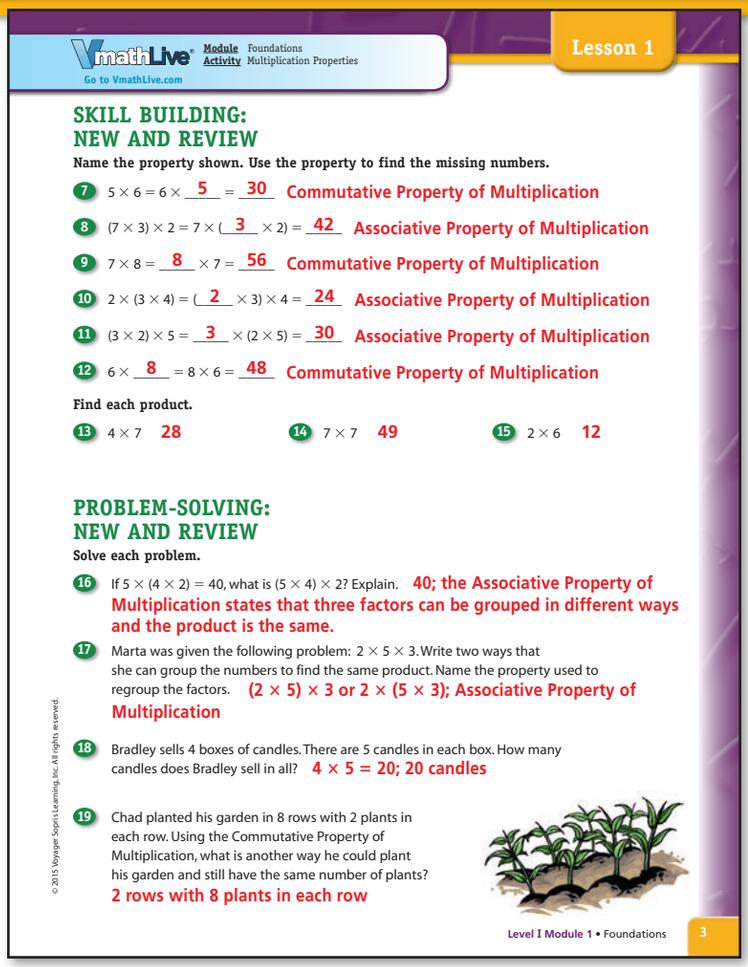
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student added instead of multiplying. Remind the student of the difference between addition and multiplication.

If student answered 1b or 1d: The student did not know the basic multiplication facts. Review how to multiply with factors of 7, 8, or 9.

If student answered problem 2 incorrectly: The student did not know the multiplication properties. Use Additional Resources in I.1.1 to review the properties of multiplication.



VmathLive Module Activity Foundations Multiplication Properties
Go to VmathLive.com

Lesson 1

SKILL BUILDING: NEW AND REVIEW

Name the property shown. Use the property to find the missing numbers.

7. $5 \times 6 = 6 \times \underline{5} = \underline{30}$ Commutative Property of Multiplication
8. $(7 \times 3) \times 2 = 7 \times (\underline{3} \times 2) = \underline{42}$ Associative Property of Multiplication
9. $7 \times 8 = \underline{8} \times 7 = \underline{56}$ Commutative Property of Multiplication
10. $2 \times (3 \times 4) = (\underline{2} \times 3) \times 4 = \underline{24}$ Associative Property of Multiplication
11. $(3 \times 2) \times 5 = \underline{3} \times (2 \times 5) = \underline{30}$ Associative Property of Multiplication
12. $6 \times \underline{8} = 8 \times 6 = \underline{48}$ Commutative Property of Multiplication

Find each product.

13. 4×7 28
14. 7×7 49
15. 2×6 12

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

16. If $5 \times (4 \times 2) = 40$, what is $(5 \times 4) \times 2$? Explain. 40; the Associative Property of Multiplication states that three factors can be grouped in different ways and the product is the same.
17. Marta was given the following problem: $2 \times 5 \times 3$. Write two ways that she can group the numbers to find the same product. Name the property used to regroup the factors. $(2 \times 5) \times 3$ or $2 \times (5 \times 3)$; Associative Property of Multiplication
18. Bradley sells 4 boxes of candles. There are 5 candles in each box. How many candles does Bradley sell in all? $4 \times 5 = 20$; 20 candles
19. Chad planted his garden in 8 rows with 2 plants in each row. Using the Commutative Property of Multiplication, what is another way he could plant his garden and still have the same number of plants? 2 rows with 8 plants in each row

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Level I Module 1 • Foundations 3

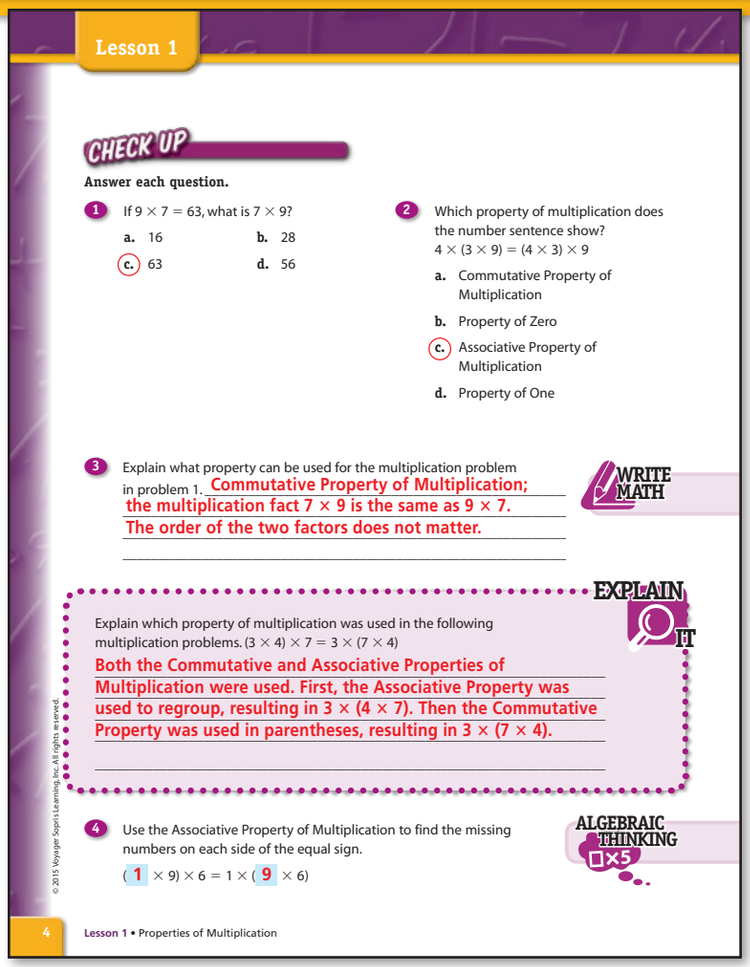


WRITE MATH

If student answered problem 3 incorrectly: While students work together, make sure the student understands the different properties of multiplication. Use Additional Resources in I.1.1 to review the properties of multiplication.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 1

CHECK UP

Answer each question.

1. If $9 \times 7 = 63$, what is 7×9?
   a. 16 b. 28
   c. 63 d. 56

2. Which property of multiplication does the number sentence show?
   $4 \times (3 \times 9) = (4 \times 3) \times 9$
   a. Commutative Property of Multiplication
   b. Property of Zero
   c. Associative Property of Multiplication
   d. Property of One

3. Explain what property can be used for the multiplication problem in problem 1. Commutative Property of Multiplication; the multiplication fact 7×9 is the same as 9×7. The order of the two factors does not matter.

WRITE MATH

EXPLAIN IT

Explain which property of multiplication was used in the following multiplication problems. $(3 \times 4) \times 7 = 3 \times (7 \times 4)$
Both the Commutative and Associative Properties of Multiplication were used. First, the Associative Property was used to regroup, resulting in $3 \times (4 \times 7)$. Then the Commutative Property was used in parentheses, resulting in $3 \times (7 \times 4)$.

4. Use the Associative Property of Multiplication to find the missing numbers on each side of the equal sign.
   $(1 \times 9) \times 6 = 1 \times (9 \times 6)$

ALGEBRAIC THINKING $\square \times 5$

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DIFFERENTIATION

Additional Resources

VmathLive
Module: Foundations
Activity: Multiplication Properties

Vmath Reteach
Reteach Student Book Module 1 Lesson 1
Reteach Teacher Edition Module 1 Lesson 1

Extra Practice
Student Book page 53

English Language Learners

Use the VmathLive Animated Glossary to review the terms *Commutative Property* and *Associative Property*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

To help students understand the differences between the Commutative and Associative Properties of Multiplication, review the meanings of the words *commute* and *associate*. First, ask for student definitions of the words. Then explain that a commuter is a person who commutes, or travels from one place to another for work. In math, the Commutative Property of Multiplication is used to show that factors can commute, or move, and the answer will not change.

To associate means to be around or near someone or something. In math, the Associative Property of Multiplication shows that associations, or groupings, of numbers can change without changing the answer. Have students pair up to practice solving problems using both the Commutative and Associative Properties of Multiplication.

Students with Special Needs

Help students internalize the meanings of *associative* and *commutative* by having them act out problems. Write $(2 \times 3) \times 4 = 4 \times (2 \times 3)$ on the board. Assign a student to each number. Have them stand in the correct order for the left side of the problem. Have students link arms to represent parentheses. Then have them change to the order of the right side of the problem. Explain that the students moved, or commuted, to show the second side. Write $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ on the board. Have the students act out the problem and discuss what changed for each side. Explain that the parentheses, or associations, changed.

Lesson 2

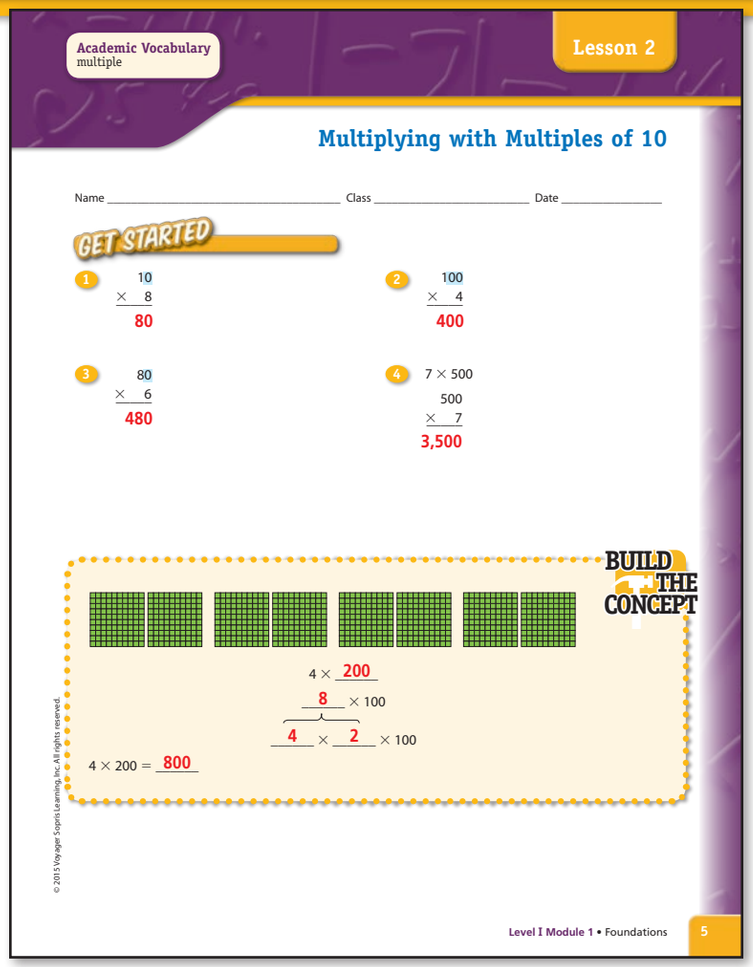
Objective

To multiply with a multiple of 10

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **multiple**
  the product of the whole number and another whole number
  (Example: 4 × 10 = 40; 40 is the fourth multiple of 10.)



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What is the ones digit of the product when multiplying by 10? (0) **What is 8 times 10?** (80) **How many 0s are in 10?** (1) **How many 0s are in the product?** (1) **The product and the factors have the same number of 0s.**

Problem 2

In 100, both the ones digit and the tens digit are 0. A multiple is the product of a given whole number and another whole number. Four times 100 is the same as skip counting four times by 100. Skip count together: *100, 200, 300, 400*. These are the first four multiples of 100. The product of 4 and 100 is 400.

How many 0s are in 100? (2) **How many 0s are in the product?** (2) **The product and the factors have the same number of 0s.**

MODEL NEW SKILLS

Problem 3

Eighty is a multiple of 10. When multiplying with a multiple of 10, there is a fast method of finding the product. Count the number of 0s in the multiple of 10, and write the same number of 0s in the answer. Next, multiply the nonzero numbers.

Nonzero numbers are numbers that are not 0. How many 0s are in 80? (1) **Write one 0 under the equal bar in the ones place. Next, multiply the nonzero numbers. What is 6 times 8?** (48) **Write 48 to the left of the 0 in the answer.**

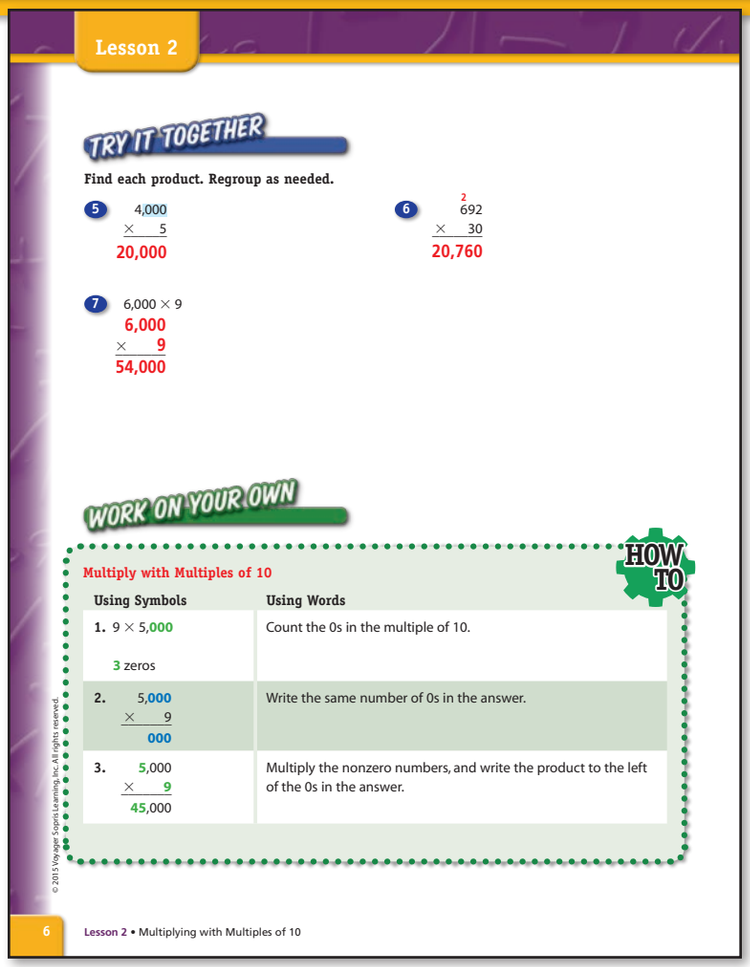
What is the product of 6 and 80? (480) **There is one 0 in 80, and there is one 0 in 480.**

Problem 4

To solve this problem, first write the problem vertically, making sure the number with the greater number of digits is on top. This already has been done. What is the multiple of 10 in this problem? (500) **How many 0s are in 500?** (2) **Write two 0s under the equal bar.**

Multiply the nonzero numbers. What are the nonzero numbers? (7 and 5) **What is 7 times 5?** (35) **Write 35 in the answer to the left of the 0s. Remember to write a comma between the ones period and the thousands period.**

What is 7 times 500? (3,500)



Lesson 2

TRY IT TOGETHER

Find each product. Regroup as needed.

5 4,000 × 5 = 20,000

6 692 × 30 = 20,760

7 6,000 × 9
6,000
× 9
54,000

WORK ON YOUR OWN

HOW TO

Multiply with Multiples of 10

Using Symbols	Using Words
1. 9 × 5,000 3 zeros	Count the 0s in the multiple of 10.
2. 5,000 × 9 = 000	Write the same number of 0s in the answer.
3. 5,000 × 9 = 45,000	Multiply the nonzero numbers, and write the product to the left of the 0s in the answer.

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BUILD THE CONCEPT

Model how to use base-10 pieces to multiply multiples of 10.

The base-10 pieces show 4 times 200, or 4 groups of 200. How many hundreds flats are there in all? (8) **So, 4 times 200 is the same as 8 groups of 100, or 8 times 100. The number 8 can be written as the product of 4 and what number?** (2) **So, 8 times 100 is the same as 4 times 2 times 100. What is 4 times 200?** (800)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What is the multiple of 10 in this problem? (4,000) **How many 0s are in 4,000?** (3) **Write three 0s under the equal bar.**

Next, multiply the nonzero numbers. What are the nonzero numbers? (5 and 4) **What is 5 times 4?** (20) **Write 20 and a comma in the answer. What is the product of 5 and 4,000?** (20,000)

There are three 0s in 4,000. How many 0s are in 20,000? (4) **Three 0s come from 4,000, and one 0 comes from the product of 5 and 4.**

Problem 6

What is the multiple of 10 in this problem? (30) **How many 0s are in 30?** (1) **Write one 0 in the answer.**

What is the next step? (Multiply the nonzero numbers.) **What are the nonzero numbers?** (3 and 692) **What is 3 times 2?** (6) **Write 6 in the answer. What is 3 times 9?** (27) **Write 7 in the answer, and write 2 above the hundreds digit of the top number. What is 3 times 6?** (18) **Now, add the regrouped number. What is 18 plus 2?** (20) **Write 20 in the answer. Place a comma between the ones period and the thousands period.**

What is the product of 30 and 692? (20,760)

Problem 7

What is the multiple of 10 in this problem? (6,000) **How many 0s are in 6,000?** (3) **First, write the numbers vertically. Next, write three 0s in the answer and multiply the nonzero numbers.**

What are the nonzero numbers? (9 and 6) **What is 9 times 6?** (54) **What is 6,000 times 9?** (54,000)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using a 4-Step Plan** strategy. Students are shown how the four-step problem-solving process is used to solve a two-step word problem involving multiplication and addition. The instruction is immediately followed by application of the strategy in problem 17.

The problem presents information about sales of hot dogs at two different festivals.

Have students identify the number of hot dogs sold each day at each festival.

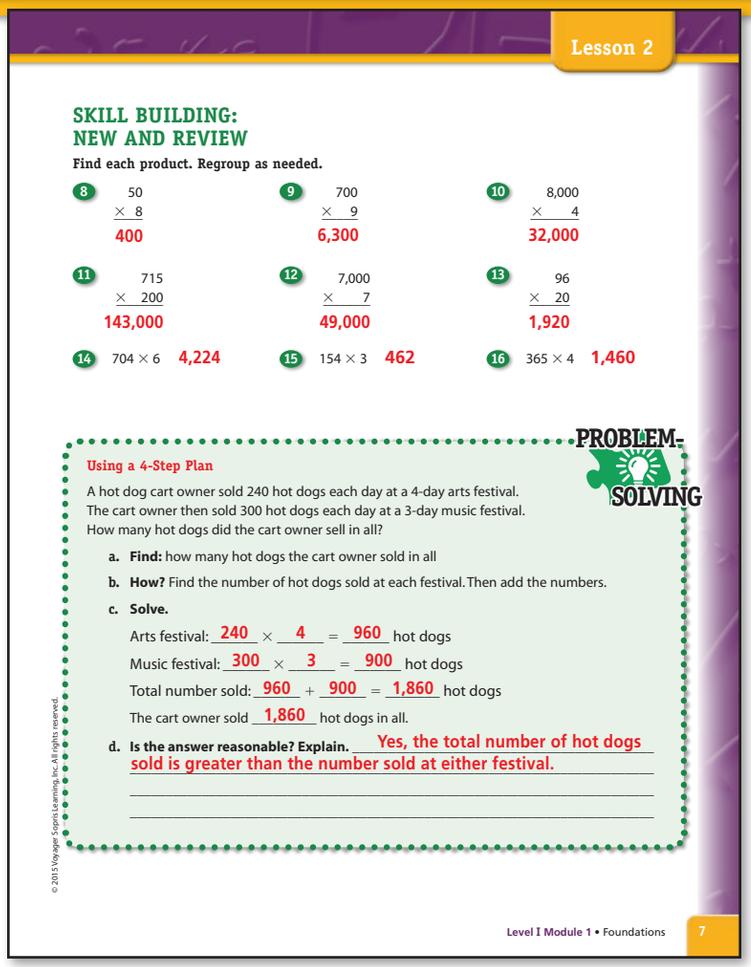
Ask students to find the number of hot dogs sold at each festival.

Next, ask students to find the total sold at both festivals.

Have students explain why the answer is reasonable.

Problems 8–20

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 2

SKILL BUILDING: NEW AND REVIEW

Find each product. Regroup as needed.

8. 50 × 8 = 400
9. 700 × 9 = 6,300
10. 8,000 × 4 = 32,000
11. 715 × 200 = 143,000
12. 7,000 × 7 = 49,000
13. 96 × 20 = 1,920
14. 704 × 6 = 4,224
15. 154 × 3 = 462
16. 365 × 4 = 1,460

PROBLEM-SOLVING

Using a 4-Step Plan

A hot dog cart owner sold 240 hot dogs each day at a 4-day arts festival. The cart owner then sold 300 hot dogs each day at a 3-day music festival. How many hot dogs did the cart owner sell in all?

a. **Find:** how many hot dogs the cart owner sold in all

b. **How?** Find the number of hot dogs sold at each festival. Then add the numbers.

c. **Solve.**

Arts festival: 240 × 4 = 960 hot dogs

Music festival: 300 × 3 = 900 hot dogs

Total number sold: 960 + 900 = 1,860 hot dogs

The cart owner sold 1,860 hot dogs in all.

d. **Is the answer reasonable? Explain.** Yes, the total number of hot dogs sold is greater than the number sold at either festival.

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Problem 17 Reminder

Be sure students first find the number of hours each person worked, then add to find the total number of hours they both worked.

CHECK UP

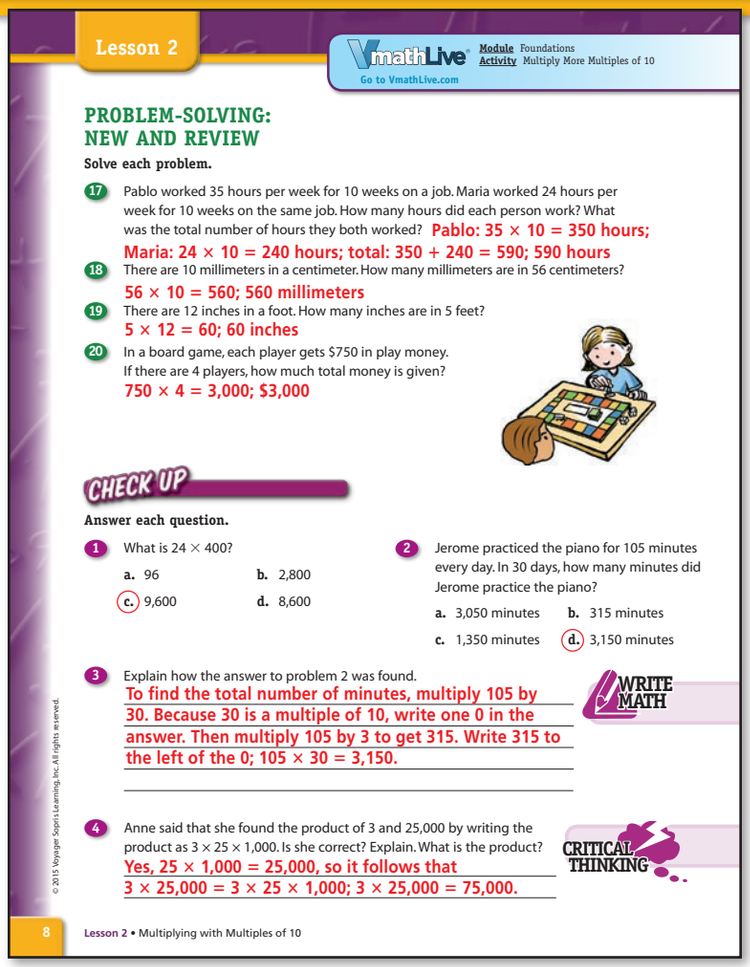
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2b: The student did not write 0s in the product for the multiple of 10. Use Additional Resources in I.1.2 to reteach how to multiply with multiples of 10.

If student answered 1d or 2a: The student wrote the proper number of 0s in the product but did not regroup when multiplying. Review how to multiply by a 1-digit number with regrouping.



Lesson 2

VmathLive Module Foundations Activity Multiply More Multiples of 10
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

17 Pablo worked 35 hours per week for 10 weeks on a job. Maria worked 24 hours per week for 10 weeks on the same job. How many hours did each person work? What was the total number of hours they both worked? Pablo: 35 × 10 = 350 hours; Maria: 24 × 10 = 240 hours; total: 350 + 240 = 590; 590 hours

18 There are 10 millimeters in a centimeter. How many millimeters are in 56 centimeters? 56 × 10 = 560; 560 millimeters

19 There are 12 inches in a foot. How many inches are in 5 feet? 5 × 12 = 60; 60 inches

20 In a board game, each player gets $750 in play money. If there are 4 players, how much total money is given? 750 × 4 = 3,000; $3,000

CHECK UP

Answer each question.

1 What is 24 × 400?
a. 96 b. 2,800
(c.) 9,600 d. 8,600

2 Jerome practiced the piano for 105 minutes every day. In 30 days, how many minutes did Jerome practice the piano?
a. 3,050 minutes b. 315 minutes
c. 1,350 minutes (d.) 3,150 minutes

3 Explain how the answer to problem 2 was found. WRITE MATH
To find the total number of minutes, multiply 105 by 30. Because 30 is a multiple of 10, write one 0 in the answer. Then multiply 105 by 3 to get 315. Write 315 to the left of the 0; 105 × 30 = 3,150.

4 Anne said that she found the product of 3 and 25,000 by writing the product as 3 × 25 × 1,000. Is she correct? Explain. What is the product? CRITICAL THINKING
Yes, 25 × 1,000 = 25,000, so it follows that 3 × 25,000 = 3 × 25 × 1,000; 3 × 25,000 = 75,000.

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8 Lesson 2 • Multiplying with Multiples of 10



WRITE MATH **If student answered problem 3 incorrectly:** While students work together, review with the student the process of multiplying a whole number by a multiple of 10.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Foundations
Activity: Multiply More Multiples of 10

Vmath Reteach
Reteach Student Book Module 1 Lesson 2
Reteach Teacher Edition Module 1 Lesson 2

Extra Practice
Student Book page 53

English Language Learners

Have students form small groups. Have each group of students sit in a circle on the floor. One student in each group starts by saying "10" aloud. Have students take turns going around the circle saying the next multiple of 10 until the group reaches 100. Allow groups to practice for a couple of minutes. Then have groups switch to counting by multiples of 100.

Students with Special Needs

Students may count the number of 0s incorrectly when multiplying with multiples of 10. Encourage students to cross out each 0 in the factors as they count. For example, in the problem 715 × 200, have students cross out the 0s as they count: 2ØØ. There are two marks, so there should be two 0s in the answer before multiplying the nonzero numbers.

Lesson 3

Objective

To add fractions with like and unlike denominators

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To write the improper fraction $\frac{8}{6}$ as a mixed number, divide 8 by 6. What is 8 divided by 6? (1 R2) The whole number part of the quotient is the whole number part of the mixed number. The remainder is the numerator in the fraction part of the mixed number. The denominator of the mixed number is the same as the denominator of the improper fraction. The improper fraction $\frac{8}{6}$ is equal to the mixed number $1\frac{2}{6}$. Is $\frac{2}{6}$ written in simplest form? (no) What is $1\frac{2}{6}$ written in simplest form? ($1\frac{1}{3}$)

Problem 2

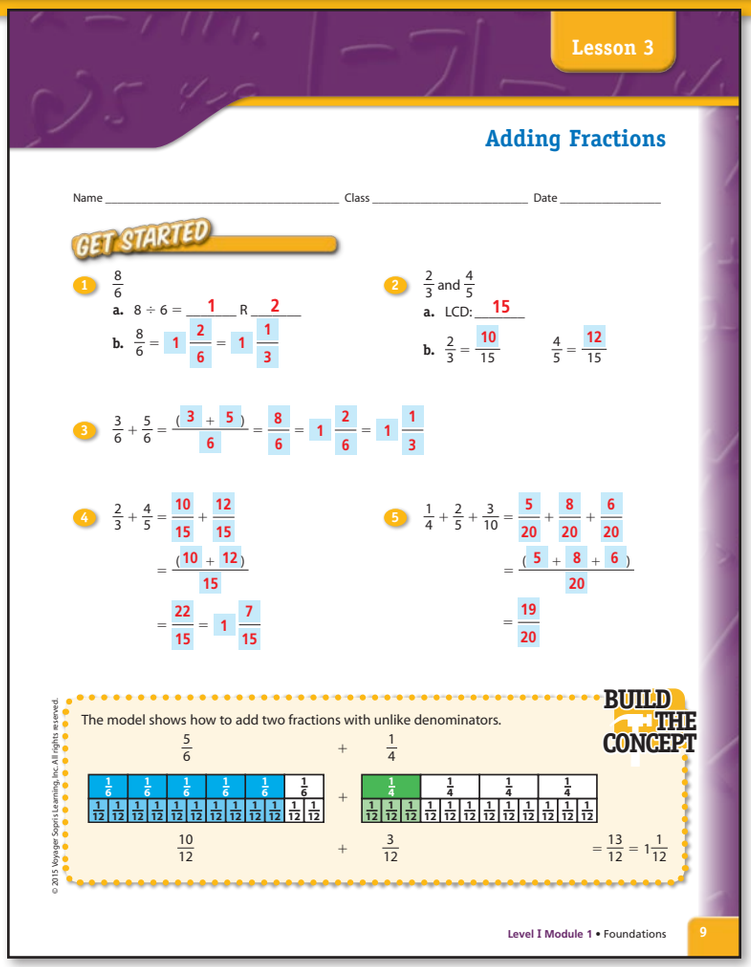
To change the fractions $\frac{2}{3}$ and $\frac{4}{5}$ to equivalent fractions with the same denominator, first find the least common denominator, or LCD, of the two fractions. The denominators of the fractions are 3 and 5. What is the least common multiple, LCM, of 3 and 5? (15) Then the LCD of $\frac{2}{3}$ and $\frac{4}{5}$ is 15.

Look at problem 2b. Now, find the fraction with a denominator of 15 that is equivalent to $\frac{2}{3}$. What is 3 multiplied by to get 15? (5) Multiply 2 and 5. What is the product? (10) The equivalent fraction is $\frac{10}{15}$. What is 5 multiplied by to get 15? (3) What is 4 times 3? (12) What fraction with a denominator of 15 is equivalent to $\frac{4}{5}$? ($\frac{12}{15}$)

MODEL NEW SKILLS

Problem 3

The denominators of $\frac{3}{6}$ and $\frac{5}{6}$ are the same, so the fractions have like denominators. To add fractions with like denominators, add the numerators and write the sum over the like denominator. What is 3 plus 5? (8) What is the like denominator? (6) So, $\frac{3}{6}$ plus $\frac{5}{6}$ equals $\frac{8}{6}$. The fraction $\frac{8}{6}$ is an improper fraction.


Lesson 3

Adding Fractions

Name _____ Class _____ Date _____

GET STARTED

1 $\frac{8}{6}$

a. $8 \div 6 =$ 1 R 2

b. $\frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

2 $\frac{2}{3}$ and $\frac{4}{5}$

a. LCD: 15

b. $\frac{2}{3} = \frac{10}{15}$ $\frac{4}{5} = \frac{12}{15}$

3 $\frac{3}{6} + \frac{5}{6} = \frac{(3+5)}{6} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

4 $\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{(10+12)}{15} = \frac{22}{15} = 1\frac{7}{15}$

5 $\frac{1}{4} + \frac{2}{5} + \frac{3}{10} = \frac{5}{20} + \frac{8}{20} + \frac{6}{20} = \frac{(5+8+6)}{20} = \frac{19}{20}$

BUILD THE CONCEPT

The model shows how to add two fractions with unlike denominators.

$\frac{5}{6} + \frac{1}{4}$

$\frac{10}{12} + \frac{3}{12} = \frac{13}{12} = 1\frac{1}{12}$

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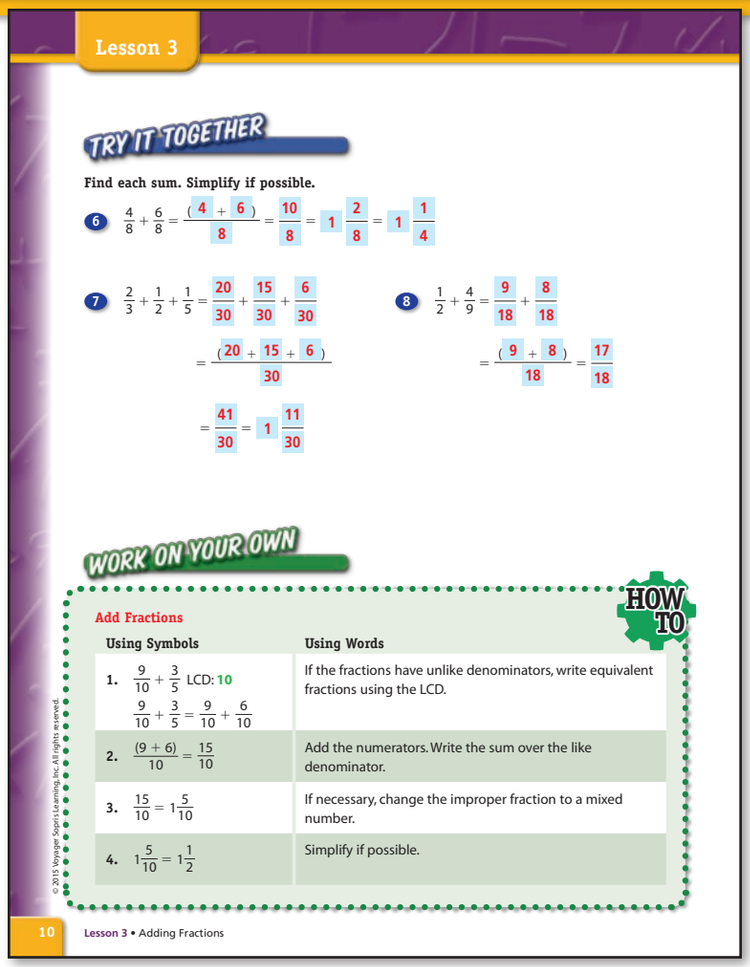
When the sum of fractions is an improper fraction, change the improper fraction to a whole number or mixed number written in simplest form. In problem 1b, $\frac{8}{6}$ was written as a mixed number in simplest form.

Problem 4

Do the fractions $\frac{2}{3}$ and $\frac{4}{5}$ have like or unlike denominators? (unlike) To add fractions with unlike denominators, first write equivalent fractions using the LCD. Then add the numerators of the equivalent fractions and write the sum over the LCD.

Equivalent fractions for $\frac{2}{3}$ and $\frac{4}{5}$ with an LCD of 15 were found in problem 2b. What fraction is equivalent to $\frac{2}{3}$? ($\frac{10}{15}$) What fraction is equivalent to $\frac{4}{5}$? ($\frac{12}{15}$) Add the numerators and write the sum over the LCD. What is 10 plus 12? (22) What is $\frac{10}{15}$ plus $\frac{12}{15}$? ($\frac{22}{15}$)

Change the improper fraction $\frac{22}{15}$ to a mixed number in simplest form. What is 22 divided by 15? (1 R7) What is the mixed number? ($1\frac{7}{15}$) Is $1\frac{7}{15}$ written in simplest form? (yes) The sum of $\frac{2}{3}$ and $\frac{4}{5}$ is $1\frac{7}{15}$.


Lesson 3

TRY IT TOGETHER

Find each sum. Simplify if possible.

6. $\frac{4}{8} + \frac{6}{8} = \frac{(4 + 6)}{8} = \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}$

7. $\frac{2}{3} + \frac{1}{2} + \frac{1}{5} = \frac{20}{30} + \frac{15}{30} + \frac{6}{30} = \frac{(20 + 15 + 6)}{30} = \frac{41}{30} = 1\frac{11}{30}$

8. $\frac{1}{2} + \frac{4}{9} = \frac{9}{18} + \frac{8}{18} = \frac{(9 + 8)}{18} = \frac{17}{18}$

WORK ON YOUR OWN

HOW TO

Add Fractions

Using Symbols	Using Words
1. $\frac{9}{10} + \frac{3}{5}$ LCD: 10 $\frac{9}{10} + \frac{3}{5} = \frac{9}{10} + \frac{6}{10}$	If the fractions have unlike denominators, write equivalent fractions using the LCD.
2. $\frac{(9 + 6)}{10} = \frac{15}{10}$	Add the numerators. Write the sum over the like denominator.
3. $\frac{15}{10} = 1\frac{5}{10}$	If necessary, change the improper fraction to a mixed number.
4. $1\frac{5}{10} = 1\frac{1}{2}$	Simplify if possible.

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10 Lesson 3 • Adding Fractions

Problem 5

This problem has three fractions. Begin by writing equivalent fractions using the LCD. What is the LCD of $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{3}{10}$? (20) **What are the equivalent fractions?** ($\frac{5}{20}$, $\frac{8}{20}$, $\frac{6}{20}$) **Now, add the numerators of all three fractions and write the sum over the LCD, 20. What is 5 plus 8 plus 6?** (19) **Write 19 over the LCD, 20. What is $\frac{1}{4}$ plus $\frac{2}{5}$ plus $\frac{3}{10}$?** ($\frac{19}{20}$) **Is $\frac{19}{20}$ written in simplest form?** (yes)

BUILD THE CONCEPT

Model how to add fractions with unlike denominators.

Are the denominators like or unlike? (unlike) **The denominators must be like denominators before adding. What is the LCD of 6 and 4?** (12) **How many $\frac{1}{12}$-fraction bars equal $\frac{5}{6}$?** (10) **How many $\frac{1}{12}$-fraction bars equal $\frac{1}{4}$?** (3) **So, $\frac{5}{6}$ is equivalent to $\frac{10}{12}$ and $\frac{1}{4}$ is equivalent to $\frac{3}{12}$.**

Now, add the numerators. What is 10 plus 3? (13) **When adding fractions with like denominators, the denominator stays the same. What is the denominator of the sum?** (12) **The bottom model shows the sum is $\frac{13}{12}$. This is an improper fraction. What is $\frac{13}{12}$ written as a mixed number?** ($1\frac{1}{12}$) **Is this written in simplest form?** (yes)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

Are the denominators of $\frac{4}{8}$ and $\frac{6}{8}$ like or unlike? (like) **How are fractions with like denominators added?** (Add the numerators of the fractions and write the sum over the like denominator.) **What is $\frac{4}{8}$ plus $\frac{6}{8}$?** ($\frac{10}{8}$) **What kind of fraction is $\frac{10}{8}$?** (improper fraction)

What is 10 divided by 8? (1 R2) **What is $\frac{10}{8}$ written as a mixed number?** ($1\frac{2}{8}$) **Is $\frac{2}{8}$ written in simplest form?** (no) **What is the simplest form of $\frac{2}{8}$?** ($\frac{1}{4}$) **What is $\frac{4}{8}$ plus $\frac{6}{8}$ written as a mixed number in simplest form?** ($1\frac{1}{4}$)

Problem 7

What is the first step in adding fractions with unlike denominators? (Write equivalent fractions using the LCD.) **What is the LCD?** (30) **What are the equivalent fractions?** ($\frac{20}{30}$, $\frac{15}{30}$, $\frac{6}{30}$) **What is the next step?** (Add the three numerators of the equivalent fractions and write the sum over the LCD.) **What is 20 plus 15 plus 6?** (41) **What is $\frac{20}{30}$ plus $\frac{15}{30}$ plus $\frac{6}{30}$?** ($\frac{41}{30}$)

What kind of fraction is $\frac{41}{30}$? (improper fraction) **What is $\frac{41}{30}$ written as a mixed number?** ($1\frac{11}{30}$) **Is $1\frac{11}{30}$ written in simplest form?** (yes) **What is $\frac{2}{3}$ plus $\frac{1}{2}$ plus $\frac{1}{5}$ as a mixed number in simplest form?** ($1\frac{11}{30}$)

Problem 8

What is the LCD of $\frac{1}{2}$ and $\frac{4}{9}$? (18) **What are the equivalent fractions?** ($\frac{9}{18}$ and $\frac{8}{18}$) **What is $\frac{1}{2}$ plus $\frac{4}{9}$?** ($\frac{17}{18}$) **Is the fraction written in simplest form?** (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using a Pattern** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a pattern. The instruction is immediately followed by application of the strategy in problem 21.

The problem states that the fractions form a pattern. Have students first find a rule for the pattern. To find the rule, start by listing the fractions with a common denominator. The LCD of 10, 2, and 10 is 10. Convert $\frac{1}{2}$ to an equivalent fraction with a denominator of 10 by multiplying both the numerator and denominator by 5: $\frac{1}{2} = \frac{5}{10}$. Now list the fractions.

$\frac{3}{10}, \frac{5}{10}, \frac{7}{10}$

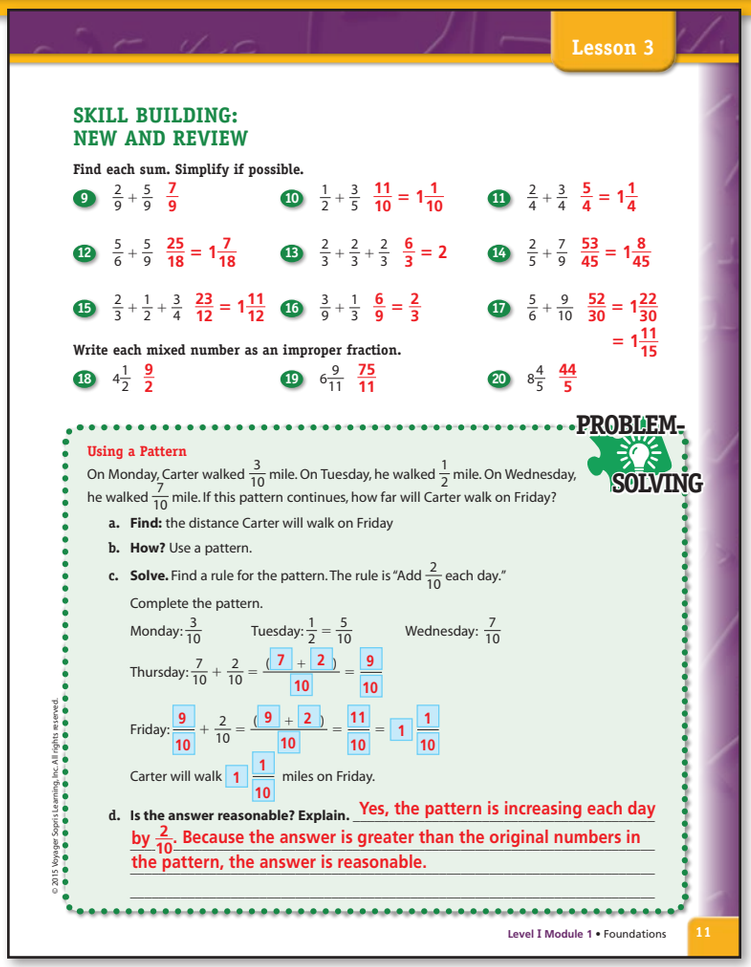
The pattern is increasing. The rule is "Add $\frac{2}{10}$ each day."

To find how far Carter walked on Thursday, add $\frac{7}{10}$ and $\frac{2}{10}$. Add the numerators, and use the same denominator: $\frac{7}{10} + \frac{2}{10} = \frac{9}{10}$. Carter walked $\frac{9}{10}$ mile on Thursday.

To find how far Carter walked on Friday, add $\frac{9}{10}$ and $\frac{2}{10}$. Add the numerators, and use the same denominator: $\frac{9}{10} + \frac{2}{10} = \frac{11}{10}$. Convert $\frac{11}{10}$ from an improper fraction to a mixed number. Divide the numerator by the denominator. Eleven divided by 10 equals 1 R1. The quotient becomes the whole number part, and the remainder becomes the denominator. Carter walked $1\frac{1}{10}$ miles on Friday.

Problems 9–24

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 3

SKILL BUILDING: NEW AND REVIEW

Find each sum. Simplify if possible.

9. $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$
10. $\frac{1}{2} + \frac{3}{5} = \frac{11}{10} = 1\frac{1}{10}$
11. $\frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$
12. $\frac{5}{6} + \frac{5}{9} = \frac{25}{18} = 1\frac{7}{18}$
13. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{6}{3} = 2$
14. $\frac{2}{5} + \frac{7}{9} = \frac{53}{45} = 1\frac{8}{45}$
15. $\frac{2}{3} + \frac{1}{2} + \frac{3}{4} = \frac{23}{12} = 1\frac{11}{12}$
16. $\frac{3}{9} + \frac{1}{3} = \frac{6}{9} = \frac{2}{3}$
17. $\frac{5}{6} + \frac{9}{10} = \frac{52}{30} = 1\frac{22}{30} = 1\frac{11}{15}$

Write each mixed number as an improper fraction.

18. $4\frac{1}{2} = \frac{9}{2}$
19. $6\frac{9}{11} = \frac{75}{11}$
20. $8\frac{4}{5} = \frac{44}{5}$

PROBLEM-SOLVING

Using a Pattern

On Monday, Carter walked $\frac{3}{10}$ mile. On Tuesday, he walked $\frac{1}{2}$ mile. On Wednesday, he walked $\frac{7}{10}$ mile. If this pattern continues, how far will Carter walk on Friday?

a. **Find:** the distance Carter will walk on Friday

b. **How?** Use a pattern.

c. **Solve.** Find a rule for the pattern. The rule is "Add $\frac{2}{10}$ each day."

Complete the pattern.

Monday: $\frac{3}{10}$ Tuesday: $\frac{1}{2} = \frac{5}{10}$ Wednesday: $\frac{7}{10}$

Thursday: $\frac{7}{10} + \frac{2}{10} = \frac{(7+2)}{10} = \frac{9}{10}$

Friday: $\frac{9}{10} + \frac{2}{10} = \frac{(9+2)}{10} = \frac{11}{10} = 1\frac{1}{10}$

Carter will walk $1\frac{1}{10}$ miles on Friday.

d. **Is the answer reasonable? Explain.** Yes, the pattern is increasing each day by $\frac{2}{10}$. Because the answer is greater than the original numbers in the pattern, the answer is reasonable.

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Problem 24 Reminder

Students must add the fractions, then change the answer from an improper fraction to a mixed number in simplest form. Last, students must compare the total with 1.

CHECK UP

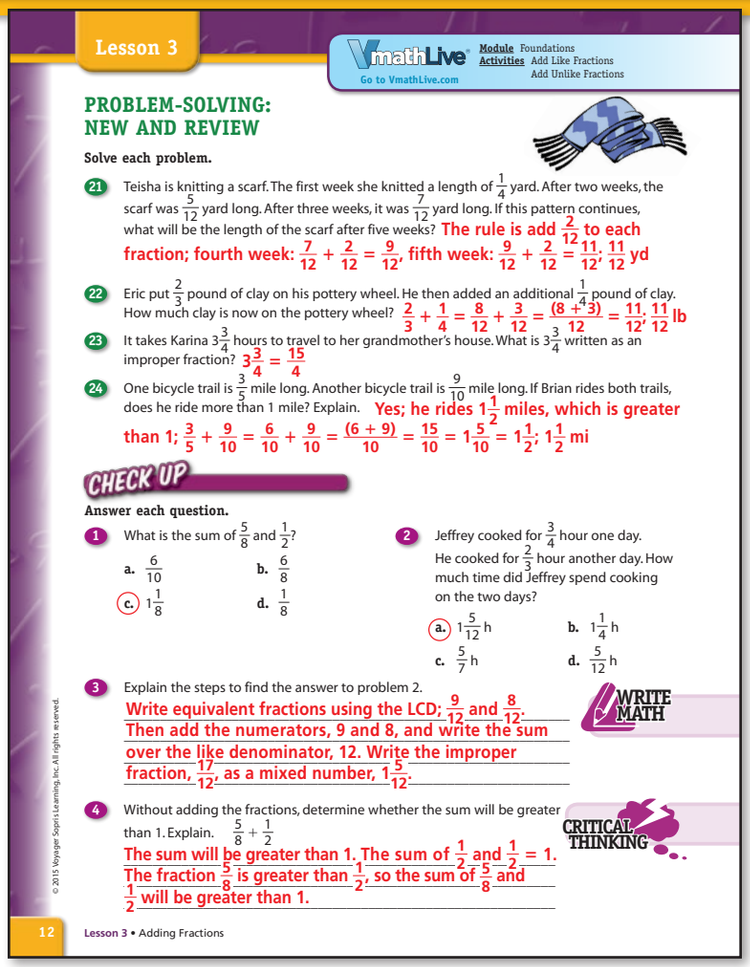
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2c: The student added the numerators and denominators instead of finding equivalent fractions using the LCD. Review how to add fractions with unlike denominators.

If student answered 1b or 2d: The student found the LCD but did not change the numerators. Review how to find equivalent fractions.



Lesson 3

VmathLive Go to VmathLive.com
Module Foundations
Activities Add Like Fractions
Add Unlike Fractions

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

21 Teisha is knitting a scarf. The first week she knitted a length of $\frac{1}{4}$ yard. After two weeks, the scarf was $\frac{5}{12}$ yard long. After three weeks, it was $\frac{7}{12}$ yard long. If this pattern continues, what will be the length of the scarf after five weeks? The rule is add $\frac{2}{12}$ to each fraction; fourth week: $\frac{7}{12} + \frac{2}{12} = \frac{9}{12}$, fifth week: $\frac{9}{12} + \frac{2}{12} = \frac{11}{12}$; $\frac{11}{12}$ yd

22 Eric put $\frac{2}{3}$ pound of clay on his pottery wheel. He then added an additional $\frac{1}{4}$ pound of clay. How much clay is now on the pottery wheel? $\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{(8+3)}{12} = \frac{11}{12}$; $\frac{11}{12}$ lb

23 It takes Karina $3\frac{3}{4}$ hours to travel to her grandmother's house. What is $3\frac{3}{4}$ written as an improper fraction? $3\frac{3}{4} = \frac{15}{4}$

24 One bicycle trail is $\frac{3}{5}$ mile long. Another bicycle trail is $\frac{9}{10}$ mile long. If Brian rides both trails, does he ride more than 1 mile? Explain. Yes; he rides $1\frac{1}{2}$ miles, which is greater than 1; $\frac{3}{5} + \frac{9}{10} = \frac{6}{10} + \frac{9}{10} = \frac{(6+9)}{10} = \frac{15}{10} = 1\frac{5}{10} = 1\frac{1}{2}$; $1\frac{1}{2}$ mi

CHECK UP

Answer each question.

1 What is the sum of $\frac{5}{8}$ and $\frac{1}{2}$?

a. $\frac{6}{10}$ b. $\frac{6}{8}$ (c.) $1\frac{1}{8}$ d. $\frac{1}{8}$

2 Jeffrey cooked for $\frac{3}{4}$ hour one day. He cooked for $\frac{2}{3}$ hour another day. How much time did Jeffrey spend cooking on the two days?

(a.) $1\frac{5}{12}$ h b. $1\frac{1}{4}$ h c. $\frac{5}{7}$ h d. $\frac{5}{12}$ h

3 Explain the steps to find the answer to problem 2. WRITE MATH

Write equivalent fractions using the LCD; $\frac{9}{12}$ and $\frac{8}{12}$. Then add the numerators, 9 and 8, and write the sum over the like denominator, 12. Write the improper fraction, $\frac{17}{12}$, as a mixed number, $1\frac{5}{12}$.

4 Without adding the fractions, determine whether the sum will be greater than 1. Explain. $\frac{5}{8} + \frac{1}{2}$ CRITICAL THINKING

The sum will be greater than 1. The sum of $\frac{1}{2}$ and $\frac{1}{2}$ = 1. The fraction $\frac{5}{8}$ is greater than $\frac{1}{2}$, so the sum of $\frac{5}{8}$ and $\frac{1}{2}$ will be greater than 1.

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12 Lesson 3 • Adding Fractions

If student answered 1d: The student forgot to put the whole number in the answer when changing from an improper fraction to a mixed number. Review how to change an improper fraction to a mixed number.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student of the steps to add fractions with unlike denominators. The fractions must have like denominators to add the numerators. This is achieved by using the LCD to write equivalent fractions.

Technology



Have students use an online search engine to discover other ways to apply adding fractions in real-life situations. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Foundations
Activities: Add Like Fractions
Add Unlike Fractions

Vmath Reteach
Reteach Student Book Module 1 Lesson 3
Reteach Teacher Edition Module 1 Lesson 3

Extra Practice
Student Book page 54

English Language Learners

Use the VmathLive Animated Glossary to review the terms *numerator* and *denominator*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Discuss the meaning of the words *like* and *unlike*. Explain that *like* means "same" and *unlike* means "different." Use the terms to describe items in the classroom. Hold up two sheets of white paper and say "like." Hold up a sheet of white paper and a sheet of colored paper and say "unlike." Transition to fractions, labeling denominators as *like* or *unlike*. Write pairs of fractions on the board and ask students to use the terms *like* and *unlike* to describe the denominators of the fraction pairs.

Students with Special Needs

Write various fractions on index cards. Distribute the cards to students. Have students pair up and work together to find the sum of the fractions on their index cards. Guide students to use fraction strips and model each step of the addition process. Once students have solved a few problems this way, have them write the steps they followed and use that as a resource.

Lesson 4

Objective

To subtract fractions with like and unlike denominators



Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The denominators of $\frac{5}{8}$ and $\frac{2}{6}$ are not the same, so the fractions have unlike denominators. To change the fractions to equivalent fractions with the same denominator, first find the least common denominator, or LCD, of the two fractions.

The denominators of the fractions are 8 and 6. What is the LCM of 8 and 6? (24) So, the LCD of $\frac{5}{8}$ and $\frac{2}{6}$ is 24. Now find the fraction with a denominator of 24 that is equivalent to $\frac{5}{8}$. What is 8 multiplied by to get 24? (3) Multiply 5 and 3. What is the product? (15) The equivalent fraction is $\frac{15}{24}$. What fraction with a denominator of 24 is equivalent to $\frac{2}{6}$? ($\frac{8}{24}$)

MODEL NEW SKILLS

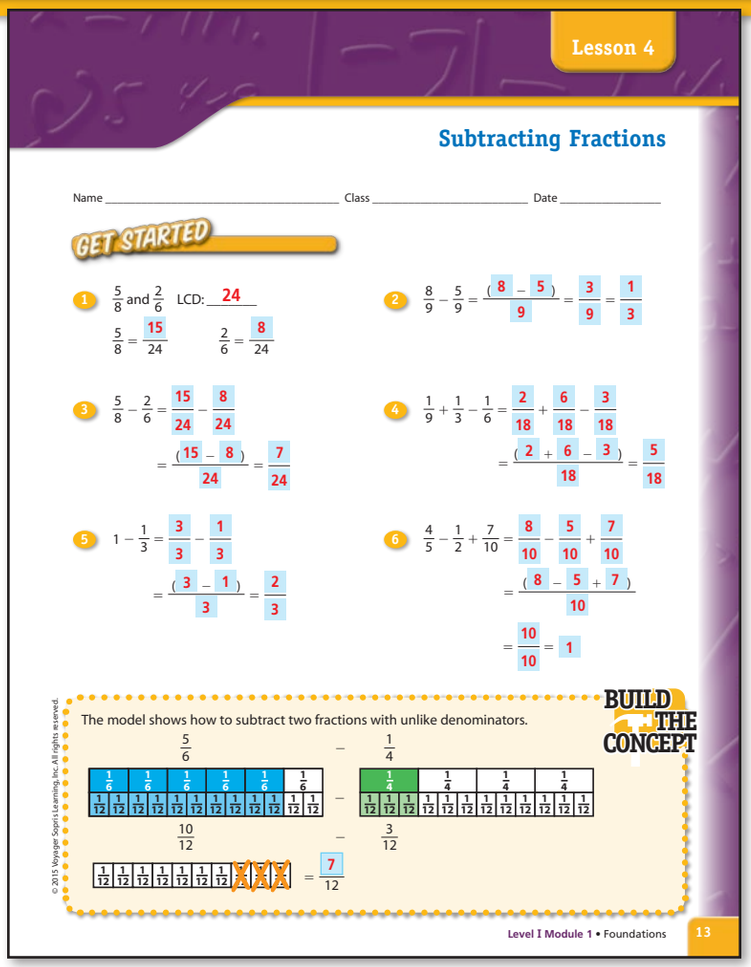
Problem 2

The denominators of $\frac{8}{9}$ and $\frac{5}{9}$ are the same, so the fractions have like denominators. To subtract fractions with like denominators, subtract their numerators and write the difference over the like denominator. What is 8 minus 5? (3) What is the like denominator? (9) So, $\frac{8}{9}$ minus $\frac{5}{9}$ equals $\frac{3}{9}$. Is $\frac{3}{9}$ written in simplest form? (no) What is $\frac{8}{9}$ minus $\frac{5}{9}$ written in simplest form? ($\frac{1}{3}$)

Problem 3

Do the fractions $\frac{5}{8}$ and $\frac{2}{6}$ have like or unlike denominators? (unlike) To subtract fractions with unlike denominators, first write equivalent fractions using the LCD. Then subtract the numerators of the equivalent fractions and write the difference over the LCD.

Equivalent fractions with an LCD of 24 were found in problem 1. What fraction is equivalent to $\frac{5}{8}$? ($\frac{15}{24}$) What fraction is equivalent to $\frac{2}{6}$? ($\frac{8}{24}$) Subtract the numerators and write the difference over the LCD, 24. What is 15 minus 8? (7) What is $\frac{15}{24}$ minus $\frac{8}{24}$? ($\frac{7}{24}$) Is $\frac{7}{24}$ written in simplest form? (yes) What is $\frac{5}{8}$ minus $\frac{2}{6}$? ($\frac{7}{24}$)



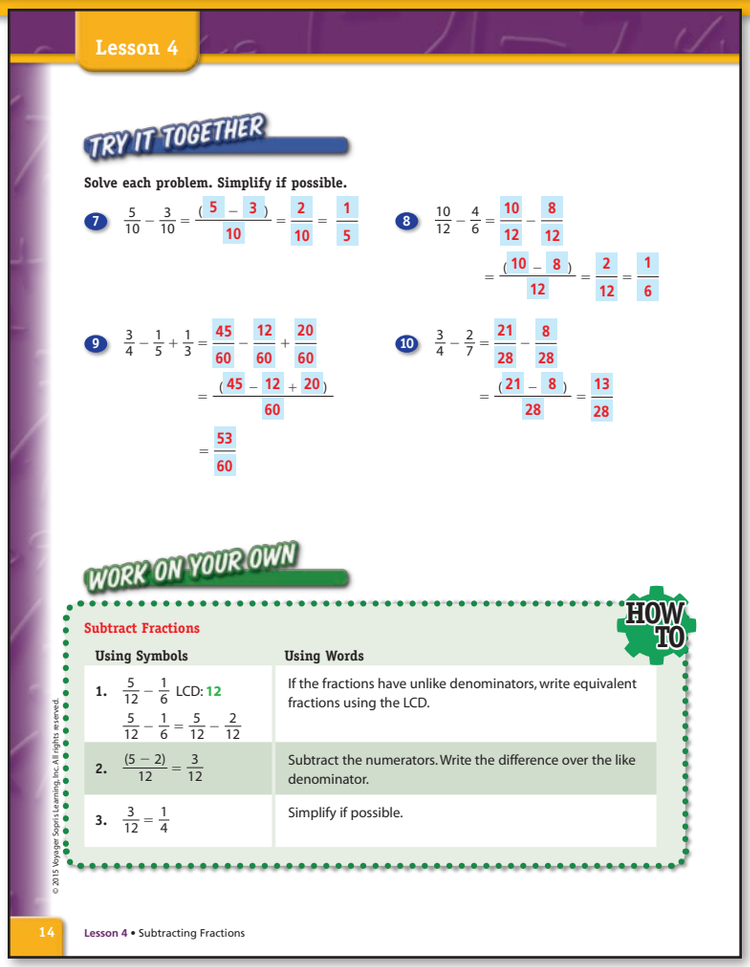
Problem 4

This problem has three fractions. Notice that the first operation is addition and the second operation is subtraction. Remember, the order of operations states that addition and subtraction are performed from left to right. Find the sum of the first two fractions, then subtract the third fraction from that sum. Begin by writing equivalent fractions using the LCD. What is the LCD of $\frac{1}{9}$, $\frac{1}{3}$, and $\frac{1}{6}$? (18) What are the equivalent fractions? ($\frac{2}{18}$, $\frac{6}{18}$, $\frac{3}{18}$)

Now, add the first two numerators, then subtract the third numerator from that sum. What is 2 plus 6 minus 3? (5) Write this answer over the LCD, 18. What is $\frac{1}{9}$ plus $\frac{1}{3}$ minus $\frac{1}{6}$? ($\frac{5}{18}$) Is $\frac{5}{18}$ written in simplest form? (yes)

Problem 5

In this problem, a fraction is being subtracted from a whole number. The whole number 1 can be written as a fraction with the same numerator and denominator. What is the fraction in this problem? ($\frac{1}{3}$) What is the denominator? (3)



Lesson 4

TRY IT TOGETHER

Solve each problem. Simplify if possible.

7 $\frac{5}{10} - \frac{3}{10} = \frac{(5 - 3)}{10} = \frac{2}{10} = \frac{1}{5}$

8 $\frac{10}{12} - \frac{4}{6} = \frac{10}{12} - \frac{8}{12} = \frac{(10 - 8)}{12} = \frac{2}{12} = \frac{1}{6}$

9 $\frac{3}{4} - \frac{1}{5} + \frac{1}{3} = \frac{45}{60} - \frac{12}{60} + \frac{20}{60} = \frac{(45 - 12 + 20)}{60} = \frac{53}{60}$

10 $\frac{3}{4} - \frac{2}{7} = \frac{21}{28} - \frac{8}{28} = \frac{(21 - 8)}{28} = \frac{13}{28}$

WORK ON YOUR OWN

HOW TO

Subtract Fractions

Using Symbols	Using Words
1. $\frac{5}{12} - \frac{1}{6}$ LCD: 12 $\frac{5}{12} - \frac{1}{6} = \frac{5}{12} - \frac{2}{12}$	If the fractions have unlike denominators, write equivalent fractions using the LCD.
2. $\frac{(5 - 2)}{12} = \frac{3}{12}$	Subtract the numerators. Write the difference over the like denominator.
3. $\frac{3}{12} = \frac{1}{4}$	Simplify if possible.

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Rewrite the whole number 1 as a fraction with the same numerator and denominator as the denominator in the fraction part of the problem. What is 1 written as a fraction? ($\frac{3}{3}$) **Now, subtract the fractions. What is $\frac{3}{3}$ minus $\frac{1}{3}$?** ($\frac{2}{3}$) **Is $\frac{2}{3}$ written in simplest form?** (yes)

Problem 6

What is the LCD of $\frac{4}{5}$, $\frac{1}{2}$, and $\frac{7}{10}$? (10) **What are the equivalent fractions with a denominator of 10?** ($\frac{8}{10}$, $\frac{5}{10}$, $\frac{7}{10}$) **Notice that the first operation is subtraction and the second operation is addition. What is 8 minus 5 plus 7?** (10) **What is $\frac{8}{10}$ minus $\frac{5}{10}$ plus $\frac{7}{10}$?** ($\frac{10}{10}$) **Is the answer written in simplest form?** (no) **How is $\frac{10}{10}$ written in simplest form?** (as the whole number 1) **What is $\frac{4}{5}$ minus $\frac{1}{2}$ plus $\frac{7}{10}$?** (1)

BUILD THE CONCEPT

Model how to subtract fractions with unlike denominators.

Models of each fraction can be used to find the difference. The fraction $\frac{5}{6}$ is modeled by the five shaded $\frac{1}{6}$-fraction bars. The fraction $\frac{1}{4}$ is modeled by the one shaded $\frac{1}{4}$-fraction bar. The denominators must be the same to subtract. What is the LCD of 6 and 4? (12)

Looking at the fraction bars, how many $\frac{1}{12}$-fraction bars equal $\frac{5}{6}$? (10) **What fraction is equivalent to $\frac{5}{6}$ and has a denominator of 12?** ($\frac{10}{12}$) **How many $\frac{1}{12}$-fraction bars equal $\frac{1}{4}$?** (3) **The subtraction is shown by crossing out, or taking away, three $\frac{1}{12}$-fraction bars from the ten $\frac{1}{12}$-fraction bars. This is the same as subtracting the numerators. How many $\frac{1}{12}$-fraction bars are left?** (7) **What is $\frac{5}{6}$ minus $\frac{1}{4}$ as shown by the models?** ($\frac{7}{12}$)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 7

Are the denominators of $\frac{5}{10}$ and $\frac{3}{10}$ like or unlike? (like) **What is $\frac{5}{10}$ minus $\frac{3}{10}$?** ($\frac{2}{10}$) **Is $\frac{2}{10}$ written in simplest form?** (no) **What is $\frac{5}{10}$ minus $\frac{3}{10}$ written in simplest form?** ($\frac{1}{5}$)

Problem 8

What is the first step in subtracting fractions with unlike denominators? (Write equivalent fractions using the LCD.) **What is the LCD of $\frac{10}{12}$ and $\frac{4}{6}$?** (12) **Keep $\frac{10}{12}$ the same. What fraction with a denominator of 12 is equivalent to $\frac{4}{6}$?** ($\frac{8}{12}$)

What is the next step? (Subtract the numerators of the equivalent fractions and write the difference over the LCD.) **What is 10 minus 8?** (2) **What is $\frac{10}{12}$ minus $\frac{8}{12}$?** ($\frac{2}{12}$) **Is $\frac{2}{12}$ written in simplest form?** (no) **What is $\frac{10}{12}$ minus $\frac{4}{6}$ written in simplest form?** ($\frac{1}{6}$)

Problem 9

What is the first step in subtracting and adding fractions with unlike denominators? (Write equivalent fractions using the LCD.) **What are the equivalent fractions?** ($\frac{45}{60}$, $\frac{12}{60}$, $\frac{20}{60}$) **What operations are in this problem?** (subtraction and addition) **What is 45 minus 12 plus 20?** (53) **What is $\frac{3}{4}$ minus $\frac{1}{5}$ plus $\frac{1}{3}$?** ($\frac{53}{60}$) **Is $\frac{53}{60}$ written in simplest form?** (yes)

Problem 10

What is the LCD of $\frac{3}{4}$ and $\frac{2}{7}$? (28) **What is $\frac{3}{4}$ minus $\frac{2}{7}$?** ($\frac{13}{28}$) **Is the fraction written in simplest form?** (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 11–26

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 25 Reminder

This problem asks for a difference between two quantities and also for the sum of the same two quantities.

CHECK UP

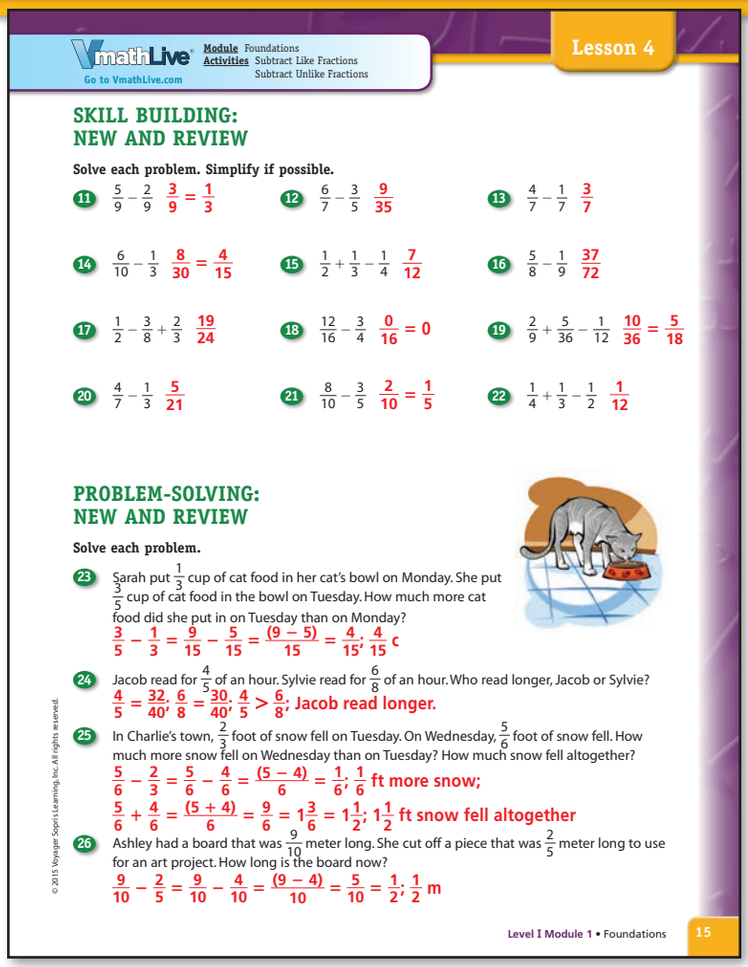
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2d: The student subtracted the numerators and denominators instead of finding equivalent fractions using the LCD. Use Additional Resources in I.1.4 to reteach how to subtract fractions with unlike denominators.

If student answered 1d or 2a: The student added instead of subtracting the fractions. Remind the student how to determine when to add and when to subtract.



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Module Activities Foundations
Subtract Like Fractions
Subtract Unlike Fractions

Lesson 4

SKILL BUILDING: NEW AND REVIEW

Solve each problem. Simplify if possible.

11. $\frac{5}{9} - \frac{2}{9}$ $\frac{3}{9} = \frac{1}{3}$
12. $\frac{6}{7} - \frac{3}{5}$ $\frac{9}{35}$
13. $\frac{4}{7} - \frac{1}{7}$ $\frac{3}{7}$
14. $\frac{6}{10} - \frac{1}{3}$ $\frac{8}{30} = \frac{4}{15}$
15. $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$ $\frac{7}{12}$
16. $\frac{5}{8} - \frac{1}{9}$ $\frac{37}{72}$
17. $\frac{1}{2} - \frac{3}{8} + \frac{2}{3}$ $\frac{19}{24}$
18. $\frac{12}{16} - \frac{3}{4}$ $\frac{0}{16} = 0$
19. $\frac{2}{9} + \frac{5}{36} - \frac{1}{12}$ $\frac{10}{36} = \frac{5}{18}$
20. $\frac{4}{7} - \frac{1}{3}$ $\frac{5}{21}$
21. $\frac{8}{10} - \frac{3}{5}$ $\frac{2}{10} = \frac{1}{5}$
22. $\frac{1}{4} + \frac{1}{3} - \frac{1}{2}$ $\frac{1}{12}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

23. Sarah put $\frac{1}{3}$ cup of cat food in her cat's bowl on Monday. She put $\frac{3}{5}$ cup of cat food in the bowl on Tuesday. How much more cat food did she put in on Tuesday than on Monday?
$\frac{3}{5} - \frac{1}{3} = \frac{9}{15} - \frac{5}{15} = \frac{(9-5)}{15} = \frac{4}{15}$; $\frac{4}{15}$ c
24. Jacob read for $\frac{4}{5}$ of an hour. Sylvie read for $\frac{6}{8}$ of an hour. Who read longer, Jacob or Sylvie?
$\frac{4}{5} = \frac{32}{40}$, $\frac{6}{8} = \frac{30}{40}$; $\frac{4}{5} > \frac{6}{8}$; Jacob read longer.
25. In Charlie's town, $\frac{2}{3}$ foot of snow fell on Tuesday. On Wednesday, $\frac{5}{6}$ foot of snow fell. How much more snow fell on Wednesday than on Tuesday? How much snow fell altogether?
$\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{(5-4)}{6} = \frac{1}{6}$; $\frac{1}{6}$ ft more snow;
$\frac{5}{6} + \frac{4}{6} = \frac{(5+4)}{6} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$; $1\frac{1}{2}$ ft snow fell altogether
26. Ashley had a board that was $\frac{9}{10}$ meter long. She cut off a piece that was $\frac{2}{5}$ meter long to use for an art project. How long is the board now?
$\frac{9}{10} - \frac{2}{5} = \frac{9}{10} - \frac{4}{10} = \frac{(9-4)}{10} = \frac{5}{10} = \frac{1}{2}$; $\frac{1}{2}$ m

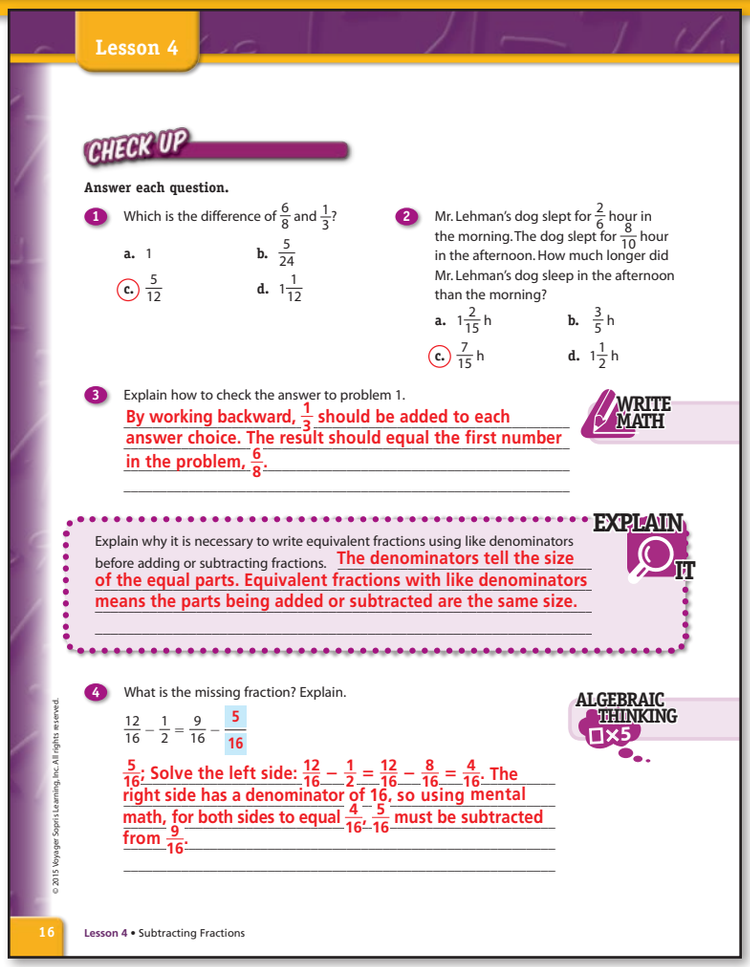
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Level I Module 1 • Foundations 15



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that addition is the opposite operation of subtraction, and addition can be used to check a subtraction problem. Show how to add the answer to the number being subtracted. The result should be equal to the number subtracted from.



Lesson 4

CHECK UP

Answer each question.

1. Which is the difference of $\frac{6}{8}$ and $\frac{1}{3}$?

   a. 1 b. $\frac{5}{24}$

   c. $\frac{5}{12}$ d. $1\frac{1}{12}$

2. Mr. Lehman's dog slept for $\frac{2}{6}$ hour in the morning. The dog slept for $\frac{8}{10}$ hour in the afternoon. How much longer did Mr. Lehman's dog sleep in the afternoon than the morning?

   a. $1\frac{2}{15}$ h b. $\frac{3}{5}$ h

   c. $\frac{7}{15}$ h d. $1\frac{1}{2}$ h

3. Explain how to check the answer to problem 1.

   By working backward, $\frac{1}{3}$ should be added to each answer choice. The result should equal the first number in the problem, $\frac{6}{8}$.

WRITE MATH

EXPLAIN IT

Explain why it is necessary to write equivalent fractions using like denominators before adding or subtracting fractions. The denominators tell the size of the equal parts. Equivalent fractions with like denominators means the parts being added or subtracted are the same size.

4. What is the missing fraction? Explain.

   $\frac{12}{16} - \frac{1}{2} = \frac{9}{16} - \frac{5}{16}$

ALGEBRAIC THINKING □×5

$\frac{5}{16}$; Solve the left side: $\frac{12}{16} - \frac{1}{2} = \frac{12}{16} - \frac{8}{16} = \frac{4}{16}$. The right side has a denominator of 16, so using mental math, for both sides to equal $\frac{4}{16}$, $\frac{5}{16}$ must be subtracted from $\frac{9}{16}$.

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16 Lesson 4 • Subtracting Fractions

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive

Module: Foundations

Activities: Subtract Like Fractions
Subtract Unlike Fractions

Vmath Reteach

Reteach Student Book Module 1 Lesson 4

Reteach Teacher Edition Module 1 Lesson 4

Extra Practice

Student Book page 54

English Language Learners

To help students with the word problems, discuss key phrases that signify addition and those that signify subtraction as the correct operation to solve a problem. Terms or phrases that signify addition are *altogether, how many in all,* and *total*. Terms or phrases that signify subtraction are *how many more* and *how many left*. Have students make a chart for reference as they work through the problems.

Check understanding by presenting a word problem and having students tell if it can be solved using addition or subtraction and explain how they know.

Students with Special Needs

Write various fractions on index cards. Distribute the cards to students. Have students pair up and work together to find the difference of the fractions on their index cards. Guide students to use fraction strips and model each step of the subtraction process.

Lesson 5

Objective

To add mixed numbers

Preskills	Lesson
Adding Fractions	I.1.3

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To add fractions with unlike denominators, write each fraction as an equivalent fraction using the LCD. What is the LCD of $\frac{4}{6}$ and $\frac{2}{3}$? (6) What are the equivalent fractions? ($\frac{4}{6}$ and $\frac{4}{6}$) What is $\frac{4}{6}$ plus $\frac{4}{6}$? ($\frac{8}{6}$) The sum is an improper fraction and should be written as a mixed number. What is $\frac{8}{6}$ written as a mixed number? ($1\frac{2}{6}$) Is $\frac{2}{6}$ written in simplest form? (no) The sum of $\frac{4}{6}$ and $\frac{2}{3}$ written as a mixed number in simplest form is $1\frac{1}{3}$.

MODEL NEW SKILLS

Problem 2

To add mixed numbers, first write the problem vertically. This already has been done in problem 2. In this problem, the fraction parts of the mixed numbers have like denominators. Next, add the fraction parts. The sum of the fractions, $\frac{4}{8}$ and $\frac{5}{8}$, is $\frac{9}{8}$. Now, add the whole number parts. What is 5 plus 3? (8) The sum of the mixed numbers is $8\frac{9}{8}$. The fraction, $\frac{9}{8}$, is an improper fraction and should be changed to a mixed number. The fraction $\frac{9}{8}$ equals $1\frac{1}{8}$. Last, add $1\frac{1}{8}$ to the sum of the whole number parts, 8. Eight plus $1\frac{1}{8}$ equals $9\frac{1}{8}$. The sum of $5\frac{4}{8}$ and $3\frac{5}{8}$ is $9\frac{1}{8}$.

Problem 3

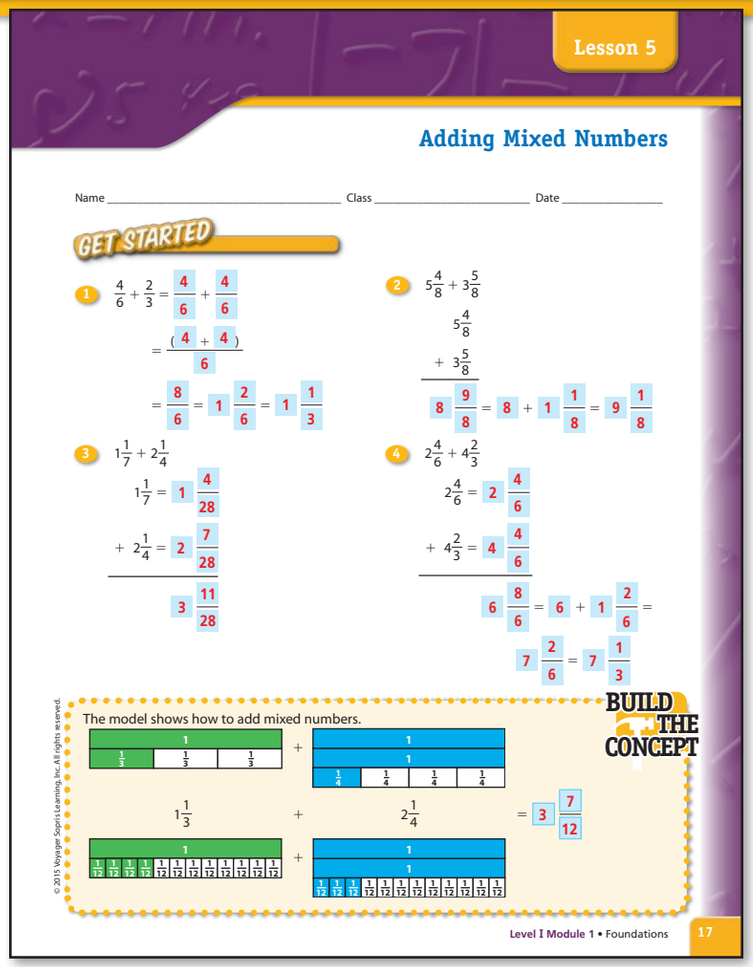
To add mixed numbers, first write the problem vertically. This already has been done in problem 3. Now, make sure the denominators of the fraction parts are the same. In this problem, the fraction parts need to be written as equivalent fractions with the same denominator. Use the LCD as the new denominator. What is the LCD of $\frac{1}{7}$ and $\frac{1}{4}$? (28) What are the equivalent fractions? ($\frac{4}{28}$ and $\frac{7}{28}$)

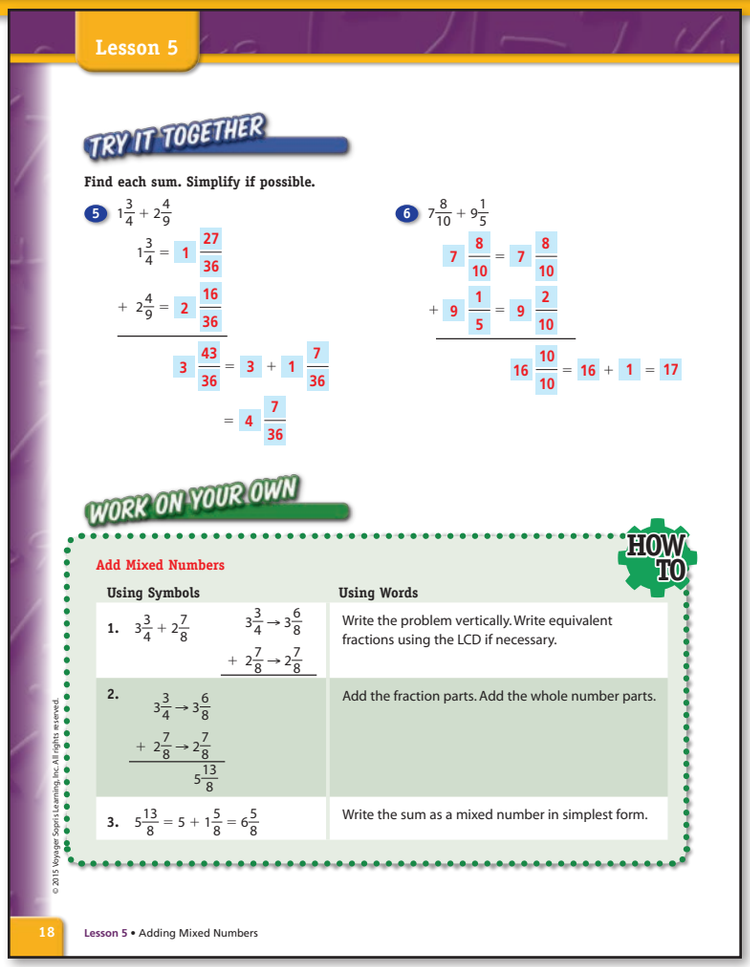
To add the mixed numbers $1\frac{4}{28}$ and $2\frac{7}{28}$, first add the fraction parts, then add the whole number parts, and write the answer as a mixed number in simplest form. What is the sum of the fraction parts $\frac{4}{28}$ and $\frac{7}{28}$? ($\frac{11}{28}$) The sum of the whole number parts, 1 and 2, is 3. So, the sum of $1\frac{1}{7}$ and $2\frac{1}{4}$ equals $3\frac{11}{28}$. Is $\frac{11}{28}$ written in simplest form? (yes) Then the answer is a mixed number written in simplest form, $3\frac{11}{28}$.

Problem 4

First, write the problem vertically. This already has been done in problem 4. The fraction parts of the mixed numbers have unlike denominators, so write the fraction parts as equivalent fractions using the LCD. What is the LCD of $\frac{4}{6}$ and $\frac{2}{3}$? (6) The equivalent fractions are $\frac{4}{6}$ and $\frac{4}{6}$. Now, add the fraction parts. What is the sum of $\frac{4}{6}$ and $\frac{4}{6}$? ($\frac{8}{6}$) Next, add the whole number parts. What is the sum of 2 and 4? (6) The sum of $2\frac{4}{6}$ and $4\frac{4}{6}$ is $6\frac{8}{6}$.

The improper fraction needs to be changed to a mixed number. What is $\frac{8}{6}$ written as a mixed number? ($1\frac{2}{6}$)





Lesson 5

TRY IT TOGETHER

Find each sum. Simplify if possible.

5 $1\frac{3}{4} + 2\frac{4}{9}$

$1\frac{3}{4} = 1\frac{27}{36}$

$+ \; 2\frac{4}{9} = 2\frac{16}{36}$

$3\frac{43}{36} = 3 + 1\frac{7}{36}$

$= 4\frac{7}{36}$

6 $7\frac{8}{10} + 9\frac{1}{5}$

$7\frac{8}{10} = 7\frac{8}{10}$

$+ \; 9\frac{1}{5} = 9\frac{2}{10}$

$16\frac{10}{10} = 16 + 1 = 17$

WORK ON YOUR OWN

HOW TO

Add Mixed Numbers

	Using Symbols	Using Words
1.	$3\frac{3}{4} + 2\frac{7}{8}$ $3\frac{3}{4} \rightarrow 3\frac{6}{8}$ $+ \; 2\frac{7}{8} \rightarrow 2\frac{7}{8}$	Write the problem vertically. Write equivalent fractions using the LCD if necessary.
2.	$3\frac{3}{4} \rightarrow 3\frac{6}{8}$ $+ \; 2\frac{7}{8} \rightarrow 2\frac{7}{8}$ $5\frac{13}{8}$	Add the fraction parts. Add the whole number parts.
3.	$5\frac{13}{8} = 5 + 1\frac{5}{8} = 6\frac{5}{8}$	Write the sum as a mixed number in simplest form.

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What is 6 plus $1\frac{2}{6}$? ($7\frac{2}{6}$) **Is $\frac{2}{6}$ written in simplest form?** (no) **What is $\frac{2}{6}$ written in simplest form?** ($\frac{1}{3}$) **The sum of $2\frac{4}{6}$ and $4\frac{2}{3}$ written in simplest form is $7\frac{1}{3}$.**

BUILD THE CONCEPT

Model how to add mixed numbers.

This problem requires adding two mixed numbers whose denominators are different. A model of the two mixed numbers can be used to find the sum. The mixed number $1\frac{1}{3}$ is modeled using 1 whole strip and one $\frac{1}{3}$-fraction bar shaded, and the mixed number $2\frac{1}{4}$ is modeled using 2 whole strips and one $\frac{1}{4}$-fraction bar shaded. How many total whole fraction strips are there? (3) **The denominators of the fractions must be the same to add. What is the LCD of $\frac{1}{3}$ and $\frac{1}{4}$?** (12)

Now, look at the second model. How many $\frac{1}{12}$-fraction bars equal $\frac{1}{3}$? (4) **How many $\frac{1}{12}$-fraction bars equal $\frac{1}{4}$?** (3) **Now, combine the fractions. How many total $\frac{1}{12}$-fraction bars are shaded?** (7) **The sum is $3\frac{7}{12}$. Is $3\frac{7}{12}$ written in simplest form?** (yes)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

This problem already has been written vertically. How are mixed numbers added when the fraction parts of the mixed numbers have unlike denominators? (Write equivalent fractions using the LCD; add the fraction parts; add the whole number parts; write the answer as a mixed number in simplest form.) **What is the LCD of $\frac{3}{4}$ and $\frac{4}{9}$?** (36) **What fraction with a denominator of 36 is equivalent to $\frac{3}{4}$?** ($\frac{27}{36}$) **What fraction is equivalent to $\frac{4}{9}$?** ($\frac{16}{36}$) **What is the sum of the fraction parts?** ($\frac{43}{36}$) **What is the sum of the whole number parts?** (3) **Next, change the improper fraction to a mixed number. What is $\frac{43}{36}$ written as a mixed number?** ($1\frac{7}{36}$) **What is the last step?** (Add $1\frac{7}{36}$ to the sum of the whole numbers.) **What is $1\frac{7}{36}$ plus 3?** ($4\frac{7}{36}$) **Is $4\frac{7}{36}$ written in simplest form?** (yes) **What is the sum of $1\frac{3}{4}$ and $2\frac{4}{9}$?** ($4\frac{7}{36}$)

Problem 6

What should be done first? (Write the problem vertically.) **What is the LCD of $\frac{8}{10}$ and $\frac{1}{5}$?** (10) **What is the equivalent fraction for $\frac{8}{10}$?** ($\frac{8}{10}$) **Why?** (It already has a denominator of 10.) **What is the equivalent fraction for $\frac{1}{5}$?** ($\frac{2}{10}$) **What is the sum of the equivalent fraction parts?** ($\frac{10}{10}$) **What is the sum of the whole number parts?** (16) **What does $\frac{10}{10}$ equal?** (1) **What is 16 plus 1?** (17) **What is the sum of $7\frac{8}{10}$ and $9\frac{1}{5}$?** (17)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 7–22

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 20 Reminder

Students must find the sum of the mixed numbers, then compare the sum with $3\frac{1}{2}$ to answer the question.

CHECK UP

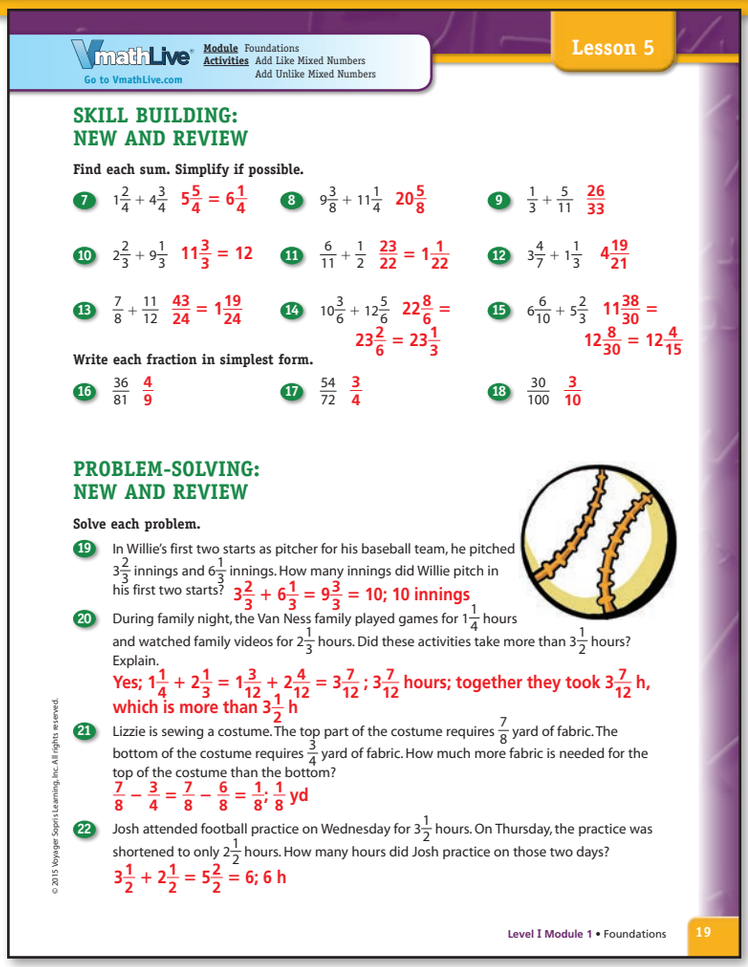
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b or 2d: The student did not add the whole number part to the sum of the whole numbers when simplifying the improper fraction. Use Additional Resources in I.1.5 to reteach how to add mixed numbers.

If student answered 1c or 2c: The student did not change the numerator when finding equivalent fractions. Review how to find equivalent fractions.



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Module Activities Foundations
Add Like Mixed Numbers
Add Unlike Mixed Numbers

Lesson 5

SKILL BUILDING: NEW AND REVIEW

Find each sum. Simplify if possible.

7. $1\frac{2}{4} + 4\frac{3}{4}$ $5\frac{5}{4} = 6\frac{1}{4}$
8. $9\frac{3}{8} + 11\frac{1}{4}$ $20\frac{5}{8}$
9. $\frac{1}{3} + \frac{5}{11}$ $\frac{26}{33}$
10. $2\frac{2}{3} + 9\frac{1}{3}$ $11\frac{3}{3} = 12$
11. $\frac{6}{11} + \frac{1}{2}$ $\frac{23}{22} = 1\frac{1}{22}$
12. $3\frac{4}{7} + 1\frac{1}{3}$ $4\frac{19}{21}$
13. $\frac{7}{8} + \frac{11}{12}$ $\frac{43}{24} = 1\frac{19}{24}$
14. $10\frac{3}{6} + 12\frac{5}{6}$ $22\frac{8}{6} = 23\frac{2}{6} = 23\frac{1}{3}$
15. $6\frac{6}{10} + 5\frac{2}{3}$ $11\frac{38}{30} = 12\frac{8}{30} = 12\frac{4}{15}$

Write each fraction in simplest form.

16. $\frac{36}{81}$ $\frac{4}{9}$
17. $\frac{54}{72}$ $\frac{3}{4}$
18. $\frac{30}{100}$ $\frac{3}{10}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

19. In Willie's first two starts as pitcher for his baseball team, he pitched $3\frac{2}{3}$ innings and $6\frac{1}{3}$ innings. How many innings did Willie pitch in his first two starts? $3\frac{2}{3} + 6\frac{1}{3} = 9\frac{3}{3} = 10$; 10 innings
20. During family night, the Van Ness family played games for $1\frac{1}{4}$ hours and watched family videos for $2\frac{1}{3}$ hours. Did these activities take more than $3\frac{1}{2}$ hours? Explain. Yes; $1\frac{1}{4} + 2\frac{1}{3} = 1\frac{3}{12} + 2\frac{4}{12} = 3\frac{7}{12}$; $3\frac{7}{12}$ hours; together they took $3\frac{7}{12}$ h, which is more than $3\frac{1}{2}$ h
21. Lizzie is sewing a costume. The top part of the costume requires $\frac{7}{8}$ yard of fabric. The bottom of the costume requires $\frac{3}{4}$ yard of fabric. How much more fabric is needed for the top of the costume than the bottom? $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}$; $\frac{1}{8}$ yd
22. Josh attended football practice on Wednesday for $3\frac{1}{2}$ hours. On Thursday, the practice was shortened to only $2\frac{1}{2}$ hours. How many hours did Josh practice on those two days? $3\frac{1}{2} + 2\frac{1}{2} = 5\frac{2}{2} = 6$; 6 h

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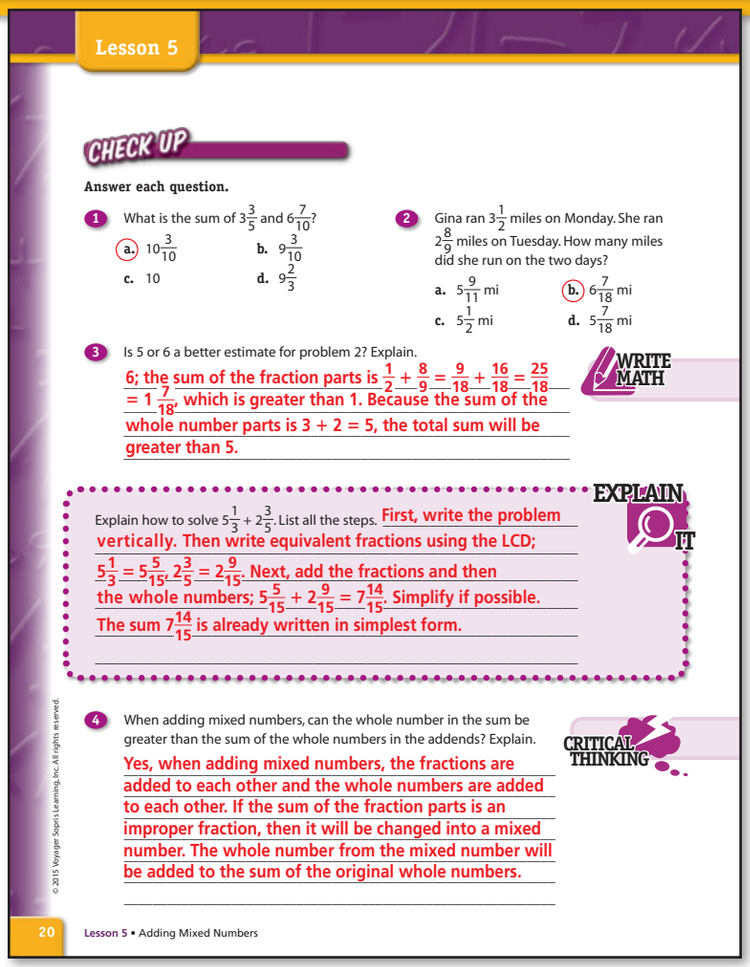
Level I Module 1 • Foundations 19

If student answered 1d or 2a: When adding the fraction parts of the mixed numbers, the student added the numerators and denominators instead of finding equivalent fractions using the LCD. Use Additional Resources in I.1.3 to reteach how to add fractions with unlike denominators.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that because the sum of the fraction parts is greater than 1, the total sum will be greater than the sum of just the whole number parts.



Lesson 5

CHECK UP

Answer each question.

1. What is the sum of $3\frac{3}{5}$ and $6\frac{7}{10}$?
   - (a.) $10\frac{3}{10}$
   - b. $9\frac{3}{10}$
   - c. 10
   - d. $9\frac{2}{3}$

2. Gina ran $3\frac{1}{2}$ miles on Monday. She ran $2\frac{8}{9}$ miles on Tuesday. How many miles did she run on the two days?
   - a. $5\frac{9}{11}$ mi
   - (b.) $6\frac{7}{18}$ mi
   - c. $5\frac{1}{2}$ mi
   - d. $5\frac{7}{18}$ mi

3. Is 5 or 6 a better estimate for problem 2? Explain.

WRITE MATH

6; the sum of the fraction parts is $\frac{1}{2} + \frac{8}{9} = \frac{9}{18} + \frac{16}{18} = \frac{25}{18} = 1\frac{7}{18}$, which is greater than 1. Because the sum of the whole number parts is 3 + 2 = 5, the total sum will be greater than 5.

EXPLAIN IT

Explain how to solve $5\frac{1}{3} + 2\frac{3}{5}$. List all the steps. First, write the problem vertically. Then write equivalent fractions using the LCD; $5\frac{1}{3} = 5\frac{5}{15}$, $2\frac{3}{5} = 2\frac{9}{15}$. Next, add the fractions and then the whole numbers; $5\frac{5}{15} + 2\frac{9}{15} = 7\frac{14}{15}$. Simplify if possible. The sum $7\frac{14}{15}$ is already written in simplest form.

4. When adding mixed numbers, can the whole number in the sum be greater than the sum of the whole numbers in the addends? Explain.

CRITICAL THINKING

Yes, when adding mixed numbers, the fractions are added to each other and the whole numbers are added to each other. If the sum of the fraction parts is an improper fraction, then it will be changed into a mixed number. The whole number from the mixed number will be added to the sum of the original whole numbers.

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EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

Technology

Using an online search engine, have students or partners research careers that would use adding mixed numbers. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Foundations
Activities: Add Like Mixed Numbers
Add Unlike Mixed Numbers

Vmath Reteach
Reteach Student Book Module 1 Lesson 5
Reteach Teacher Edition Module 1 Lesson 5

Extra Practice
Student Book page 55

English Language Learners

Use the VmathLive Animated Glossary to review *mixed number, improper fraction, whole number, fraction,* and *sum*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Work through an addition problem involving two mixed numbers with unlike denominators. Use math terms to describe each step of the process. Label the whole number and the fraction part of each mixed number. Label any improper fractions or fractions in simplest form. Then have students complete a problem on their own and explain the steps.

Students with Special Needs

Write a mixed number addition problem on the board that requires students to regroup a fraction sum because it is an improper fraction. Emphasize that it is important to remember to add the whole number from the fraction sum after regrouping to the whole number sum. Write another problem on the board, such as $5\frac{2}{3} + 2\frac{1}{2}$. Now, write two possible answers, $8\frac{1}{6}$ (the correct sum) and $7\frac{1}{6}$ (the whole number of the fractional sum not added to the whole number sum), and have students choose and explain which answer is correct.

Lesson 6

Objective

To subtract a mixed number

Preskills	Lesson
Subtracting Fractions	I.1.4

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To subtract $\frac{2}{8}$ from $\frac{5}{6}$, first find equivalent fractions using the LCD. What is the LCD of $\frac{5}{6}$ and $\frac{2}{8}$? (24) What are the equivalent fractions? ($\frac{20}{24}$ and $\frac{6}{24}$) What is $\frac{20}{24}$ minus $\frac{6}{24}$? ($\frac{14}{24}$) Is $\frac{14}{24}$ written in simplest form? (no) What is $\frac{5}{6}$ minus $\frac{2}{8}$ written in simplest form? ($\frac{7}{12}$)

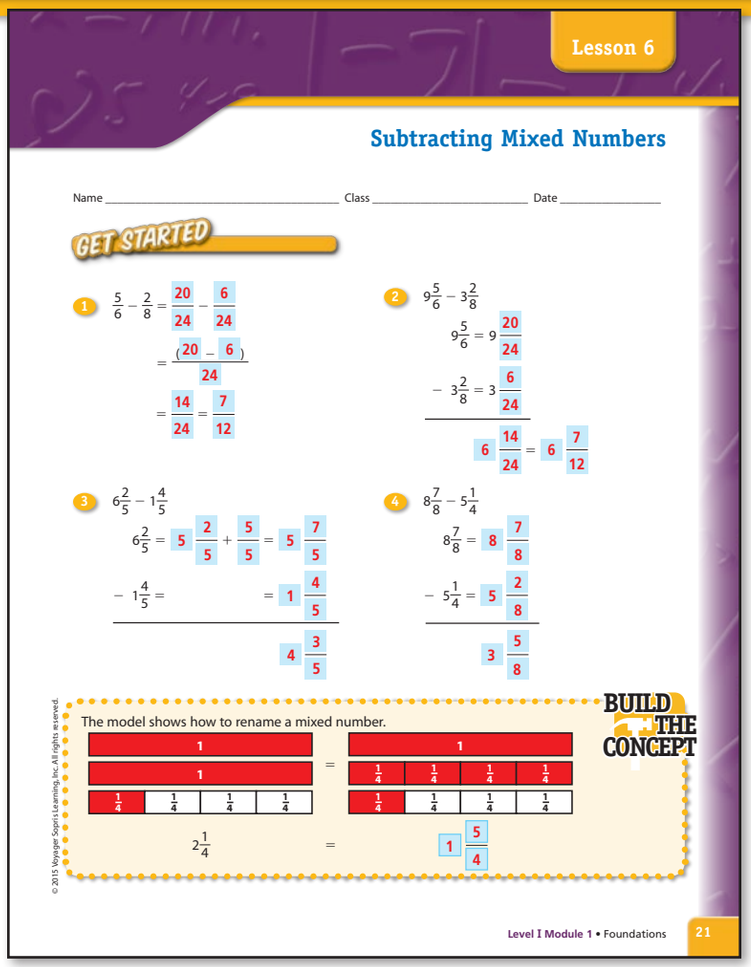
MODEL NEW SKILLS

Problem 2

To subtract a mixed number from another mixed number, first write the problem vertically. Next, subtract the fraction part, then subtract the whole number part, and finally simplify the answer if possible. This problem already has been written vertically, so to find the difference of $9\frac{5}{6}$ and $3\frac{2}{8}$, find the equivalent fractions for $\frac{5}{6}$ and $\frac{2}{8}$. These were found in problem 1. What is $\frac{20}{24}$ minus $\frac{6}{24}$? ($\frac{14}{24}$) Now, subtract the whole number part. What is 9 minus 3? (6) Is $\frac{14}{24}$ written in simplest form? (no) The difference of $9\frac{5}{6}$ and $3\frac{2}{8}$ written in simplest form is $6\frac{7}{12}$.

Problem 3

When a problem is written vertically, sometimes the fraction part in the bottom mixed number is greater than the fraction part in the top mixed number. When this problem is written vertically, what is the top fraction part? ($\frac{2}{5}$) What is the bottom fraction part? ($\frac{4}{5}$) Because $\frac{2}{5}$ is less than $\frac{4}{5}$, $\frac{4}{5}$ cannot be subtracted from $\frac{2}{5}$. The top mixed number must be renamed before the difference can be found.

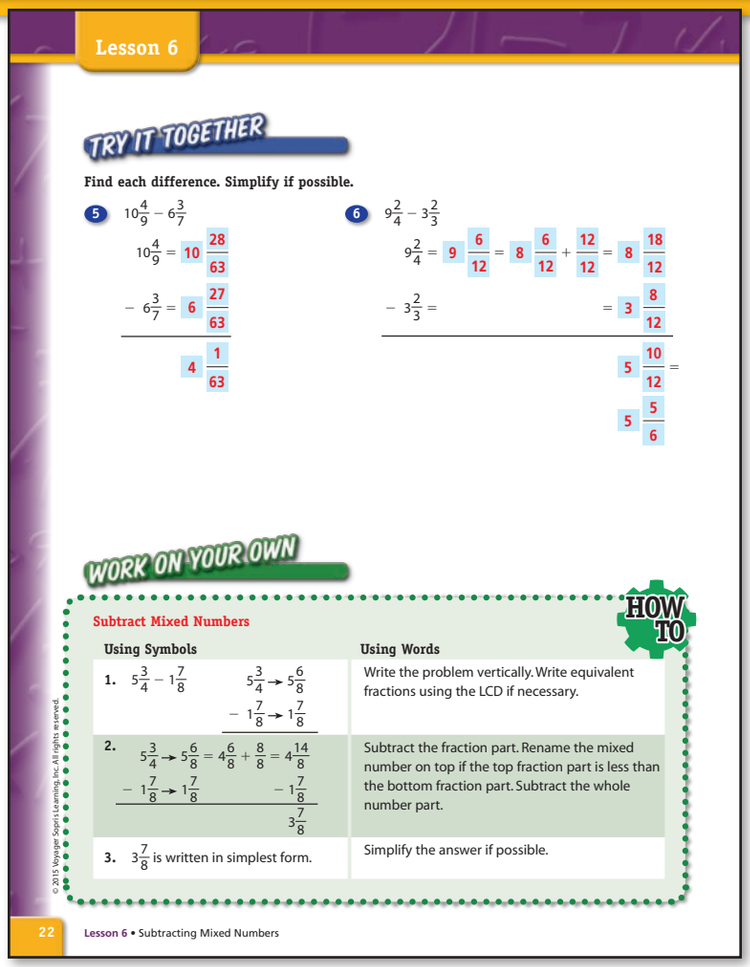


To rename $6\frac{2}{5}$, first subtract 1 from the whole number part. What is 6 minus 1? (5) Then add 1 in the form of a fraction to the $\frac{2}{5}$. The fraction form of 1 should have the same numerator and denominator as the denominator of $\frac{2}{5}$. What is the denominator of $\frac{2}{5}$? (5) So, the fraction $\frac{5}{5}$ should be added to $\frac{2}{5}$. What is $\frac{5}{5}$ plus $\frac{2}{5}$? ($\frac{7}{5}$) The mixed number $6\frac{2}{5}$ is renamed as $5\frac{7}{5}$.

Now that the top mixed number has been renamed, the top fraction part is greater than the bottom fraction part and the mixed number can be subtracted. First, subtract the fraction part. What is $\frac{7}{5}$ minus $\frac{4}{5}$? ($\frac{3}{5}$) Now, subtract the whole number part. What is 5 minus 1? (4) Is $4\frac{3}{5}$ written in simplest form? (yes) So, the difference of $6\frac{2}{5}$ and $1\frac{4}{5}$ written in simplest form is $4\frac{3}{5}$.

Problem 4

What is the LCD of $\frac{7}{8}$ and $\frac{1}{4}$? (8) What is the equivalent fraction for $\frac{1}{4}$ using a denominator of 8? ($\frac{2}{8}$) Does the top fraction part have to be renamed before subtracting? (no) Why not? (The top fraction part, $\frac{7}{8}$, is greater than the bottom fraction part, $\frac{2}{8}$.) What is $\frac{7}{8}$ minus $\frac{2}{8}$? ($\frac{5}{8}$) What is 8 minus 5? (3) What is the difference of $8\frac{7}{8}$ and $5\frac{1}{4}$ written in simplest form? ($3\frac{5}{8}$)



Lesson 6

TRY IT TOGETHER

Find each difference. Simplify if possible.

5 $10\frac{4}{9} - 6\frac{3}{7}$

$10\frac{4}{9} = 10\frac{28}{63}$

$-\ 6\frac{3}{7} = 6\frac{27}{63}$

$4\frac{1}{63}$

6 $9\frac{2}{4} - 3\frac{2}{3}$

$9\frac{2}{4} = 9\frac{6}{12} = 8\frac{6}{12} + \frac{12}{12} = 8\frac{18}{12}$

$-\ 3\frac{2}{3} = 3\frac{8}{12}$

$5\frac{10}{12} = 5\frac{5}{6}$

WORK ON YOUR OWN

Subtract Mixed Numbers

HOW TO

Using Symbols	Using Words
1. $5\frac{3}{4} - 1\frac{7}{8}$ $5\frac{3}{4} \rightarrow 5\frac{6}{8}$ $-\ 1\frac{7}{8} \rightarrow 1\frac{7}{8}$	Write the problem vertically. Write equivalent fractions using the LCD if necessary.
2. $5\frac{3}{4} \rightarrow 5\frac{6}{8} = 4\frac{6}{8} + \frac{8}{8} = 4\frac{14}{8}$ $-\ 1\frac{7}{8} \rightarrow 1\frac{7}{8}$ $\quad -\ 1\frac{7}{8}$ $3\frac{7}{8}$	Subtract the fraction part. Rename the mixed number on top if the top fraction part is less than the bottom fraction part. Subtract the whole number part.
3. $3\frac{7}{8}$ is written in simplest form.	Simplify the answer if possible.

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BUILD THE CONCEPT

Model how to rename a mixed number.

When subtracting mixed numbers, the fraction part of the number being subtracted may be greater than the fraction part of the other mixed number. In these cases, one of the whole numbers can be regrouped to get more fraction parts. The number 1 can be written as a fraction with any denominator. If the denominator and the numerator are the same, the fraction equals 1.

Look at the mixed number $2\frac{1}{4}$. One whole can be traded for fourths. How many $\frac{1}{4}$-fraction bars are in 1 whole? (4) **Add $\frac{4}{4}$ and $\frac{1}{4}$ to find the new fraction part. What is $\frac{4}{4}$ plus $\frac{1}{4}$?** ($\frac{5}{4}$) **The mixed number $2\frac{1}{4}$ is the same as $1\frac{5}{4}$.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

This problem already has been written vertically. To subtract $10\frac{4}{9}$ and $6\frac{3}{7}$, do equivalent fractions using the LCD need to be written? (yes) **Why?** (The fraction parts have unlike denominators.) **What are the equivalent fractions?** ($\frac{28}{63}$ and $\frac{27}{63}$) **Does the top mixed number need to be renamed?** (no) **Why not?** (The top fraction part is greater than the bottom fraction part.)

What is the next step in subtracting the mixed number? (Subtract the fraction part.) **What is $\frac{28}{63}$ minus $\frac{27}{63}$?** ($\frac{1}{63}$) **What is the next step?** (Subtract the whole number part.) **What is 10 minus 6?** (4) **What is $10\frac{4}{9}$ minus $6\frac{3}{7}$?** ($4\frac{1}{63}$) **Is the answer written in simplest form?** (yes)

Problem 6

This problem already has been written vertically. What are the equivalent fractions for $\frac{2}{4}$ and $\frac{2}{3}$ using the LCD? ($\frac{6}{12}$ and $\frac{8}{12}$) **Does the top mixed number need to be renamed?** (yes) **Why?** (The fraction $\frac{6}{12}$ is less than $\frac{8}{12}$.) **How is the mixed number renamed?** (Subtract 1 from the whole number and add 1 in fraction form to the fraction part.)

What is 9 minus 1? (8) **What fraction that equals 1 should be added to $\frac{6}{12}$?** ($\frac{12}{12}$) **What is the renamed mixed number?** ($8\frac{18}{12}$) **How is the subtraction problem completed?** (Subtract the fraction parts; subtract the whole number part; simplify the answer if possible.)

What is $\frac{18}{12}$ minus $\frac{8}{12}$? ($\frac{10}{12}$) **What is 8 minus 3?** (5) **What is $9\frac{2}{4}$ minus $3\frac{2}{3}$?** ($5\frac{10}{12}$) **Is $5\frac{10}{12}$ written in simplest form?** (no) **What is the difference of $9\frac{2}{4}$ and $3\frac{2}{3}$ written in simplest form?** ($5\frac{5}{6}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using Reasonableness and Estimation** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving estimation. The instruction is immediately followed by application of the strategy in problem 19.

The problem states how much flour Mrs. Edna had, how much she used, and how much she says she has left. Students are asked whether Mrs. Edna's result is reasonable.

First, students should round the numbers used in the problem. Six and one-eighth is about equal to 6. Two and three-fourths is about equal to 3. Subtract the rounded numbers: 6 minus 3 equals 3. Compare this estimate with Mrs. Edna's result.

Mrs. Edna estimated she has $4\frac{7}{8}$ cups of flour left. Is the estimate of 3 close to $4\frac{7}{8}$? Because the estimate is not close, Mrs. Edna's answer is not reasonable.

Ask students to check the answer another way.

Problems 7–22

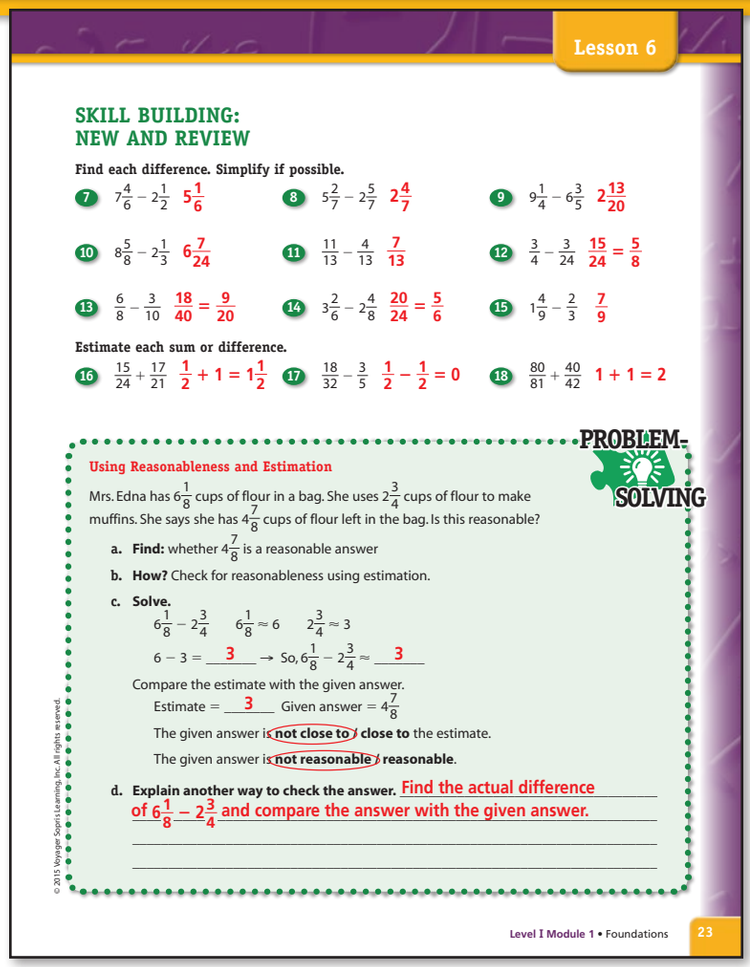
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 8, 9, 14, and 15 Reminder

Students need to rename the mixed number before they subtract.

Problem 21 Reminder

Students need to find the sum of the mixed numbers in the problem.



Lesson 6

SKILL BUILDING: NEW AND REVIEW

Find each difference. Simplify if possible.

7. $7\frac{4}{6} - 2\frac{1}{2}$ $5\frac{1}{6}$
8. $5\frac{2}{7} - 2\frac{5}{7}$ $2\frac{4}{7}$
9. $9\frac{1}{4} - 6\frac{3}{5}$ $2\frac{13}{20}$
10. $8\frac{5}{8} - 2\frac{1}{3}$ $6\frac{7}{24}$
11. $\frac{11}{13} - \frac{4}{13}$ $\frac{7}{13}$
12. $\frac{3}{4} - \frac{3}{24}$ $\frac{15}{24} = \frac{5}{8}$
13. $\frac{6}{8} - \frac{3}{10}$ $\frac{18}{40} = \frac{9}{20}$
14. $3\frac{2}{6} - 2\frac{4}{8}$ $\frac{20}{24} = \frac{5}{6}$
15. $1\frac{4}{9} - \frac{2}{3}$ $\frac{7}{9}$

Estimate each sum or difference.

16. $\frac{15}{24} + \frac{17}{21}$ $\frac{1}{2} + 1 = 1\frac{1}{2}$
17. $\frac{18}{32} - \frac{3}{5}$ $\frac{1}{2} - \frac{1}{2} = 0$
18. $\frac{80}{81} + \frac{40}{42}$ $1 + 1 = 2$

PROBLEM-SOLVING

Using Reasonableness and Estimation

Mrs. Edna has $6\frac{1}{8}$ cups of flour in a bag. She uses $2\frac{3}{4}$ cups of flour to make muffins. She says she has $4\frac{7}{8}$ cups of flour left in the bag. Is this reasonable?

a. **Find:** whether $4\frac{7}{8}$ is a reasonable answer

b. **How?** Check for reasonableness using estimation.

c. **Solve.**

$6\frac{1}{8} - 2\frac{3}{4}$ $6\frac{1}{8} \approx 6$ $2\frac{3}{4} \approx 3$

$6 - 3 =$ 3 → So, $6\frac{1}{8} - 2\frac{3}{4} \approx$ 3

Compare the estimate with the given answer.

Estimate = 3 Given answer = $4\frac{7}{8}$

The given answer is ~~not close to~~ / **close to** the estimate.

The given answer is ~~not reasonable~~ / **reasonable**.

d. **Explain another way to check the answer.** Find the actual difference of $6\frac{1}{8} - 2\frac{3}{4}$ and compare the answer with the given answer.

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Problem 22 Reminder

Make sure that students realize they have to subtract to find the brother's height, then compare this result with the sister's height.

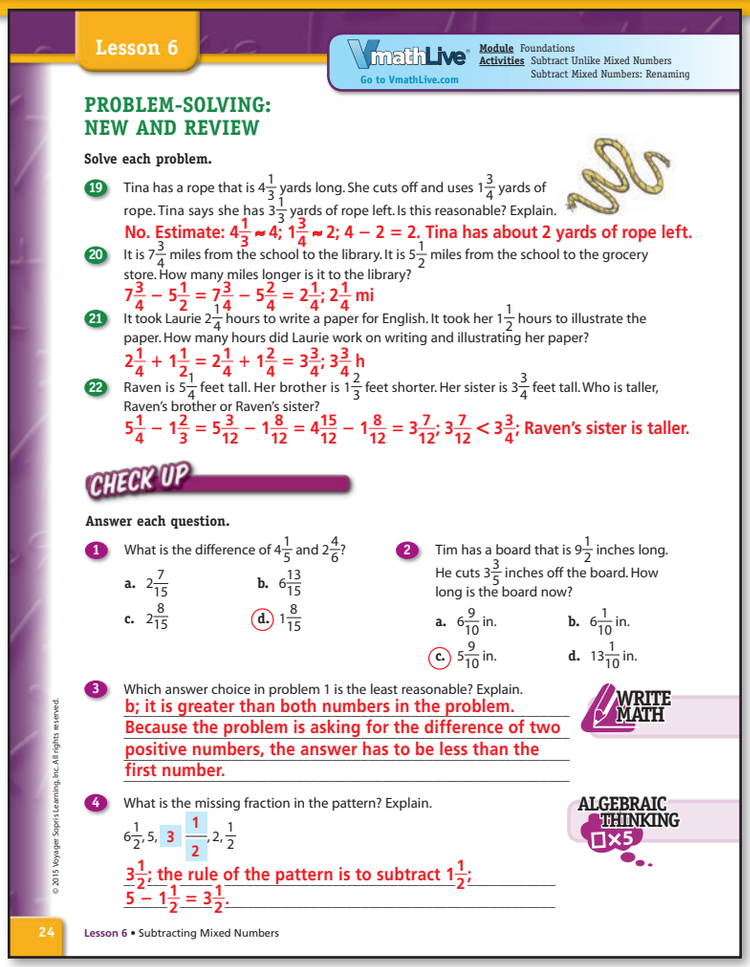
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b or 2d: The student added instead of subtracting. Remind the student that the word *difference* in problem 1 indicates subtraction and the word *cuts* in problem 2 means the board is getting shorter.



Lesson 6

VmathLive Go to VmathLive.com
Module Activities Foundations
Subtract Unlike Mixed Numbers
Subtract Mixed Numbers: Renaming

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

19 Tina has a rope that is $4\frac{1}{3}$ yards long. She cuts off and uses $1\frac{3}{4}$ yards of rope. Tina says she has $3\frac{1}{3}$ yards of rope left. Is this reasonable? Explain.

No. Estimate: $4\frac{1}{3} \approx 4$; $1\frac{3}{4} \approx 2$; $4 - 2 = 2$. Tina has about 2 yards of rope left.

20 It is $7\frac{3}{4}$ miles from the school to the library. It is $5\frac{1}{2}$ miles from the school to the grocery store. How many miles longer is it to the library?

$7\frac{3}{4} - 5\frac{1}{2} = 7\frac{3}{4} - 5\frac{2}{4} = 2\frac{1}{4}$; $2\frac{1}{4}$ mi

21 It took Laurie $2\frac{1}{4}$ hours to write a paper for English. It took her $1\frac{1}{2}$ hours to illustrate the paper. How many hours did Laurie work on writing and illustrating her paper?

$2\frac{1}{4} + 1\frac{1}{2} = 2\frac{1}{4} + 1\frac{2}{4} = 3\frac{3}{4}$; $3\frac{3}{4}$ h

22 Raven is $5\frac{1}{4}$ feet tall. Her brother is $1\frac{2}{3}$ feet shorter. Her sister is $3\frac{3}{4}$ feet tall. Who is taller, Raven's brother or Raven's sister?

$5\frac{1}{4} - 1\frac{2}{3} = 5\frac{3}{12} - 1\frac{8}{12} = 4\frac{15}{12} - 1\frac{8}{12} = 3\frac{7}{12}$; $3\frac{7}{12} < 3\frac{3}{4}$; Raven's sister is taller.

CHECK UP

Answer each question.

1 What is the difference of $4\frac{1}{5}$ and $2\frac{4}{6}$?

a. $2\frac{7}{15}$ b. $6\frac{13}{15}$ c. $2\frac{8}{15}$ (d.) $1\frac{8}{15}$

2 Tim has a board that is $9\frac{1}{2}$ inches long. He cuts $3\frac{3}{5}$ inches off the board. How long is the board now?

a. $6\frac{9}{10}$ in. b. $6\frac{1}{10}$ in. (c.) $5\frac{9}{10}$ in. d. $13\frac{1}{10}$ in.

3 Which answer choice in problem 1 is the least reasonable? Explain.

WRITE MATH

b; it is greater than both numbers in the problem. Because the problem is asking for the difference of two positive numbers, the answer has to be less than the first number.

4 What is the missing fraction in the pattern? Explain.

ALGEBRAIC THINKING □×5

$6\frac{1}{2}, 5, 3\frac{1}{2}, 2, 1\frac{1}{2}$

$3\frac{1}{2}$; the rule of the pattern is to subtract $1\frac{1}{2}$; $5 - 1\frac{1}{2} = 3\frac{1}{2}$.

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If student answered 1c or 2a: The student did not subtract 1 from the whole number when renaming the top mixed number. Use Additional Resources in I.1.6 to reteach how to subtract a mixed number when the top mixed number needs to be renamed.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that when subtracting a positive number, the difference is always less than the first number.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive

Module: Foundations

Activities: Subtract Unlike Mixed Numbers
Subtract Mixed Numbers: Renaming

Vmath Reteach

Reteach Student Book Module 1 Lesson 6

Reteach Teacher Edition Module 1 Lesson 6

Extra Practice

Student Book page 55

English Language Learners

Define the word *rename*. Show that the same number can have different names. To help students understand this, use fraction strips to model $3\frac{1}{4}$, $2\frac{5}{4}$, and $1\frac{9}{4}$. This activity illustrates that each mixed number names the same amount.

Students with Special Needs

Have students use fraction strips to model one whole in different ways. Have students place a whole fraction strip on the table, then model one whole using halves, thirds, fourths, fifths, sixths, and so on. Have students practice renaming mixed numbers by trading a whole for fractional parts.

Make a set of cards with mixed numbers written on them. Shuffle the cards and have a student pick two cards. Use the mixed numbers on the cards to write a subtraction expression, subtracting the lesser mixed number from the greater mixed number. Have students tell if they need to rename before subtracting and explain why.

Gizmos Guided Discovery

Lesson 7

Objective

To use an area model to multiply fractions

WITH CONCEPTUAL DEVELOPMENT TECHNOLOGY

Materials

- Gizmo: Multiplying Fractions

Gizmos Log In Instructions

- Log in to vmath.voyagersopris.com using your Username and Password provided in VPORT®.
- Select the Gizmos poster.
- Click on the Multiplying Fractions Gizmo link.

Lesson Notes

Before beginning the lesson, be sure students have their Student Books and are ready to work at the computers.

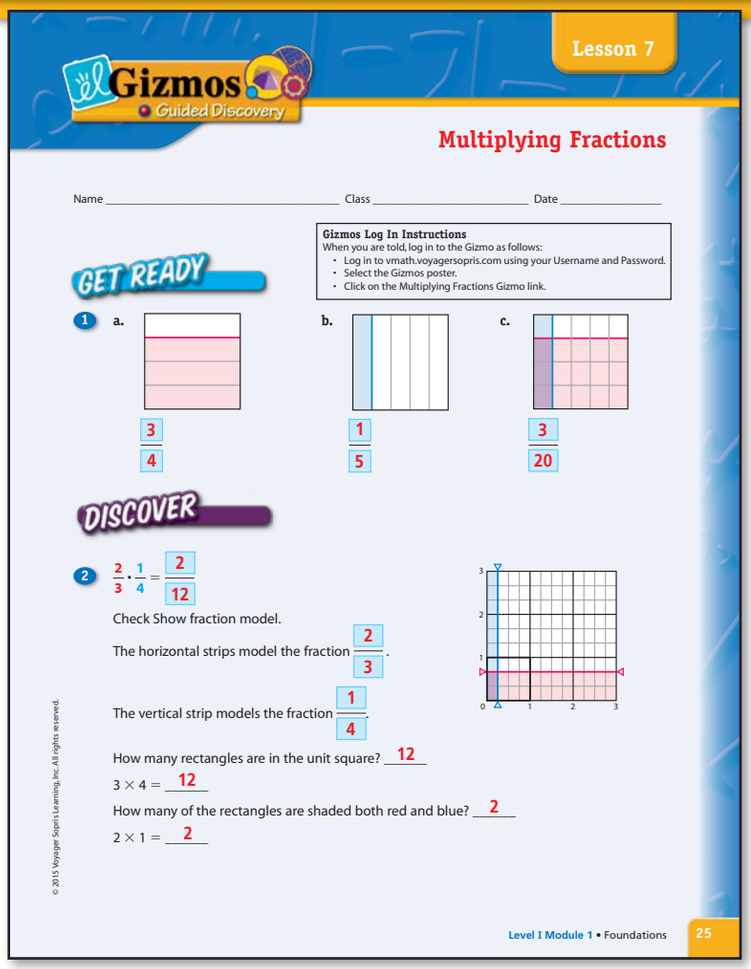
Complete problem 1 before students log in to the Multiplying Fractions Gizmo.

If students have additional time, have them answer Assessment Question 1 in the Gizmo. They can click on the Check Your Answers button to see how well they did on the assessment.

GET READY

Problem 1

Look at the diagram in problem 1a. How many horizontal strips are shaded? (3) **Because there are 4 horizontal strips and 3 are shaded, the diagram models the fraction $\frac{3}{4}$. Look at the diagram in problem 1b. What fraction is modeled by the diagram?** ($\frac{1}{5}$) **How many rectangles are in the diagram in problem 1c?** (20) **How many rectangles are shaded both red and blue?** (3) **The diagram models the fraction $\frac{3}{20}$.**



Gizmos Guided Discovery — Lesson 7

Multiplying Fractions

Name _____ Class _____ Date _____

Gizmos Log In Instructions
When you are told, log in to the Gizmo as follows:
- Log in to vmath.voyagersopris.com using your Username and Password.
- Select the Gizmos poster.
- Click on the Multiplying Fractions Gizmo link.

GET READY

1 a. 3/4 b. 1/5 c. 3/20

DISCOVER

2 $\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12}$

Check Show fraction model.

The horizontal strips model the fraction 2/3.

The vertical strip models the fraction 1/4.

How many rectangles are in the unit square? 12

$3 \times 4 =$ 12

How many of the rectangles are shaded both red and blue? 2

$2 \times 1 =$ 2

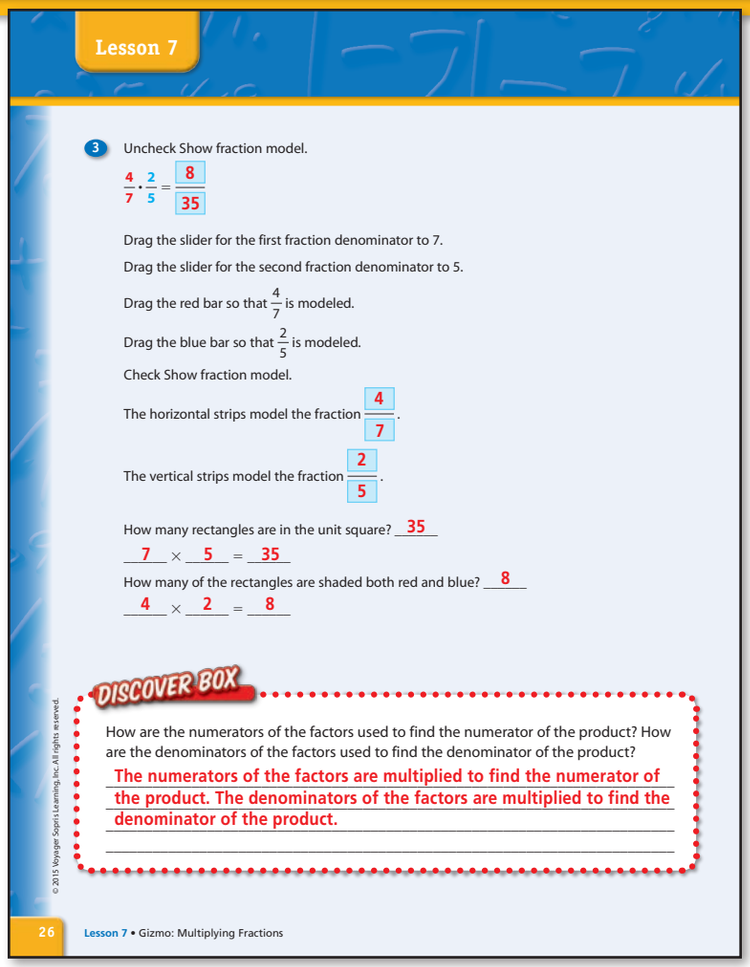
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DISCOVER

Problem 2

Have students log in to the Multiplying Fractions Gizmo using the instructions in the box. **What product is shown in the Gizmo for problem 2?** ($\frac{2}{3} \cdot \frac{1}{4}$) **Notice that a solid dot is used to indicate multiplication rather than the ×. A model can be used to find this product. Check Show fraction model. In the model, a unit square is shown in the lower left corner. The unit square is divided into rectangles by the horizontal strips and vertical strips. How many horizontal strips are in the unit square?** (3) **How many horizontal strips are shaded?** (2) **The horizontal strips model the fraction $\frac{2}{3}$. How many vertical strips are in the unit square?** (4) **How many vertical strips are shaded?** (1) **What fraction is modeled by the vertical strip?** ($\frac{1}{4}$) **The product is the portion of the model that is shaded both red and blue. How many rectangles are in the unit square?** (12) **Three horizontal strips times 4 vertical strips equals 12 rectangles. How many rectangles are shaded both red and blue?** (2) **Two red horizontal strips times 1 blue vertical strip is equal to 2 shaded rectangles. The product of $\frac{2}{3}$ and $\frac{1}{4}$ is $\frac{2}{12}$.**



Lesson 7

3 Uncheck Show fraction model.

$\frac{4}{7} \cdot \frac{2}{5} = \frac{8}{35}$

Drag the slider for the first fraction denominator to 7.

Drag the slider for the second fraction denominator to 5.

Drag the red bar so that $\frac{4}{7}$ is modeled.

Drag the blue bar so that $\frac{2}{5}$ is modeled.

Check Show fraction model.

The horizontal strips model the fraction $\frac{4}{7}$.

The vertical strips model the fraction $\frac{2}{5}$.

How many rectangles are in the unit square? 35

7 × 5 = 35

How many of the rectangles are shaded both red and blue? 8

4 × 2 = 8

DISCOVER BOX

How are the numerators of the factors used to find the numerator of the product? How are the denominators of the factors used to find the denominator of the product?

The numerators of the factors are multiplied to find the numerator of the product. The denominators of the factors are multiplied to find the denominator of the product.

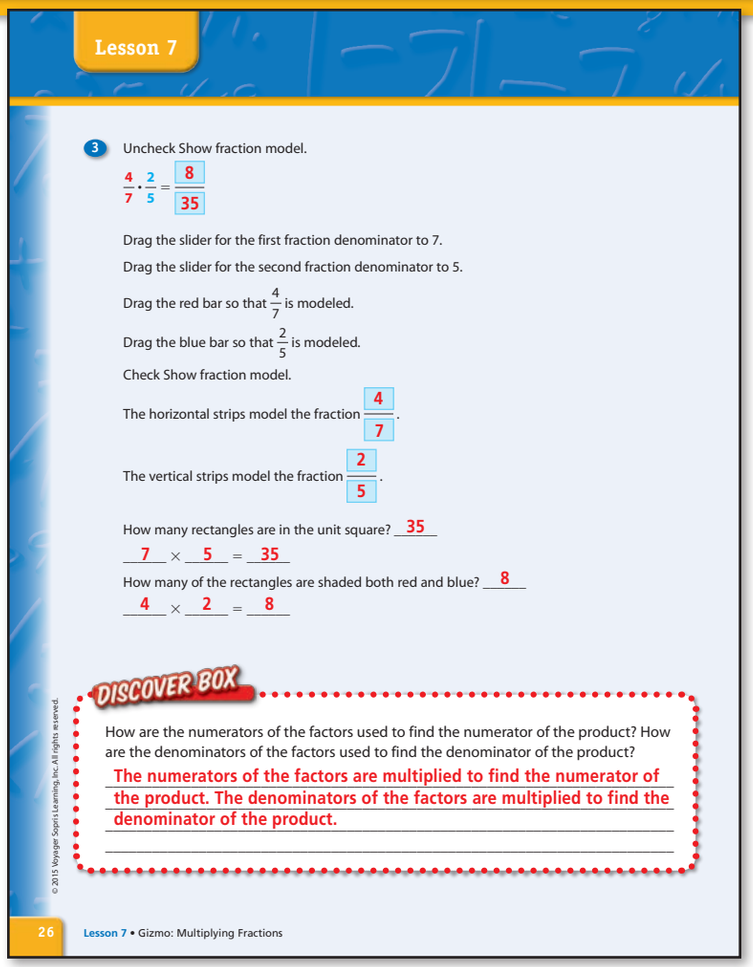
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Problem 3

Uncheck Show fraction model before starting problem 3. To model the product of $\frac{4}{7}$ and $\frac{2}{5}$, first set the denominators. Drag the first fraction denominator slider to 7 and drag the second fraction denominator slider to 5. Note: Students could also enter the denominators in the boxes beside the sliders. **Now, model each fraction in the product. Drag the red horizontal bar on the model until the fraction at the end of the bar is $\frac{4}{7}$. Drag the blue vertical bar on the model until the fraction at the top of the bar is $\frac{2}{5}$. Check Show fraction model. Look at the unit square in the model. How many horizontal strips are in the unit square?** (7) **How many horizontal strips are shaded?** (4) **What fraction is modeled by the horizontal strips?** ($\frac{4}{7}$) **How many vertical strips are in the unit square?** (5) **How many vertical strips are shaded?** (2) **What fraction is modeled by the vertical strips?** ($\frac{2}{5}$) **The product is the portion of the model that is shaded both red and blue. How many rectangles are in the unit square?** (35) **The number of rectangles, 35, is equal to the number of horizontal strips, 7, times the number of vertical strips, 5. How many rectangles are shaded both red and blue?** (8) **Four red horizontal strips times 2 blue vertical strips is equal to 8 shaded rectangles. The product of $\frac{4}{7}$ and $\frac{2}{5}$ is $\frac{8}{35}$.**

DISCOVER BOX

In problem 3, what are the numerators of the factors? (4 and 2) **What is the numerator of the product?** (8) **What operation can be used with 4 and 2 to find 8?** (multiplication) **How are the numerators of the factors used to find the numerator of the product?** (The numerators of the factors are multiplied to find the numerator of the product.) **In problem 3, what are the denominators of the factors?** (7 and 5) **What is the denominator of the product?** (35) **What operation can be used with 7 and 5 to find 35?** (multiplication) **How are the denominators of the factors used to find the denominator of the product?** (The denominators of the factors are multiplied to find the denominator of the product.)



Lesson 7

3 Uncheck Show fraction model.

$\frac{4}{7} \cdot \frac{2}{5} = \frac{8}{35}$

Drag the slider for the first fraction denominator to 7.

Drag the slider for the second fraction denominator to 5.

Drag the red bar so that $\frac{4}{7}$ is modeled.

Drag the blue bar so that $\frac{2}{5}$ is modeled.

Check Show fraction model.

The horizontal strips model the fraction $\frac{4}{7}$.

The vertical strips model the fraction $\frac{2}{5}$.

How many rectangles are in the unit square? 35

7 × 5 = 35

How many of the rectangles are shaded both red and blue? 8

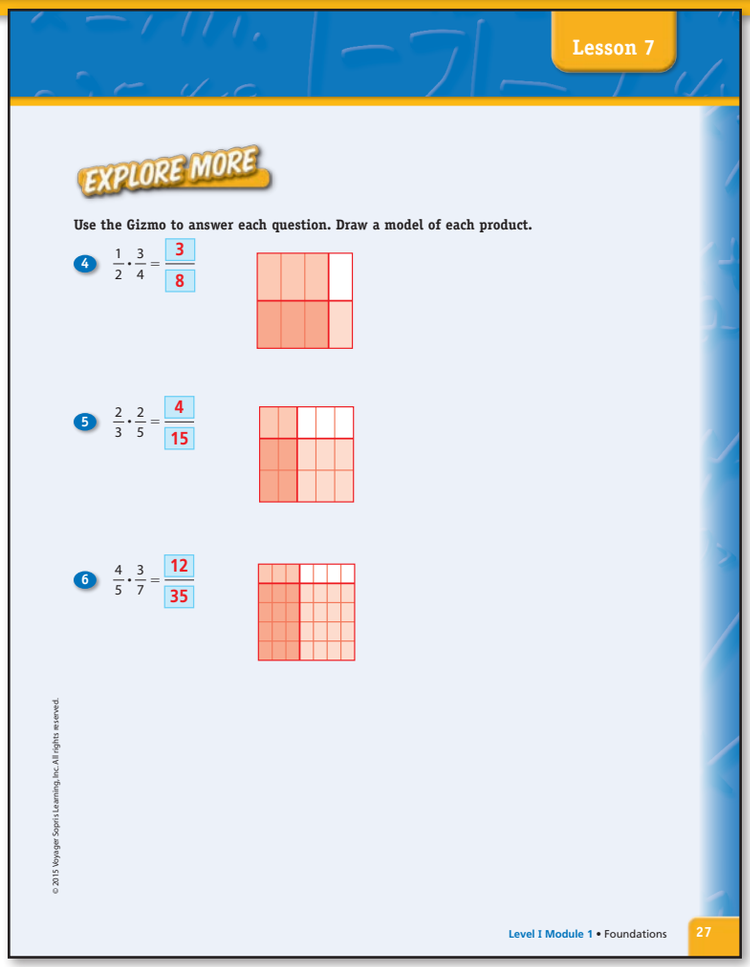
4 × 2 = 8

DISCOVER BOX

How are the numerators of the factors used to find the numerator of the product? How are the denominators of the factors used to find the denominator of the product?

The numerators of the factors are multiplied to find the numerator of the product. The denominators of the factors are multiplied to find the denominator of the product.

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EXPLORE MORE

Problems 4–6

Have students work individually to use the Gizmo to model products of fractions. Students will draw a model of the product in their Student Books. If possible, have them use a different color for each fraction in the model.

Extension: Have partners work together. One partner rolls two number cubes. Use the numbers rolled to write a fraction, using the greater number for the denominator. The other partner rolls the number cubes again to write another fraction. Then they multiply the fractions.

Lesson 8

Objective

To multiply a fraction and a mixed number or multiply two mixed numbers



Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To multiply two fractions, multiply the numerators, then the denominators. What is 2 times 3? (6) **What is 5 times 4?** (20) **What is $\frac{2}{5}$ times $\frac{3}{4}$?** ($\frac{6}{20}$) **Is $\frac{6}{20}$ written in simplest form?** (no) **What is the product of $\frac{2}{5}$ and $\frac{3}{4}$ written in simplest form?** ($\frac{3}{10}$)

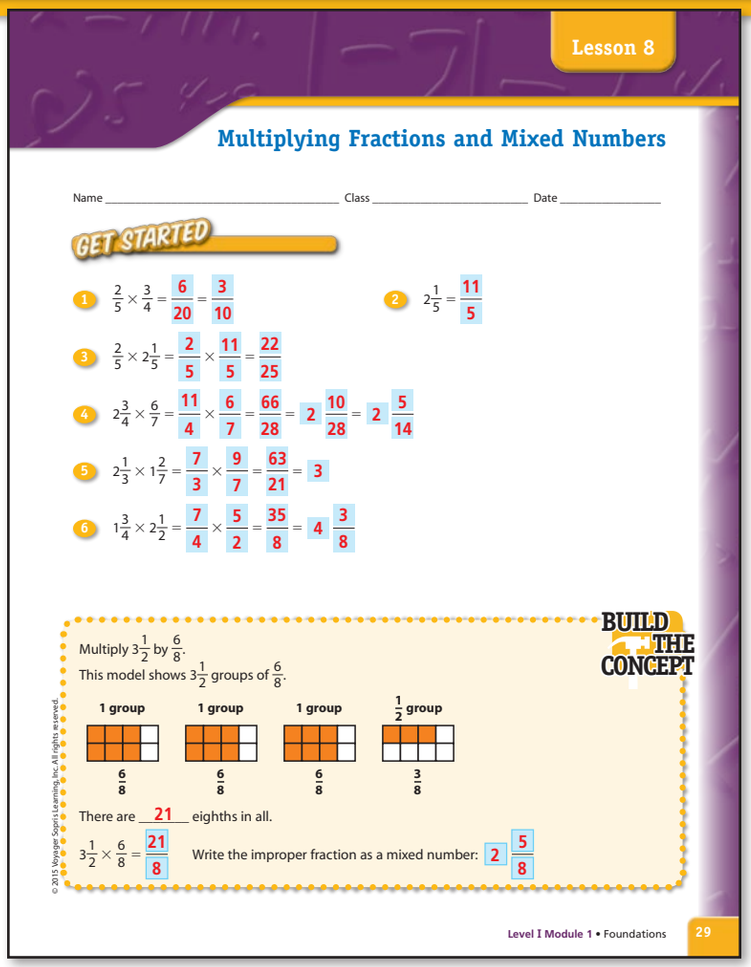
Problem 2

To change a mixed number to an improper fraction, multiply the denominator of the fraction part by the whole number part and add the numerator of the fraction part to the product. This number is the numerator of the improper fraction. What is 5 times 2? (10) **What is 10 plus 1?** (11) **Eleven is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the original fraction part, 5. What is $2\frac{1}{5}$ written as an improper fraction?** ($\frac{11}{5}$)

MODEL NEW SKILLS

Problem 3

To multiply a mixed number and a fraction, write the mixed number as an improper fraction and multiply the two fractions. The mixed number $2\frac{1}{5}$ was changed to the improper fraction $\frac{11}{5}$ in problem 2. Now, multiply as with fractions. The product of $\frac{2}{5}$ and $\frac{11}{5}$ is $\frac{22}{25}$. Is $\frac{22}{25}$ written in simplest form? (yes)



Lesson 8

Multiplying Fractions and Mixed Numbers

Name _____ Class _____ Date _____

GET STARTED

1. $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$
2. $2\frac{1}{5} = \frac{11}{5}$
3. $\frac{2}{5} \times 2\frac{1}{5} = \frac{2}{5} \times \frac{11}{5} = \frac{22}{25}$
4. $2\frac{3}{4} \times \frac{6}{7} = \frac{11}{4} \times \frac{6}{7} = \frac{66}{28} = 2\frac{10}{28} = 2\frac{5}{14}$
5. $2\frac{1}{3} \times 1\frac{2}{7} = \frac{7}{3} \times \frac{9}{7} = \frac{63}{21} = 3$
6. $1\frac{3}{4} \times 2\frac{1}{2} = \frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$

BUILD THE CONCEPT

Multiply $3\frac{1}{2}$ by $\frac{6}{8}$.

This model shows $3\frac{1}{2}$ groups of $\frac{6}{8}$.

1 group, 1 group, 1 group, $\frac{1}{2}$ group

$\frac{6}{8}$ $\frac{6}{8}$ $\frac{6}{8}$ $\frac{3}{8}$

There are 21 eighths in all.

$3\frac{1}{2} \times \frac{6}{8} = \frac{21}{8}$ Write the improper fraction as a mixed number: $2\frac{5}{8}$

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Problem 4

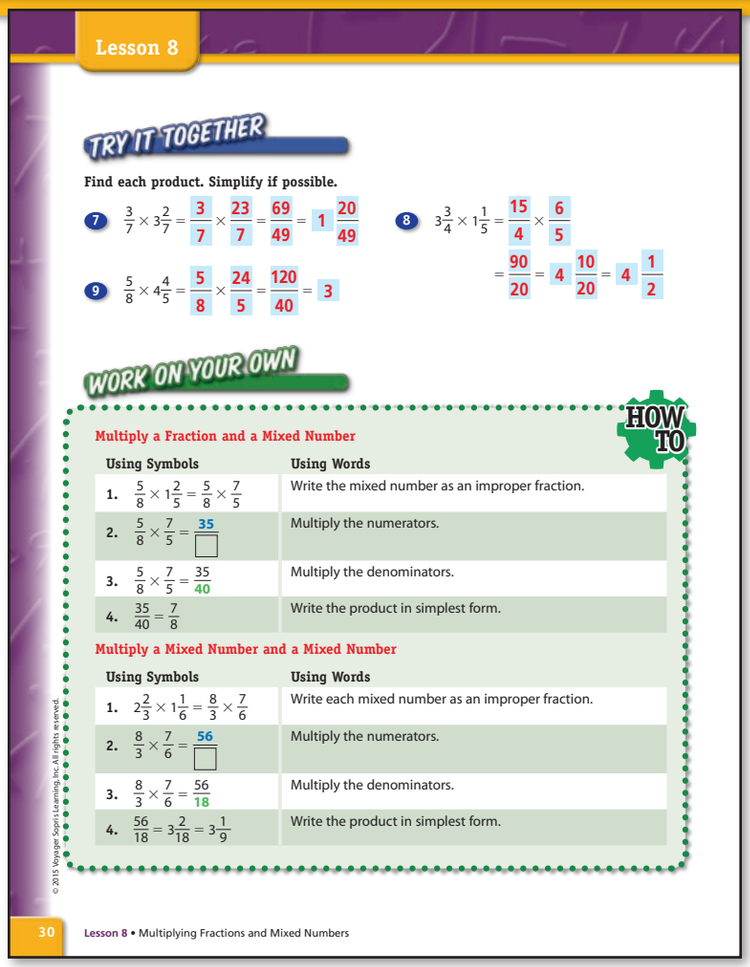
To multiply $2\frac{3}{4}$ and $\frac{6}{7}$, write $2\frac{3}{4}$ as an improper fraction. What is $2\frac{3}{4}$ written as an improper fraction? ($\frac{11}{4}$) **What is the product of $\frac{11}{4}$ and $\frac{6}{7}$?** ($\frac{66}{28}$) **The product is an improper fraction and needs to be changed to a mixed number in simplest form. What is 66 divided by 28?** (2 R10) **What is $\frac{66}{28}$ written as a mixed number?** ($2\frac{10}{28}$) **Is $\frac{10}{28}$ written in simplest form?** (no) **What is the product of $2\frac{3}{4}$ and $\frac{6}{7}$ written in simplest form?** ($2\frac{5}{14}$)

Problem 5

To multiply two mixed numbers, write both mixed numbers as improper fractions and multiply the two improper fractions. What is $2\frac{1}{3}$ written as an improper fraction? ($\frac{7}{3}$) **What is $1\frac{2}{7}$ written as an improper fraction?** ($\frac{9}{7}$) **What is the product of $\frac{7}{3}$ and $\frac{9}{7}$?** ($\frac{63}{21}$) **To write the improper fraction $\frac{63}{21}$ as a mixed number, divide the numerator by the denominator. What is 63 divided by 21?** (3) **The improper fraction simplifies to a whole number. What is the product of $2\frac{1}{3}$ and $1\frac{2}{7}$?** (3)

Problem 6

What is $1\frac{3}{4}$ written as an improper fraction? ($\frac{7}{4}$) **What is $2\frac{1}{2}$ written as an improper fraction?** ($\frac{5}{2}$) **Now, multiply**



Lesson 8

TRY IT TOGETHER

Find each product. Simplify if possible.

7. $\frac{3}{7} \times 3\frac{2}{7} = \frac{3}{7} \times \frac{23}{7} = \frac{69}{49} = 1\frac{20}{49}$

8. $3\frac{3}{4} \times 1\frac{1}{5} = \frac{15}{4} \times \frac{6}{5} = \frac{90}{20} = 4\frac{10}{20} = 4\frac{1}{2}$

9. $\frac{5}{8} \times 4\frac{4}{5} = \frac{5}{8} \times \frac{24}{5} = \frac{120}{40} = 3$

WORK ON YOUR OWN

HOW TO

Multiply a Fraction and a Mixed Number

	Using Symbols	Using Words
1.	$\frac{5}{8} \times 1\frac{2}{5} = \frac{5}{8} \times \frac{7}{5}$	Write the mixed number as an improper fraction.
2.	$\frac{5}{8} \times \frac{7}{5} = \frac{35}{\square}$	Multiply the numerators.
3.	$\frac{5}{8} \times \frac{7}{5} = \frac{35}{40}$	Multiply the denominators.
4.	$\frac{35}{40} = \frac{7}{8}$	Write the product in simplest form.

Multiply a Mixed Number and a Mixed Number

	Using Symbols	Using Words
1.	$2\frac{2}{3} \times 1\frac{1}{6} = \frac{8}{3} \times \frac{7}{6}$	Write each mixed number as an improper fraction.
2.	$\frac{8}{3} \times \frac{7}{6} = \frac{56}{\square}$	Multiply the numerators.
3.	$\frac{8}{3} \times \frac{7}{6} = \frac{56}{18}$	Multiply the denominators.
4.	$\frac{56}{18} = 3\frac{2}{18} = 3\frac{1}{9}$	Write the product in simplest form.

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as with fractions. What is the product of $\frac{7}{4}$ and $\frac{5}{2}$? ($\frac{35}{8}$) The product is an improper fraction and needs to be written as a mixed number. What is 35 divided by 8? (4 R3) What is $\frac{35}{8}$ written as a mixed number? ($4\frac{3}{8}$) Is this written in simplest form? (yes) So, the problem is complete.

BUILD THE CONCEPT

Model how to multiply a mixed number and a fraction.

This model shows the multiplication problem $3\frac{1}{2}$ times $\frac{6}{8}$ as $3\frac{1}{2}$ groups of $\frac{6}{8}$. Each of the first three rectangles shows one group of $\frac{6}{8}$. How many groups of $\frac{6}{8}$ are shown? (3) The last rectangle shows only 3 parts shaded because it models half of $\frac{6}{8}$, and half of 6 is 3.

How many shaded eighths are there in all? (21) What is the product of $3\frac{1}{2}$ and $\frac{6}{8}$? ($\frac{21}{8}$) What is $\frac{21}{8}$ written as a mixed number in simplest form? ($2\frac{5}{8}$)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 7

What is the first step in multiplying a mixed number and a fraction? (Write the mixed number as an improper fraction.) What is $3\frac{2}{7}$ written as an improper fraction? ($\frac{23}{7}$) What fractions should be multiplied? ($\frac{3}{7}$ and $\frac{23}{7}$) What is $\frac{3}{7}$ times $\frac{23}{7}$? ($\frac{69}{49}$) How is $\frac{69}{49}$ changed to a mixed number? (Divide the numerator by the denominator.) What is 69 divided by 49? (1 R20) What is $\frac{69}{49}$ written as a mixed number? ($1\frac{20}{49}$) Is $\frac{20}{49}$ written in simplest form? (yes) Why? (The GCF of 20 and 49 is 1.) What is the product of $\frac{3}{7}$ and $3\frac{2}{7}$ written in simplest form? ($1\frac{20}{49}$)

Problem 8

What is $3\frac{3}{4}$ written as an improper fraction? ($\frac{15}{4}$) What is $1\frac{1}{5}$ written as an improper fraction? ($\frac{6}{5}$) What is the product of $\frac{15}{4}$ and $\frac{6}{5}$? ($\frac{90}{20}$) What is 90 divided by 20? (4 R10) What is $\frac{90}{20}$ written as a mixed number? ($4\frac{10}{20}$) Is $\frac{10}{20}$ written in simplest form? (no) What is $\frac{10}{20}$ written in simplest form? ($\frac{1}{2}$) What is the product of $3\frac{3}{4}$ and $1\frac{1}{5}$ written in simplest form? ($4\frac{1}{2}$)

Problem 9

What is $4\frac{4}{5}$ written as an improper fraction? ($\frac{24}{5}$) What fractions should be multiplied? ($\frac{5}{8}$ and $\frac{24}{5}$) What is the product? ($\frac{120}{40}$) What is the next step? (Simplify $\frac{120}{40}$ by dividing the numerator by the denominator.) What is 120 divided by 40? (3) What is $\frac{5}{8}$ times $4\frac{4}{5}$? (3)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 10–25

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 12, 13, and 16 **Reminder**

Simplifying the improper fraction in each of these problems will result in a whole number.

Problem 22 **Reminder**

Students must first find the number of hours Jocelyn worked on her homework. Then they must subtract the number of hours spent reading from the result.

CHECK UP

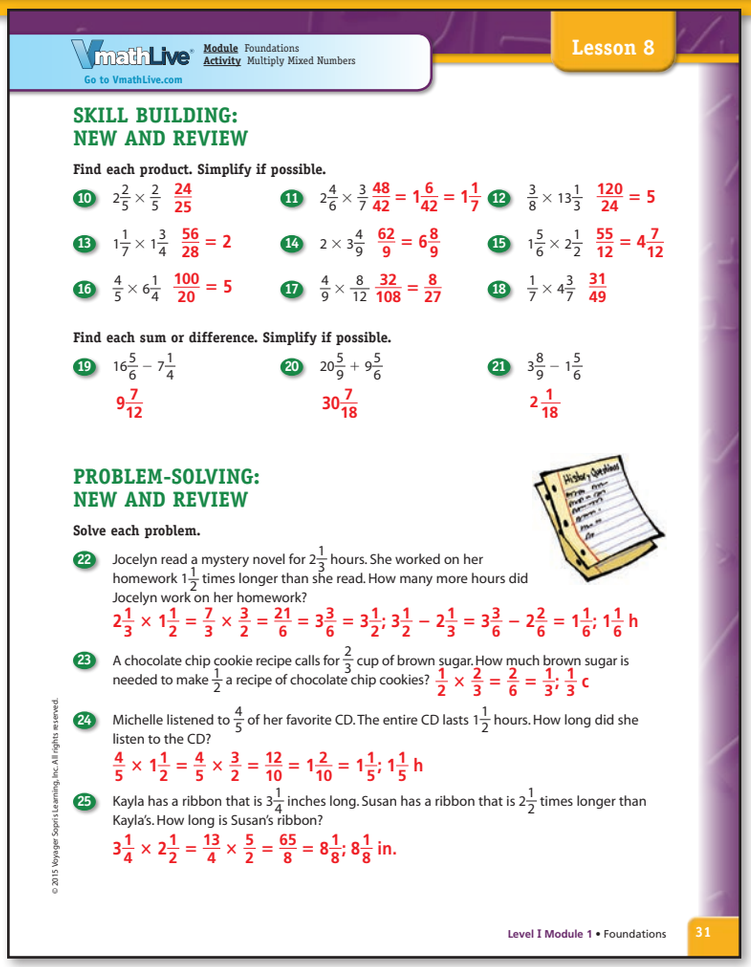
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2d: The student multiplied the fractions, then multiplied that product by the whole number. Remind the student that the mixed number needs to be changed to an improper fraction before the numbers can be multiplied. Use Additional Resources in I.1.8 to review how to multiply a fraction and a mixed number.

If student answered 1c or 2b: The student did not change the mixed number to an improper fraction, but rather multiplied the fractions and added the whole number. Review how to write a mixed number as an improper fraction. Use Additional Resources in I.1.8 to review how to multiply a fraction and a mixed number.



VmathLive Module Foundations Activity Multiply Mixed Numbers
Go to VmathLive.com

Lesson 8

SKILL BUILDING: NEW AND REVIEW

Find each product. Simplify if possible.

10. $2\frac{2}{5} \times \frac{2}{5} = \frac{24}{25}$
11. $2\frac{4}{6} \times \frac{3}{7} = \frac{48}{42} = 1\frac{6}{42} = 1\frac{1}{7}$
12. $\frac{3}{8} \times 13\frac{1}{3} = \frac{120}{24} = 5$
13. $1\frac{1}{7} \times 1\frac{3}{4} = \frac{56}{28} = 2$
14. $2 \times 3\frac{4}{9} = \frac{62}{9} = 6\frac{8}{9}$
15. $1\frac{5}{6} \times 2\frac{1}{2} = \frac{55}{12} = 4\frac{7}{12}$
16. $\frac{4}{5} \times 6\frac{1}{4} = \frac{100}{20} = 5$
17. $\frac{4}{9} \times \frac{8}{12} = \frac{32}{108} = \frac{8}{27}$
18. $\frac{1}{7} \times 4\frac{3}{7} = \frac{31}{49}$

Find each sum or difference. Simplify if possible.

19. $16\frac{5}{6} - 7\frac{1}{4} = 9\frac{7}{12}$
20. $20\frac{5}{9} + 9\frac{5}{6} = 30\frac{7}{18}$
21. $3\frac{8}{9} - 1\frac{5}{6} = 2\frac{1}{18}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

22. Jocelyn read a mystery novel for $2\frac{1}{3}$ hours. She worked on her homework $1\frac{1}{2}$ times longer than she read. How many more hours did Jocelyn work on her homework?
$2\frac{1}{3} \times 1\frac{1}{2} = \frac{7}{3} \times \frac{3}{2} = \frac{21}{6} = 3\frac{3}{6} = 3\frac{1}{2}$; $3\frac{1}{2} - 2\frac{1}{3} = 3\frac{3}{6} - 2\frac{2}{6} = 1\frac{1}{6}$; $1\frac{1}{6}$ h
23. A chocolate chip cookie recipe calls for $\frac{2}{3}$ cup of brown sugar. How much brown sugar is needed to make $\frac{1}{2}$ a recipe of chocolate chip cookies? $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$; $\frac{1}{3}$ c
24. Michelle listened to $\frac{4}{5}$ of her favorite CD. The entire CD lasts $1\frac{1}{2}$ hours. How long did she listen to the CD?
$\frac{4}{5} \times 1\frac{1}{2} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$; $1\frac{1}{5}$ h
25. Kayla has a ribbon that is $3\frac{1}{4}$ inches long. Susan has a ribbon that is $2\frac{1}{2}$ times longer than Kayla's. How long is Susan's ribbon?
$3\frac{1}{4} \times 2\frac{1}{2} = \frac{13}{4} \times \frac{5}{2} = \frac{65}{8} = 8\frac{1}{8}$; $8\frac{1}{8}$ in.

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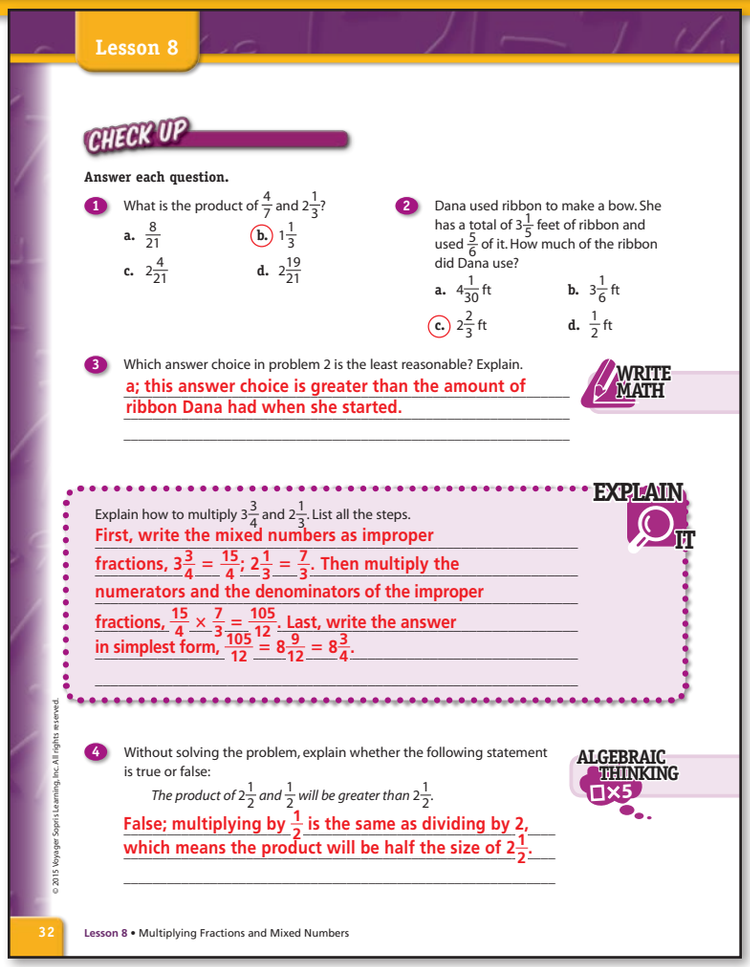
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If student answered 1d or 2a: The student added the fraction and mixed number rather than multiplying. Remind the student to read the problem carefully.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, read problem 2 with the student. Show the student that Dana starts with $3\frac{1}{5}$ feet of ribbon and that if she uses a part of it, that amount cannot be greater than what she started with.



Lesson 8

CHECK UP

Answer each question.

1. What is the product of $\frac{4}{7}$ and $2\frac{1}{3}$?
   a. $\frac{8}{21}$
   b. $1\frac{1}{3}$
   c. $2\frac{4}{21}$
   d. $2\frac{19}{21}$

2. Dana used ribbon to make a bow. She has a total of $3\frac{1}{5}$ feet of ribbon and used $\frac{5}{6}$ of it. How much of the ribbon did Dana use?
   a. $4\frac{1}{30}$ ft
   b. $3\frac{1}{6}$ ft
   c. $2\frac{2}{3}$ ft
   d. $\frac{1}{2}$ ft

3. Which answer choice in problem 2 is the least reasonable? Explain.
   a; this answer choice is greater than the amount of ribbon Dana had when she started.

WRITE MATH

EXPLAIN IT

Explain how to multiply $3\frac{3}{4}$ and $2\frac{1}{3}$. List all the steps.
First, write the mixed numbers as improper fractions, $3\frac{3}{4} = \frac{15}{4}$; $2\frac{1}{3} = \frac{7}{3}$. Then multiply the numerators and the denominators of the improper fractions, $\frac{15}{4} \times \frac{7}{3} = \frac{105}{12}$. Last, write the answer in simplest form, $\frac{105}{12} = 8\frac{9}{12} = 8\frac{3}{4}$.

4. Without solving the problem, explain whether the following statement is true or false:
   The product of $2\frac{1}{2}$ and $\frac{1}{2}$ will be greater than $2\frac{1}{2}$.
   False; multiplying by $\frac{1}{2}$ is the same as dividing by 2, which means the product will be half the size of $2\frac{1}{2}$.

ALGEBRAIC THINKING □×5

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32 Lesson 8 • Multiplying Fractions and Mixed Numbers

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Foundations
Activity: Multiply Mixed Numbers

Vmath Reteach
Reteach Student Book Module 1 Lesson 8
Reteach Teacher Edition Module 1 Lesson 8

Extra Practice
Student Book page 56

English Language Learners

Write a multiplication sign (×) on the board. Review the meaning of this symbol. Make a chart of words related to multiplication such as *factor, product, times,* and *multiply.*

Present word problems to students and ask them if multiplication can be used to solve the problem. Ask students to explain how they know.

Students with Special Needs

Students may struggle with remembering the steps for writing a mixed number as an improper fraction, which is an essential preskill for this lesson. Review these steps with students by writing a mixed number on the board such as $2\frac{3}{8}$. Walk students through the steps and have them work out the same problem on their paper to use as a later reference.

Now, tell students to look at each problem before solving and identify any mixed numbers that need to be changed to improper fractions before multiplying. Then give partners a few problems, some with mixed numbers and some without mixed numbers, and ask students to identify which need to be changed before multiplying.

Lesson 9

Objective

To find the reciprocal of a whole number, a fraction, or a mixed number

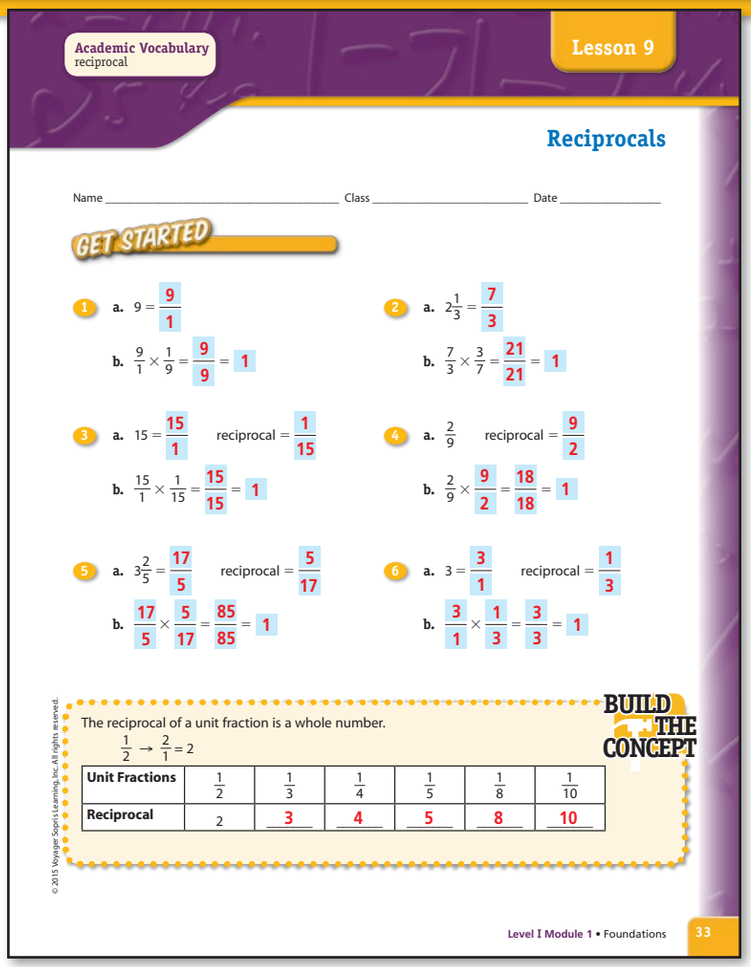
Preskills	Lesson
Multiplying Fractions and Mixed Numbers	I.1.8

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **reciprocal**

  for any nonzero number x, the reciprocal is $\frac{1}{x}$; the number 0 has no reciprocal; the reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$



Academic Vocabulary
reciprocal

Lesson 9

Reciprocals

Name _____ Class _____ Date _____

GET STARTED

1 a. $9 = \frac{9}{1}$

b. $\frac{9}{1} \times \frac{1}{9} = \frac{9}{9} = 1$

2 a. $2\frac{1}{3} = \frac{7}{3}$

b. $\frac{7}{3} \times \frac{3}{7} = \frac{21}{21} = 1$

3 a. $15 = \frac{15}{1}$ reciprocal $= \frac{1}{15}$

b. $\frac{15}{1} \times \frac{1}{15} = \frac{15}{15} = 1$

4 a. $\frac{2}{9}$ reciprocal $= \frac{9}{2}$

b. $\frac{2}{9} \times \frac{9}{2} = \frac{18}{18} = 1$

5 a. $3\frac{2}{5} = \frac{17}{5}$ reciprocal $= \frac{5}{17}$

b. $\frac{17}{5} \times \frac{5}{17} = \frac{85}{85} = 1$

6 a. $3 = \frac{3}{1}$ reciprocal $= \frac{1}{3}$

b. $\frac{3}{1} \times \frac{1}{3} = \frac{3}{3} = 1$

BUILD THE CONCEPT

The reciprocal of a unit fraction is a whole number.

$\frac{1}{2} \rightarrow \frac{2}{1} = 2$

Unit Fractions	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$
Reciprocal	2	3	4	5	8	10

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To write a whole number as an improper fraction, write the whole number as the numerator and write 1 as the denominator. How is 9 written as an improper fraction? ($\frac{9}{1}$)

In problem 1b, multiply $\frac{9}{1}$ and $\frac{1}{9}$. What is the product? ($\frac{9}{9}$) What is $\frac{9}{9}$ written in simplest form? (1)

Problem 2

What is $2\frac{1}{3}$ written as an improper fraction? ($\frac{7}{3}$) In problem 2b, multiply $\frac{7}{3}$ and $\frac{3}{7}$. What is the product? ($\frac{21}{21}$) What is $\frac{21}{21}$ written in simplest form? (1)

MODEL NEW SKILLS

Problem 3

Every number except 0 has a reciprocal. To write the reciprocal of a whole number, first write the whole number as a fraction with 1 as the denominator and the whole number as the numerator. How is 15 written as a fraction? ($\frac{15}{1}$) Now, switch the numerator and the denominator. The reciprocal of 15 is $\frac{1}{15}$.

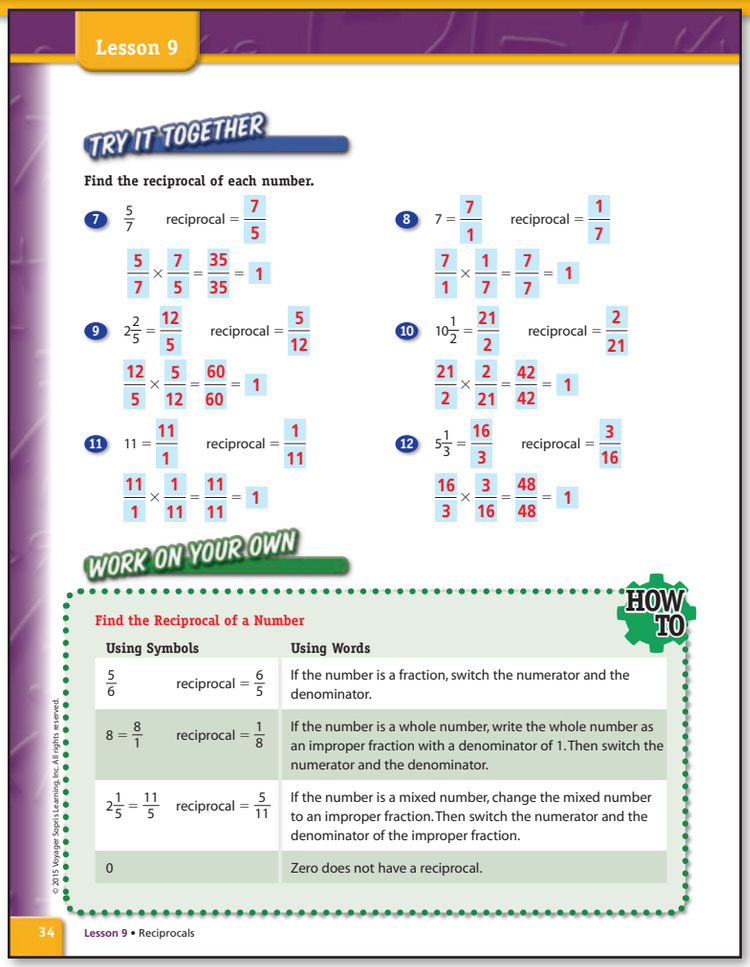
The product of a number and its reciprocal is always 1. In problem 3b, find the product of 15 and its reciprocal. Multiply $\frac{15}{1}$ and $\frac{1}{15}$. What is the product? ($\frac{15}{15}$) What is $\frac{15}{15}$ written in simplest form? (1) Because the product is 1, 15 and $\frac{1}{15}$ are reciprocals.

Problem 4

To write the reciprocal of a fraction, switch the numerator and the denominator. What is the numerator in $\frac{2}{9}$? (2) What is the denominator? (9) These are switched to make the reciprocal. The reciprocal of $\frac{2}{9}$ is $\frac{9}{2}$. The product of a number and its reciprocal is always 1. Multiply $\frac{2}{9}$ and $\frac{9}{2}$. What is the product? ($\frac{18}{18}$) What is $\frac{18}{18}$ written in simplest form? (1)

Problem 5

To write the reciprocal of a mixed number, first write the mixed number as an improper fraction. What is $3\frac{2}{5}$ written as an improper fraction? ($\frac{17}{5}$) Now, switch the numerator and denominator to write the reciprocal of the fraction $\frac{17}{5}$. The reciprocal of $\frac{17}{5}$ is $\frac{5}{17}$.



Lesson 9

TRY IT TOGETHER

Find the reciprocal of each number.

7. $\frac{5}{7}$ reciprocal = $\frac{7}{5}$

$\frac{5}{7} \times \frac{7}{5} = \frac{35}{35} = 1$

8. $7 = \frac{7}{1}$ reciprocal = $\frac{1}{7}$

$\frac{7}{1} \times \frac{1}{7} = \frac{7}{7} = 1$

9. $2\frac{2}{5} = \frac{12}{5}$ reciprocal = $\frac{5}{12}$

$\frac{12}{5} \times \frac{5}{12} = \frac{60}{60} = 1$

10. $10\frac{1}{2} = \frac{21}{2}$ reciprocal = $\frac{2}{21}$

$\frac{21}{2} \times \frac{2}{21} = \frac{42}{42} = 1$

11. $11 = \frac{11}{1}$ reciprocal = $\frac{1}{11}$

$\frac{11}{1} \times \frac{1}{11} = \frac{11}{11} = 1$

12. $5\frac{1}{3} = \frac{16}{3}$ reciprocal = $\frac{3}{16}$

$\frac{16}{3} \times \frac{3}{16} = \frac{48}{48} = 1$

WORK ON YOUR OWN

HOW TO

Find the Reciprocal of a Number

Using Symbols	Using Words
$\frac{5}{6}$ reciprocal = $\frac{6}{5}$	If the number is a fraction, switch the numerator and the denominator.
$8 = \frac{8}{1}$ reciprocal = $\frac{1}{8}$	If the number is a whole number, write the whole number as an improper fraction with a denominator of 1. Then switch the numerator and the denominator.
$2\frac{1}{5} = \frac{11}{5}$ reciprocal = $\frac{5}{11}$	If the number is a mixed number, change the mixed number to an improper fraction. Then switch the numerator and the denominator of the improper fraction.
0	Zero does not have a reciprocal.

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Multiply $\frac{17}{5}$ and $\frac{5}{17}$ to show that the product is 1. What is the product of $\frac{17}{5}$ and $\frac{5}{17}$? ($\frac{85}{85}$) What is $\frac{85}{85}$ written in simplest form? (1)

Problem 6

To write the reciprocal of a whole number, first write the whole number as an improper fraction. How is 3 written as a fraction? ($\frac{3}{1}$) Now, switch the numerator and the denominator to write the reciprocal. What is the numerator? (3) What is the denominator? (1) What is the reciprocal of 3? ($\frac{1}{3}$)

It is also true that 3 is the reciprocal of $\frac{1}{3}$. To show that 3 and $\frac{1}{3}$ are reciprocals, multiply $\frac{3}{1}$ and $\frac{1}{3}$. What is $\frac{3}{1}$ times $\frac{1}{3}$? ($\frac{3}{3}$) What is $\frac{3}{3}$ written in simplest form? (1)

BUILD THE CONCEPT

Model how to understand reciprocals of unit fractions.

A unit fraction has a 1 as the numerator and any whole number as the denominator. To find the reciprocal of a fraction, switch the numerator and denominator. For the unit fraction $\frac{1}{2}$, what is the numerator? (1) What is the denominator? (2) So, 1 is the denominator of its reciprocal and 2 is the numerator of its reciprocal. When the denominator of a fraction is 1, the fraction represents a whole number. That whole number is the same as the numerator. What whole number is equal to $\frac{2}{1}$? (2)

Look at the other unit fractions in the table. What is the reciprocal of $\frac{1}{3}$? (3) Of $\frac{1}{4}$? (4) Of $\frac{1}{5}$? (5) Of $\frac{1}{8}$? (8) Of $\frac{1}{10}$? (10) All the reciprocals are whole numbers.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 7

How is the reciprocal of a fraction written? (by switching the numerator and the denominator) What is the reciprocal of $\frac{5}{7}$? ($\frac{7}{5}$) What is the product of $\frac{5}{7}$ and $\frac{7}{5}$? ($\frac{35}{35}$) What is $\frac{35}{35}$ written in simplest form? (1)

Problem 8

How is 7 written as an improper fraction? ($\frac{7}{1}$) Switch the numerator and denominator. What is the reciprocal of 7? ($\frac{1}{7}$) What is the product of $\frac{7}{1}$ and $\frac{1}{7}$? ($\frac{7}{7}$) What is $\frac{7}{7}$ written in simplest form? (1)

Problem 9

What is the first step in finding the reciprocal of a mixed number? (Change the mixed number to an improper fraction.) What is $2\frac{2}{5}$ written as an improper fraction? ($\frac{12}{5}$) What is the reciprocal of $\frac{12}{5}$? ($\frac{5}{12}$) Multiply $\frac{12}{5}$ and $\frac{5}{12}$. What is the product? ($\frac{60}{60}$) What is $\frac{60}{60}$ written in simplest form? (1)

Problem 10

What is $10\frac{1}{2}$ written as an improper fraction? ($\frac{21}{2}$) What is the reciprocal of $\frac{21}{2}$? ($\frac{2}{21}$) What is $\frac{21}{2}$ times $\frac{2}{21}$? ($\frac{42}{42}$) What is $\frac{42}{42}$ written in simplest form? (1)

Problem 11

What is 11 written as an improper fraction? ($\frac{11}{1}$) What is the reciprocal of $\frac{11}{1}$? ($\frac{1}{11}$) What is $\frac{11}{1}$ times $\frac{1}{11}$ written in simplest form? (1)

Problem 12

What is $5\frac{1}{3}$ written as an improper fraction? ($\frac{16}{3}$) What is the reciprocal of $\frac{16}{3}$? ($\frac{3}{16}$) What is $\frac{16}{3}$ times $\frac{3}{16}$? ($\frac{48}{48}$) What is $\frac{48}{48}$ written in simplest form? (1)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 13–31

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 25–27 Reminder

Students will apply the concept that the product of a number and its reciprocal is 1 by writing the missing reciprocal in each problem.

Problem 31 Reminder

Students must first correctly change the mixed number to an improper fraction before finding the reciprocal. Then they compare the result with Tanya's answer.

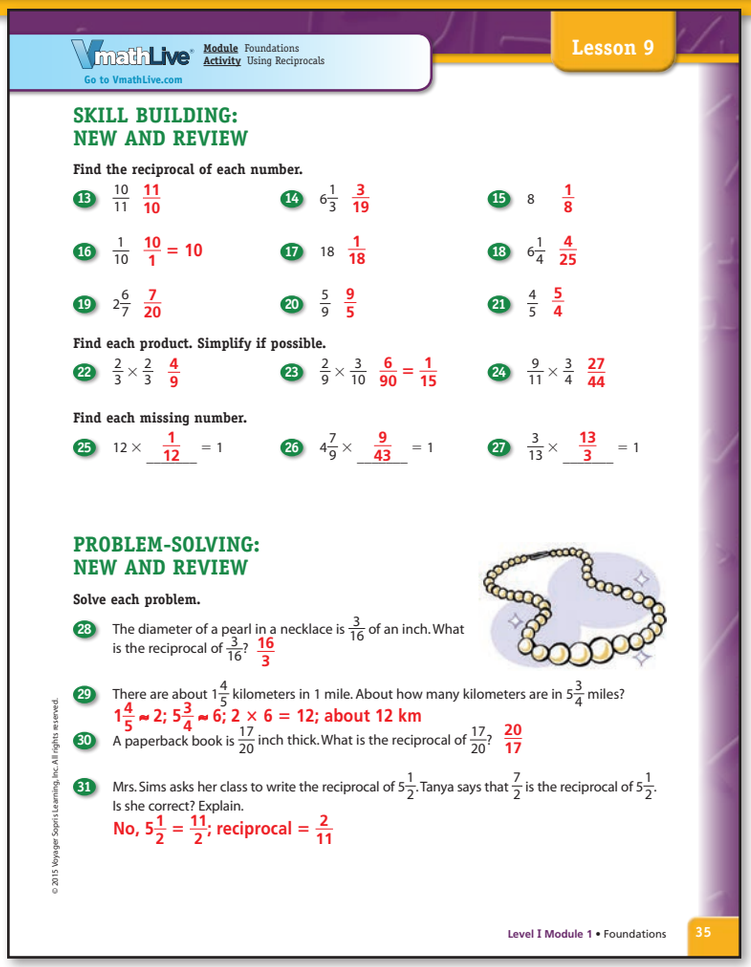
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly: The student did not know how to find the reciprocal. Use Additional Resources in I.1.9 to reteach how to find the reciprocal of a fraction.



VmathLive Module Foundations Activity Using Reciprocals
Go to VmathLive.com

Lesson 9

SKILL BUILDING: NEW AND REVIEW

Find the reciprocal of each number.

13. $\frac{10}{11}$ $\frac{11}{10}$
14. $6\frac{1}{3}$ $\frac{3}{19}$
15. 8 $\frac{1}{8}$
16. $\frac{1}{10}$ $\frac{10}{1} = 10$
17. 18 $\frac{1}{18}$
18. $6\frac{1}{4}$ $\frac{4}{25}$
19. $2\frac{6}{7}$ $\frac{7}{20}$
20. $\frac{5}{9}$ $\frac{9}{5}$
21. $\frac{4}{5}$ $\frac{5}{4}$

Find each product. Simplify if possible.

22. $\frac{2}{3} \times \frac{2}{3}$ $\frac{4}{9}$
23. $\frac{2}{9} \times \frac{3}{10}$ $\frac{6}{90} = \frac{1}{15}$
24. $\frac{9}{11} \times \frac{3}{4}$ $\frac{27}{44}$

Find each missing number.

25. $12 \times \underline{\frac{1}{12}} = 1$
26. $4\frac{7}{9} \times \underline{\frac{9}{43}} = 1$
27. $\frac{3}{13} \times \underline{\frac{13}{3}} = 1$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

28. The diameter of a pearl in a necklace is $\frac{3}{16}$ of an inch. What is the reciprocal of $\frac{3}{16}$? $\frac{16}{3}$

29. There are about $1\frac{4}{5}$ kilometers in 1 mile. About how many kilometers are in $5\frac{3}{4}$ miles?
$1\frac{4}{5} \approx 2$; $5\frac{3}{4} \approx 6$; $2 \times 6 = 12$; about 12 km

30. A paperback book is $\frac{17}{20}$ inch thick. What is the reciprocal of $\frac{17}{20}$? $\frac{20}{17}$

31. Mrs. Sims asks her class to write the reciprocal of $5\frac{1}{2}$. Tanya says that $\frac{7}{2}$ is the reciprocal of $5\frac{1}{2}$. Is she correct? Explain.
No, $5\frac{1}{2} = \frac{11}{2}$; reciprocal $= \frac{2}{11}$

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Level I Module 1 • Foundations 35

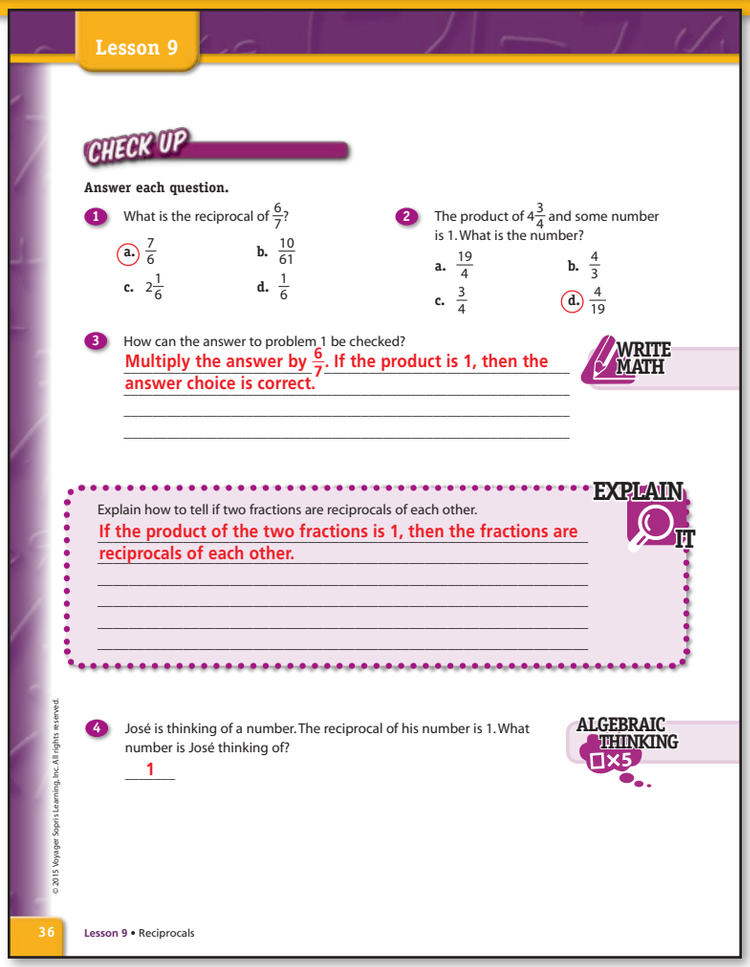
If student answered 2a: The student found the improper fraction but did not switch the numerator and denominator to find the reciprocal. Use Additional Resources in I.1.9 to reteach how to find the reciprocal of a mixed number.

If student answered 2b: The student found the reciprocal of the fraction part of the mixed number only. Remind the student that the mixed number must be changed to an improper fraction first before a reciprocal can be found.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the answer to problem 1 is the reciprocal of $\frac{6}{7}$. The student should multiply the answer by $\frac{6}{7}$. If the product is 1, then the answer choice is correct.



Lesson 9

CHECK UP

Answer each question.

1. What is the reciprocal of $\frac{6}{7}$?
   (a.) $\frac{7}{6}$ b. $\frac{10}{61}$ c. $2\frac{1}{6}$ d. $\frac{1}{6}$

2. The product of $4\frac{3}{4}$ and some number is 1. What is the number?
   a. $\frac{19}{4}$ b. $\frac{4}{3}$ c. $\frac{3}{4}$ (d.) $\frac{4}{19}$

3. How can the answer to problem 1 be checked?
   Multiply the answer by $\frac{6}{7}$. If the product is 1, then the answer choice is correct.

WRITE MATH

EXPLAIN IT

Explain how to tell if two fractions are reciprocals of each other.
If the product of the two fractions is 1, then the fractions are reciprocals of each other.

4. José is thinking of a number. The reciprocal of his number is 1. What number is José thinking of?
   1

ALGEBRAIC THINKING □×5

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36 Lesson 9 • Reciprocals

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Foundations
Activity: Using Reciprocals

Vmath Reteach
Reteach Student Book Module 1 Lesson 9
Reteach Teacher Edition Module 1 Lesson 9

Extra Practice
Student Book page 56

English Language Learners

Use the VmathLive Animated Glossary to review the term *reciprocal*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write the fraction $\frac{2}{5}$ on the board. Ask students to identify the numerator and denominator. Then write the fraction $\frac{5}{2}$ on the board. Again, have students identify the numerator and denominator. Show that these two fractions are reciprocals by drawing a line from the numerator of $\frac{2}{5}$ to the denominator of $\frac{5}{2}$ and from the denominator of $\frac{2}{5}$ to the numerator of $\frac{5}{2}$. Check understanding by writing pairs of fractions on the board and asking students to identify which pairs are reciprocals.

Students with Special Needs

Provide pairs of students with 20 index cards and a piece of string or yarn. Have students work in pairs to make two sets of digits cards from 1 to 10. Have students shuffle the cards and place them facedown. Tell students that the string or yarn represents the fraction bar in a fraction. One student selects two cards and places one card above the yarn and one card below the line to form a fraction. The other student switches the cards to show the reciprocal. Ask students to record the fraction and its reciprocal on a sheet of paper.

Lesson 10

Objective

To divide a fraction by a fraction or a whole number by a fraction

Preskills	Lesson
Reciprocals	I.1.9

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To multiply fractions, first multiply numerators. What is 6 times 3? (18) Then multiply denominators. What is 11 times 7? (77) What is the product of $\frac{6}{11}$ and $\frac{3}{7}$? ($\frac{18}{77}$) Is $\frac{18}{77}$ written in simplest form? (yes) Why? (The GCF of 18 and 77 is 1.)

Problem 2

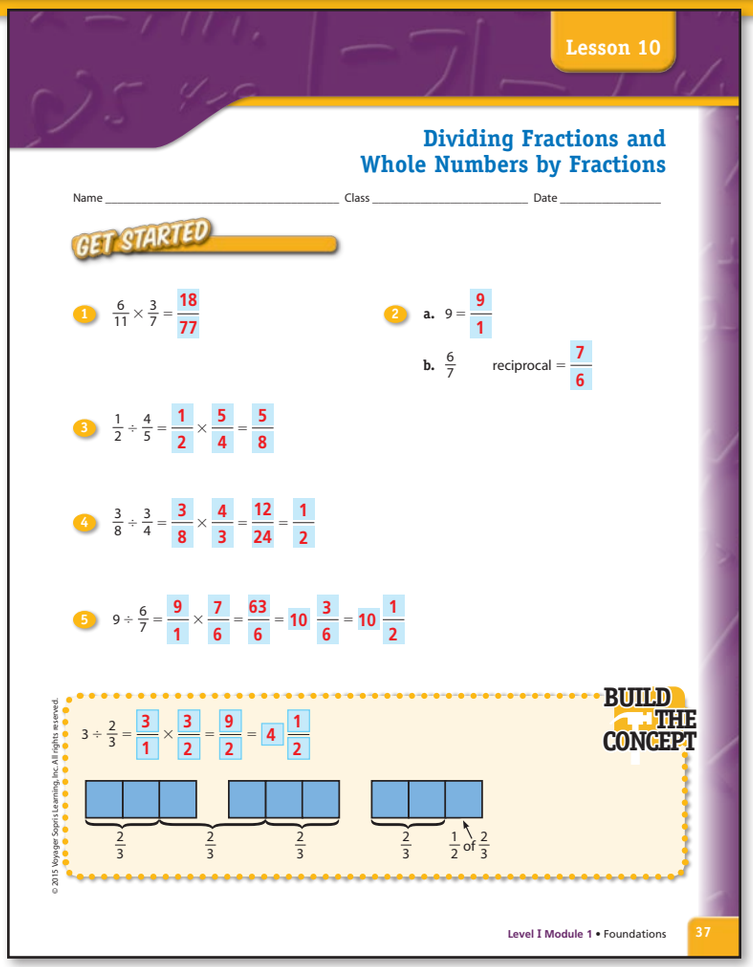
To write a whole number as an improper fraction, write the whole number as the numerator and 1 as the denominator. What is 9 written as an improper fraction? ($\frac{9}{1}$)

Look at problem 2b. To find the reciprocal of a fraction, switch the numerator and denominator. What is the numerator in $\frac{6}{7}$? (6) What is the denominator? (7) What is the reciprocal of $\frac{6}{7}$? ($\frac{7}{6}$)

MODEL NEW SKILLS

Problem 3

In a division problem such as $\frac{1}{2}$ divided by $\frac{4}{5}$, the second fraction is the divisor. To divide a fraction by a fraction, multiply the first fraction by the reciprocal of the second fraction. What is the second fraction? ($\frac{4}{5}$) What is the reciprocal of $\frac{4}{5}$? ($\frac{5}{4}$) The fractions to be multiplied are $\frac{1}{2}$ and $\frac{5}{4}$. What is $\frac{1}{2}$ times $\frac{5}{4}$? ($\frac{5}{8}$) What is the GCF of 5 and 8 (1) The fraction is written in simplest form because it is a proper fraction and the GCF of the numerator and denominator is 1. So, $\frac{1}{2}$ divided by $\frac{4}{5}$ is $\frac{5}{8}$.

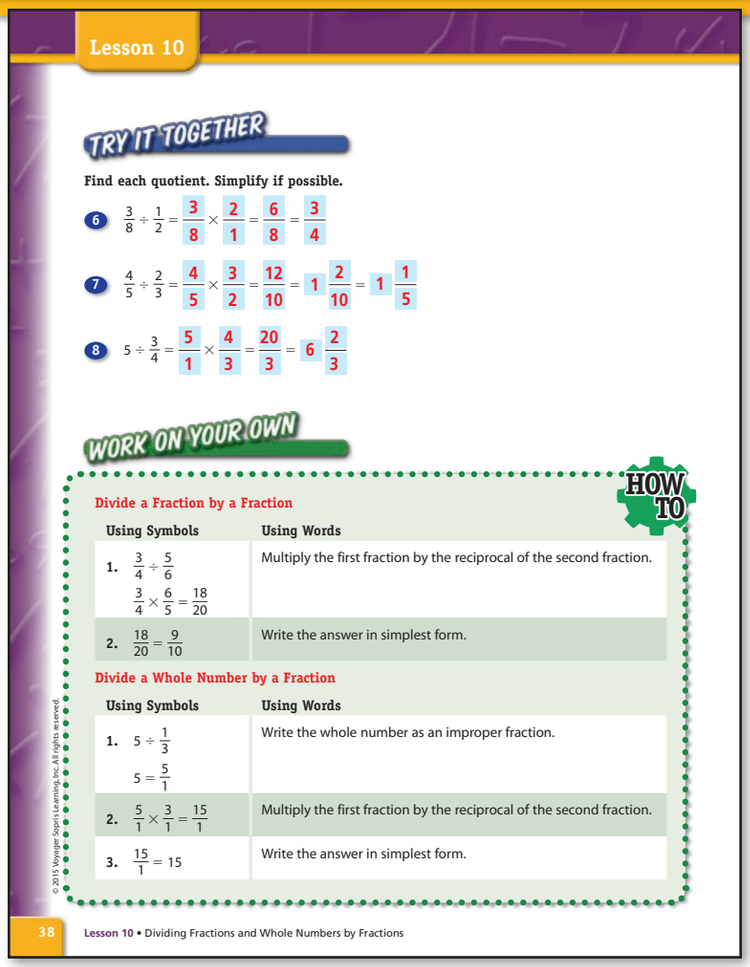


Problem 4

To divide $\frac{3}{8}$ by $\frac{3}{4}$, multiply $\frac{3}{8}$ by the reciprocal of $\frac{3}{4}$. What is the reciprocal of $\frac{3}{4}$? ($\frac{4}{3}$) What fractions should be multiplied? ($\frac{3}{8}$ and $\frac{4}{3}$) What is $\frac{3}{8}$ times $\frac{4}{3}$? ($\frac{12}{24}$) Is $\frac{12}{24}$ written in simplest form? (no) Why not? (The GCF of 12 and 24 is 12.) What is $\frac{3}{8}$ divided by $\frac{3}{4}$ written in simplest form? ($\frac{1}{2}$)

Problem 5

To divide 9 by $\frac{6}{7}$, begin by writing the whole number as an improper fraction. What is 9 written as an improper fraction? ($\frac{9}{1}$) What is the reciprocal of $\frac{6}{7}$? ($\frac{7}{6}$) Next, multiply the fractions. What is $\frac{9}{1}$ times $\frac{7}{6}$? ($\frac{63}{6}$) The answer is an improper fraction, so divide to change it to a mixed number. What is 63 divided by 6? (10 R3) What is $\frac{63}{6}$ written as a mixed number? ($10\frac{3}{6}$) Is $\frac{3}{6}$ written in simplest form? (no) What is $\frac{3}{6}$ written in simplest form? ($\frac{1}{2}$) So, 9 divided by $\frac{6}{7}$ is $10\frac{1}{2}$.



Lesson 10

TRY IT TOGETHER

Find each quotient. Simplify if possible.

6. $\frac{3}{8} \div \frac{1}{2} = \frac{3}{8} \times \frac{2}{1} = \frac{6}{8} = \frac{3}{4}$

7. $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$

8. $5 \div \frac{3}{4} = \frac{5}{1} \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3}$

WORK ON YOUR OWN

HOW TO

Divide a Fraction by a Fraction

	Using Symbols	Using Words
1.	$\frac{3}{4} \div \frac{5}{6}$ $\frac{3}{4} \times \frac{6}{5} = \frac{18}{20}$	Multiply the first fraction by the reciprocal of the second fraction.
2.	$\frac{18}{20} = \frac{9}{10}$	Write the answer in simplest form.

Divide a Whole Number by a Fraction

	Using Symbols	Using Words
1.	$5 \div \frac{1}{3}$ $5 = \frac{5}{1}$	Write the whole number as an improper fraction.
2.	$\frac{5}{1} \times \frac{3}{1} = \frac{15}{1}$	Multiply the first fraction by the reciprocal of the second fraction.
3.	$\frac{15}{1} = 15$	Write the answer in simplest form.

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38 Lesson 10 • Dividing Fractions and Whole Numbers by Fractions

BUILD THE CONCEPT

Model how to divide a whole number by a fraction.

The division problem 3 divided by $\frac{2}{3}$ is the same as dividing 3 into groups of $\frac{2}{3}$ each. Look at the model below the problem. How many fully shaded rectangles are there? (3) **How many groups of $\frac{2}{3}$ can be formed by the shaded area?** (4) **How many sections are left over?** (1)

Notice that the one leftover section represents half of a $\frac{2}{3}$ group. So, the entire model represents 3 divided by $\frac{2}{3}$ is equal to $4\frac{1}{2}$.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

How is a fraction divided by a fraction? (by multiplying the first fraction by the reciprocal of the second fraction) **What is the reciprocal of the second fraction?** ($\frac{2}{1}$) **What is the next step?** (Multiply the fractions.) **What fractions should be multiplied?** ($\frac{3}{8}$ and $\frac{2}{1}$) **What is the product?** ($\frac{6}{8}$) **Is $\frac{6}{8}$ written in simplest form?** (no) **What is $\frac{3}{8}$ divided by $\frac{1}{2}$ written in simplest form?** ($\frac{3}{4}$)

Problem 7

What is the reciprocal of $\frac{2}{3}$? ($\frac{3}{2}$) **What is $\frac{4}{5}$ times $\frac{3}{2}$?** ($\frac{12}{10}$) **Is the product written in simplest form?** (no) **Why not?** (It is an improper fraction.) **Divide to change the improper fraction to a mixed number. What is 12 divided by 10?** (1 R2) **What is the mixed number?** ($1\frac{2}{10}$) **Is $\frac{2}{10}$ written in simplest form?** (no) **What is $\frac{4}{5}$ divided by $\frac{2}{3}$ written in simplest form?** ($1\frac{1}{5}$)

Problem 8

What is the first step when dividing a whole number by a fraction? (Write the whole number as an improper fraction.) **What is 5 written as an improper fraction?** ($\frac{5}{1}$) **What is the next step?** (Write the reciprocal of the second fraction.) **What is the reciprocal of $\frac{3}{4}$?** ($\frac{4}{3}$) **What is the next step?** (Multiply $\frac{5}{1}$ and $\frac{4}{3}$.) **What is the product of $\frac{5}{1}$ and $\frac{4}{3}$?** ($\frac{20}{3}$) **What is $\frac{20}{3}$ written as a mixed number?** ($6\frac{2}{3}$) **What is 5 divided by $\frac{3}{4}$ written in simplest form?** ($6\frac{2}{3}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Drawing a Picture** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a picture. The instruction is immediately followed by application of the strategy in problem 21.

The problem states that Dante has $\frac{3}{4}$ pound of clay. Students are asked whether he can make at least 3 ornaments that weigh $\frac{1}{4}$ pound each.

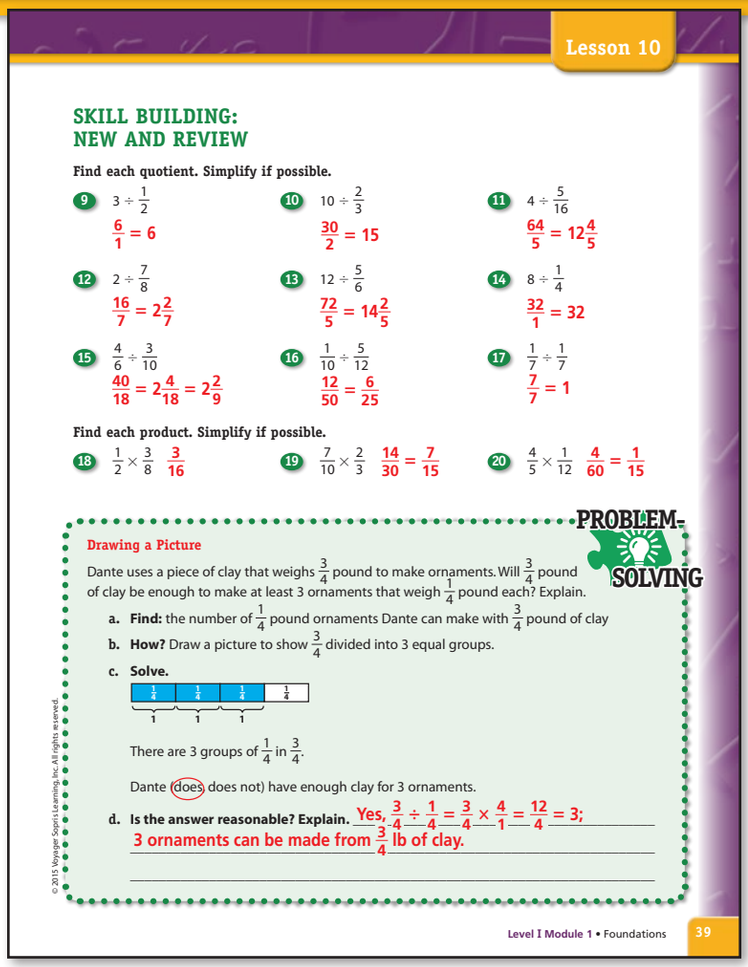
Have students draw a picture to solve the problem. The picture should represent $\frac{3}{4}$ by showing 4 equal parts with 3 parts shaded. Next, have students divide the shaded parts into 3 equal groups. They should draw brackets to mark each group of $\frac{1}{4}$. Have students count the groups. Because there are 3 groups of $\frac{1}{4}$, Dante does have enough clay to make 3 ornaments.

Problems 9–24

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 9, 10, 14, 17, 21, and 24 **Reminder**

Students should write each quotient as a whole number for these problems.



Lesson 10

SKILL BUILDING: NEW AND REVIEW

Find each quotient. Simplify if possible.

9. $3 \div \frac{1}{2}$ $\frac{6}{1} = 6$
10. $10 \div \frac{2}{3}$ $\frac{30}{2} = 15$
11. $4 \div \frac{5}{16}$ $\frac{64}{5} = 12\frac{4}{5}$
12. $2 \div \frac{7}{8}$ $\frac{16}{7} = 2\frac{2}{7}$
13. $12 \div \frac{5}{6}$ $\frac{72}{5} = 14\frac{2}{5}$
14. $8 \div \frac{1}{4}$ $\frac{32}{1} = 32$
15. $\frac{4}{6} \div \frac{3}{10}$ $\frac{40}{18} = 2\frac{4}{18} = 2\frac{2}{9}$
16. $\frac{1}{10} \div \frac{5}{12}$ $\frac{12}{50} = \frac{6}{25}$
17. $\frac{1}{7} \div \frac{1}{7}$ $\frac{7}{7} = 1$

Find each product. Simplify if possible.

18. $\frac{1}{2} \times \frac{3}{8}$ $\frac{3}{16}$
19. $\frac{7}{10} \times \frac{2}{3}$ $\frac{14}{30} = \frac{7}{15}$
20. $\frac{4}{5} \times \frac{1}{12}$ $\frac{4}{60} = \frac{1}{15}$

PROBLEM-SOLVING

Drawing a Picture

Dante uses a piece of clay that weighs $\frac{3}{4}$ pound to make ornaments. Will $\frac{3}{4}$ pound of clay be enough to make at least 3 ornaments that weigh $\frac{1}{4}$ pound each? Explain.

a. **Find:** the number of $\frac{1}{4}$ pound ornaments Dante can make with $\frac{3}{4}$ pound of clay

b. **How?** Draw a picture to show $\frac{3}{4}$ divided into 3 equal groups.

c. **Solve.**

There are 3 groups of $\frac{1}{4}$ in $\frac{3}{4}$.

Dante (does) does not) have enough clay for 3 ornaments.

d. **Is the answer reasonable? Explain.** Yes, $\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = \frac{12}{4} = 3$; 3 ornaments can be made from $\frac{3}{4}$ lb of clay.

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CHECK UP

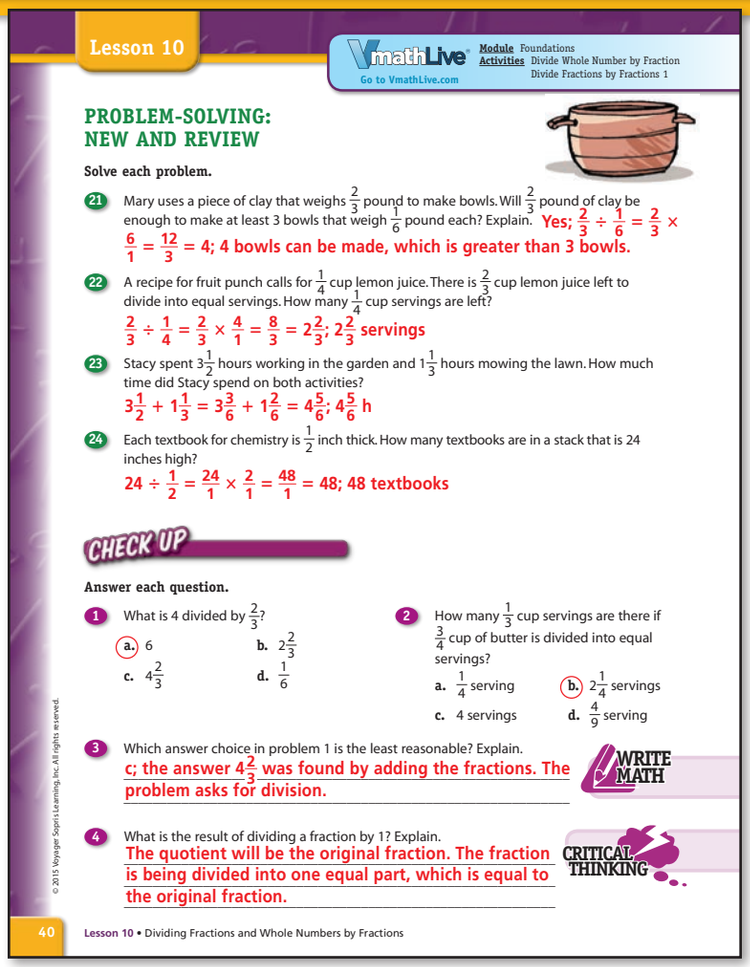
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b or 2a: The student did not find the reciprocal of the fraction before multiplying. Remind the student that to divide, the whole number must be multiplied by the reciprocal of the fraction.

If student answered 1d: The student used the reciprocal of the whole number rather than the reciprocal of the second fraction. Use Additional Resources in I.1.10 to reteach how to divide a whole number by a fraction.



Lesson 10

VmathLive Go to VmathLive.com
Module Activities Foundations
Divide Whole Number by Fraction
Divide Fractions by Fractions 1

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

21 Mary uses a piece of clay that weighs $\frac{2}{3}$ pound to make bowls. Will $\frac{2}{3}$ pound of clay be enough to make at least 3 bowls that weigh $\frac{1}{6}$ pound each? Explain. Yes; $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4$; 4 bowls can be made, which is greater than 3 bowls.

22 A recipe for fruit punch calls for $\frac{1}{4}$ cup lemon juice. There is $\frac{2}{3}$ cup lemon juice left to divide into equal servings. How many $\frac{1}{4}$ cup servings are left?
$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$; $2\frac{2}{3}$ servings

23 Stacy spent $3\frac{1}{2}$ hours working in the garden and $1\frac{1}{3}$ hours mowing the lawn. How much time did Stacy spend on both activities?
$3\frac{1}{2} + 1\frac{1}{3} = 3\frac{3}{6} + 1\frac{2}{6} = 4\frac{5}{6}$; $4\frac{5}{6}$ h

24 Each textbook for chemistry is $\frac{1}{2}$ inch thick. How many textbooks are in a stack that is 24 inches high?
$24 \div \frac{1}{2} = \frac{24}{1} \times \frac{2}{1} = \frac{48}{1} = 48$; 48 textbooks

CHECK UP

Answer each question.

1 What is 4 divided by $\frac{2}{3}$?
(a.) 6 b. $2\frac{2}{3}$ c. $4\frac{2}{3}$ d. $\frac{1}{6}$

2 How many $\frac{1}{3}$ cup servings are there if $\frac{3}{4}$ cup of butter is divided into equal servings?
a. $\frac{1}{4}$ serving (b.) $2\frac{1}{4}$ servings c. 4 servings d. $\frac{4}{9}$ serving

3 Which answer choice in problem 1 is the least reasonable? Explain.
c; the answer $4\frac{2}{3}$ was found by adding the fractions. The problem asks for division.

WRITE MATH

4 What is the result of dividing a fraction by 1? Explain.
The quotient will be the original fraction. The fraction is being divided into one equal part, which is equal to the original fraction.

CRITICAL THINKING

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If student answered 2c: The student used the reciprocal of both the first and the second fraction. Use Additional Resources in I.1.10 to reteach how to divide a fraction by a fraction.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student to read the problem carefully. The words *divided into* indicate division is needed, not multiplication.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Foundations
Activities: Divide Whole Number by Fraction
Divide Fractions by Fractions 1

Vmath Reteach
Reteach Student Book Module 1 Lesson 10
Reteach Teacher Edition Module 1 Lesson 10

Extra Practice
Student Book page 57

English Language Learners

Use the VmathLive Animated Glossary to review the terms *fraction* and *division*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write the problem $2 \div \frac{2}{3}$. Help students understand that $2 \div \frac{2}{3}$ is asking how many groups of $\frac{2}{3}$ are in 2. Have students model the number 2 with 2 whole fraction strips. Then have them use $\frac{1}{3}$-fraction bars to represent 2, placing the $\frac{1}{3}$-fraction bars beneath the 2 whole fraction strips.

Students with Special Needs

Write a problem that shows a fraction divided by a fraction. Have a volunteer come to the board and explain how to solve the division problem. As the student verbally explains each step, he or she should write that step on the board. Pause after each step and ask other students if the step is correct. If it is not correct, students should help correct the step. The volunteer then proceeds to the next step. Continue until the problem is solved.

Lesson 11

Objective

To divide a fraction by a mixed number or a mixed number by a mixed number

Preskills	Lesson
Reciprocals	I.1.9
Dividing Fractions and Whole Numbers by Fractions	I.1.10

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To write the reciprocal of a mixed number, first change the mixed number to an improper fraction. What is $4\frac{3}{5}$ written as an improper fraction? ($\frac{23}{5}$) Then switch the numerator and denominator. What is the reciprocal of $\frac{23}{5}$? ($\frac{5}{23}$) What is the reciprocal of $4\frac{3}{5}$? ($\frac{5}{23}$)

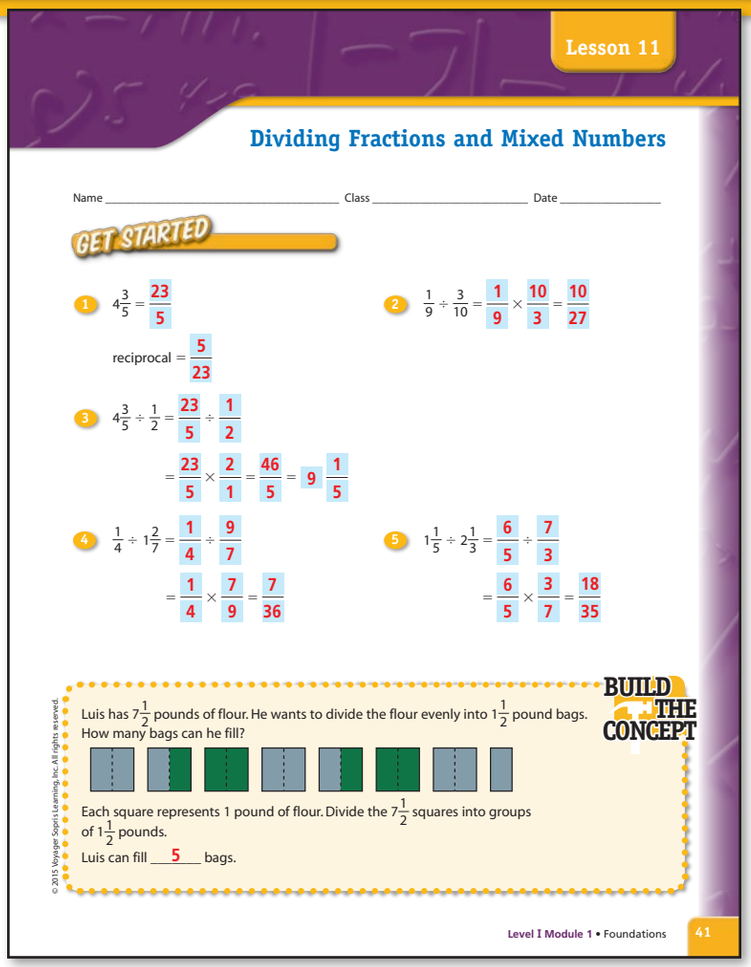
Problem 2

To divide a fraction by a fraction, multiply the first fraction by the reciprocal of the second fraction. What is the second fraction? ($\frac{3}{10}$) What is the reciprocal of $\frac{3}{10}$? ($\frac{10}{3}$) What fractions should be multiplied? ($\frac{1}{9}$ and $\frac{10}{3}$) What is $\frac{1}{9}$ times $\frac{10}{3}$? ($\frac{10}{27}$) Is $\frac{10}{27}$ written in simplest form? (yes) So, $\frac{1}{9}$ divided by $\frac{3}{10}$ is $\frac{10}{27}$.

MODEL NEW SKILLS

Problem 3

To divide a mixed number by a fraction, first write the mixed number as an improper fraction. The mixed number $4\frac{3}{5}$ was written as an improper fraction in problem 1. What is the improper fraction? ($\frac{23}{5}$) Rewrite the problem using the improper fraction, but keep the division sign. The division problem is $\frac{23}{5} \div \frac{1}{2}$. Now, write the problem as a multiplication problem. Multiply the first fraction by the reciprocal of the second fraction.



Lesson 11

Dividing Fractions and Mixed Numbers

Name _____ Class _____ Date _____

GET STARTED

1. $4\frac{3}{5} = \frac{23}{5}$

reciprocal = $\frac{5}{23}$

2. $\frac{1}{9} \div \frac{3}{10} = \frac{1}{9} \times \frac{10}{3} = \frac{10}{27}$

3. $4\frac{3}{5} \div \frac{1}{2} = \frac{23}{5} \div \frac{1}{2}$

$= \frac{23}{5} \times \frac{2}{1} = \frac{46}{5} = 9\frac{1}{5}$

4. $\frac{1}{4} \div 1\frac{2}{7} = \frac{1}{4} \div \frac{9}{7}$

$= \frac{1}{4} \times \frac{7}{9} = \frac{7}{36}$

5. $1\frac{1}{5} \div 2\frac{1}{3} = \frac{6}{5} \div \frac{7}{3}$

$= \frac{6}{5} \times \frac{3}{7} = \frac{18}{35}$

BUILD THE CONCEPT

Luis has $7\frac{1}{2}$ pounds of flour. He wants to divide the flour evenly into $1\frac{1}{2}$ pound bags. How many bags can he fill?

Each square represents 1 pound of flour. Divide the $7\frac{1}{2}$ squares into groups of $1\frac{1}{2}$ pounds.

Luis can fill 5 bags.

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What is the second fraction? ($\frac{1}{2}$) What is the reciprocal of $\frac{1}{2}$? ($\frac{2}{1}$) What fractions should be multiplied? ($\frac{23}{5}$ and $\frac{2}{1}$) What is the product of $\frac{23}{5}$ and $\frac{2}{1}$? ($\frac{46}{5}$) Divide to change the improper fraction to a mixed number. What is $\frac{46}{5}$ written as a mixed number? ($9\frac{1}{5}$) So, $4\frac{3}{5}$ divided by $\frac{1}{2}$ is $9\frac{1}{5}$.

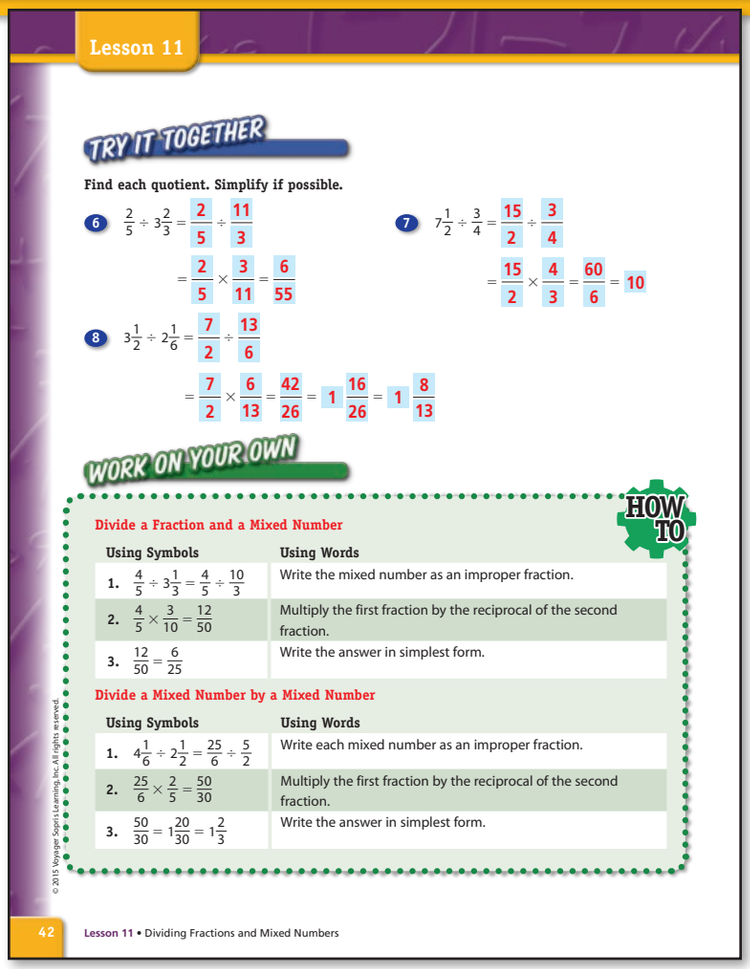
Problem 4

Begin by changing the mixed number to an improper fraction. What is $1\frac{2}{7}$ written as an improper fraction? ($\frac{9}{7}$) Rewrite the problem using the improper fraction, but keep the division sign. How is the division problem written now? ($\frac{1}{4} \div \frac{9}{7}$)

Now, write the division problem as a multiplication problem. Multiply the first fraction by the reciprocal of the second fraction. What is the second fraction? ($\frac{9}{7}$) What is the reciprocal of $\frac{9}{7}$? ($\frac{7}{9}$) What is $\frac{1}{4}$ times $\frac{7}{9}$? ($\frac{7}{36}$) Because $\frac{7}{36}$ is a proper fraction and the GCF of the numerator and denominator is 1, the answer is written in simplest form. So, $\frac{1}{4}$ divided by $1\frac{2}{7}$ is $\frac{7}{36}$.

Problem 5

To divide a mixed number by a mixed number, first change each mixed number to an improper fraction. What is $1\frac{1}{5}$ written as an improper fraction? ($\frac{6}{5}$) What is



Lesson 11

TRY IT TOGETHER

Find each quotient. Simplify if possible.

6 $\frac{2}{5} \div 3\frac{2}{3} = \frac{2}{5} \div \frac{11}{3}$

$= \frac{2}{5} \times \frac{3}{11} = \frac{6}{55}$

7 $7\frac{1}{2} \div \frac{3}{4} = \frac{15}{2} \div \frac{3}{4}$

$= \frac{15}{2} \times \frac{4}{3} = \frac{60}{6} = 10$

8 $3\frac{1}{2} \div 2\frac{1}{6} = \frac{7}{2} \div \frac{13}{6}$

$= \frac{7}{2} \times \frac{6}{13} = \frac{42}{26} = 1\frac{16}{26} = 1\frac{8}{13}$

WORK ON YOUR OWN

HOW TO

Divide a Fraction and a Mixed Number

	Using Symbols	Using Words
1.	$\frac{4}{5} \div 3\frac{1}{3} = \frac{4}{5} \div \frac{10}{3}$	Write the mixed number as an improper fraction.
2.	$\frac{4}{5} \times \frac{3}{10} = \frac{12}{50}$	Multiply the first fraction by the reciprocal of the second fraction.
3.	$\frac{12}{50} = \frac{6}{25}$	Write the answer in simplest form.

Divide a Mixed Number by a Mixed Number

	Using Symbols	Using Words
1.	$4\frac{1}{6} \div 2\frac{1}{2} = \frac{25}{6} \div \frac{5}{2}$	Write each mixed number as an improper fraction.
2.	$\frac{25}{6} \times \frac{2}{5} = \frac{50}{30}$	Multiply the first fraction by the reciprocal of the second fraction.
3.	$\frac{50}{30} = 1\frac{20}{30} = 1\frac{2}{3}$	Write the answer in simplest form.

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What is $2\frac{1}{3}$ written as an improper fraction? ($\frac{7}{3}$) **The division problem now is $\frac{6}{5} \div \frac{7}{3}$.**

Now, write the division problem as a multiplication problem. Multiply the first fraction by the reciprocal of the second fraction. What is the reciprocal of $\frac{7}{3}$? ($\frac{3}{7}$) **What fractions should be multiplied?** ($\frac{6}{5}$ and $\frac{3}{7}$) **What is $\frac{6}{5}$ times $\frac{3}{7}$? Is $\frac{18}{35}$ written in simplest form?** (yes) **So, $1\frac{1}{5}$ divided by $2\frac{1}{3}$ is $\frac{18}{35}$.**

BUILD THE CONCEPT

Model how to divide mixed numbers.

What does Luis want to do? (divide $7\frac{1}{2}$ lb of flour into $1\frac{1}{2}$ lb bags) **What does one square in the model represent?** (1 lb of flour) **The model has groups of $1\frac{1}{2}$ alternating in color.**

How many total groups are divided into $1\frac{1}{2}$ groups? ($7\frac{1}{2}$) **This model represents $7\frac{1}{2}$ divided by $1\frac{1}{2}$. How many equal groups of $1\frac{1}{2}$ are there?** (5) **How many bags can Luis fill?** (5 bags)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

What is the first step when dividing a fraction by a mixed number? (Change the mixed number to an improper fraction.) **What is $3\frac{2}{3}$ written as an improper fraction?** ($\frac{11}{3}$) **Rewrite the division problem using the improper fraction. What is the next step?** (Multiply the first fraction by the reciprocal of the second fraction.) **What fractions should be multiplied?** ($\frac{2}{5}$ and $\frac{3}{11}$) **What is the product?** ($\frac{6}{55}$) **Does the answer need to be simplified?** (no) **Why not?** (The answer is a proper fraction and the GCF of the numerator and the denominator is 1.) **What is $\frac{2}{5}$ divided by $3\frac{2}{3}$?** ($\frac{6}{55}$)

Problem 7

What is $7\frac{1}{2}$ written as an improper fraction? ($\frac{15}{2}$) **What is the reciprocal of the second fraction?** ($\frac{4}{3}$) **What is the product of the first fraction and the reciprocal of the second fraction?** ($\frac{60}{6}$) **Divide to simplify. What is 60 divided by 6?** (10) **What is $7\frac{1}{2}$ divided by $\frac{3}{4}$?** (10)

Problem 8

What must be done first to divide a mixed number by a mixed number? (Write each mixed number as an improper fraction.) **What is $3\frac{1}{2}$ written as an improper fraction?** ($\frac{7}{2}$) **What is $2\frac{1}{6}$ written as an improper fraction?** ($\frac{13}{6}$) **What is the reciprocal of $\frac{13}{6}$?** ($\frac{6}{13}$) **What fractions should be multiplied?** ($\frac{7}{2}$ and $\frac{6}{13}$) **What is $\frac{7}{2}$ times $\frac{6}{13}$?** ($\frac{42}{26}$) **Divide to write the improper fraction as a mixed number. What is 42 divided by 26?** (1 R16) **What is $\frac{42}{26}$ written as a mixed number?** ($1\frac{16}{26}$) **Is the mixed number written in simplest form?** (no) **Simplify the fraction part in the mixed number. What is $3\frac{1}{2}$ divided by $2\frac{1}{6}$ written in simplest form?** ($1\frac{8}{13}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 9–24

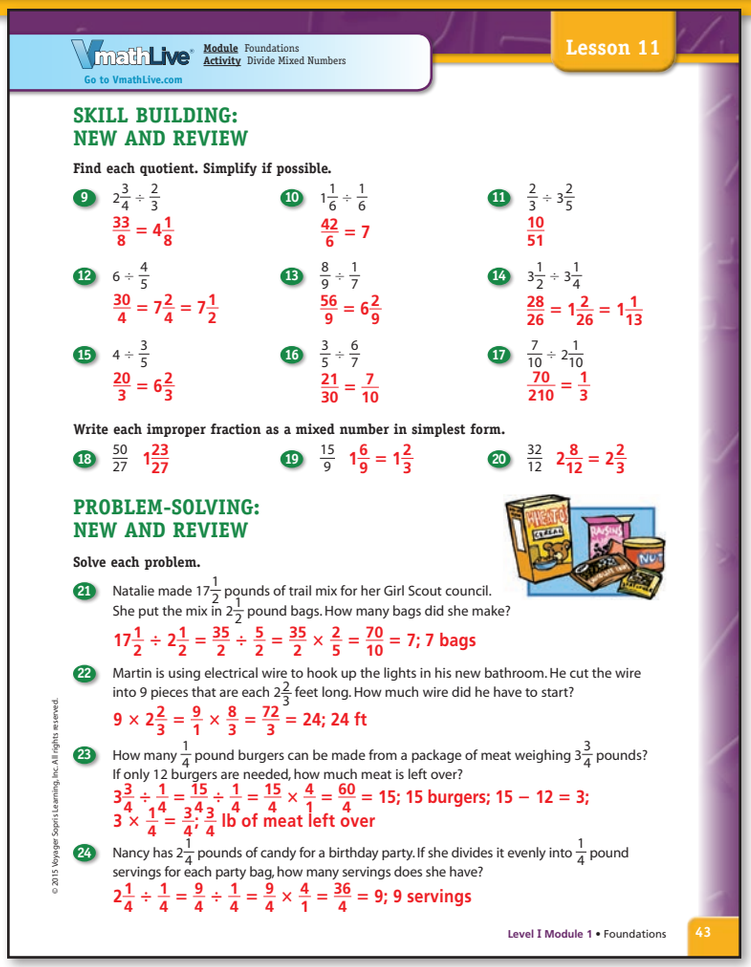
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 21 Reminder

Errors may occur in deciding which number is the first fraction (dividend) and which is the second fraction (divisor) in the division problem. Tell students to determine which of the mixed numbers is being divided into smaller, equal groups. This number should be written first in the division problem.

Problem 23 Reminder

To find the number of $\frac{1}{4}$ pound burgers, students must divide the weight of the meat, $3\frac{3}{4}$ pounds, by the weight of each burger, $\frac{1}{4}$ pound. In this division problem, $3\frac{3}{4}$ is the first fraction and $\frac{1}{4}$ is the divisor. Students will then subtract the number of burgers, 12, from 15 and multiply by $\frac{1}{4}$ to find how much meat is left over.



VmathLive Module Foundations Activity Divide Mixed Numbers
Go to VmathLive.com

Lesson 11

SKILL BUILDING: NEW AND REVIEW

Find each quotient. Simplify if possible.

9. $2\frac{3}{4} \div \frac{2}{3}$ $\frac{33}{8} = 4\frac{1}{8}$
10. $1\frac{1}{6} \div \frac{1}{6}$ $\frac{42}{6} = 7$
11. $\frac{2}{3} \div 3\frac{2}{5}$ $\frac{10}{51}$
12. $6 \div \frac{4}{5}$ $\frac{30}{4} = 7\frac{2}{4} = 7\frac{1}{2}$
13. $\frac{8}{9} \div \frac{1}{7}$ $\frac{56}{9} = 6\frac{2}{9}$
14. $3\frac{1}{2} \div 3\frac{1}{4}$ $\frac{28}{26} = 1\frac{2}{26} = 1\frac{1}{13}$
15. $4 \div \frac{3}{5}$ $\frac{20}{3} = 6\frac{2}{3}$
16. $\frac{3}{5} \div \frac{6}{7}$ $\frac{21}{30} = \frac{7}{10}$
17. $\frac{7}{10} \div 2\frac{1}{10}$ $\frac{70}{210} = \frac{1}{3}$

Write each improper fraction as a mixed number in simplest form.

18. $\frac{50}{27}$ $1\frac{23}{27}$
19. $\frac{15}{9}$ $1\frac{6}{9} = 1\frac{2}{3}$
20. $\frac{32}{12}$ $2\frac{8}{12} = 2\frac{2}{3}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

21. Natalie made $17\frac{1}{2}$ pounds of trail mix for her Girl Scout council. She put the mix in $2\frac{1}{2}$ pound bags. How many bags did she make?
$17\frac{1}{2} \div 2\frac{1}{2} = \frac{35}{2} \div \frac{5}{2} = \frac{35}{2} \times \frac{2}{5} = \frac{70}{10} = 7$; 7 bags
22. Martin is using electrical wire to hook up the lights in his new bathroom. He cut the wire into 9 pieces that are each $2\frac{2}{3}$ feet long. How much wire did he have to start?
$9 \times 2\frac{2}{3} = \frac{9}{1} \times \frac{8}{3} = \frac{72}{3} = 24$; 24 ft
23. How many $\frac{1}{4}$ pound burgers can be made from a package of meat weighing $3\frac{3}{4}$ pounds? If only 12 burgers are needed, how much meat is left over?
$3\frac{3}{4} \div \frac{1}{4} = \frac{15}{4} \div \frac{1}{4} = \frac{15}{4} \times \frac{4}{1} = \frac{60}{4} = 15$; 15 burgers; $15 - 12 = 3$; $3 \times \frac{1}{4} = \frac{3}{4}$; $\frac{3}{4}$ lb of meat left over
24. Nancy has $2\frac{1}{4}$ pounds of candy for a birthday party. If she divides it evenly into $\frac{1}{4}$ pound servings for each party bag, how many servings does she have?
$2\frac{1}{4} \div \frac{1}{4} = \frac{9}{4} \div \frac{1}{4} = \frac{9}{4} \times \frac{4}{1} = \frac{36}{4} = 9$; 9 servings

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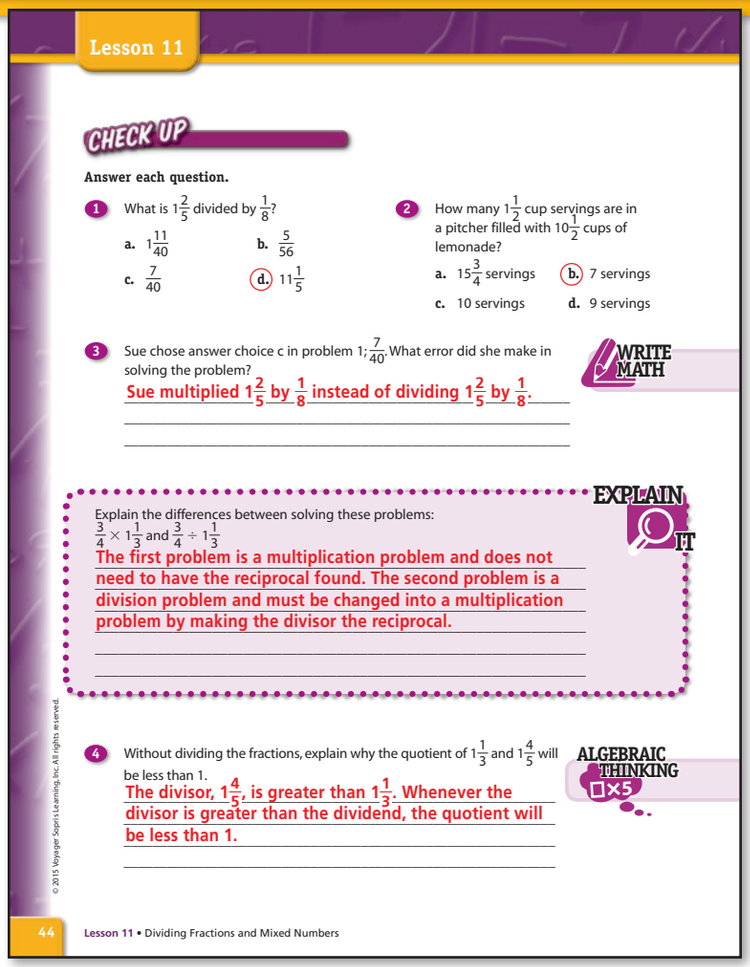
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b: The student used the reciprocal of the first fraction instead of the second fraction when multiplying. Use Additional Resources in I.1.11 to reteach how to divide fractions and mixed numbers.



Lesson 11

CHECK UP

Answer each question.

1. What is $1\frac{2}{5}$ divided by $\frac{1}{8}$?
   a. $1\frac{11}{40}$ b. $\frac{5}{56}$ c. $\frac{7}{40}$ d. $11\frac{1}{5}$

2. How many $1\frac{1}{2}$ cup servings are in a pitcher filled with $10\frac{1}{2}$ cups of lemonade?
   a. $15\frac{3}{4}$ servings b. 7 servings c. 10 servings d. 9 servings

3. Sue chose answer choice c in problem 1: $\frac{7}{40}$. What error did she make in solving the problem?
   Sue multiplied $1\frac{2}{5}$ by $\frac{1}{8}$ instead of dividing $1\frac{2}{5}$ by $\frac{1}{8}$.

WRITE MATH

EXPLAIN IT

Explain the differences between solving these problems:
$\frac{3}{4} \times 1\frac{1}{3}$ and $\frac{3}{4} \div 1\frac{1}{3}$
The first problem is a multiplication problem and does not need to have the reciprocal found. The second problem is a division problem and must be changed into a multiplication problem by making the divisor the reciprocal.

4. Without dividing the fractions, explain why the quotient of $1\frac{1}{3}$ and $1\frac{4}{5}$ will be less than 1.
   The divisor, $1\frac{4}{5}$, is greater than $1\frac{1}{3}$. Whenever the divisor is greater than the dividend, the quotient will be less than 1.

ALGEBRAIC THINKING □×5

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If student answered 1c or 2a: The student did not find the reciprocal of the second fraction. Remind the student that when dividing fractions and mixed numbers, the first fraction must be multiplied by the reciprocal of the second fraction. If necessary, use Additional Resources in I.1.11 to reteach how to divide fractions and mixed numbers.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that dividing by a fraction is carried out by multiplying by the reciprocal of the fraction.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Foundations
Activity: Divide Mixed Numbers

Vmath Reteach
Reteach Student Book Module 1 Lesson 11
Reteach Teacher Edition Module 1 Lesson 11

Extra Practice
Student Book page 57

English Language Learners

Use the VmathLive Animated Glossary to review the terms *mixed number* and *fraction*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write some proper fractions, improper fractions, and mixed numbers on the board. Have students make a chart with the following column headings: *Proper Fractions, Improper Fractions,* and *Mixed Numbers*. Direct students to copy the numbers on the board into the proper columns.

Students with Special Needs

Write the mixed number $2\frac{3}{4}$ on the board. Have students model the number with $\frac{1}{4}$-fraction strips. Ask students to count the total number of fourths in $2\frac{3}{4}$. Students should realize that there are 11 fourths. Have students work in pairs to write the mixed number $2\frac{3}{4}$ as an improper fraction.

Lesson 12

Objective

To write a rational number in the form $\frac{a}{b}$

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **rational number**

  a number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not 0

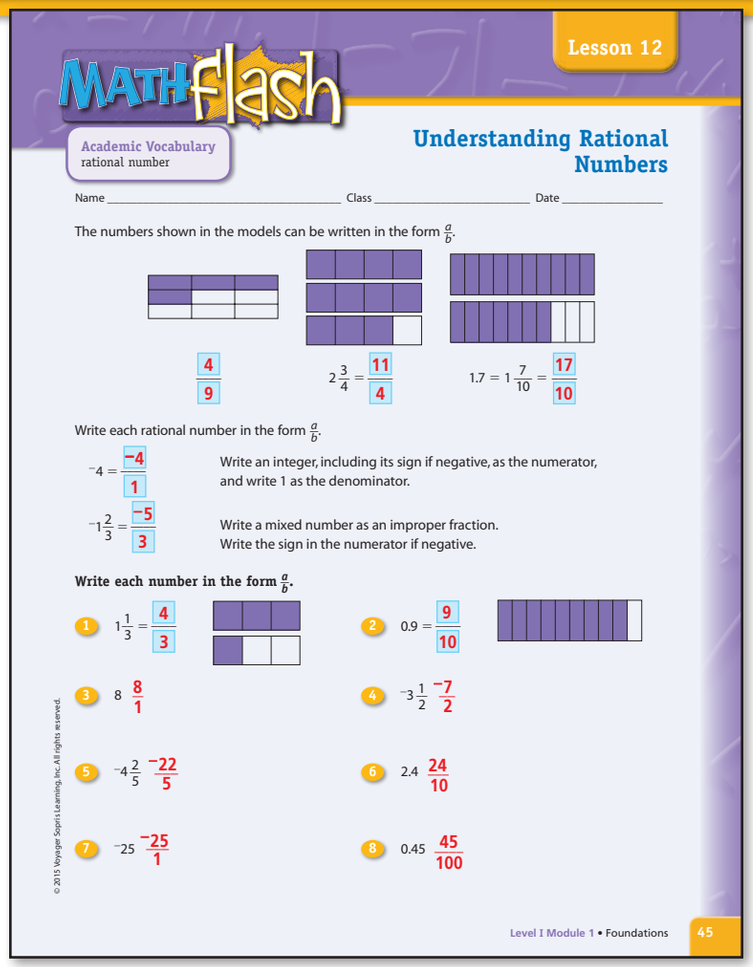
Model the following skills for students.

Look at the model on the left. How many total parts are in the model? (9) **How many parts are shaded?** (4) **What fraction is represented by the model?** ($\frac{4}{9}$) **What number is *a*?** (4) **What number is *b*?** (9) **A fraction is a rational number because it is in the form $\frac{a}{b}$. The model in the middle represents $2\frac{3}{4}$. How is $2\frac{3}{4}$ written as an improper fraction?** ($\frac{11}{4}$) **What number is *a*?** (11) **What number is *b*?** (4) **So, $2\frac{3}{4}$ is a rational number. Why?** (It can be written in the form $\frac{a}{b}$.)

The model on the right represents 1.7. The decimal 1.7 can be written as the mixed number $1\frac{7}{10}$. How is $1\frac{7}{10}$ written as an improper fraction? ($\frac{17}{10}$) **Is 1.7 a rational number?** (yes)

Work with students to complete the following skills.

To write an integer as a rational number, write the integer and its sign as the numerator, and write 1 as the denominator. What is $^-4$ written as a rational number? ($\frac{-4}{1}$) **To write a mixed number as a rational number, write the mixed number as an improper fraction. If negative, write the negative sign in the numerator. What is the numerator of the improper fraction for $^-1\frac{2}{3}$?** ($^-5$) **Why?** (3 times 1 plus 2 equals 5, and the negative sign is written in the numerator) **What is $^-1\frac{2}{3}$ written as a rational number?** ($\frac{-5}{3}$)



MATH Flash

Lesson 12

Academic Vocabulary
rational number

Understanding Rational Numbers

Name _____ Class _____ Date _____

The numbers shown in the models can be written in the form $\frac{a}{b}$.

$\frac{4}{9}$ $2\frac{3}{4} = \frac{11}{4}$ $1.7 = 1\frac{7}{10} = \frac{17}{10}$

Write each rational number in the form $\frac{a}{b}$.

$^-4 = \frac{-4}{1}$ Write an integer, including its sign if negative, as the numerator, and write 1 as the denominator.

$^-1\frac{2}{3} = \frac{-5}{3}$ Write a mixed number as an improper fraction. Write the sign in the numerator if negative.

Write each number in the form $\frac{a}{b}$.

1. $1\frac{1}{3} = \frac{4}{3}$
2. $0.9 = \frac{9}{10}$
3. $8\ \frac{8}{1}$
4. $^-3\frac{1}{2}\ \frac{-7}{2}$
5. $^-4\frac{2}{5}\ \frac{-22}{5}$
6. $2.4\ \frac{24}{10}$
7. $^-25\ \frac{-25}{1}$
8. $0.45\ \frac{45}{100}$

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Lesson 13

Objective

To add positive and negative fractions

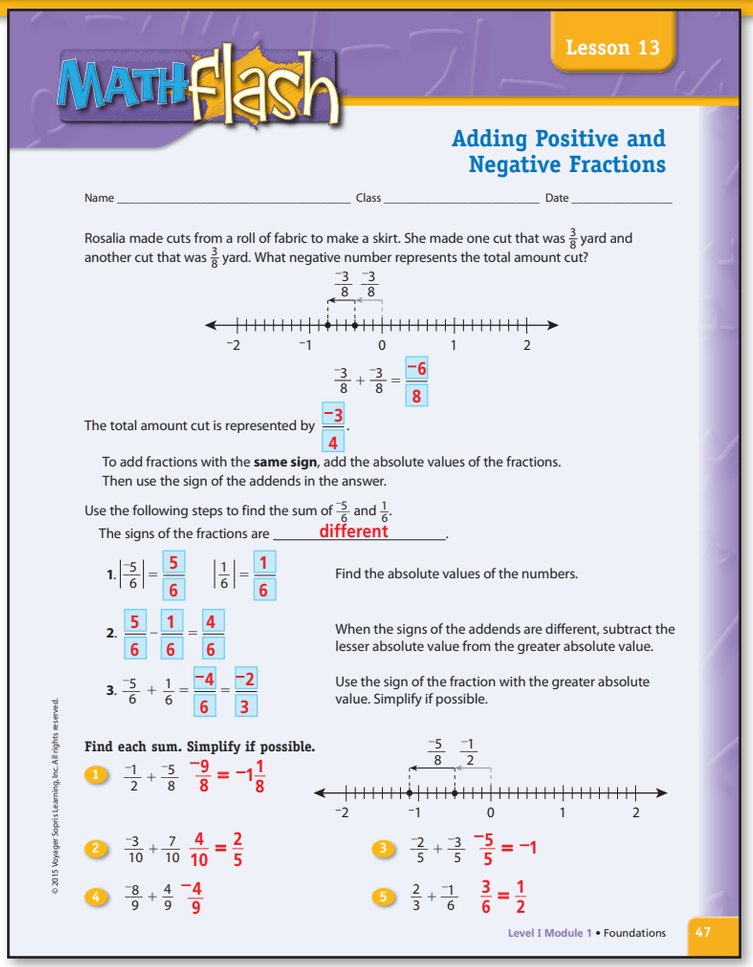
Model the following skills for students.

How can each cut of $\frac{3}{8}$ yard from the fabric be represented as a negative fraction? ($\frac{-3}{8}$) **To find the total amount cut, what operation should be used?** (addition) **The problem is modeled by $\frac{-3}{8} \cdot \frac{-3}{8}$. This sum is shown on the number line. According to the number line, what is the sum?** ($\frac{-6}{8}$) **The fraction $\frac{-6}{8}$ simplifies to $\frac{-3}{4}$. Reread the problem. Does a negative answer make sense?** (yes) **Why?** (The two cuts are represented by negative fractions, so the total amount cut should be negative.)

To add two fractions with the same sign, add their absolute values. Then use the sign of the addends in the answer.

Work with students to complete the following skill.

To find the sum of $\frac{-5}{6}$ and $\frac{1}{6}$, first look at the signs. Are the signs the same or different? (different) **When the signs of the addends are different, subtract the lesser absolute value from the greater absolute value. What are the absolute values of $\frac{-5}{6}$ and $\frac{1}{6}$?** ($\frac{5}{6}$ and $\frac{1}{6}$) **Which absolute value is greater?** ($\frac{5}{6}$) **What is $\frac{5}{6}$ minus $\frac{1}{6}$?** ($\frac{4}{6}$) **Which fraction has the greater absolute value?** ($\frac{-5}{6}$) **What is the sign of $\frac{-5}{6}$?** (negative) **What is $\frac{-5}{6}$ plus $\frac{1}{6}$?** ($\frac{-4}{6}$) **What is $\frac{-4}{6}$ written in simplest form?** ($\frac{-2}{3}$)



MATH Flash

Lesson 13

Adding Positive and Negative Fractions

Name _____ Class _____ Date _____

Rosalia made cuts from a roll of fabric to make a skirt. She made one cut that was $\frac{3}{8}$ yard and another cut that was $\frac{3}{8}$ yard. What negative number represents the total amount cut?

$\frac{-3}{8}$ $\frac{-3}{8}$

−2 −1 0 1 2

$\frac{-3}{8} + \frac{-3}{8} = \frac{-6}{8}$

The total amount cut is represented by $\frac{-3}{4}$.

To add fractions with the **same sign**, add the absolute values of the fractions. Then use the sign of the addends in the answer.

Use the following steps to find the sum of $\frac{-5}{6}$ and $\frac{1}{6}$.

The signs of the fractions are different.

1. $\left|\frac{-5}{6}\right| = \frac{5}{6}$ $\left|\frac{1}{6}\right| = \frac{1}{6}$ Find the absolute values of the numbers.
2. $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ When the signs of the addends are different, subtract the lesser absolute value from the greater absolute value.
3. $\frac{-5}{6} + \frac{1}{6} = \frac{-4}{6} = \frac{-2}{3}$ Use the sign of the fraction with the greater absolute value. Simplify if possible.

Find each sum. Simplify if possible.

1. $\frac{-1}{2} + \frac{-5}{8} = \frac{-9}{8} = -1\frac{1}{8}$

$\frac{-5}{8}$ $\frac{-1}{2}$

−2 −1 0 1 2

2. $\frac{-3}{10} + \frac{7}{10} = \frac{4}{10} = \frac{2}{5}$
3. $\frac{-2}{5} + \frac{-3}{5} = \frac{-5}{5} = -1$
4. $\frac{-8}{9} + \frac{4}{9} = \frac{-4}{9}$
5. $\frac{2}{3} + \frac{-1}{6} = \frac{3}{6} = \frac{1}{2}$

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Technology

Using an online search engine, have students or partners research careers that would need to add positive and negative fractions. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

Lesson 14

Objective

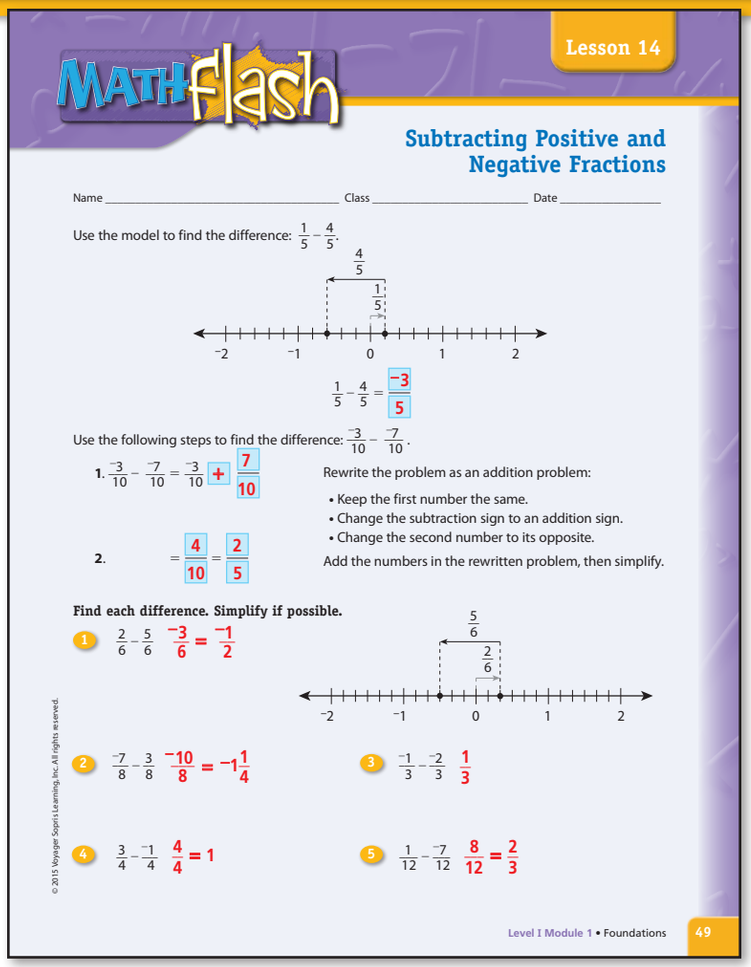
To subtract positive and negative fractions

Model the following skill for students.

A number line can be used to subtract fractions. Subtracting a positive fraction on a number line is the same as adding a negative fraction on a number line. To subtract a positive fraction on a number line, move to the left on the number line. To subtract $\frac{4}{5}$ from $\frac{1}{5}$, first find $\frac{1}{5}$ on the number line. To subtract $\frac{4}{5}$, move $\frac{4}{5}$ unit to the left. What is $\frac{1}{5}$ minus $\frac{4}{5}$? ($\frac{-3}{5}$) How does $\frac{4}{5}$ compare to $\frac{1}{5}$? ($\frac{4}{5}$ is greater than $\frac{1}{5}$.) Because the number being subtracted is greater, the result is negative.

Work with students to complete the following skill.

To subtract $\frac{-7}{10}$ from $\frac{-3}{10}$, rewrite the problem as an addition problem. To do this, keep the first number the same. Then change the subtraction sign to an addition sign. Next, change the second number to its opposite. What is the opposite of $\frac{-7}{10}$? ($\frac{7}{10}$) How is the subtraction problem rewritten as an addition problem? ($\frac{-3}{10} + \frac{7}{10}$) What is $\frac{-3}{10}$ minus $\frac{-7}{10}$? ($\frac{4}{10}$) What is the simplest form of $\frac{4}{10}$? ($\frac{2}{5}$)



MATH Flash

Lesson 14

Subtracting Positive and Negative Fractions

Name _____ Class _____ Date _____

Use the model to find the difference: $\frac{1}{5} - \frac{4}{5}$.

$\frac{1}{5} - \frac{4}{5} = \frac{-3}{5}$

Use the following steps to find the difference: $\frac{-3}{10} - \frac{-7}{10}$.

1. $\frac{-3}{10} - \frac{-7}{10} = \frac{-3}{10} + \frac{7}{10}$

Rewrite the problem as an addition problem:

- Keep the first number the same.
- Change the subtraction sign to an addition sign.
- Change the second number to its opposite.

2. $= \frac{4}{10} = \frac{2}{5}$

Add the numbers in the rewritten problem, then simplify.

Find each difference. Simplify if possible.

1. $\frac{2}{6} - \frac{5}{6}$ $\frac{-3}{6} = \frac{-1}{2}$

2. $\frac{-7}{8} - \frac{3}{8}$ $\frac{-10}{8} = -1\frac{1}{4}$

3. $\frac{-1}{3} - \frac{-2}{3}$ $\frac{1}{3}$

4. $\frac{3}{4} - \frac{-1}{4}$ $\frac{4}{4} = 1$

5. $\frac{1}{12} - \frac{-7}{12}$ $\frac{8}{12} = \frac{2}{3}$

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Lesson 15

Objective

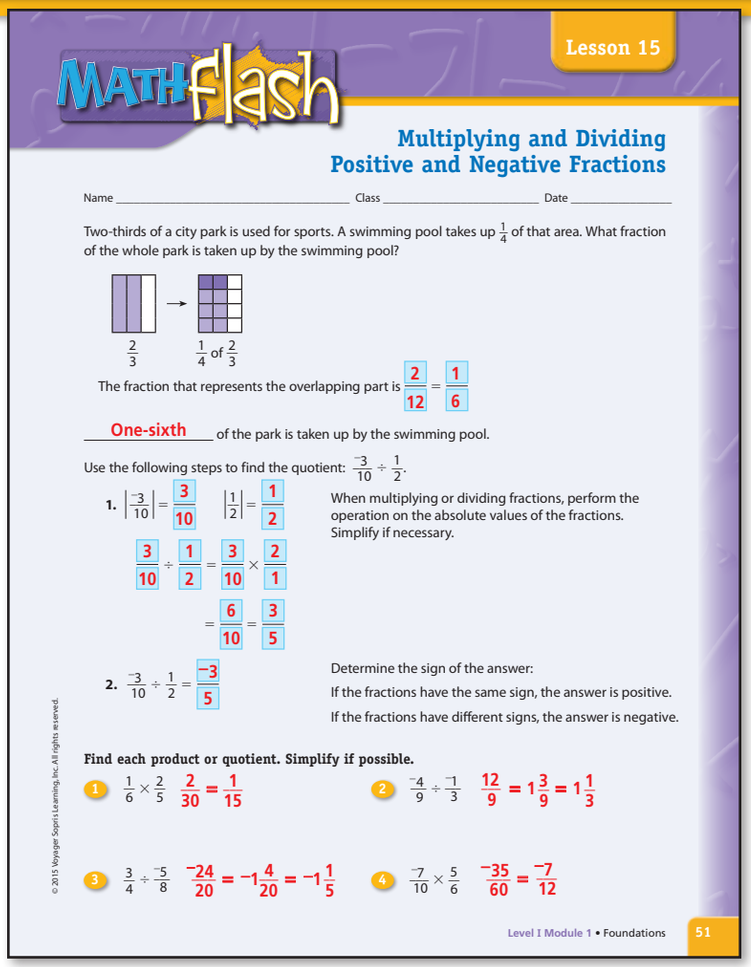
To multiply and divide positive and negative fractions

Model the following skill for students.

In the first model, the shaded area represents the $\frac{2}{3}$ of the park that is used for sports. Now look at the second model. The swimming pool takes up $\frac{1}{4}$ of the area that is used for sports. This is the dark purple area of the second model. How many sections are shaded dark purple? (2) These 2 sections represent the area of the park that is taken up by the swimming pool. How many total sections are there? (12) What fraction of the park is taken up by the swimming pool? ($\frac{2}{12}$) What is $\frac{2}{12}$ written in simplest form? ($\frac{1}{6}$) So, $\frac{1}{6}$ of the park is taken up by the swimming pool.

Work with students to complete the following skill.

To divide $\frac{-3}{10}$ by $\frac{1}{2}$, first divide the absolute values of the numbers. Then determine the sign of the quotient. What are the absolute values of $\frac{-3}{10}$ and $\frac{1}{2}$? ($\frac{3}{10}$ and $\frac{1}{2}$) Now divide using the absolute values. To divide a fraction by a fraction, multiply the first fraction by the reciprocal of the second fraction. What is the reciprocal of $\frac{1}{2}$? ($\frac{2}{1}$) What numbers should be multiplied? ($\frac{3}{10}$ and $\frac{2}{1}$) What is $\frac{3}{10}$ times $\frac{2}{1}$? ($\frac{6}{10}$) What is $\frac{6}{10}$ written in simplest form? ($\frac{3}{5}$) Are the signs of the fractions in the original problem different? (yes) So, the quotient is negative. What is $\frac{-3}{10}$ divided by $\frac{1}{2}$? ($\frac{-3}{5}$)



MATH Flash

Lesson 15

Multiplying and Dividing Positive and Negative Fractions

Name _____ Class _____ Date _____

Two-thirds of a city park is used for sports. A swimming pool takes up $\frac{1}{4}$ of that area. What fraction of the whole park is taken up by the swimming pool?

$\frac{2}{3}$ → $\frac{1}{4}$ of $\frac{2}{3}$

The fraction that represents the overlapping part is $\frac{2}{12} = \frac{1}{6}$

One-sixth of the park is taken up by the swimming pool.

Use the following steps to find the quotient: $\frac{-3}{10} \div \frac{1}{2}$.

1. $\left|\frac{-3}{10}\right| = \frac{3}{10}$ $\left|\frac{1}{2}\right| = \frac{1}{2}$

$\frac{3}{10} \div \frac{1}{2} = \frac{3}{10} \times \frac{2}{1}$

$= \frac{6}{10} = \frac{3}{5}$

When multiplying or dividing fractions, perform the operation on the absolute values of the fractions. Simplify if necessary.

2. $\frac{-3}{10} \div \frac{1}{2} = \frac{-3}{5}$

Determine the sign of the answer:

If the fractions have the same sign, the answer is positive.

If the fractions have different signs, the answer is negative.

Find each product or quotient. Simplify if possible.

1. $\frac{1}{6} \times \frac{2}{5}$ $\frac{2}{30} = \frac{1}{15}$

2. $\frac{-4}{9} \div \frac{-1}{3}$ $\frac{12}{9} = 1\frac{3}{9} = 1\frac{1}{3}$

3. $\frac{3}{4} \div \frac{-5}{8}$ $\frac{-24}{20} = -1\frac{4}{20} = -1\frac{1}{5}$

4. $\frac{-7}{10} \times \frac{5}{6}$ $\frac{-35}{60} = \frac{-7}{12}$

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Technology

Using an online search engine, have students or partners research careers that would need the fraction skills learned in this module. Encourage them to describe how and why these concepts would be needed in specific careers. Additional digital content is available through this feature in the eBook.

Extra Practice Answer Key

Extra Practice

Name _____ Class _____ Date _____

Lesson 1 Properties of Multiplication

Name the property shown. Use the property to find the missing numbers.

1. $6 \times 9 = 9 \times \underline{6} = \underline{54}$
   Commutative Property of Multiplication
2. $8 \times (2 \times 4) = (8 \times 2) \times \underline{4} = \underline{64}$
   Associative Property of Multiplication
3. $7 \times 4 = 4 \times \underline{7} = \underline{28}$
   Commutative Property of Multiplication
4. $\underline{8} \times 5 = 5 \times 8 = \underline{40}$
   Commutative Property of Multiplication
5. $(6 \times 3) \times 2 = 6 \times (3 \times \underline{2}) = \underline{36}$
   Associative Property of Multiplication
6. $4 \times (6 \times 1) = (4 \times 6) \times \underline{1} = \underline{24}$
   Associative Property of Multiplication
7. $9 \times 8 = 8 \times \underline{9} = \underline{72}$
   Commutative Property of Multiplication
8. $(7 \times 5) \times 2 = 7 \times (5 \times \underline{2}) = \underline{70}$
   Associative Property of Multiplication

Lesson 2 Multiplying with Multiples of 10

Find each product. Regroup as needed.

1. $700 \times 4 = 2{,}800$
2. $28 \times 20 = 560$
3. $4{,}000 \times 5 = 20{,}000$
4. $190 \times 9 = 1{,}710$
5. $80 \times 9 = 720$
6. $3{,}800 \times 7 = 26{,}600$
7. $69 \times 50 = 3{,}450$
8. $9 \times 900 = 8{,}100$
9. $9{,}000 \times 4 = 36{,}000$

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Extra Practice

Name _____ Class _____ Date _____

Lesson 3 Adding Fractions

Find each sum. Simplify if possible.

1. $\frac{3}{7} + \frac{5}{7} = \frac{8}{7} = 1\frac{1}{7}$
2. $\frac{3}{4} + \frac{1}{3} = \frac{13}{12} = 1\frac{1}{12}$
3. $\frac{2}{5} + \frac{7}{15} = \frac{13}{15}$
4. $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$
5. $\frac{9}{10} + \frac{1}{2} = \frac{14}{10} = 1\frac{4}{10} = 1\frac{2}{5}$
6. $\frac{4}{7} + \frac{10}{21} = \frac{22}{21} = 1\frac{1}{21}$
7. $\frac{2}{3} + \frac{5}{6} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$
8. $\frac{3}{8} + \frac{1}{4} = \frac{5}{8}$
9. $\frac{4}{5} + \frac{9}{10} = \frac{17}{10} = 1\frac{7}{10}$
10. $\frac{8}{9} + \frac{5}{9} = \frac{13}{9} = 1\frac{4}{9}$
11. $\frac{1}{6} + \frac{7}{12} = \frac{9}{12} = \frac{3}{4}$
12. $\frac{3}{10} + \frac{3}{4} = \frac{21}{20} = 1\frac{1}{20}$

Lesson 4 Subtracting Fractions

Find each difference. Simplify if possible.

1. $\frac{7}{8} - \frac{1}{4} = \frac{5}{8}$
2. $\frac{4}{6} - \frac{7}{12} = \frac{1}{12}$
3. $\frac{9}{10} - \frac{1}{5} = \frac{7}{10}$
4. $\frac{12}{15} - \frac{3}{5} = \frac{3}{15} = \frac{1}{5}$
5. $\frac{19}{20} - \frac{4}{10} = \frac{11}{20}$
6. $\frac{12}{16} - \frac{3}{8} = \frac{6}{16} = \frac{3}{8}$
7. $\frac{2}{3} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$
8. $\frac{11}{12} - \frac{3}{4} = \frac{2}{12} = \frac{1}{6}$
9. $\frac{9}{10} - \frac{3}{5} = \frac{3}{10}$
10. $\frac{8}{9} - \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$
11. $\frac{13}{16} - \frac{5}{8} = \frac{3}{16}$
12. $\frac{13}{14} - \frac{4}{7} = \frac{5}{14}$

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Extra Practice

Name _____ Class _____ Date _____

Lesson 5 Adding Mixed Numbers

Find each sum. Simplify if possible.

1. $4\frac{3}{4} + 5\frac{3}{4} = 9\frac{6}{4} = 10\frac{2}{4} = 10\frac{1}{2}$
2. $5\frac{7}{16} + 2\frac{5}{16} = 7\frac{12}{16} = 7\frac{3}{4}$
3. $6\frac{3}{7} + 1\frac{5}{7} = 7\frac{8}{7} = 8\frac{1}{7}$
4. $2\frac{3}{5} + 3\frac{1}{2} = 5\frac{11}{10} = 6\frac{1}{10}$
5. $9\frac{2}{3} + 4\frac{4}{9} = 13\frac{10}{9} = 14\frac{1}{9}$
6. $1\frac{5}{8} + 6\frac{5}{6} = 7\frac{35}{24} = 8\frac{11}{24}$
7. $4\frac{1}{3} + 5\frac{3}{4} = 9\frac{13}{12} = 10\frac{1}{12}$
8. $7\frac{2}{5} + 5\frac{5}{6} = 12\frac{37}{30} = 13\frac{7}{30}$
9. $3\frac{9}{10} + 2\frac{4}{5} = 5\frac{17}{10} = 6\frac{7}{10}$
10. $8\frac{3}{8} + 6\frac{2}{24} = 14\frac{11}{24}$
11. $2\frac{2}{3} + 4\frac{1}{7} = 6\frac{17}{21}$
12. $1\frac{13}{15} + 2\frac{1}{10} = 3\frac{29}{30}$

Lesson 6 Subtracting Mixed Numbers

Find each difference. Simplify if possible.

1. $9\frac{5}{7} - 4\frac{3}{7} = 5\frac{2}{7}$
2. $12\frac{3}{5} - 8\frac{4}{5} = 3\frac{4}{5}$
3. $15\frac{3}{4} - 9\frac{1}{2} = 6\frac{1}{4}$
4. $7\frac{1}{9} - 4\frac{7}{9} = 2\frac{3}{9} = 2\frac{1}{3}$
5. $12\frac{7}{8} - 5\frac{1}{2} = 7\frac{3}{8}$
6. $9\frac{1}{3} - 2\frac{7}{15} = 6\frac{13}{15}$
7. $7\frac{3}{4} - 3\frac{2}{3} = 4\frac{1}{12}$
8. $11\frac{7}{8} - 5\frac{1}{2} = 6\frac{3}{8}$
9. $3\frac{9}{10} - 1\frac{2}{5} = 2\frac{5}{10} = 2\frac{1}{2}$
10. $6\frac{1}{8} - 5\frac{13}{16} = \frac{5}{16}$
11. $16\frac{2}{3} - 14\frac{1}{6} = 2\frac{3}{6} = 2\frac{1}{2}$
12. $8\frac{2}{5} - 6\frac{3}{4} = 1\frac{13}{20}$

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Extra Practice

Name _____ Class _____ Date _____

Lesson 8 Multiplying Fractions and Mixed Numbers

Find each product. Simplify if possible.

1. $1\frac{1}{4} \times 1\frac{2}{3} = \frac{25}{12} = 2\frac{1}{12}$
2. $2\frac{3}{5} \times 1\frac{1}{3} = \frac{52}{15} = 3\frac{7}{15}$
3. $3\frac{1}{3} \times 1\frac{2}{7} = \frac{90}{21} = 4\frac{6}{21} = 4\frac{2}{7}$
4. $3\frac{2}{3} \times \frac{1}{4} = \frac{11}{12}$
5. $\frac{5}{7} \times 1\frac{1}{5} = \frac{30}{35} = \frac{6}{7}$
6. $5\frac{1}{3} \times \frac{2}{5} = \frac{32}{15} = 2\frac{2}{15}$
7. $7\frac{3}{5} \times \frac{2}{7} = \frac{76}{35} = 2\frac{6}{35}$
8. $\frac{3}{8} \times 3\frac{1}{6} = \frac{57}{48} = 1\frac{9}{48} = 1\frac{3}{16}$
9. $\frac{9}{10} \times 2\frac{2}{9} = \frac{180}{90} = 2$
10. $5\frac{3}{10} \times \frac{1}{4} = \frac{53}{40} = 1\frac{13}{40}$
11. $\frac{3}{8} \times 2\frac{4}{5} = \frac{42}{40} = 1\frac{2}{40} = 1\frac{1}{20}$
12. $4\frac{2}{3} \times \frac{5}{12} = \frac{70}{36} = 1\frac{34}{36} = 1\frac{17}{18}$

Lesson 9 Reciprocals

Find the reciprocal of each number.

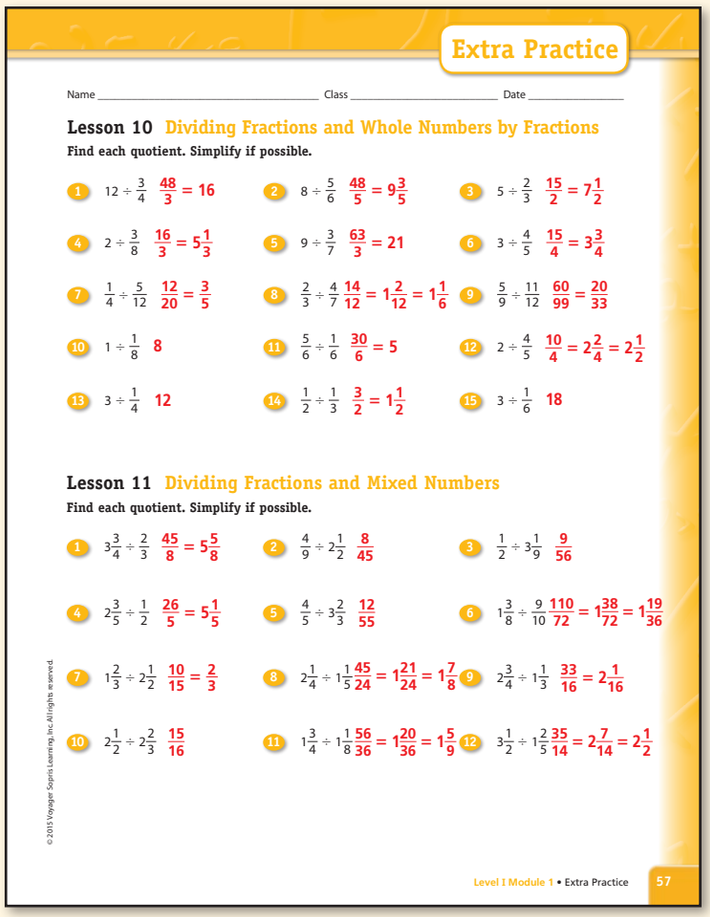
1. $6 \quad \frac{1}{6}$
2. $\frac{3}{5} \quad \frac{5}{3}$
3. $\frac{1}{12} \quad \frac{12}{1} = 12$
4. $3\frac{1}{3} \quad \frac{3}{10}$
5. $\frac{11}{12} \quad \frac{12}{11}$
6. $3\frac{4}{7} \quad \frac{7}{25}$

Find each missing number.

7. $87 \times \frac{1}{87} = 1$
8. $\frac{1}{8} \times \frac{8}{1} = 1$
9. $\frac{3}{8} \times \frac{8}{3} = 1$
10. $9\frac{3}{8} \times \frac{8}{75} = 1$
11. $\frac{11}{13} \times \frac{13}{11} = 1$
12. $3\frac{6}{9} \times \frac{9}{33} = 1$

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Extra Practice Answer Key



Extra Practice

Name _____ Class _____ Date _____

Lesson 10 Dividing Fractions and Whole Numbers by Fractions

Find each quotient. Simplify if possible.

1. $12 \div \frac{3}{4}$ $\frac{48}{3} = 16$
2. $8 \div \frac{5}{6}$ $\frac{48}{5} = 9\frac{3}{5}$
3. $5 \div \frac{2}{3}$ $\frac{15}{2} = 7\frac{1}{2}$
4. $2 \div \frac{3}{8}$ $\frac{16}{3} = 5\frac{1}{3}$
5. $9 \div \frac{3}{7}$ $\frac{63}{3} = 21$
6. $3 \div \frac{4}{5}$ $\frac{15}{4} = 3\frac{3}{4}$
7. $\frac{1}{4} \div \frac{5}{12}$ $\frac{12}{20} = \frac{3}{5}$
8. $\frac{2}{3} \div \frac{4}{7}$ $\frac{14}{12} = 1\frac{2}{12} = 1\frac{1}{6}$
9. $\frac{5}{9} \div \frac{11}{12}$ $\frac{60}{99} = \frac{20}{33}$
10. $1 \div \frac{1}{8}$ 8
11. $\frac{5}{6} \div \frac{1}{6}$ $\frac{30}{6} = 5$
12. $2 \div \frac{4}{5}$ $\frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$
13. $3 \div \frac{1}{4}$ 12
14. $\frac{1}{2} \div \frac{1}{3}$ $\frac{3}{2} = 1\frac{1}{2}$
15. $3 \div \frac{1}{6}$ 18

Lesson 11 Dividing Fractions and Mixed Numbers

Find each quotient. Simplify if possible.

1. $3\frac{3}{4} \div \frac{2}{3}$ $\frac{45}{8} = 5\frac{5}{8}$
2. $\frac{4}{9} \div 2\frac{1}{2}$ $\frac{8}{45}$
3. $\frac{1}{2} \div 3\frac{1}{9}$ $\frac{9}{56}$
4. $2\frac{3}{5} \div \frac{1}{2}$ $\frac{26}{5} = 5\frac{1}{5}$
5. $\frac{4}{5} \div 3\frac{2}{3}$ $\frac{12}{55}$
6. $1\frac{3}{8} \div \frac{9}{10}$ $\frac{110}{72} = 1\frac{38}{72} = 1\frac{19}{36}$
7. $1\frac{2}{3} \div 2\frac{1}{2}$ $\frac{10}{15} = \frac{2}{3}$
8. $2\frac{1}{4} \div 1\frac{1}{5}$ $\frac{45}{24} = 1\frac{21}{24} = 1\frac{7}{8}$
9. $2\frac{3}{4} \div 1\frac{1}{3}$ $\frac{33}{16} = 2\frac{1}{16}$
10. $2\frac{1}{2} \div 2\frac{2}{3}$ $\frac{15}{16}$
11. $1\frac{3}{4} \div 1\frac{1}{8}$ $\frac{56}{36} = 1\frac{20}{36} = 1\frac{5}{9}$
12. $3\frac{1}{2} \div 1\frac{2}{5}$ $\frac{35}{14} = 2\frac{7}{14} = 2\frac{1}{2}$

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Level I Module 1 • Extra Practice 57

Glossary

Associative Property of Multiplication
three factors can be grouped in different ways and the product will be the same

Commutative Property of Multiplication
two factors can be multiplied in any order and the product will be the same

multiple
the product of the whole number and another whole number (Example: $4 \times 10 = 40$; 40 is the fourth multiple of 10.)

rational number
a number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not 0

reciprocal
for any nonzero number x, the reciprocal is $\frac{1}{x}$; the number 0 has no reciprocal; the reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$

Real Numbers

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Lesson PL1

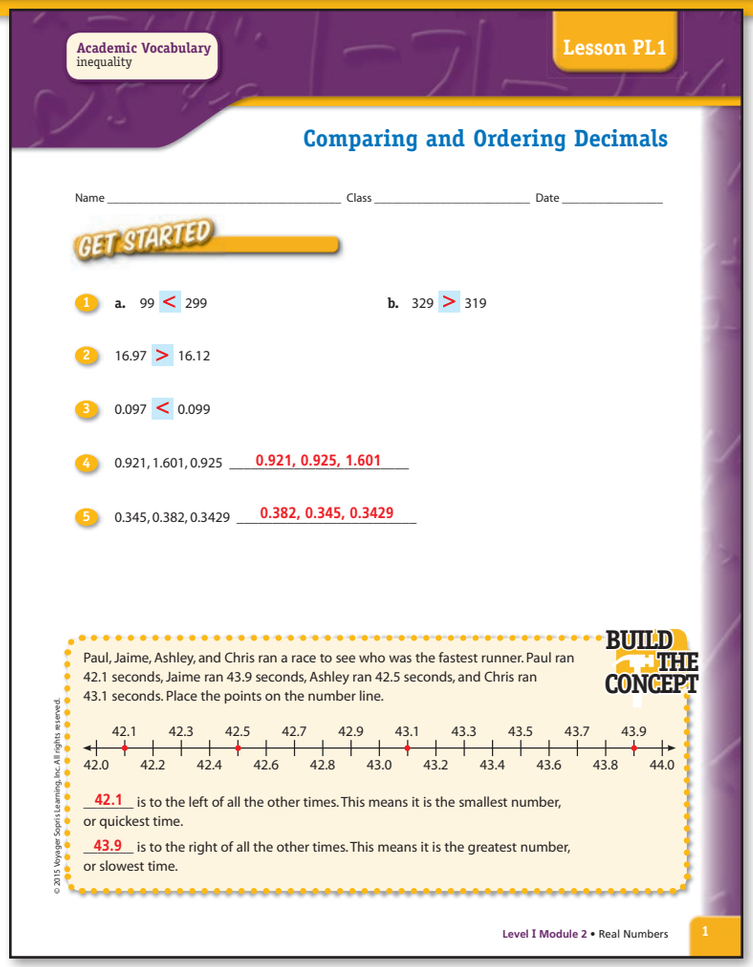
Objective

To compare and order decimal numbers

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **inequality**
  a number sentence that contains the symbol > (greater than) or < (less than)



Academic Vocabulary
inequality

Lesson PL1

Comparing and Ordering Decimals

Name _____ Class _____ Date _____

GET STARTED

1. a. 99 < 299 b. 329 > 319
2. 16.97 > 16.12
3. 0.097 < 0.099
4. 0.921, 1.601, 0.925 0.921, 0.925, 1.601
5. 0.345, 0.382, 0.3429 0.382, 0.345, 0.3429

BUILD THE CONCEPT

Paul, Jaime, Ashley, and Chris ran a race to see who was the fastest runner. Paul ran 42.1 seconds, Jaime ran 43.9 seconds, Ashley ran 42.5 seconds, and Chris ran 43.1 seconds. Place the points on the number line.

42.0 42.1 42.2 42.3 42.4 42.5 42.6 42.7 42.8 42.9 43.0 43.1 43.2 43.3 43.4 43.5 43.6 43.7 43.8 43.9 44.0

42.1 is to the left of all the other times. This means it is the smallest number, or quickest time.

43.9 is to the right of all the other times. This means it is the greatest number, or slowest time.

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Level I Module 2 • Real Numbers 1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To compare the numbers, start with the greatest place value in each number. What is the greatest place value in 99? (tens) **What is the greatest place value in 299?** (hundreds) **A number with a place value through the tens is less than a number with a place value through the hundreds. Which number is less, 99 or 299?** (99) **Write the symbol for less than between 99 and 299. Remember, the inequality symbol always points to the lesser number.**

In problem 1b, the numbers 329 and 319 have the same digit in the hundreds place. What is the digit in the tens place of 329? (2) **What digit is in the tens place of 319?** (1) **Which digit is greater, 2 or 1?** (2) **Which number is greater, 329 or 319?** (329) **Write the inequality symbol for greater than between 329 and 319.**

MODEL NEW SKILLS

Problem 2

In this problem, 16.97 and 16.12 are decimal numbers. To compare two decimal numbers, first compare the whole numbers. Are these whole numbers the same or different? (same) **Because both numbers have the same whole number, more comparisons must be made. Compare the digits to the right of the decimal point, from left to right, until the digits differ. Where is the first place in which the digits differ?** (tenths) **The digit in the tenths place of 16.97 is 9, and the digit in the tenths place of 16.12 is 1. Which digit is greater, 9 or 1?** (9) **Because 9 is greater than 1, 16.97 is greater than 16.12. Write the inequality symbol for greater than between 16.97 and 16.12.**

Problem 3

For both numbers, what is the whole number? (0) **Because the whole numbers are the same, compare the digits to the right of the decimal point, from left to right, until the digits differ. Where is the first place in which the digits differ?** (thousandths) **What is the digit in the thousandths place in 0.097?** (7) **What is the digit in the thousandths place in 0.099?** (9) **Because 7 is less than 9, 0.097 is less than 0.099. Write the less than symbol between 0.097 and 0.099.**

Problem 4

To order these decimal numbers from least to greatest, first compare the whole numbers. One of the numbers has a whole number of 1, and the other two numbers have whole numbers of 0. This means that 1.601 is the greatest decimal.


Lesson PL1

TRY IT TOGETHER

Compare. Write >, <, or =.

6 0.005 < 0.3

7 2.24922 > 2.24921

Order from greatest to least.

8 0.215, 0.125, 0.219

0.219, 0.215, 0.125

Order from least to greatest.

9 25.636, 25.615, 28.045

25.615, 25.636, 28.045

WORK ON YOUR OWN

HOW TO

Compare or Order Decimals Using Symbols

Using Symbols	Using Words
1. Order 0.63, 0.572, and 1.1. 0.63, 0.572, 1.1 1 > 0, so 1.1 is the greatest number.	Compare the whole numbers. Continue if the whole numbers are the same.
2. 0.63, 0.572 6 > 5, so 0.572 is the least number.	Compare the digits to the right of the decimal point, from left to right, until the digits differ. When the digits differ, the number with the lesser digit is the lesser number.
3. least to greatest: 0.572, 0.63, 1.1 greatest to least: 1.1, 0.63, 0.572	Write the numbers in order from least to greatest or greatest to least.

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Now, compare the two remaining decimal numbers, 0.921 and 0.925. Where is the first place in which the digits differ? (thousandths) **The digit 1 in the number 0.921 is less than 5, so 0.921 is less than 0.925. The order of the decimals from least to greatest is 0.921, 0.925, 1.601.**

Problem 5

To order these decimals from greatest to least, first compare the whole numbers. Are the whole numbers the same or different? (same) **In which place do the digits first differ?** (hundredths) **One of the numbers has an 8 in the hundredths place, and two of the numbers have a 4 in the hundredths place. So, 0.382 is the greatest number.**

In which place do the remaining two numbers differ? (thousandths) **Which digit is greater, 5 or 2?** (5) **So, 0.3429 is the least number. What is the order of these decimals from greatest to least?** (0.382, 0.345, 0.3429)

BUILD THE CONCEPT

Model how to compare decimals.

A number line can be used to order decimals. Numbers increase from left to right on a number line. Put each decimal on the number line. Which runner's number is the smallest? (42.1) **Why?** (It is to the left of all the other runners' numbers.) **Which runner's number is the next smallest?** (42.5) **Why?** (Moving left to right on the number line, it is the next labeled number.)

Which runner's number is next? (43.1) **Is this number smaller or larger than the previous number?** (larger) **What is the greatest number labeled on the number line?** (43.9) **Why?** (It is to the right of all the other runners' numbers.) **What is the least, or quickest, time?** (42.1 sec) **What is the greatest, or slowest, time?** (43.9 sec)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

What is the first step in comparing these decimals? (Compare the whole numbers.) **Are the whole numbers the same or different?** (same) **What is the next step?** (Compare the digits to the right of the decimal point, from left to right, until the digits differ.) **Where do the digits first differ?** (the tenths place) **What digit is in the tenths place in 0.005?** (0) **What digit is in the tenths place in 0.3?** (3) **Which is less, 0 or 3?** (0) **Which decimal number is less?** (0.005) **Which inequality symbol will be used to compare 0.005 and 0.3?** (less than)

Problem 7

Are the whole numbers the same or different? (same) **In which place do the digits first differ?** (hundred-thousandths) **Which number is less?** (2.24921) **Which inequality symbol will be used to compare 2.24922 and 2.24921?** (greater than)

Problem 8

Are the whole numbers the same or different? (same) **In which place do the digits first differ?** (tenths) **Which digit is less, 2 or 1?** (1) **Which decimal is the least?** (0.125) **In which place do the remaining numbers differ?** (thousandths) **Which digit is greater, 5 or 9?** (9) **What is the order of the decimals from greatest to least?** (0.219, 0.215, 0.125)

Problem 9

Are the whole numbers the same or different? (Two are the same; one is different.) **What are the whole numbers?** (25, 28) **Which decimal number is the greatest?** (28.045) **In which place value do the remaining numbers differ?** (hundredths) **Which number is the least?** (25.615) **What is the order of the decimals from least to greatest?** (25.615, 25.636, 28.045)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 10–22

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 19 Reminder

Be sure that students know that the lesser time is the faster time.

Problem 21 Reminder

Tell students that the fastest lap is the one with the least time. Suggest to students that they write the times in order from fastest to slowest. Then use the information in the problem to match each swimmer with her time.

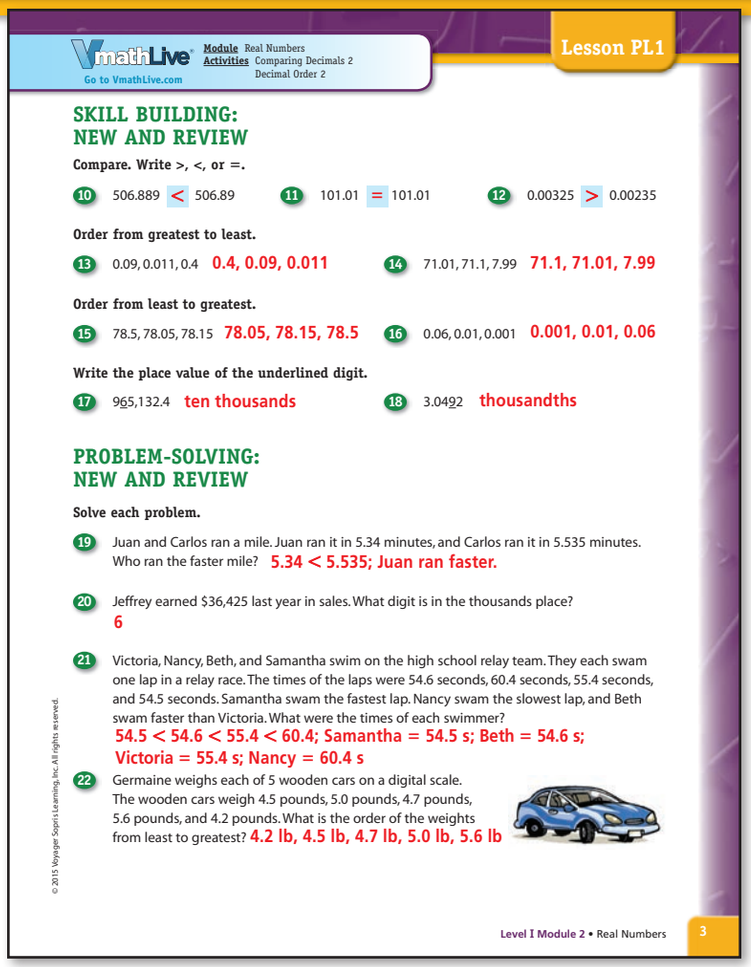
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a, 1b, 1c, 2a, or 2b: The student did not compare the decimal place values correctly. Use Additional Resources in I.2.PL1 to reteach how to compare decimal numbers.



VmathLive Go to VmathLive.com
Module Activities Real Numbers Comparing Decimals 2 Decimal Order 2

Lesson PL1

SKILL BUILDING: NEW AND REVIEW

Compare. Write >, <, or =.

10 506.889 < 506.89 11 101.01 = 101.01 12 0.00325 > 0.00235

Order from greatest to least.

13 0.09, 0.011, 0.4 0.4, 0.09, 0.011 14 71.01, 71.1, 7.99 71.1, 71.01, 7.99

Order from least to greatest.

15 78.5, 78.05, 78.15 78.05, 78.15, 78.5 16 0.06, 0.01, 0.001 0.001, 0.01, 0.06

Write the place value of the underlined digit.

17 9<u>6</u>5,132.4 ten thousands 18 3.04<u>9</u>2 thousandths

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

19 Juan and Carlos ran a mile. Juan ran it in 5.34 minutes, and Carlos ran it in 5.535 minutes. Who ran the faster mile? 5.34 < 5.535; Juan ran faster.

20 Jeffrey earned $36,425 last year in sales. What digit is in the thousands place? 6

21 Victoria, Nancy, Beth, and Samantha swim on the high school relay team. They each swam one lap in a relay race. The times of the laps were 54.6 seconds, 60.4 seconds, 55.4 seconds, and 54.5 seconds. Samantha swam the fastest lap. Nancy swam the slowest lap, and Beth swam faster than Victoria. What were the times of each swimmer? 54.5 < 54.6 < 55.4 < 60.4; Samantha = 54.5 s; Beth = 54.6 s; Victoria = 55.4 s; Nancy = 60.4 s

22 Germaine weighs each of 5 wooden cars on a digital scale. The wooden cars weigh 4.5 pounds, 5.0 pounds, 4.7 pounds, 5.6 pounds, and 4.2 pounds. What is the order of the weights from least to greatest? 4.2 lb, 4.5 lb, 4.7 lb, 5.0 lb, 5.6 lb

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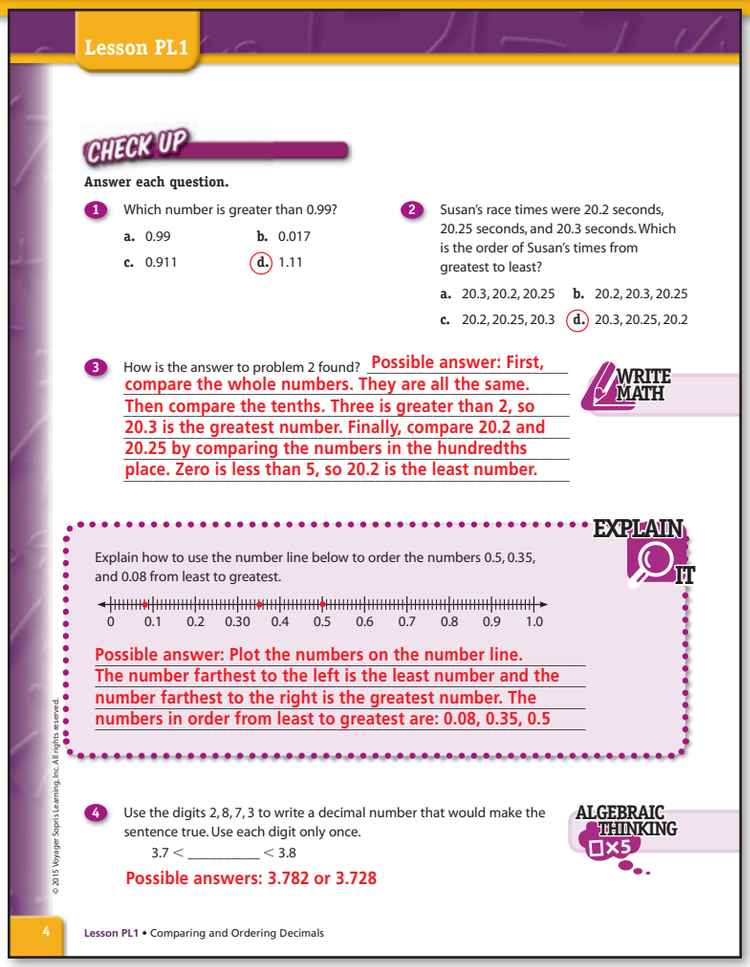
Level I Module 2 • Real Numbers 3

If student answered 2c: The student chose the numbers in order from least to greatest instead of greatest to least. Remind the student to read the question carefully.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to compare digits in each place value from left to right.



Lesson PL1

CHECK UP

Answer each question.

1. Which number is greater than 0.99?
   a. 0.99 b. 0.017
   c. 0.911 d. 1.11

2. Susan's race times were 20.2 seconds, 20.25 seconds, and 20.3 seconds. Which is the order of Susan's times from greatest to least?
   a. 20.3, 20.2, 20.25 b. 20.2, 20.3, 20.25
   c. 20.2, 20.25, 20.3 d. 20.3, 20.25, 20.2

3. How is the answer to problem 2 found? Possible answer: First, compare the whole numbers. They are all the same. Then compare the tenths. Three is greater than 2, so 20.3 is the greatest number. Finally, compare 20.2 and 20.25 by comparing the numbers in the hundredths place. Zero is less than 5, so 20.2 is the least number.

WRITE MATH

EXPLAIN IT

Explain how to use the number line below to order the numbers 0.5, 0.35, and 0.08 from least to greatest.

0 0.1 0.2 0.30 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Possible answer: Plot the numbers on the number line. The number farthest to the left is the least number and the number farthest to the right is the greatest number. The numbers in order from least to greatest are: 0.08, 0.35, 0.5

4. Use the digits 2, 8, 7, 3 to write a decimal number that would make the sentence true. Use each digit only once.
   3.7 < _____ < 3.8
   Possible answers: 3.782 or 3.728

ALGEBRAIC THINKING □×5

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4 Lesson PL1 • Comparing and Ordering Decimals

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Real Numbers
Activities: Comparing Decimals 2
Decimal Order 2

Vmath Reteach
Reteach Student Book Module 2 Lesson PL1
Reteach Teacher Edition Module 2 Lesson PL1

Extra Practice
Student Book page 43

English Language Learners

Use the VmathLive Animated Glossary to review the term *order*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Have the students form groups of 4 or 5 students. Ask each group to make a list of the date of each student's birthday. Then ask the group to order the numbers from least to greatest and greatest to least with each number listed only once. Have each group share their results with the class.

Students with Special Needs

Write various decimal numbers from 0 to 1 on index cards. The numbers should have no more than one decimal place. Give two index cards to each pair of students. Draw a number line on the board from 0 to 1. Label each tenth. To the left of the number line, write *less*. To the right of the number line, write *greater*. Ask students to tape their cards where they belong on the number line. Have each pair work together to create two comparison statements using the phrases *is less than* and *is greater than*. For example, "0.2 is less than 0.5" and "0.5 is greater than 0.2."

Lesson PL2

Objective

To use a problem-solving plan to solve an application word problem



Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Look at problem 1a. What is the sum of 8.2 and 7.32? (15.52) **Look at problem 1b. What is the difference of $\frac{3}{4}$ and $\frac{1}{4}$?** ($\frac{2}{4}$) **What is $\frac{2}{4}$ written in simplest form?** ($\frac{1}{2}$) **Look at problem 1c. What is the product of 7.6 and 9.6?** (72.96) **Look at problem 1d. What is 45.5 divided by 0.5?** (91)

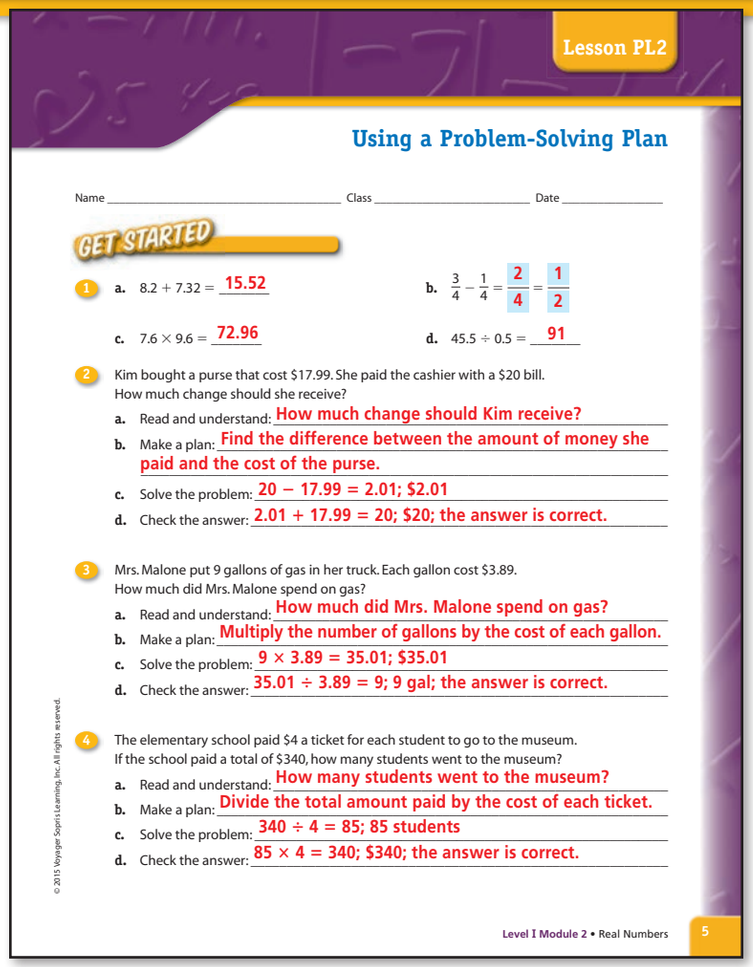
MODEL NEW SKILLS

Problem 2

An application word problem requires applying mathematical skills and knowledge to find a solution. One way to solve application word problems is to use a problem-solving plan. First, read the problem and understand what question is being asked. Second, make a plan to solve the problem. The plan may use addition, subtraction, multiplication, division, or a combination of these skills. Third, apply the mathematical skills from the plan to solve the problem. Fourth, always check the answer by performing the opposite operation of the problem.

Look at problem 2. First, read the problem and understand the question asked. What question is asked? (How much change should Kim receive?) **Next, make a plan to solve this problem. What operation could be used to solve for the amount?** (subtraction) **So, the plan to solve this problem is to find the difference between the amount of money she gave the cashier and the cost of the purse.**

Now, solve the problem. What is 20 minus 17.99? (2.01) **That means that Kim should receive $2.01 in change. Now that the answer has been found, it should be checked for accuracy. What is the opposite operation of subtraction?** (addition) **Because addition is the opposite operation, the answer can be checked by adding the amount of money she should receive in change and the cost of the purse. What is $2.01 plus $17.99?** ($20) **Because the answer, $20, is the same as the amount she gave to the cashier, the answer is correct.**



Lesson PL2

Using a Problem-Solving Plan

Name _____ Class _____ Date _____

GET STARTED

1 a. 8.2 + 7.32 = 15.52 b. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

c. 7.6 × 9.6 = 72.96 d. 45.5 ÷ 0.5 = 91

2 Kim bought a purse that cost $17.99. She paid the cashier with a $20 bill. How much change should she receive?

a. Read and understand: How much change should Kim receive?

b. Make a plan: Find the difference between the amount of money she paid and the cost of the purse.

c. Solve the problem: 20 − 17.99 = 2.01; $2.01

d. Check the answer: 2.01 + 17.99 = 20; $20; the answer is correct.

3 Mrs. Malone put 9 gallons of gas in her truck. Each gallon cost $3.89. How much did Mrs. Malone spend on gas?

a. Read and understand: How much did Mrs. Malone spend on gas?

b. Make a plan: Multiply the number of gallons by the cost of each gallon.

c. Solve the problem: 9 × 3.89 = 35.01; $35.01

d. Check the answer: 35.01 ÷ 3.89 = 9; 9 gal; the answer is correct.

4 The elementary school paid $4 a ticket for each student to go to the museum. If the school paid a total of $340, how many students went to the museum?

a. Read and understand: How many students went to the museum?

b. Make a plan: Divide the total amount paid by the cost of each ticket.

c. Solve the problem: 340 ÷ 4 = 85; 85 students

d. Check the answer: 85 × 4 = 340; $340; the answer is correct.

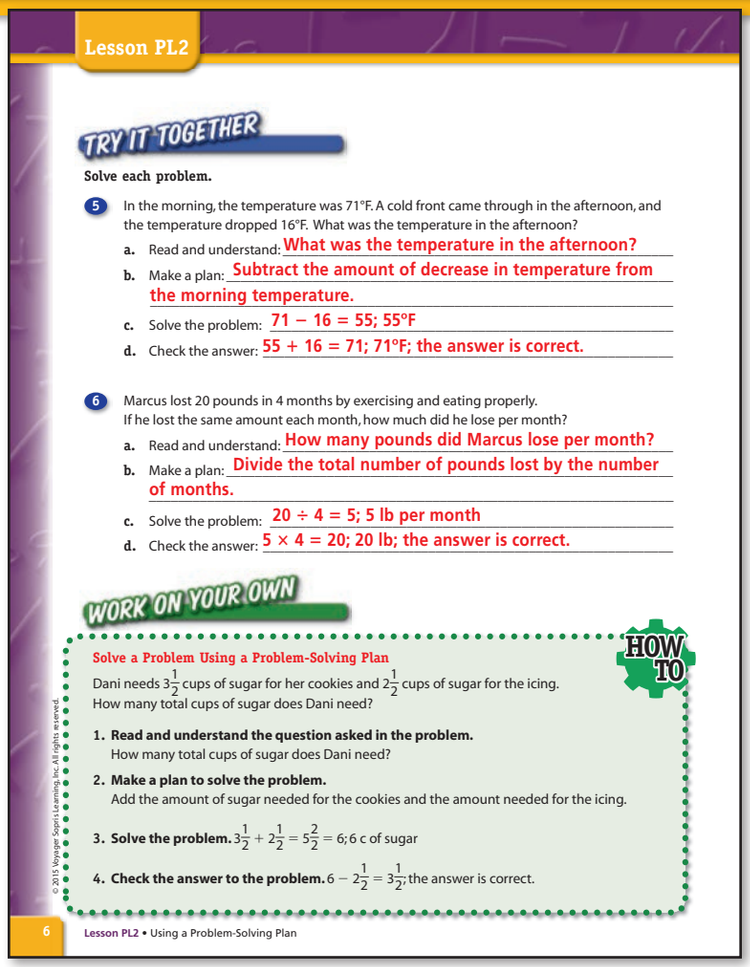
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Problem 3

First, read and understand the question asked in the problem. What is the question? (How much did Mrs. Malone spend on gas?) **Next, make a plan to solve the problem. What operation could be used to solve the problem?** (multiplication) **Multiply the number of gallons she bought by the cost of each gallon. Now, solve the problem. What is 9 times 3.89?** (35.01) **How much did Mrs. Malone spend?** ($35.01)

Check the answer by performing the opposite operation. What is the opposite operation of multiplication? (division) **Divide the total amount she spent on gas by the cost of each gallon. What is $35.01 divided by $3.89?** (9) **Because the answer, 9, is the same as the number of gallons she bought, the answer is correct.**



Lesson PL2

TRY IT TOGETHER

Solve each problem.

5 In the morning, the temperature was 71°F. A cold front came through in the afternoon, and the temperature dropped 16°F. What was the temperature in the afternoon?

a. Read and understand: What was the temperature in the afternoon?

b. Make a plan: Subtract the amount of decrease in temperature from the morning temperature.

c. Solve the problem: 71 − 16 = 55; 55°F

d. Check the answer: 55 + 16 = 71; 71°F; the answer is correct.

6 Marcus lost 20 pounds in 4 months by exercising and eating properly. If he lost the same amount each month, how much did he lose per month?

a. Read and understand: How many pounds did Marcus lose per month?

b. Make a plan: Divide the total number of pounds lost by the number of months.

c. Solve the problem: 20 ÷ 4 = 5; 5 lb per month

d. Check the answer: 5 × 4 = 20; 20 lb; the answer is correct.

WORK ON YOUR OWN

HOW TO

Solve a Problem Using a Problem-Solving Plan

Dani needs $3\frac{1}{2}$ cups of sugar for her cookies and $2\frac{1}{2}$ cups of sugar for the icing. How many total cups of sugar does Dani need?

1. **Read and understand the question asked in the problem.**
   How many total cups of sugar does Dani need?
2. **Make a plan to solve the problem.**
   Add the amount of sugar needed for the cookies and the amount needed for the icing.
3. **Solve the problem.** $3\frac{1}{2} + 2\frac{1}{2} = 5\frac{2}{2} = 6$; 6 c of sugar
4. **Check the answer to the problem.** $6 - 2\frac{1}{2} = 3\frac{1}{2}$; the answer is correct.

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Problem 4

First, read and understand the question. What is the question asking for? (How many students went to the museum?) **Next, make a plan to solve the problem. What operation could be used?** (division) **What should be divided?** (the total amount paid by the cost of each ticket) **What is 340 divided by 4?** (85) **How many students went to the museum?** (85)

Check the answer. What operation should be used to check this answer? (multiplication) **Why?** (Multiplication is the opposite operation of division.) **Multiply the total number of students by the cost of each ticket. What is 85 times $4?** ($340) **Because $340 is the same as the total amount paid, the answer is correct.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What is the question in the problem? (What was the temperature in the afternoon?) **What operation could be used to solve this problem?** (subtraction) **What should be subtracted?** (the amount of decrease in temperature from the morning temperature) **What was the morning temperature?** (71°F) **How many degrees did it drop?** (16) **What is 71 minus 16?** (55) **What was the temperature in the afternoon?** (55°F)

How can the answer be checked? (Add the temperature in the afternoon to the number of degrees it dropped.) **What is 55 plus 16?** (71) **Because 71°F is the same as the morning temperature, the answer is correct.**

Problem 6

What is the question asked in the problem? (How many pounds did Marcus lose per month?) **What is the plan to solve the problem?** (Divide the total number of pounds lost by the number of months.) **How many pounds did he lose?** (20) **How many months did it take him?** (4) **What is 20 divided by 4?** (5) **How many pounds did he lose each month?** (5)

How can this answer be checked? (Multiply 5 and 4.) **What is 5 times 4?** (20) **Because 20 pounds is the same as the number of total pounds lost, the answer is correct.**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 7–10

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 7 Reminder

Show students that this is a two-step problem. Students will first have to add the cost of the two items, then subtract the sum from $100.

Problem 9 Reminder

Show students that this is a two-step problem. Students will first have to find how many total T-shirts there are, then divide the total amount spent by the total number of T-shirts.

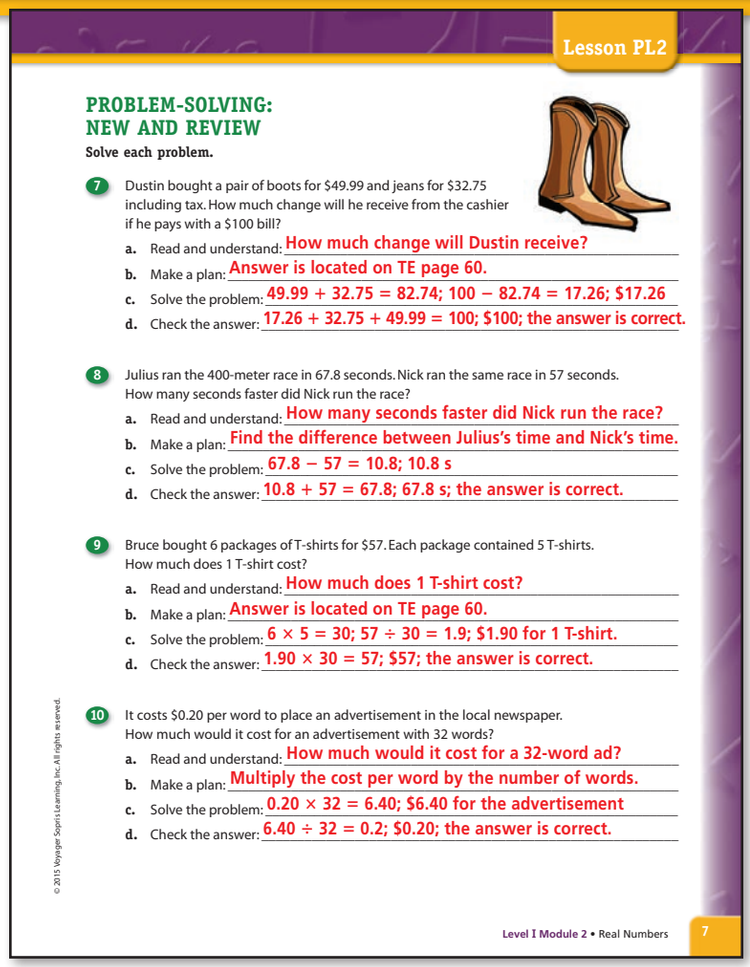
Problem-Solving: New and Review Answers, Student Book page 7

7. b. Add the amounts for the boots and the jeans, then subtract this sum from the amount he paid the cashier.

9. b. Multiply the number of packages by the number of T-shirts in each package. Then divide the total amount spent by the total number of T-shirts.

Extra Practice, Student Book page 43

1. b. Divide the total number of cups in the box by the amount in each serving.



Lesson PL2

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

7 Dustin bought a pair of boots for $49.99 and jeans for $32.75 including tax. How much change will he receive from the cashier if he pays with a $100 bill?

a. Read and understand: How much change will Dustin receive?
b. Make a plan: Answer is located on TE page 60.
c. Solve the problem: 49.99 + 32.75 = 82.74; 100 − 82.74 = 17.26; $17.26
d. Check the answer: 17.26 + 32.75 + 49.99 = 100; $100; the answer is correct.

8 Julius ran the 400-meter race in 67.8 seconds. Nick ran the same race in 57 seconds. How many seconds faster did Nick run the race?

a. Read and understand: How many seconds faster did Nick run the race?
b. Make a plan: Find the difference between Julius's time and Nick's time.
c. Solve the problem: 67.8 − 57 = 10.8; 10.8 s
d. Check the answer: 10.8 + 57 = 67.8; 67.8 s; the answer is correct.

9 Bruce bought 6 packages of T-shirts for $57. Each package contained 5 T-shirts. How much does 1 T-shirt cost?

a. Read and understand: How much does 1 T-shirt cost?
b. Make a plan: Answer is located on TE page 60.
c. Solve the problem: 6 × 5 = 30; 57 ÷ 30 = 1.9; $1.90 for 1 T-shirt.
d. Check the answer: 1.90 × 30 = 57; $57; the answer is correct.

10 It costs $0.20 per word to place an advertisement in the local newspaper. How much would it cost for an advertisement with 32 words?

a. Read and understand: How much would it cost for a 32-word ad?
b. Make a plan: Multiply the cost per word by the number of words.
c. Solve the problem: 0.20 × 32 = 6.40; $6.40 for the advertisement
d. Check the answer: 6.40 ÷ 32 = 0.2; $0.20; the answer is correct.

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CHECK UP

ASSESS INFORMALLY

Error Analysis

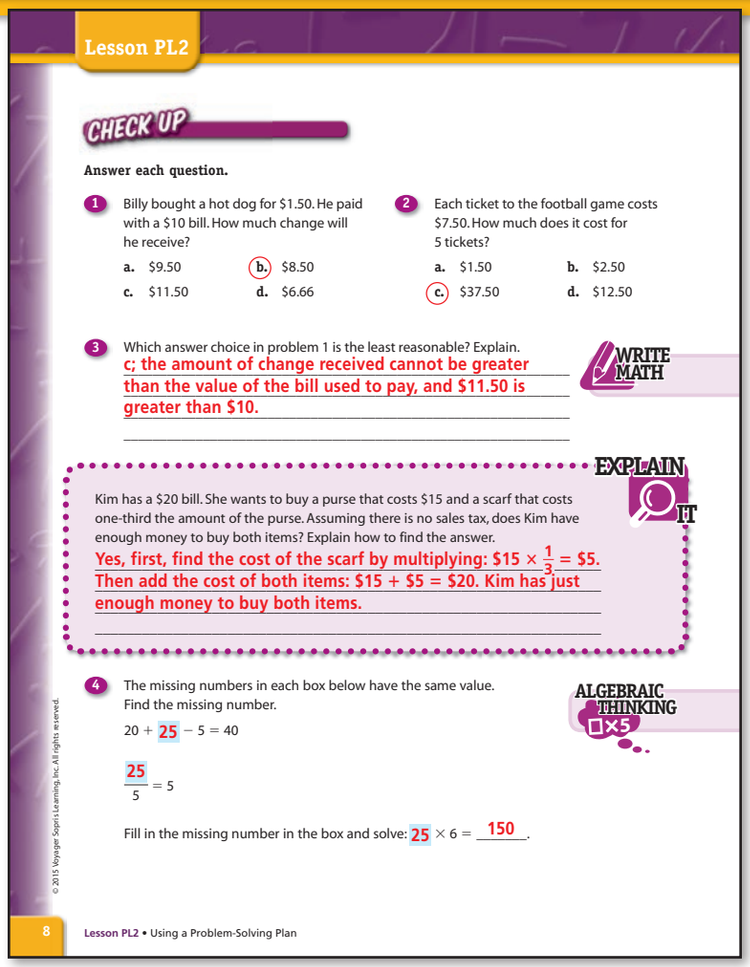
Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student subtracted incorrectly. Review with the student how to subtract decimals.

If student answered 1c or 2d: The student added instead of performing the correct operation. Review with the student the key words to look for in a word problem.

If student answered 1d or 2a: The student divided instead of performing the correct operation. Review with the student the key words to look for in a word problem.

If student answered 2b: The student subtracted instead of performing the correct operation. Review with the student the key words to look for in a word problem.



Lesson PL2

CHECK UP

Answer each question.

1. Billy bought a hot dog for $1.50. He paid with a $10 bill. How much change will he receive?
   a. $9.50 (b.) $8.50 c. $11.50 d. $6.66

2. Each ticket to the football game costs $7.50. How much does it cost for 5 tickets?
   a. $1.50 b. $2.50 (c.) $37.50 d. $12.50

3. Which answer choice in problem 1 is the least reasonable? Explain.
   c; the amount of change received cannot be greater than the value of the bill used to pay, and $11.50 is greater than $10.

WRITE MATH

EXPLAIN IT

Kim has a $20 bill. She wants to buy a purse that costs $15 and a scarf that costs one-third the amount of the purse. Assuming there is no sales tax, does Kim have enough money to buy both items? Explain how to find the answer.
Yes, first, find the cost of the scarf by multiplying: $15 × $\frac{1}{3}$ = $5. Then add the cost of both items: $15 + $5 = $20. Kim has just enough money to buy both items.

4. The missing numbers in each box below have the same value. Find the missing number.
   20 + 25 − 5 = 40

   $\frac{25}{5} = 5$

   Fill in the missing number in the box and solve: 25 × 6 = 150.

ALGEBRAIC THINKING □×5

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8 Lesson PL2 • Using a Problem-Solving Plan



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that the amount of change received should be less than the amount given to the cashier.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 2 Lesson PL2
Reteach Teacher Edition Module 2 Lesson PL2

Extra Practice
Student Book page 43

English Language Learners

Have students form small groups. Discuss their favorite activities and how they can relate them to math application problems. Encourage students to use different operations with a variety of numbers (whole numbers, fractions, and decimals). For example, some students may choose basket weaving as the activity. Have them think about the number of straws needed to weave one basket, two baskets, and so on. Consider also the length of the straws needed. Have students record their results on a poster board and share them with other groups.

Students with Special Needs

Work with students who have difficulty with operations with fractions. Review the algorithms of multiplying and dividing with mixed numbers by showing step-by-step examples of each. Remind students that the very first step to multiply or divide with mixed numbers is to change the mixed numbers to improper fractions.

Have students work in pairs. Give each pair some problems to convert mixed numbers to improper fractions, and vice versa.

Lesson 1

Objective

To write and evaluate an expression with an exponent

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **base**
  the number used as a factor in repeated multiplication
- **exponent**
  a number that indicates how many times a base is used as a factor in repeated multiplication

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

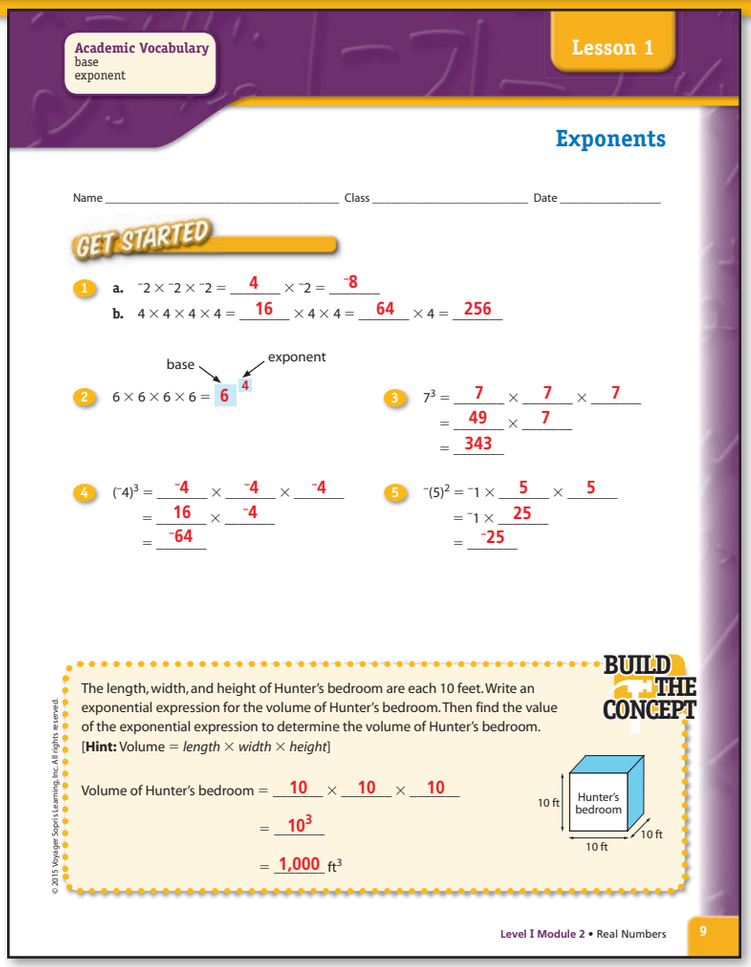
Problem 1

In problem 1a, the integer $^{-}2$ is a factor three times. To multiply more than two factors, multiply two factors at a time. Follow the rules for the sign of the product each time. What is $^{-}2$ times $^{-}2$? (4) **The product is positive because the signs of the factors are the same. What is 4 times $^{-}2$?** ($^{-}8$) **The product is negative because the signs of the factors are different. The value of $^{-}2$ times $^{-}2$ times $^{-}2$ is $^{-}8$. Look at problem 1b. To multiply 4 four times, multiply two factors at a time. What is 4 times 4?** (16) **What is 16 times 4?** (64) **What is 64 times 4?** (256)

MODEL NEW SKILLS

Problem 2

When a number is used as a factor more than once, this is called *repeated multiplication*. Exponents represent repeated multiplication. The number used as the repeated factor is the base. The exponent shows how many times the base is used as a factor. In this problem, 6 is used as a factor four times. The base is 6, and the exponent is 4. The expression is read as *six to the fourth power*.



Academic Vocabulary
base
exponent

Lesson 1

Exponents

Name _____ Class _____ Date _____

GET STARTED

1 a. $^{-}2 \times {}^{-}2 \times {}^{-}2 =$ 4 $\times {}^{-}2 =$ $^{-}8$

b. $4 \times 4 \times 4 \times 4 \times 4 =$ 16 $\times 4 \times 4 =$ 64 $\times 4 =$ 256

2 $6 \times 6 \times 6 \times 6 \times 6 =$ 6^4 (base: 6, exponent: 4)

3 $7^3 =$ 7 $\times$ 7 $\times$ 7
$=$ 49 $\times$ 7
$=$ 343

4 $(^{-}4)^3 =$ $^{-}4$ $\times$ $^{-}4$ $\times$ $^{-}4$
$=$ 16 $\times$ $^{-}4$
$=$ $^{-}64$

5 $^{-}(5)^2 = {}^{-}1 \times$ 5 $\times$ 5
$= {}^{-}1 \times$ 25
$=$ $^{-}25$

BUILD THE CONCEPT

The length, width, and height of Hunter's bedroom are each 10 feet. Write an exponential expression for the volume of Hunter's bedroom. Then find the value of the exponential expression to determine the volume of Hunter's bedroom.
[**Hint:** Volume = *length* × *width* × *height*]

Volume of Hunter's bedroom = 10 × 10 × 10
$= 10^3$
$=$ 1,000 ft^3

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Problem 3

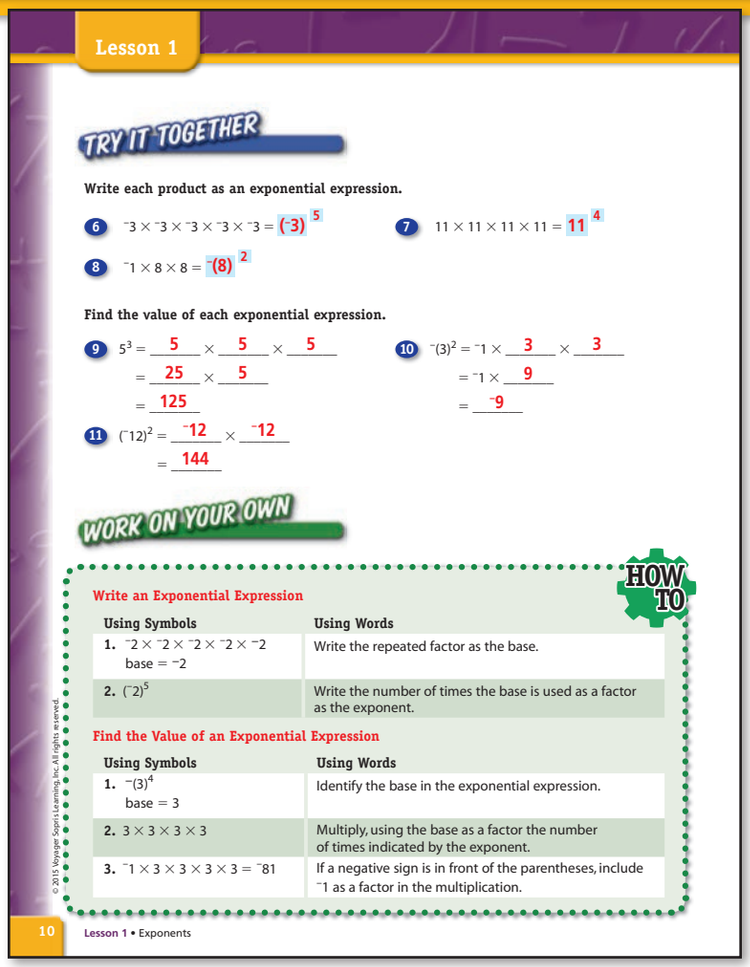
An expression with a base and an exponent is called an *exponential expression*. To evaluate an exponential expression, write the base as a factor the number of times indicated by the exponent. In this problem, 7 is the base and 3 is the exponent. This expression is read as *seven to the third power*, or *seven cubed*. Write $7 \times 7 \times 7$. Multiply the first two factors. What is 7 times 7? (49) **What is 49 times 7?** (343) **What is 7 times 7 times 7?** (343) **The value of the expression 7^3 is 343.**

Problem 4

In this problem, $^{-}4$ is the base and 3 is the exponent. This expression is read as *negative four to the third power*, or *negative four cubed*. Write $^{-}4 \times {}^{-}4 \times {}^{-}4$. Multiply the first two factors. What is $^{-}4$ times $^{-}4$? (16) **What is 16 times $^{-}4$?** ($^{-}64$) **What is $^{-}4$ times $^{-}4$ times $^{-}4$?** ($^{-}64$) **The value of the expression $(^{-}4)^3$ is $^{-}64$. Notice that when a negative base, such as $^{-}4$, is raised to an odd power, such as 3, the product is negative.**

Problem 5

In this problem, the base of the exponential expression is 5. The negative symbol in front of the parentheses means that $^{-}1$ is a separate factor in the multiplication.



Lesson 1

TRY IT TOGETHER

Write each product as an exponential expression.

6. $^{-}3 \times {}^{-}3 \times {}^{-}3 \times {}^{-}3 \times {}^{-}3 = (^{-}3)^{5}$

7. $11 \times 11 \times 11 \times 11 = 11^{4}$

8. $^{-}1 \times 8 \times 8 = {}^{-}(8)^{2}$

Find the value of each exponential expression.

9. $5^3 = 5 \times 5 \times 5$
$= 25 \times 5$
$= 125$

10. $^{-}(3)^2 = {}^{-}1 \times 3 \times 3$
$= {}^{-}1 \times 9$
$= {}^{-}9$

11. $(^{-}12)^2 = {}^{-}12 \times {}^{-}12$
$= 144$

WORK ON YOUR OWN

HOW TO

Write an Exponential Expression

Using Symbols	Using Words
1. $^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2$ base $= {}^{-}2$	Write the repeated factor as the base.
2. $(^{-}2)^5$	Write the number of times the base is used as a factor as the exponent.

Find the Value of an Exponential Expression

Using Symbols	Using Words
1. $-(3)^4$ base $= 3$	Identify the base in the exponential expression.
2. $3 \times 3 \times 3 \times 3$	Multiply, using the base as a factor the number of times indicated by the exponent.
3. $^{-}1 \times 3 \times 3 \times 3 \times 3 = {}^{-}81$	If a negative sign is in front of the parentheses, include $^{-}1$ as a factor in the multiplication.

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10 Lesson 1 • Exponents

To evaluate this expression, $^{-}1$ is written as the first factor. Write 5 as a factor twice. What is 5 times 5? (25) **What is $^{-}1$ times 25?** ($^{-}25$) **The value of the expression $-(5)^2$ is $^{-}25$.**

To evaluate any exponential expression, always identify the base of the expression first. Then write the base as a factor the number of times indicated by the exponent. If a negative sign is in front of the parentheses, that means $^{-}1$ is a factor in the multiplication.

BUILD THE CONCEPT

Model how to write and evaluate an expression with an exponent.

What is the length of Hunter's bedroom? (10 ft) **What is the width of Hunter's bedroom?** (10 ft) **What is the height of Hunter's bedroom?** (10 ft) **The formula for volume is *Volume = length × width × height*. What multiplication problem can be used to find the volume of Hunter's bedroom?** ($10 \times 10 \times 10$) **Write an exponential expression that represents the volume of the bedroom. What number is the base?** (10) **What number is the exponent?** (3) **What is the exponential expression?** (10^3) **What is the value of the exponential expression?** (1,000) **Volume is recorded in cubic units. What is the volume of Hunter's bedroom?** (1,000 ft^3)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

What factor is repeated in this product? ($^{-}3$) **What is the base?** ($^{-}3$) **How many times is $^{-}3$ used as a factor?** (5 times) **What is the exponent?** (5) **How is this expression read?** (negative three to the fifth power)

Problem 7

What is the base? (11) **What is the exponent?** (4) **How is this expression read?** (eleven to the fourth power)

Problem 8

What does $^{-}1$ as a factor mean? (A negative sign will be in front of the exponential expression.) **What is the base?** (8) **What is the exponent?** (2) **How is this expression read?** (the opposite of eight to the second power, or the opposite of eight squared)

Problem 9

How is the expression read? (five to the third power, or five cubed) **Which number is the base?** (5) **Which number is the exponent?** (3) **How is the product written as repeated factors?** ($5 \times 5 \times 5$) **What is 5 times 5?** (25) **What is 25 times 5?** (125) **What is the value of 5^3?** (125)

Problem 10

What is the base in the exponential expression? (3) **What is the exponent?** (2) **What does the negative sign in front of the parentheses indicate?** (Negative 1 is a factor in the multiplication.) **The factors are $^{-}1$, 3, and 3. What is the 3 times 3?** (9) **What is $^{-}1$ times 9?** ($^{-}9$) **What is the value of $^{-}1$ times 3^2?** ($^{-}9$)

Problem 11

What is the base in the exponential expression? ($^{-}12$) **What is the exponent?** (2) **How is the repeated multiplication written?** ($^{-}12 \times {}^{-}12$) **What is the value of the expression?** (144) **Why is the product positive?** (Both factors have the same sign.) **When a negative base is raised to an even power, the product is positive.**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Finding a Pattern** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a pattern. The instruction is immediately followed by application of the strategy in problem 24.

Have students read the problem.

Ask students what they are to find.

Have students read the problem again. The problem mentions a pattern.

Have students read the data in the table to determine a pattern.

Expression	Value	Exponent in Expression
$(^{-}1)^1$	$^{-}1$	odd
$(^{-}1)^2$	1	even
$(^{-}1)^3$	$^{-}1$	odd
$(^{-}1)^4$	1	even
$(^{-}1)^5$	$^{-}1$	odd

Odd exponents give a value of $^{-}1$.

Even exponents give a value of 1.

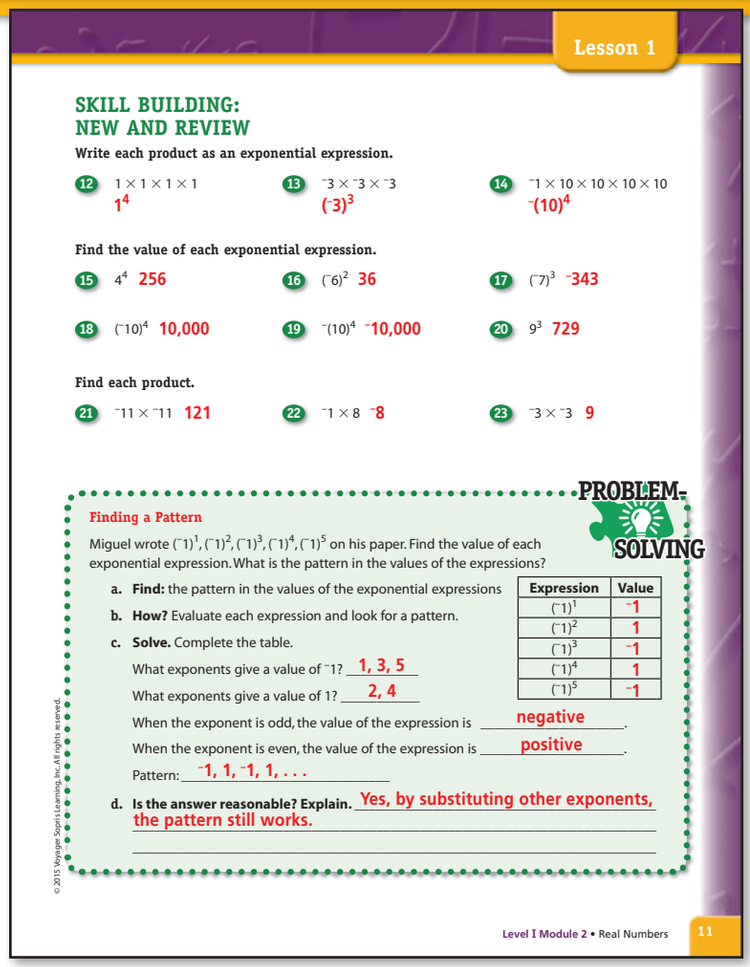
The answer is reasonable because the pattern holds when substituting other nonzero, positive even and odd numbers.

Problems 12–27

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 14 Reminder

Be sure students recognize that when $^{-}1$ is a factor, a negative sign will be in front of the parentheses in the answer.



Lesson 1

SKILL BUILDING: NEW AND REVIEW

Write each product as an exponential expression.

12. $1 \times 1 \times 1 \times 1$ 1^4
13. $^{-}3 \times {}^{-}3 \times {}^{-}3$ $(^{-}3)^3$
14. $^{-}1 \times 10 \times 10 \times 10 \times 10$ $^{-}(10)^4$

Find the value of each exponential expression.

15. 4^4 256
16. $(^{-}6)^2$ 36
17. $(^{-}7)^3$ $^{-}343$
18. $(^{-}10)^4$ 10,000
19. $^{-}(10)^4$ $^{-}10{,}000$
20. 9^3 729

Find each product.

21. $^{-}11 \times {}^{-}11$ 121
22. $^{-}1 \times 8$ $^{-}8$
23. $^{-}3 \times {}^{-}3$ 9

PROBLEM-SOLVING

Finding a Pattern

Miguel wrote $(^{-}1)^1, (^{-}1)^2, (^{-}1)^3, (^{-}1)^4, (^{-}1)^5$ on his paper. Find the value of each exponential expression. What is the pattern in the values of the expressions?

a. **Find:** the pattern in the values of the exponential expressions

b. **How?** Evaluate each expression and look for a pattern.

c. **Solve.** Complete the table.

Expression	Value
$(^{-}1)^1$	$^{-}1$
$(^{-}1)^2$	1
$(^{-}1)^3$	$^{-}1$
$(^{-}1)^4$	1
$(^{-}1)^5$	$^{-}1$

What exponents give a value of $^{-}1$? 1, 3, 5

What exponents give a value of 1? 2, 4

When the exponent is odd, the value of the expression is negative.

When the exponent is even, the value of the expression is positive.

Pattern: $^{-}1, 1, {}^{-}1, 1, \ldots$

d. **Is the answer reasonable? Explain.** Yes, by substituting other exponents, the pattern still works.

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Problem 19 Reminder

Be sure students recognize that the base of the expression is a positive number and that $^{-}1$ is also a factor in the multiplication.

CHECK UP

ASSESS INFORMALLY

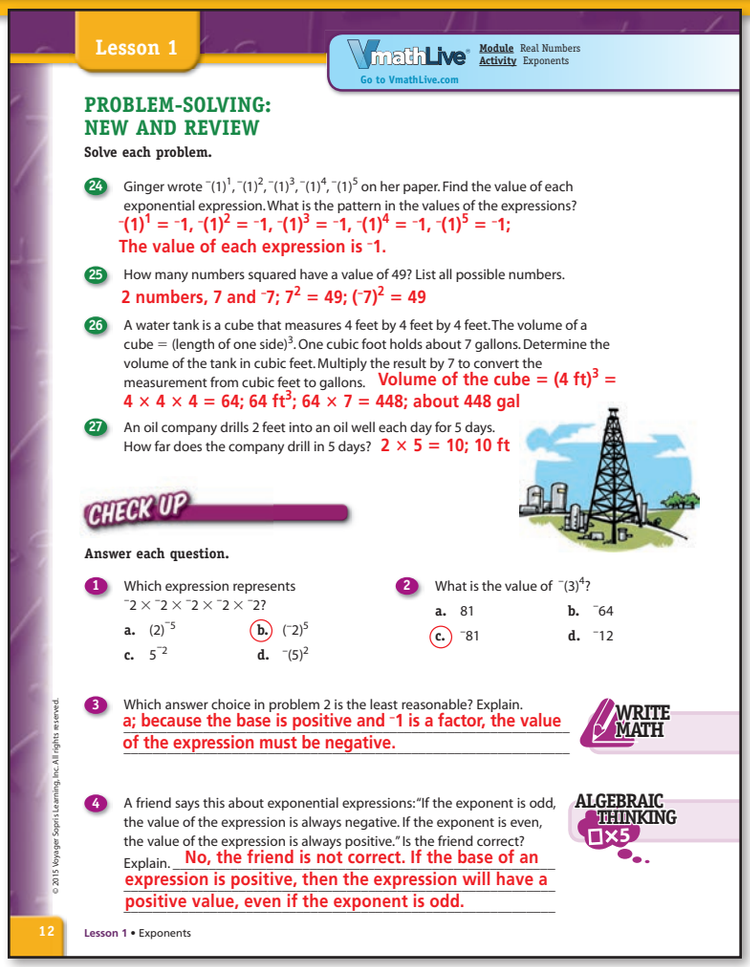
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2a: The student confused the placement of the negative sign. Use Additional Resources in I.2.1 to reteach how to evaluate exponential expressions.

If student answered 1c, 1d, or 2b: The student confused the base and the exponent. Use Additional Resources in I.2.1 to reteach how to evaluate exponential expressions.

If student answered 2d: The student multiplied the base by the exponent. Use Additional Resources in I.2.1 to reteach how to evaluate exponential expressions.



Lesson 1

VmathLive Module Real Numbers Activity Exponents
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

24 Ginger wrote $^{-}(1)^1, ^{-}(1)^2, ^{-}(1)^3, ^{-}(1)^4, ^{-}(1)^5$ on her paper. Find the value of each exponential expression. What is the pattern in the values of the expressions?
$^{-}(1)^1 = ^{-}1$, $^{-}(1)^2 = ^{-}1$, $^{-}(1)^3 = ^{-}1$, $^{-}(1)^4 = ^{-}1$, $^{-}(1)^5 = ^{-}1$;
The value of each expression is $^{-}1$.

25 How many numbers squared have a value of 49? List all possible numbers.
2 numbers, 7 and $^{-}7$; $7^2 = 49$; $(^{-}7)^2 = 49$

26 A water tank is a cube that measures 4 feet by 4 feet by 4 feet. The volume of a cube = (length of one side)^3. One cubic foot holds about 7 gallons. Determine the volume of the tank in cubic feet. Multiply the result by 7 to convert the measurement from cubic feet to gallons. Volume of the cube = $(4 \text{ ft})^3 = 4 \times 4 \times 4 = 64$; 64 ft^3; $64 \times 7 = 448$; about 448 gal

27 An oil company drills 2 feet into an oil well each day for 5 days. How far does the company drill in 5 days? $2 \times 5 = 10$; 10 ft

CHECK UP

Answer each question.

1 Which expression represents $^{-}2 \times ^{-}2 \times ^{-}2 \times ^{-}2 \times ^{-}2$?
a. $(2)^{-5}$ (b.) $(^{-}2)^5$
c. 5^{-2} d. $^{-}(5)^2$

2 What is the value of $^{-}(3)^4$?
a. 81 b. $^{-}64$
(c.) $^{-}81$ d. $^{-}12$

3 Which answer choice in problem 2 is the least reasonable? Explain.
a; because the base is positive and $^{-}1$ is a factor, the value of the expression must be negative.

WRITE MATH

4 A friend says this about exponential expressions: "If the exponent is odd, the value of the expression is always negative. If the exponent is even, the value of the expression is always positive." Is the friend correct? Explain. No, the friend is not correct. If the base of an expression is positive, then the expression will have a positive value, even if the exponent is odd.

ALGEBRAIC THINKING □×5

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12 Lesson 1 • Exponents



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the value of the expression in problem 2 must be negative. Have the student write all the factors of the expression. The final product will be the product of $^{-}1$ and a positive number; therefore, it will be negative.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Real Numbers
Activity: Exponents

Vmath Reteach
Reteach Student Book Module 2 Lesson 1
Reteach Teacher Edition Module 2 Lesson 1

Extra Practice
Student Book page 44

English Language Learners

Use the VmathLive Animated Glossary to review the terms *base (of a power)* and *exponent*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Review the multiple meanings of the term *base* with students. Tell students that outside of mathematics, *base* is used to describe the bottom or lowest part of an object such as a lamp or a statue. Allow students to practice writing various exponential expressions given the value of the base. Make sure to include both positive and negative numbers as bases.

Students with Special Needs

Use sheets of construction paper to create number cards from $^{-}9$ to 9. Select a volunteer to create a "human" exponential expression.

- Have the volunteer pick a number card and hold it up. Tell students that this number is the base.
- Have the volunteer raise his or her left hand and hold up 1, 2, 3, 4, or 5 fingers. Tell the students that the number of fingers held up represents the exponent.
- Show students how to write the base as a factor. Each time the base is written in the expression, have the volunteer put one finger down. Continue until all fingers are down. Explain that the exponent tells how many times the base is written as a factor.
- Have students multiply to find the product. Repeat until each student has formed a "human" exponential expression.

Lesson 2

Objective

To use the order of operations to evaluate an expression

Preskills	Lesson
Exponents	I.2.1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To subtract integers, write the problem as an addition problem by changing the subtraction sign to addition and changing the second integer to its opposite. What is the addition problem? ($^{-}3 + 5$) **What is $^{-}3$ plus 5?** (2) **What is $^{-}3$ minus $^{-}5$?** (2)

Problem 2

What is the base of this exponential expression? ($^{-}6$) **What is the exponent?** (2) **How is the expression written as repeated multiplication?** ($^{-}6 \times {}^{-}6$) **What is $^{-}6$ times $^{-}6$?** (36) **So, $^{-}6$ to the second power, or $^{-}6$ squared, is 36.**

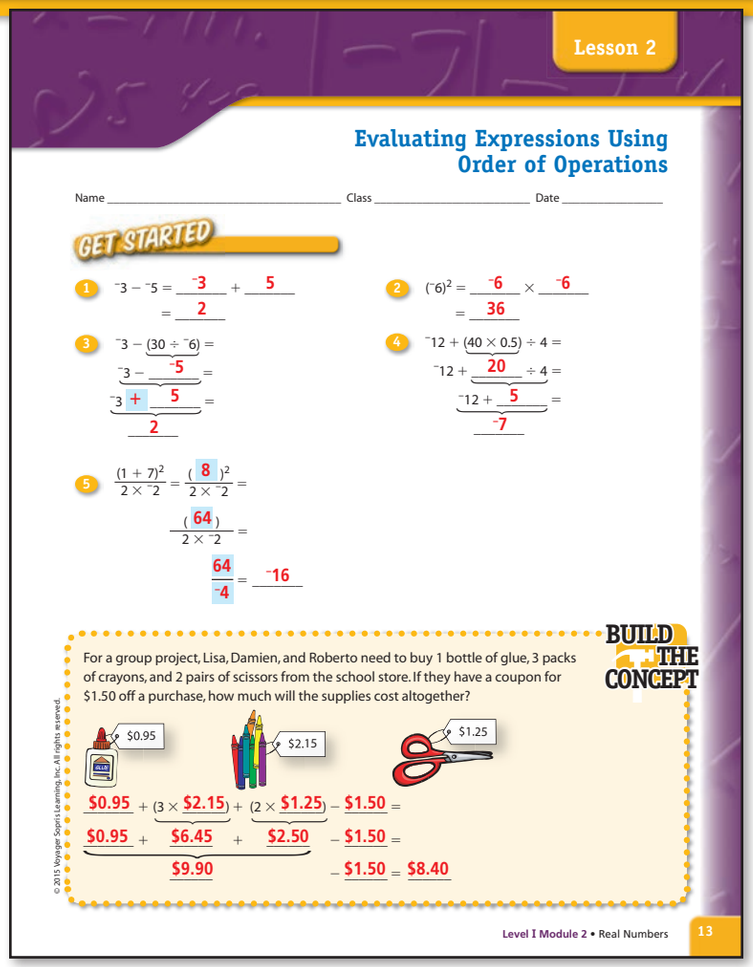
MODEL NEW SKILLS

Problem 3

The expressions in problems 1 and 2 have only one operation each. Some expressions have more than one operation. To evaluate an expression with two or more operations, it is necessary to evaluate the expression using the order of operations.

Always perform operations in parentheses first. Next, evaluate exponents. Then perform multiplication and division from left to right, and finally, perform addition and subtraction from left to right.

Does the expression in this problem contain parentheses? (yes) **What is the operation in parentheses?** ($30 \div {}^{-}6$) **What is 30 divided by $^{-}6$?** ($^{-}5$) **What must be done next to complete the problem?** (Subtract $^{-}5$ from $^{-}3$.) **How is $^{-}3$ minus $^{-}5$ written as an addition problem?** ($^{-}3 + 5$) **What is $^{-}3$ plus 5?** (2) **The value of the expression is 2.**


Lesson 2

Evaluating Expressions Using Order of Operations

Name _____ Class _____ Date _____

GET STARTED

1. $^{-}3 - {}^{-}5 = {}^{-}3 + 5 = 2$

2. $(^{-}6)^2 = {}^{-}6 \times {}^{-}6 = 36$

3. $^{-}3 - (30 \div {}^{-}6) = {}^{-}3 - {}^{-}5 = {}^{-}3 + 5 = 2$

4. $^{-}12 + (40 \times 0.5) \div 4 = {}^{-}12 + 20 \div 4 = {}^{-}12 + 5 = {}^{-}7$

5. $\frac{(1+7)^2}{2 \times {}^{-}2} = \frac{(8)^2}{2 \times {}^{-}2} = \frac{(64)}{2 \times {}^{-}2} = \frac{64}{{}^{-}4} = {}^{-}16$

BUILD THE CONCEPT

For a group project, Lisa, Damien, and Roberto need to buy 1 bottle of glue, 3 packs of crayons, and 2 pairs of scissors from the school store. If they have a coupon for $1.50 off a purchase, how much will the supplies cost altogether?

$\$0.95 + (3 \times \$2.15) + (2 \times \$1.25) - \$1.50 =$

$\$0.95 + \$6.45 + \$2.50 - \$1.50 =$

$\$9.90 - \$1.50 = \$8.40$

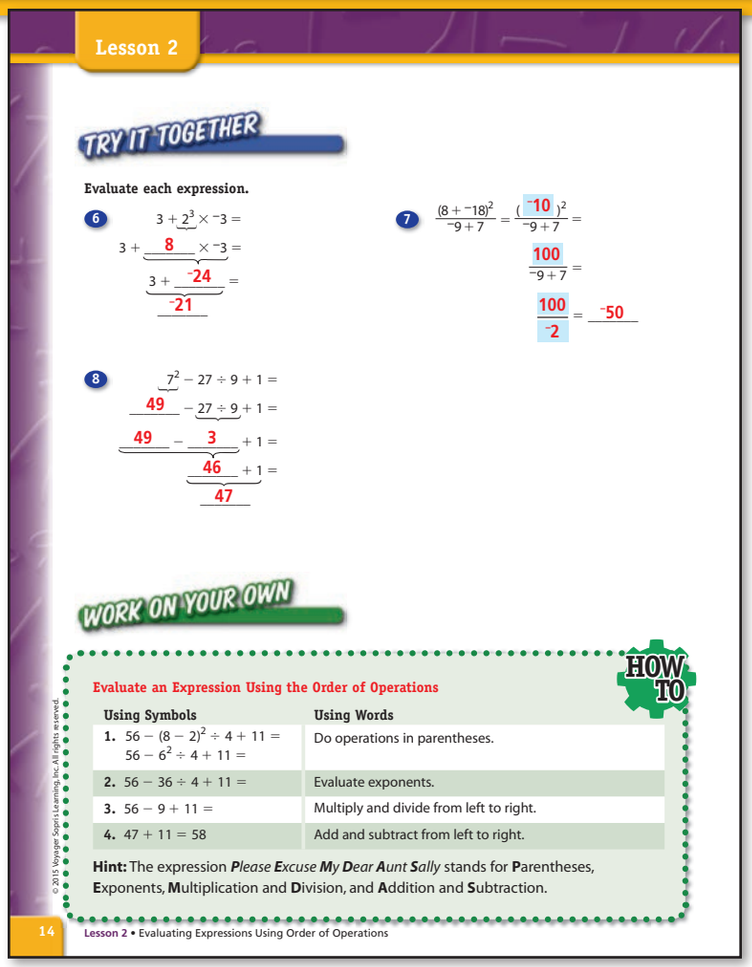
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Problem 4

The expression in this problem contains parentheses, so perform the operation in parentheses first. What is 40 times 0.5? (20) **No exponents are in the expression, so the next step is to perform multiplication and division from left to right. That means the next step is to divide 20 by 4. What is 20 divided by 4?** (5) **The last step is to add $^{-}12$ and 5. What is $^{-}12$ plus 5?** ($^{-}7$) **The value of the expression is $^{-}7$.**

Problem 5

When an expression is given in the form of a fraction, evaluate the numerator, or top part, of the fraction completely, then evaluate the denominator, or bottom part, of the fraction completely. Look at the numerator of the expression. What is 1 plus 7? (8) **What is 8^2?** (64) **Now, evaluate the denominator. What is 2 times $^{-}2$?** ($^{-}4$) **In a fraction, the fraction bar represents division. What is 64 divided by $^{-}4$?** ($^{-}16$) **The value of the expression is $^{-}16$.**



Lesson 2

TRY IT TOGETHER

Evaluate each expression.

6 $3 + 2^3 \times {}^-3 =$

$3 + \underbrace{8 \times {}^-3}_{{}^-24} =$

$\underbrace{3 + {}^-24}_{{}^-21} =$

7 $\frac{(8 + {}^-18)^2}{{}^-9 + 7} = \frac{({}^-10)^2}{{}^-9 + 7} =$

$\frac{100}{{}^-9 + 7} =$

$\frac{100}{{}^-2} = {}^-50$

8 $\underbrace{7^2}_{49} - 27 \div 9 + 1 =$

$49 - \underbrace{27 \div 9}_{3} + 1 =$

$\underbrace{49 - 3}_{46} + 1 =$

$\underbrace{46 + 1}_{47} =$

WORK ON YOUR OWN

HOW TO

Evaluate an Expression Using the Order of Operations

Using Symbols	Using Words
1. $56 - (8 - 2)^2 \div 4 + 11 =$ $56 - 6^2 \div 4 + 11 =$	Do operations in parentheses.
2. $56 - 36 \div 4 + 11 =$	Evaluate exponents.
3. $56 - 9 + 11 =$	Multiply and divide from left to right.
4. $47 + 11 = 58$	Add and subtract from left to right.

Hint: The expression *Please Excuse My Dear Aunt Sally* stands for **P**arentheses, **E**xponents, **M**ultiplication and **D**ivision, and **A**ddition and **S**ubtraction.

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14 Lesson 2 • Evaluating Expressions Using Order of Operations

BUILD THE CONCEPT

Model how to use the order of operations to evaluate an expression.

How much does a pack of crayons cost? ($2.15) **How many packs does the group need to buy?** (3) **Where does $2.15 belong in the number sentence?** (the second line) **Why?** (Three packs need to be bought, so the price should be multiplied by 3.) **How much does a bottle of glue cost?** ($0.95) **How many bottles does the group need to buy?** (1) **Where does $0.95 belong in the number sentence?** (the first line) **Why?** (One bottle needs to be bought, so the price does not need to be multiplied by another number.) **How much does a pair of scissors cost?** ($1.25) **How many pairs does the group need to buy?** (2) **Where does $1.25 belong in the number sentence?** (the third line) **Why?** (Two pairs need to be bought, so the price should be multiplied by 2.) **The students have a coupon for $1.50 off the purchase. Where does $1.50 belong in the number sentence?** (the last line) **Why?** (This amount is being subtracted from the total cost.) **What should be done first to evaluate the expression?** (Work in parentheses.) **What is 3 times $2.15?** ($6.45) **What is 2 times $1.25?** ($2.50) **What operations should be completed next?** (addition and subtraction) **Add the three amounts on the left. What is the sum?** ($9.90) **Now subtract the remaining amount. What is the difference?** ($8.40) **How much will the supplies cost altogether?** ($8.40)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

What should be done first to evaluate this expression? (Evaluate the exponential expression.) **What is 2^3?** (8) **What operation should be done next?** (multiplication) **What is 8 times $^-3$?** ($^-24$) **What operation is left?** (addition) **What is 3 plus $^-24$?** ($^-21$)

Problem 7

The expression in this problem is in the form of a fraction. What should be done first to evaluate this expression? (Evaluate the numerator completely.) **Do the operation in parentheses first. What is 8 plus $^-18$?** ($^-10$) **What should be done next?** (Evaluate the exponential expression.) **What is $(^-10)^2$?** (100) **Now, evaluate the denominator. What is $^-9$ plus 7?** ($^-2$) **The fraction bar is a division symbol. What is 100 divided by $^-2$?** ($^-50$)

Problem 8

What is 7^2? (49) **What is 27 divided by 9?** (3) **What is 49 minus 3?** (46) **What is 46 plus 1?** (47) **What is the value of the expression?** (47)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 9–21

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 13 **Reminder**

This problem involves solving an expression in the form of a fraction. Remind students to solve the numerator first, then the denominator. Then students should divide the numerator by the denominator.

CHECK UP

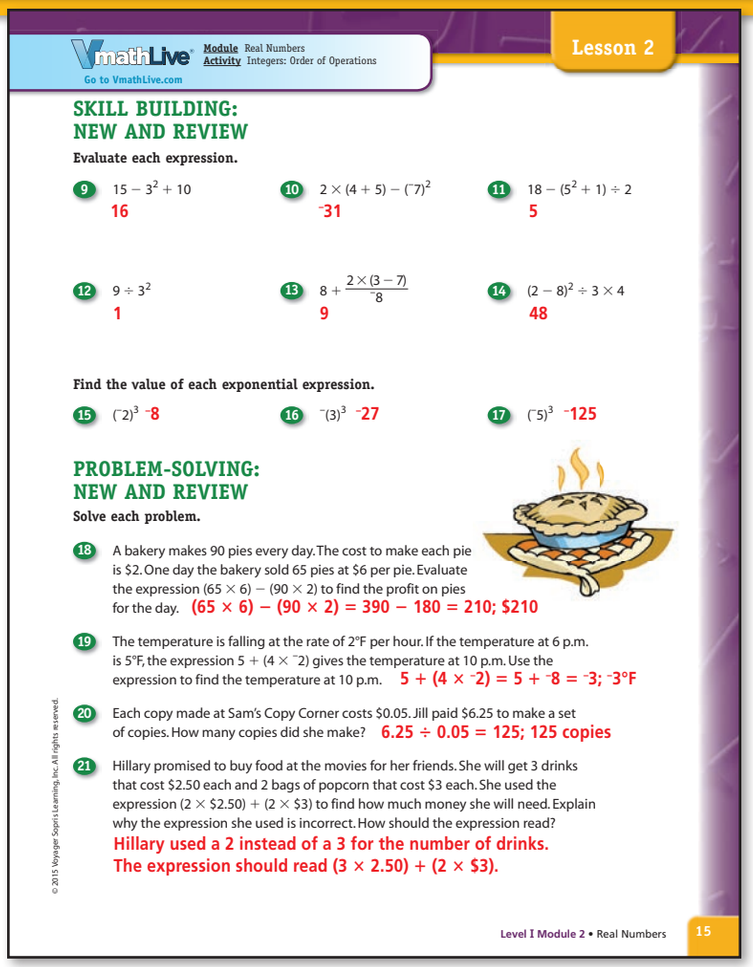
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1d: The student subtracted before multiplying. Use Additional Resources in I.2.2 to reteach how to evaluate the expression using the order of operations.

If student answered 1b or 2a: The student chose an answer with the correct absolute value but incorrect sign. Remind the student to follow the sign rules for working with integers when evaluating expressions.



VmathLive Module Real Numbers Activity Integers: Order of Operations Go to VmathLive.com

Lesson 2

SKILL BUILDING: NEW AND REVIEW

Evaluate each expression.

9. $15 - 3^2 + 10$ **16**
10. $2 \times (4 + 5) - (^-7)^2$ **$^-31$**
11. $18 - (5^2 + 1) \div 2$ **5**
12. $9 \div 3^2$ **1**
13. $8 + \frac{2 \times (3 - 7)}{^-8}$ **9**
14. $(2 - 8)^2 \div 3 \times 4$ **48**

Find the value of each exponential expression.

15. $(^-2)^3$ **$^-8$**
16. $^-(3)^3$ **$^-27$**
17. $(^-5)^3$ **$^-125$**

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

18. A bakery makes 90 pies every day. The cost to make each pie is $2. One day the bakery sold 65 pies at $6 per pie. Evaluate the expression $(65 \times 6) - (90 \times 2)$ to find the profit on pies for the day. **$(65 \times 6) - (90 \times 2) = 390 - 180 = 210$; $210**
19. The temperature is falling at the rate of 2°F per hour. If the temperature at 6 p.m. is 5°F, the expression $5 + (4 \times ^-2)$ gives the temperature at 10 p.m. Use the expression to find the temperature at 10 p.m. **$5 + (4 \times ^-2) = 5 + ^-8 = ^-3$; $^-3$°F**
20. Each copy made at Sam's Copy Corner costs $0.05. Jill paid $6.25 to make a set of copies. How many copies did she make? **$6.25 \div 0.05 = 125$; 125 copies**
21. Hillary promised to buy food at the movies for her friends. She will get 3 drinks that cost $2.50 each and 2 bags of popcorn that cost $3 each. She used the expression $(2 \times \$2.50) + (2 \times \$3)$ to find how much money she will need. Explain why the expression she used is incorrect. How should the expression read?
Hillary used a 2 instead of a 3 for the number of drinks. The expression should read $(3 \times 2.50) + (2 \times \$3)$.

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Level I Module 2 • Real Numbers 15

If student answered 2b or 2c: The student did not evaluate the denominator completely before completing the problem. Use Additional Resources in I.2.2 to reteach how to evaluate the expression using the order of operations.

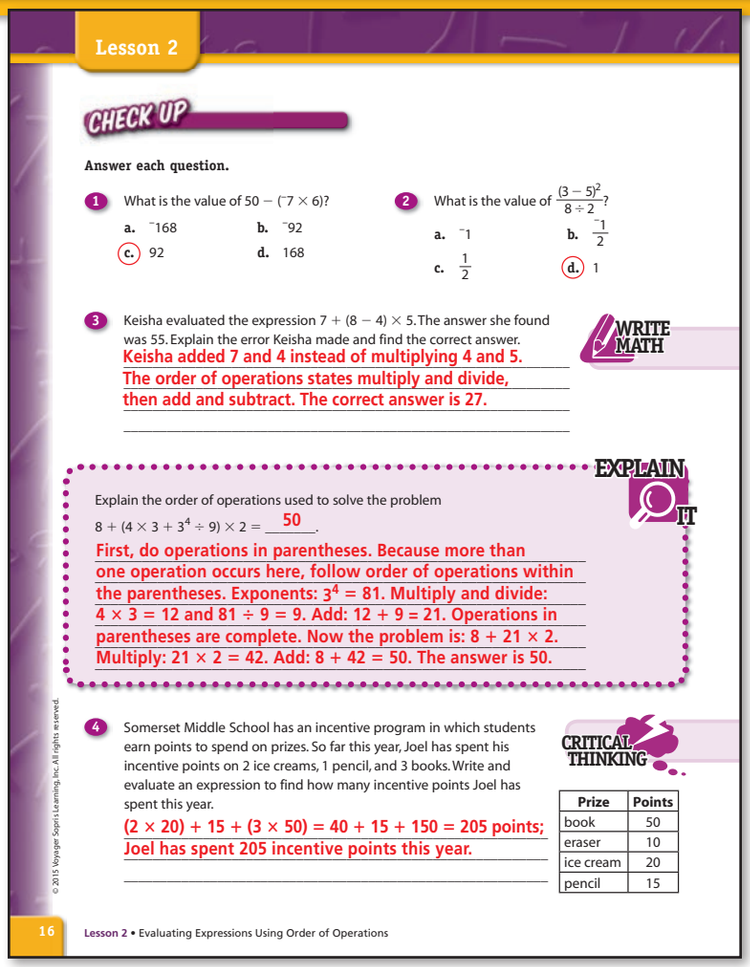


WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student the mnemonic ***P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally* to remind the student of the various operations and the order in which they should be performed.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 2

CHECK UP

Answer each question.

1 What is the value of $50 - (^{-}7 \times 6)$?

a. $^{-}168$ b. $^{-}92$

c. 92 d. 168

2 What is the value of $\frac{(3-5)^2}{8 \div 2}$?

a. $^{-}1$ b. $\frac{^{-}1}{2}$

c. $\frac{1}{2}$ d. 1

3 Keisha evaluated the expression $7 + (8 - 4) \times 5$. The answer she found was 55. Explain the error Keisha made and find the correct answer.

WRITE MATH

Keisha added 7 and 4 instead of multiplying 4 and 5. The order of operations states multiply and divide, then add and subtract. The correct answer is 27.

EXPLAIN IT

Explain the order of operations used to solve the problem $8 + (4 \times 3 + 3^4 \div 9) \times 2 =$ 50.

First, do operations in parentheses. Because more than one operation occurs here, follow order of operations within the parentheses. Exponents: $3^4 = 81$. Multiply and divide: $4 \times 3 = 12$ and $81 \div 9 = 9$. Add: $12 + 9 = 21$. Operations in parentheses are complete. Now the problem is: $8 + 21 \times 2$. Multiply: $21 \times 2 = 42$. Add: $8 + 42 = 50$. The answer is 50.

4 Somerset Middle School has an incentive program in which students earn points to spend on prizes. So far this year, Joel has spent his incentive points on 2 ice creams, 1 pencil, and 3 books. Write and evaluate an expression to find how many incentive points Joel has spent this year.

CRITICAL THINKING

Prize	Points
book	50
eraser	10
ice cream	20
pencil	15

$(2 \times 20) + 15 + (3 \times 50) = 40 + 15 + 150 = 205$ points; Joel has spent 205 incentive points this year.

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16 Lesson 2 • Evaluating Expressions Using Order of Operations

DIFFERENTIATION

Additional Resources

VmathLive
Module: Real Numbers
Activity: Integers: Order of Operations

Vmath Reteach
Reteach Student Book Module 2 Lesson 2
Reteach Teacher Edition Module 2 Lesson 2

Extra Practice
Student Book page 44

English Language Learners

Use the VmathLive Animated Glossary to review the terms *order* and *operations*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Explain to students that operations can be ordered just like they learned to order numbers. Encourage English Language Learners to create a mnemonic device similar to ***P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally* in their native language, or using a combination of English and their native language. Be sure students understand that a direct translation of the phrase into their native language might not be helpful because the initial letters of the words could change in the translation. Encourage students to adapt the idea of a mnemonic device to what works best for them.

Students with Special Needs

Provide students with four index cards, a sheet of colored construction paper, and a glue stick. Write the problem $1{,}000 - 60 \div (8 + 2) + 2^2 \times 4^3$ on the board and have students write it at the top of their papers. Guide students through the problem according to the order of operations. Students should write and complete the steps for an operation, then rewrite the problem on a single index card. Repeat for each operation until the problem is solved. Once all four steps are complete, have students glue the index cards in order on their sheet of construction paper. Allow students to reference the paper throughout the lesson as needed.

Lesson 3

Objective

To evaluate an expression with positive, negative, and zero exponents

Preskills	Lesson
Exponents	I.2.1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The expression 6^3 is an exponential expression. What is the base? (6) **What is the exponent?** (3) **To evaluate an exponential expression, write the base as a factor the number of times indicated by the exponent. To show the base used as a factor 3 times, write 6 times 6 times 6. Now multiply. What is 6 times 6?** (36) **What is 36 times 6?** (216) **Then 6^3 is 216.**

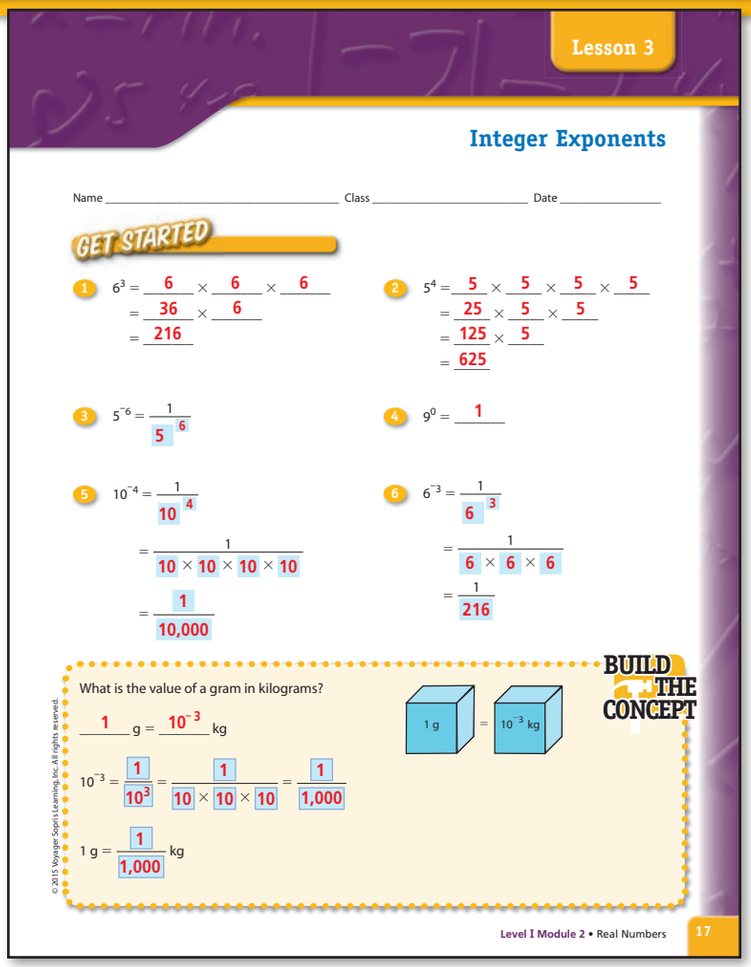
Problem 2

What is the base of the expression 5^4? (5) **What is the exponent?** (4) **Evaluate the expression by writing 5 as a factor 4 times. What is 5 times 5?** (25) **What is 25 times 5?** (125) **What is 125 times 5?** (625) **The value of 5^4 is 625.**

MODEL NEW SKILLS

Problem 3

The exponential expressions in problems 1 and 2 have positive exponents. An exponent also can be negative. The expression 5^{-6} has a negative exponent. An exponential expression with a negative exponent can be rewritten with a positive exponent by writing the expression as a fraction. The numerator of the fraction is 1. The denominator uses the original expression, 5^{-6}, but changes the negative exponent to a positive number. What is the new exponential expression in the denominator? (5^6) **The expression 5^{-6} written with a positive exponent is $\frac{1}{5^6}$.**



Lesson 3

Integer Exponents

Name _____ Class _____ Date _____

GET STARTED

1. $6^3 = \frac{6}{} \times \frac{6}{} \times \frac{6}{}$
$= \frac{36}{} \times \frac{6}{}$
$= \frac{216}{}$

2. $5^4 = \frac{5}{} \times \frac{5}{} \times \frac{5}{} \times \frac{5}{}$
$= \frac{25}{} \times \frac{5}{} \times \frac{5}{}$
$= \frac{125}{} \times \frac{5}{}$
$= \frac{625}{}$

3. $5^{-6} = \frac{1}{5^6}$

4. $9^0 = \frac{1}{}$

5. $10^{-4} = \frac{1}{10^4}$
$= \frac{1}{10 \times 10 \times 10 \times 10}$
$= \frac{1}{10{,}000}$

6. $6^{-3} = \frac{1}{6^3}$
$= \frac{1}{6 \times 6 \times 6}$
$= \frac{1}{216}$

BUILD THE CONCEPT

What is the value of a gram in kilograms?

$\frac{1}{}$ g = $\frac{10^{-3}}{}$ kg

$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1{,}000}$

1 g = $\frac{1}{1{,}000}$ kg

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Problem 4

The expression in this problem is read as *nine to the zero power.* For any nonzero base, if the exponent is zero, the value of the expression is 1. What is the base of 9^0? (9) **What is the exponent?** (0) **What is the value of 9^0?** (1)

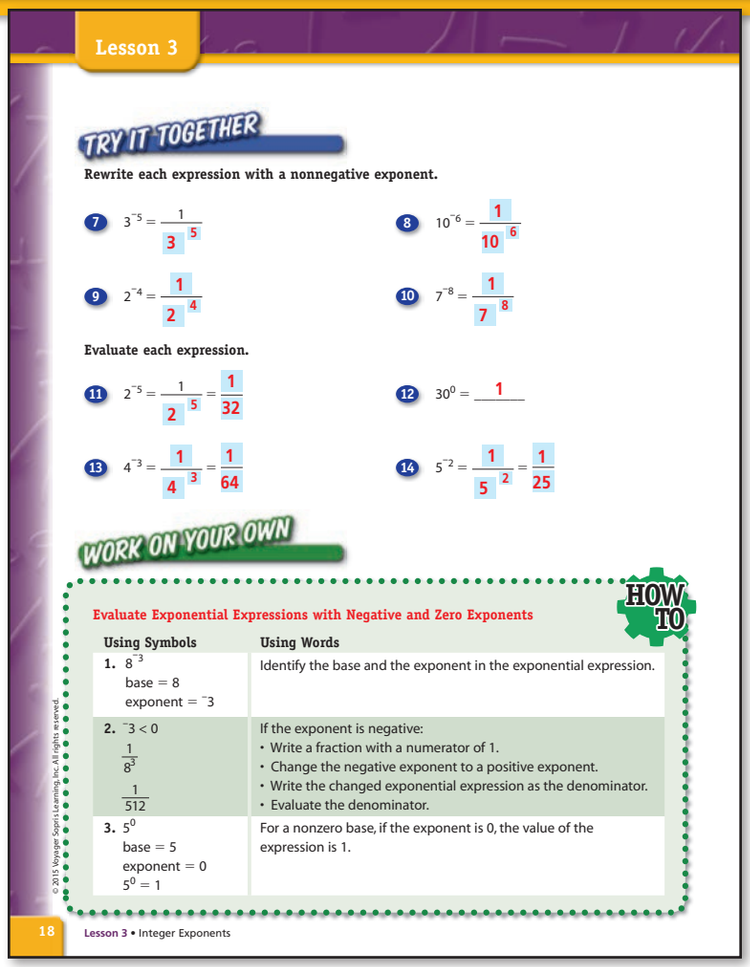
Problem 5

To evaluate an expression with a negative exponent, rewrite the expression with a positive exponent. To write 10^{-4} with a positive exponent, write a fraction with 1 as the numerator. Then change the negative exponent to a positive exponent and write the changed expression as the denominator.

What is 10^{-4} written with a positive exponent? ($\frac{1}{10^4}$) **Now, evaluate the denominator. What is 10 times 10 times 10 times 10?** (10,000) **What is $\frac{1}{10^4}$?** ($\frac{1}{10{,}000}$) **The value of 10^{-4} is $\frac{1}{10{,}000}$.**

Problem 6

To evaluate 6^{-3}, first write the expression with a positive exponent. To write 6^{-3} with a positive exponent, write a fraction with 1 as the numerator. Then change the



Lesson 3

TRY IT TOGETHER

Rewrite each expression with a nonnegative exponent.

7. $3^{-5} = \frac{1}{3^{5}}$

8. $10^{-6} = \frac{1}{10^{6}}$

9. $2^{-4} = \frac{1}{2^{4}}$

10. $7^{-8} = \frac{1}{7^{8}}$

Evaluate each expression.

11. $2^{-5} = \frac{1}{2^{5}} = \frac{1}{32}$

12. $30^{0} = 1$

13. $4^{-3} = \frac{1}{4^{3}} = \frac{1}{64}$

14. $5^{-2} = \frac{1}{5^{2}} = \frac{1}{25}$

WORK ON YOUR OWN

HOW TO

Evaluate Exponential Expressions with Negative and Zero Exponents

Using Symbols	Using Words
1. 8^{-3} base = 8 exponent = $^{-}3$	Identify the base and the exponent in the exponential expression.
2. $^{-}3 < 0$ $\frac{1}{8^{3}}$ $\frac{1}{512}$	If the exponent is negative: • Write a fraction with a numerator of 1. • Change the negative exponent to a positive exponent. • Write the changed exponential expression as the denominator. • Evaluate the denominator.
3. 5^{0} base = 5 exponent = 0 $5^{0} = 1$	For a nonzero base, if the exponent is 0, the value of the expression is 1.

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negative exponent to a positive exponent and write the changed expression as the denominator. What is 6^{-3} written with a positive exponent? ($\frac{1}{6^3}$) **Now, evaluate the denominator. This was evaluated in problem 1. What is 6^3?** (216) **What is $\frac{1}{6^3}$?** ($\frac{1}{216}$) **The value of 6^{-3} is $\frac{1}{216}$.**

BUILD THE CONCEPT

Model how to evaluate an expression with a negative exponent.

What exponential expression equates the value of a gram in kilograms? (10^{-3} kg) **What is the base of this expression?** (10) **What is the exponent?** ($^{-}3$) **Is the exponent positive, negative, or zero?** (negative) **The exponential expression should be written as a fraction. What is the numerator of the fraction?** (1) **What is the denominator of the fraction?** (10^3) **What multiplication sentence represents 10^3?** ($10 \times 10 \times 10$) **What is 10 times 10 times 10?** (1,000) **What is the new denominator?** (1,000) **What fraction equals 10^{-3}?** ($\frac{1}{1{,}000}$) **What is the value of 1 gram in kilograms?** ($\frac{1}{1{,}000}$ kg)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 7

What is the first step when changing an exponential expression with a negative exponent to an expression with a positive exponent? (Write a fraction with 1 as the numerator.) **What is the next step?** (Change the negative exponent to a positive exponent and write the changed expression as the denominator.) **What is 3^{-5} written with a positive exponent?** ($\frac{1}{3^5}$)

Problem 8

What number should be written as the numerator of the fraction? (1) **What expression should be written as the denominator of the fraction?** (10^6) **What is 10^{-6} written with a positive exponent?** ($\frac{1}{10^6}$)

Problem 9

What expression should be written as the denominator of the fraction? (2^4) **What is 2^{-4} written with a positive exponent?** ($\frac{1}{2^4}$)

Problem 10

What is 7^{-8} written with a positive exponent? ($\frac{1}{7^8}$)

Problem 11

What is the first step when evaluating an expression with a negative exponent? (Write the expression with a positive exponent.) **What is 2^{-5} written with a positive exponent?** ($\frac{1}{2^5}$) **What is the value of 2^5?** (32) **What is the value of $\frac{1}{2^5}$?** ($\frac{1}{32}$) **What is the value of 2^{-5}?** ($\frac{1}{32}$)

Problem 12

What is the base of 30^0? (30) **What is the exponent?** (0) **What is the value of an expression with a nonzero base and an exponent of zero?** (1) **What is the value of 30^0?** (1)

Problem 13

What is 4^{-3} written with a positive exponent? ($\frac{1}{4^3}$) **What is the value of 4^3?** (64) **What is the value of 4^{-3}?** ($\frac{1}{64}$)

Problem 14

What is the value of 5^{-2}? ($\frac{1}{25}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Pattern** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a pattern. The instruction is immediately followed by application of the strategy in problem 27.

Have students read the problem.

Ask students what they are to find.

Have students read the problem again. The problem mentions a pattern.

Have students list the numbers in the problem:

$2^{-6}, 2^{-5}, 2^{-4}, ?, ?$

Have students describe the pattern using words:

The base is the same, but each subsequent exponent is 1 more than the previous exponent.

The rule of the pattern is: Add 1 to the exponent each week.

Have student complete the pattern:

$2^{-4+1} = 2^{-3} = \frac{1}{8}$

$2^{-3+1} = 2^{-2} = \frac{1}{4}$

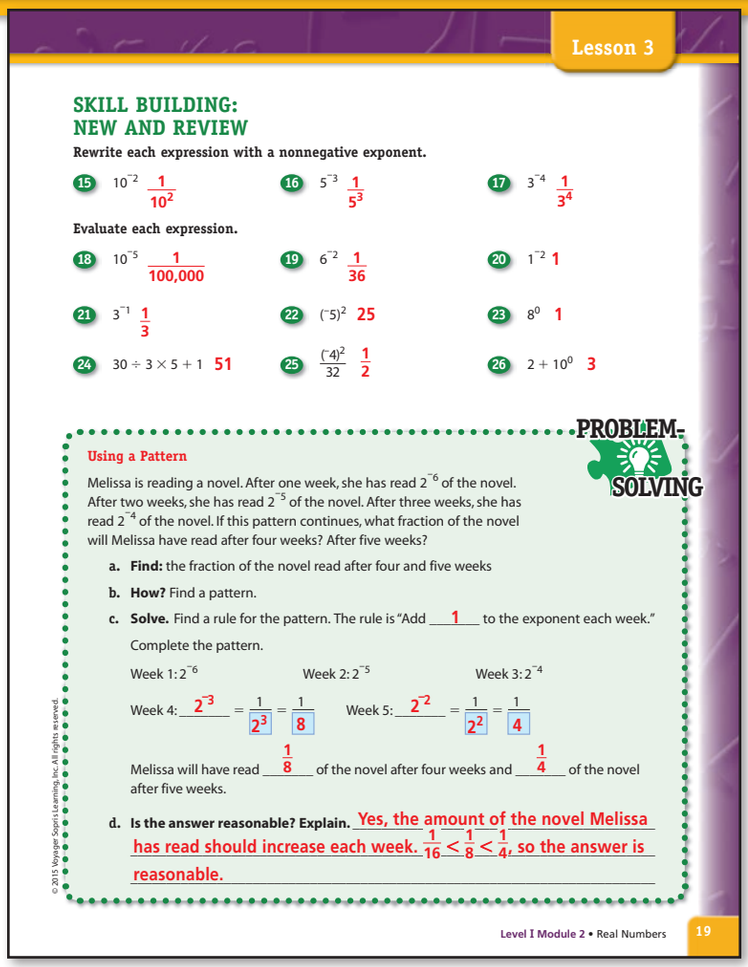
The answer is reasonable because each week, the amount of the novel read increases.

Problems 15–30

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 20 Reminder

Review with the students that the value of 1 raised to any power is 1.



Lesson 3

SKILL BUILDING: NEW AND REVIEW

Rewrite each expression with a nonnegative exponent.

15. 10^{-2} $\frac{1}{10^2}$
16. 5^{-3} $\frac{1}{5^3}$
17. 3^{-4} $\frac{1}{3^4}$

Evaluate each expression.

18. 10^{-5} $\frac{1}{100{,}000}$
19. 6^{-2} $\frac{1}{36}$
20. 1^{-2} 1
21. 3^{-1} $\frac{1}{3}$
22. $(-5)^2$ 25
23. 8^0 1
24. $30 \div 3 \times 5 + 1$ 51
25. $\frac{(-4)^2}{32}$ $\frac{1}{2}$
26. $2 + 10^0$ 3

PROBLEM-SOLVING

Using a Pattern

Melissa is reading a novel. After one week, she has read 2^{-6} of the novel. After two weeks, she has read 2^{-5} of the novel. After three weeks, she has read 2^{-4} of the novel. If this pattern continues, what fraction of the novel will Melissa have read after four weeks? After five weeks?

a. **Find:** the fraction of the novel read after four and five weeks

b. **How?** Find a pattern.

c. **Solve.** Find a rule for the pattern. The rule is "Add 1 to the exponent each week."

Complete the pattern.

Week 1: 2^{-6} Week 2: 2^{-5} Week 3: 2^{-4}

Week 4: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ Week 5: $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Melissa will have read $\frac{1}{8}$ of the novel after four weeks and $\frac{1}{4}$ of the novel after five weeks.

d. **Is the answer reasonable? Explain.** Yes, the amount of the novel Melissa has read should increase each week. $\frac{1}{16} < \frac{1}{8} < \frac{1}{4}$, so the answer is reasonable.

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Problem 30 Reminder

Tell students to identify the base and the exponent in each expression. For a positive exponent, write the base as a factor the number of times indicated by the exponent. For a negative exponent, first rewrite the expression with a positive exponent, then evaluate.

CHECK UP

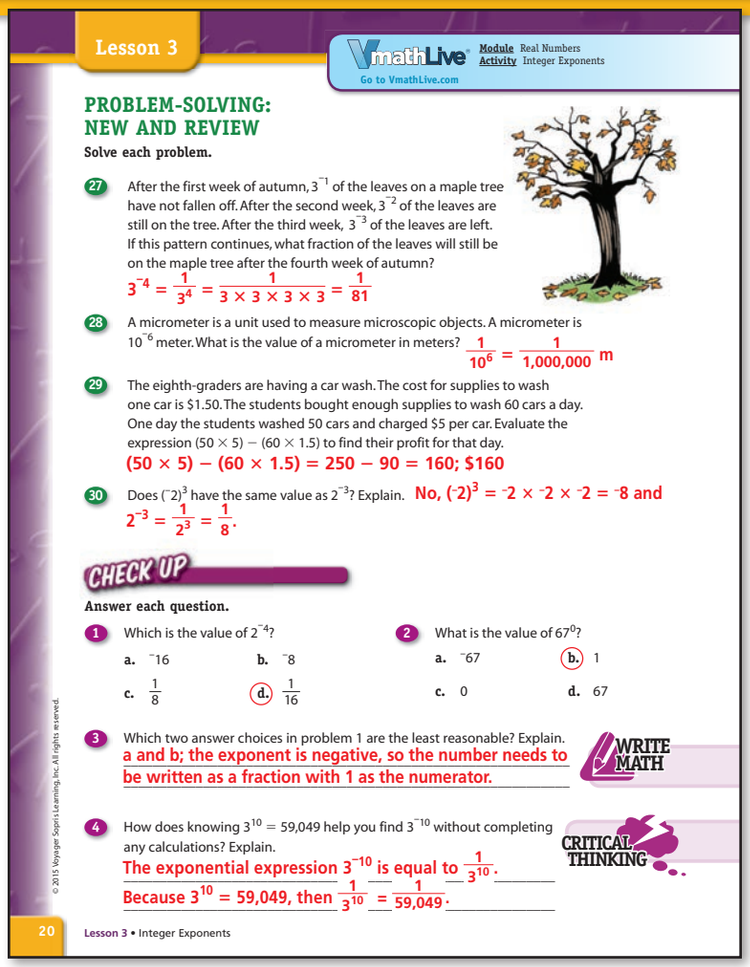
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student associated a negative exponent with a negative base. Use Additional Resources in I.2.3 to reteach how to evaluate expressions with integer exponents.

If student answered 1b or 1c: The student multiplied the exponent by the base. Review the steps to rewrite the expression with a positive exponent, then review the steps to evaluate the rewritten expression.



Lesson 3

VmathLive Module Real Numbers Activity Integer Exponents
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

27. After the first week of autumn, 3^{-1} of the leaves on a maple tree have not fallen off. After the second week, 3^{-2} of the leaves are still on the tree. After the third week, 3^{-3} of the leaves are left. If this pattern continues, what fraction of the leaves will still be on the maple tree after the fourth week of autumn?
$3^{-4} = \frac{1}{3^4} = \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}$

28. A micrometer is a unit used to measure microscopic objects. A micrometer is 10^{-6} meter. What is the value of a micrometer in meters? $\frac{1}{10^6} = \frac{1}{1{,}000{,}000}$ m

29. The eighth-graders are having a car wash. The cost for supplies to wash one car is $1.50. The students bought enough supplies to wash 60 cars a day. One day the students washed 50 cars and charged $5 per car. Evaluate the expression $(50 \times 5) - (60 \times 1.5)$ to find their profit for that day.
$(50 \times 5) - (60 \times 1.5) = 250 - 90 = 160$; $160

30. Does $(^-2)^3$ have the same value as 2^{-3}? Explain. No, $(^-2)^3 = {}^-2 \times {}^-2 \times {}^-2 = {}^-8$ and $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

CHECK UP

Answer each question.

1. Which is the value of 2^{-4}?
   a. $^-16$ b. $^-8$ c. $\frac{1}{8}$ (d.) $\frac{1}{16}$

2. What is the value of 67^0?
   a. $^-67$ (b.) 1 c. 0 d. 67

3. Which two answer choices in problem 1 are the least reasonable? Explain. WRITE MATH
a and b; the exponent is negative, so the number needs to be written as a fraction with 1 as the numerator.

4. How does knowing $3^{10} = 59{,}049$ help you find 3^{-10} without completing any calculations? Explain. CRITICAL THINKING
The exponential expression 3^{-10} is equal to $\frac{1}{3^{10}}$.
Because $3^{10} = 59{,}049$, then $\frac{1}{3^{10}} = \frac{1}{59{,}049}$.

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If student answered problem 2 incorrectly: The student did not remember that the value of a nonzero base with a power of zero is 1. Use Additional Resources in I.2.3 to reteach how to evaluate expressions with integer exponents.



If student answered problem 3 incorrectly: While students work together, review with the student that if the exponent of an expression is negative, then the value of the expression will be a fraction with 1 as the numerator.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Real Numbers
Activity: Integer Exponents

Vmath Reteach
Reteach Student Book Module 2 Lesson 3
Reteach Teacher Edition Module 2 Lesson 3

Extra Practice
Student Book page 45

English Language Learners

Introducing negative exponents might cause confusion when evaluating exponential expressions. Remind students that if the base is negative, it remains unchanged. However, if the exponent is negative, the expression will need to be changed according to the steps outlined in the lesson.

Students with Special Needs

Students with special needs might have trouble following the multistep procedure presented in this lesson for evaluating expressions with negative exponents. To help minimize student perception of the number of steps involved, relate this skill to one they have already mastered. Explain to students that evaluating expressions with negative exponents follows the same process as positive exponents, with one additional step. They can ignore the negative sign in an exponent, and solve the expression as if it had a positive exponent. Then they create a fraction with 1 as the numerator, and the value they found as the denominator.

Lesson 4

Objective

To use properties of exponents to simplify an expression

Preskills	Lesson
Exponents	I.2.1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The expression 6^3 is an exponential expression. What is the base? (6) **What is the exponent?** (3) **Write 6 times 6 times 6 to show that 6 is used as a factor 3 times.**

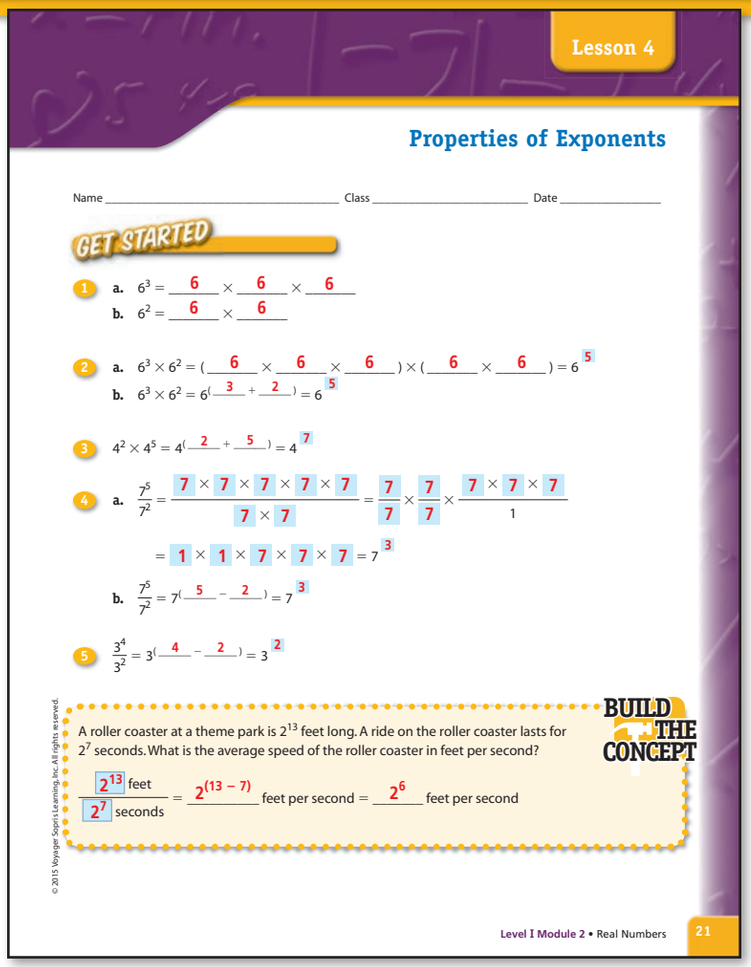
The expression shown in problem 1b is also an exponential expression. What is the base? (6) **What is the exponent?** (2) **How is the product written as repeated multiplication?** (6×6)

MODEL NEW SKILLS

Problem 2

The expression 6^3 times 6^2 can be written with a single exponent because both factors have the same base, 6. Writing the expression with a single exponent is called *simplifying the expression*.

In problem 2a, write the number of times 6 is used as a factor as repeated multiplication. In problem 1, it was determined that 6^3 equals 6 times 6 times 6 and 6^2 equals 6 times 6. Then the product of 6^3 and 6^2 is the product of 6 times 6 times 6, and the product of 6 times 6. How many times is 6 used as a factor in 6^3 times 6^2? (5) **To simplify 6^3 times 6^2, write it as 6^5.**



Lesson 4

Properties of Exponents

Name _____ Class _____ Date _____

GET STARTED

1 a. $6^3 = \underline{6} \times \underline{6} \times \underline{6}$

b. $6^2 = \underline{6} \times \underline{6}$

2 a. $6^3 \times 6^2 = (\underline{6} \times \underline{6} \times \underline{6}) \times (\underline{6} \times \underline{6}) = 6^{\boxed{5}}$

b. $6^3 \times 6^2 = 6^{(\underline{3} + \underline{2})} = 6^{\boxed{5}}$

3 $4^2 \times 4^5 = 4^{(\underline{2} + \underline{5})} = 4^{\boxed{7}}$

4 a. $\frac{7^5}{7^2} = \frac{\boxed{7} \times \boxed{7} \times \boxed{7} \times \boxed{7} \times \boxed{7}}{\boxed{7} \times \boxed{7}} = \frac{\boxed{7}}{\boxed{7}} \times \frac{\boxed{7}}{\boxed{7}} \times \frac{\boxed{7} \times \boxed{7} \times \boxed{7}}{1}$

$= \boxed{1} \times \boxed{1} \times \boxed{7} \times \boxed{7} \times \boxed{7} = 7^{\boxed{3}}$

b. $\frac{7^5}{7^2} = 7^{(\underline{5} - \underline{2})} = 7^{\boxed{3}}$

5 $\frac{3^4}{3^2} = 3^{(\underline{4} - \underline{2})} = 3^{\boxed{2}}$

BUILD THE CONCEPT

A roller coaster at a theme park is 2^{13} feet long. A ride on the roller coaster lasts for 2^7 seconds. What is the average speed of the roller coaster in feet per second?

$\frac{\boxed{2^{13}} \text{ feet}}{\boxed{2^7} \text{ seconds}} = \underline{2^{(13-7)}}$ feet per second $= \underline{2^6}$ feet per second

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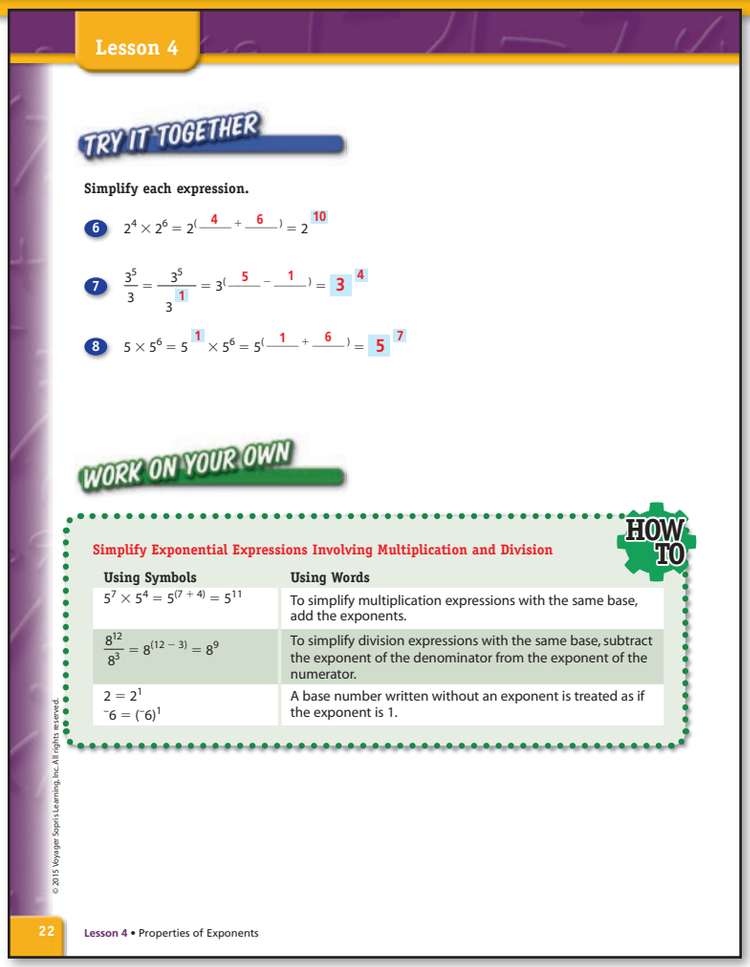
In problem 2b, notice that the number of times 6 is used as a factor in 6^3 times 6^2 is the same as the sum of the exponents in 6^3 and 6^2. What is 3 plus 2? (5) **To simplify multiplication expressions with the same base, keep the same base and add the exponents. So, 6^3 times 6^2 is $6^{(3+2)}$, which is 6^5.**

Problem 3

Are the bases in 4^2 and 4^5 the same? (yes) **Then the expression can be simplified. To simplify the multiplication expression $4^2 \times 4^5$, keep the same base and add the exponents. What is the exponent of 4^2?** (2) **What is the exponent of 4^5?** (5) **What is 2 plus 5?** (7) **Then 4^2 times 4^5 is 4^7.**

Problem 4

The expression $\frac{7^5}{7^2}$ is the quotient of two exponential expressions that have the same base. To simplify this expression, divide 7^5 by 7^2. To divide 7^5 by 7^2, first write it as 7^5 over 7^2. How is 7^5 written as a product? ($7 \times 7 \times 7 \times 7 \times 7$) **How is 7^2 written as a product?** (7×7)



Lesson 4

TRY IT TOGETHER

Simplify each expression.

6. $2^4 \times 2^6 = 2^{(\underline{4} + \underline{6})} = 2^{10}$

7. $\frac{3^5}{3} = \frac{3^5}{3^{\underline{1}}} = 3^{(\underline{5} - \underline{1})} = 3^4$

8. $5 \times 5^6 = 5^{\underline{1}} \times 5^6 = 5^{(\underline{1} + \underline{6})} = 5^7$

WORK ON YOUR OWN

HOW TO

Simplify Exponential Expressions Involving Multiplication and Division

Using Symbols	Using Words
$5^7 \times 5^4 = 5^{(7+4)} = 5^{11}$	To simplify multiplication expressions with the same base, add the exponents.
$\frac{8^{12}}{8^3} = 8^{(12-3)} = 8^9$	To simplify division expressions with the same base, subtract the exponent of the denominator from the exponent of the numerator.
$2 = 2^1$ $^-6 = (^-6)^1$	A base number written without an exponent is treated as if the exponent is 1.

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This means that $\frac{7^5}{7^2}$ can be written as $\frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7}$. The expression can be regrouped and rewritten as $\frac{7}{7} \times \frac{7}{7} \times \frac{7 \times 7 \times 7}{1}$. Notice that the first two factors in the fraction are 7 divided by 7. What is the value of the first 7 divided by 7? (1) **What is the value of the second 7 divided by 7?** (1) **How many factors of 7 are left?** (3) **What is 1 times 1 times 7 times 7 times 7 written with an exponent?** (7^3) **To simplify 7^5 divided by 7^2, write 7^3. Notice that the number of times 7 is used as a factor in 7^5 divided by 7^2 is the same as the difference of the exponents in 7^5 and 7^2. What is 5 minus 2?** (3)

To simplify division expressions with the same base, subtract the exponent of the denominator from the exponent of the numerator and keep the same base. So, $\frac{7^5}{7^2}$ is 7^3.

Problem 5

To simplify this expression, keep the same base and subtract the exponent of the denominator from the exponent of the numerator. What is 4 minus 2? (2) **When $\frac{3^4}{3^2}$ is simplified, what is the result?** (3^2)

BUILD THE CONCEPT

Model how to simplify an expression using the properties of exponents.

What does the problem ask to find? (the average speed of the roller coaster) **The formula for speed is *distance divided by time*. What is the distance of the roller coaster?** (2^{13} ft) **What is the time of a roller coaster ride?** (2^7 sec) **What expression represents the average speed of the roller coaster?** ($\frac{2^{13}}{2^7}$) **Can this expression be simplified?** (yes) **Why?** (the bases are the same) **What is the common base?** (2) **What is the exponent of the numerator?** (13) **What is the exponent of the denominator?** (7) **What is 13 minus 7?** (6) **What is the new exponential expression?** (2^6) **What is the average speed of the roller coaster?** (2^6 ft per sec)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

Are the bases the same? (yes) **Are the expressions being multiplied or divided?** (multiplied) **How are multiplication expressions with the same base simplified?** (by adding the exponents of the expressions) **What numbers should be added?** (4 and 6) **What is 4 plus 6?** (10) **What is the result when 2^4 times 2^6 is simplified?** (2^{10})

Problem 7

When a base has no written exponent present, it implies an exponent of 1. What is the exponent of the base in the denominator? (1) **Should the exponents be added or subtracted?** (subtracted) **Why?** (The expressions are being divided.) **What is 5 minus 1?** (4) **What is the simplified expression?** (3^4)

Problem 8

Should the exponents be added or subtracted? (added) **Why?** (The expressions are multiplied.) **What is 1 plus 6?** (7) **What is the simplified expression?** (5^7)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 9–24

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

CHECK UP

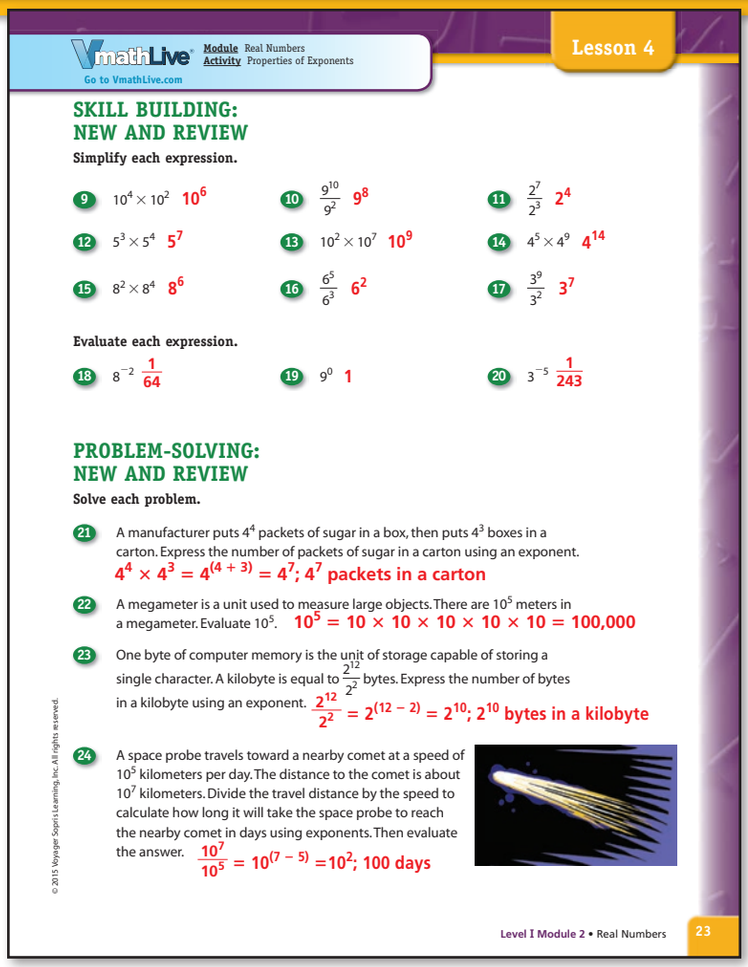
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly:
The student either divided, subtracted, or multiplied the exponents instead of adding. Use Additional Resources in I.2.4 to reteach how to simplify exponential expressions.

If student answered problem 2 incorrectly: The student either divided, multiplied, or added the exponents instead of subtracting. Use Additional Resources in I.2.4 to reteach how to simplify exponential expressions.



VmathLive Module Real Numbers Activity Properties of Exponents
Go to VmathLive.com

Lesson 4

SKILL BUILDING: NEW AND REVIEW

Simplify each expression.

9. $10^4 \times 10^2$ 10^6
10. $\frac{9^{10}}{9^2}$ 9^8
11. $\frac{2^7}{2^3}$ 2^4
12. $5^3 \times 5^4$ 5^7
13. $10^2 \times 10^7$ 10^9
14. $4^5 \times 4^9$ 4^{14}
15. $8^2 \times 8^4$ 8^6
16. $\frac{6^5}{6^3}$ 6^2
17. $\frac{3^9}{3^2}$ 3^7

Evaluate each expression.

18. 8^{-2} $\frac{1}{64}$
19. 9^0 1
20. 3^{-5} $\frac{1}{243}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

21. A manufacturer puts 4^4 packets of sugar in a box, then puts 4^3 boxes in a carton. Express the number of packets of sugar in a carton using an exponent.
$4^4 \times 4^3 = 4^{(4+3)} = 4^7$; 4^7 packets in a carton

22. A megameter is a unit used to measure large objects. There are 10^5 meters in a megameter. Evaluate 10^5. $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$

23. One byte of computer memory is the unit of storage capable of storing a single character. A kilobyte is equal to 2^{12} bytes. Express the number of bytes in a kilobyte using an exponent. $\frac{2^{12}}{2^2} = 2^{(12-2)} = 2^{10}$; 2^{10} bytes in a kilobyte

24. A space probe travels toward a nearby comet at a speed of 10^5 kilometers per day. The distance to the comet is about 10^7 kilometers. Divide the travel distance by the speed to calculate how long it will take the space probe to reach the nearby comet in days using exponents. Then evaluate the answer. $\frac{10^7}{10^5} = 10^{(7-5)} = 10^2$; 100 days

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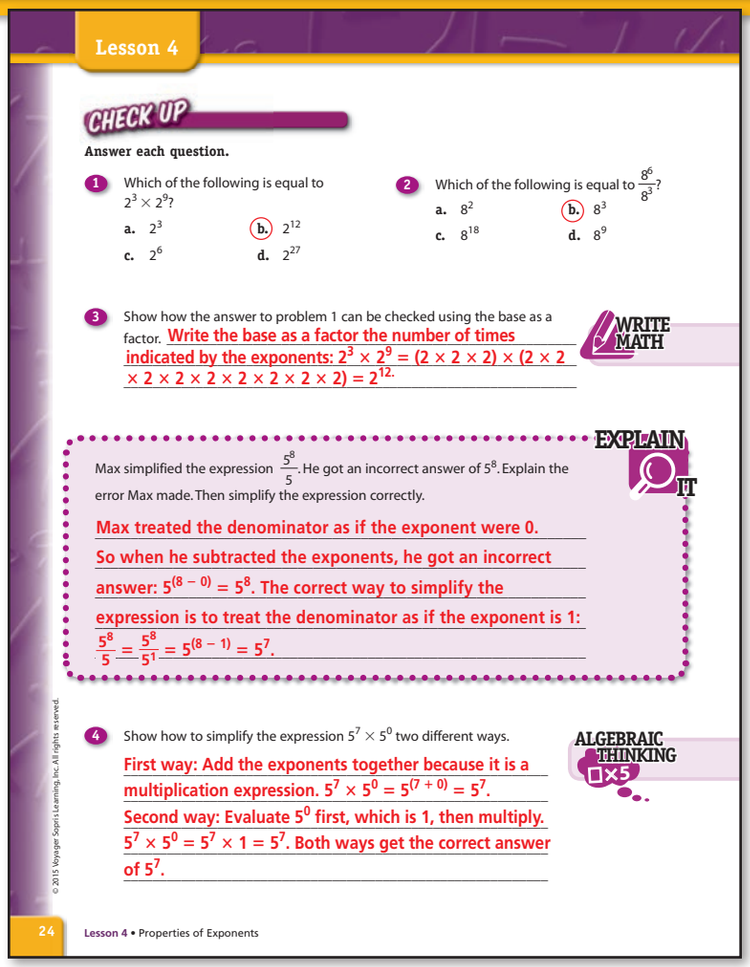


WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to write an exponential expression using repeated multiplication.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 4

CHECK UP

Answer each question.

1. Which of the following is equal to $2^3 \times 2^9$?
   a. 2^3
   b. 2^{12}
   c. 2^6
   d. 2^{27}

2. Which of the following is equal to $\frac{8^6}{8^3}$?
   a. 8^2
   b. 8^3
   c. 8^{18}
   d. 8^9

3. Show how the answer to problem 1 can be checked using the base as a factor. Write the base as a factor the number of times indicated by the exponents: $2^3 \times 2^9 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) = 2^{12}$.

WRITE MATH

EXPLAIN IT

Max simplified the expression $\frac{5^8}{5}$. He got an incorrect answer of 5^8. Explain the error Max made. Then simplify the expression correctly.

Max treated the denominator as if the exponent were 0. So when he subtracted the exponents, he got an incorrect answer: $5^{(8-0)} = 5^8$. The correct way to simplify the expression is to treat the denominator as if the exponent is 1: $\frac{5^8}{5} = \frac{5^8}{5^1} = 5^{(8-1)} = 5^7$.

4. Show how to simplify the expression $5^7 \times 5^0$ two different ways.
   First way: Add the exponents together because it is a multiplication expression. $5^7 \times 5^0 = 5^{(7+0)} = 5^7$.
   Second way: Evaluate 5^0 first, which is 1, then multiply. $5^7 \times 5^0 = 5^7 \times 1 = 5^7$. Both ways get the correct answer of 5^7.

ALGEBRAIC THINKING

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Technology

Have students use an online search engine to discover other ways to apply evaluating exponents in real-life situations. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Real Numbers
Activity: Properties of Exponents

Vmath Reteach
Reteach Student Book Module 2 Lesson 4
Reteach Teacher Edition Module 2 Lesson 4

Extra Practice
Student Book page 45

English Language Learners

Students might be unfamiliar with some of the terminology used in the application problems for this lesson. Demonstrate the concepts of speed, time, and distance for students. For example, walk across the room and explain to students that this is a slow speed. Then run across the room to show students a fast speed. Demonstrate the concept of time by having students stand for 30 seconds, then sit back down. Focus on the clock together as a class to emphasize the time the students stood was 30 seconds. Demonstrate the concept of distance in a similar way. Continue to interactively demonstrate any terminology students seem questionable about throughout the lesson.

Students with Special Needs

Students might confuse when to add the exponents of an expression and when to subtract the exponents of an expression. Show students how the multiplication sign looks similar to an addition sign, except it is rotated. Then show students how a fraction bar looks similar to a minus sign, except it is larger.

Lesson 5

Objective

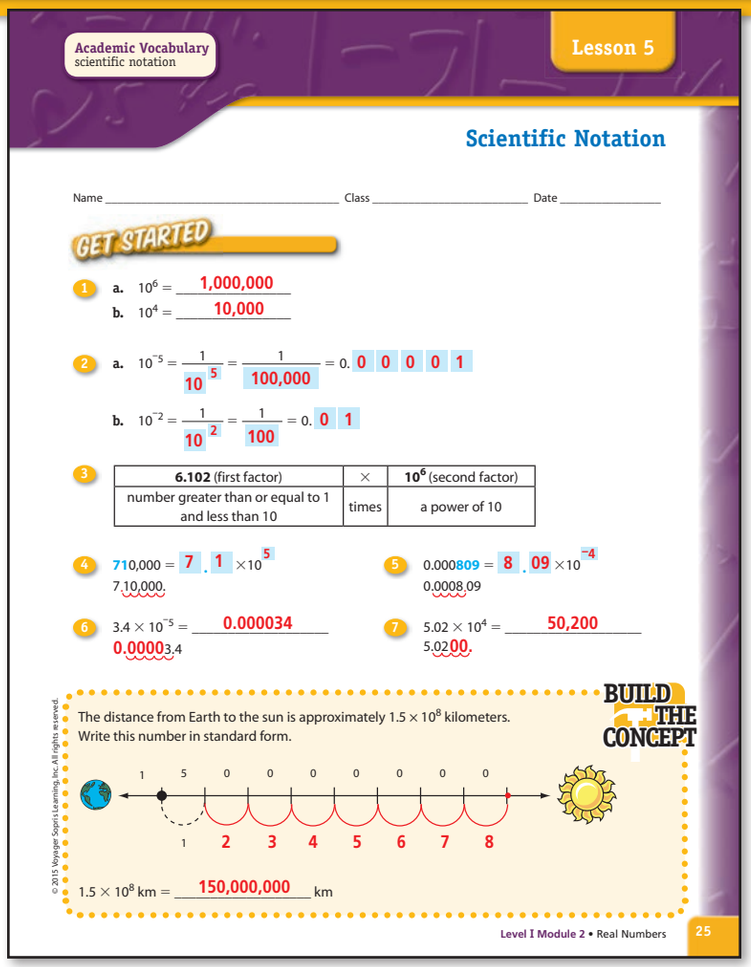
To write a number using scientific notation

Preskills	Lesson
Exponents	I.2.1
Integer Exponents	I.2.3

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **scientific notation**
  a number expressed as the product of two factors; the first factor is a number greater than or equal to 1 and less than 10, and the second is a power of 10


Academic Vocabulary
scientific notation

Lesson 5

Scientific Notation

Name _____ Class _____ Date _____

GET STARTED

1 a. 10^6 = 1,000,000
b. 10^4 = 10,000

2 a. $10^{-5} = \frac{1}{10^5} = \frac{1}{100,000} = 0.00001$

b. $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

3

6.102 (first factor)	×	10^6 (second factor)
number greater than or equal to 1 and less than 10	times	a power of 10

4 710,000 = 7.1 × 10^5
7.10,000.

5 0.000809 = 8.09 × 10^{-4}
0.0008.09

6 3.4 × 10^{-5} = 0.000034
0.00003.4

7 5.02 × 10^4 = 50,200
5.02000.

BUILD THE CONCEPT

The distance from Earth to the sun is approximately 1.5 × 10^8 kilometers. Write this number in standard form.

1 5 0 0 0 0 0 0 0 0
1 2 3 4 5 6 7 8

1.5 × 10^8 km = 150,000,000 km

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What is the value of 10^6? (1,000,000) **What is the value of 10^4?** (10,000)

Problem 2

To evaluate 10^{-5}, first write the expression with a positive exponent. What is 10^{-5} written with a positive exponent? ($\frac{1}{10^5}$) **Now evaluate the denominator. What is the value of 10^5?** (100,000) **Then 10^{-5} is equal to $\frac{1}{100,000}$. How is this fraction read?** (one one hundred-thousandth) **How is this number written as a decimal?** (0.00001) **Then 10^{-5} is equal to 0.00001.**

Look at problem 2b. To write 10^{-2} as a decimal, first write it with a positive exponent. What is 10^{-2} written with a positive exponent? ($\frac{1}{10^2}$) **Now evaluate. What is the value of $\frac{1}{10^2}$?** ($\frac{1}{100}$) **How is the fraction written as a decimal?** (0.01) **Then 10^{-2} equals 0.01.**

MODEL NEW SKILLS

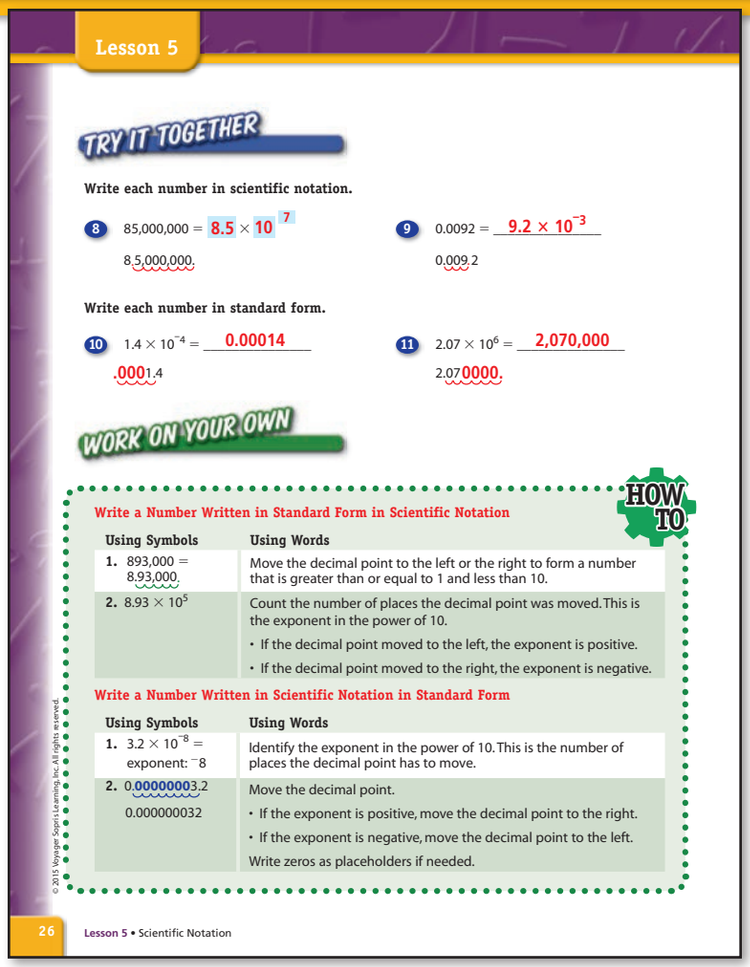
Problem 3

The expression 6.102 × 10^6 is the product of two factors. The first factor is a number greater than or equal to 1 and less than 10. The second factor is a power of 10. A number expressed in this form is in scientific notation.

Problem 4

Scientific notation is used to easily write very large numbers or very small numbers that otherwise would require many zeros to write in standard form. Positive exponents are used to represent large numbers, and negative exponents are used to represent small numbers less than 1.

To express 710,000 in scientific notation, move the decimal point to the left to form a number greater than or equal to 1 and less than 10. Then multiply that number by a power of 10. The decimal point in 710,000 is to the right of the ones place. Move it five places to the left to form the number 7.1, dropping the zeros. Because the decimal point was moved five places to the left, the exponent of the power of 10 is 5. Written in scientific notation, 710,000 is 7.1 × 10^5.



Lesson 5

TRY IT TOGETHER

Write each number in scientific notation.

8 85,000,000 = 8.5 × 10^7

8.5,000,000.

9 0.0092 = 9.2 × 10^−3

0.009.2

Write each number in standard form.

10 1.4 × 10^−4 = 0.00014

.0001.4

11 2.07 × 10^6 = 2,070,000

2.070000.

WORK ON YOUR OWN

HOW TO

Write a Number Written in Standard Form in Scientific Notation

Using Symbols	Using Words
1. 893,000 = 8.93,000.	Move the decimal point to the left or the right to form a number that is greater than or equal to 1 and less than 10.
2. 8.93 × 10^5	Count the number of places the decimal point was moved. This is the exponent in the power of 10. • If the decimal point moved to the left, the exponent is positive. • If the decimal point moved to the right, the exponent is negative.

Write a Number Written in Scientific Notation in Standard Form

Using Symbols	Using Words
1. 3.2 × 10^−8 = exponent: −8	Identify the exponent in the power of 10. This is the number of places the decimal point has to move.
2. 0.00000003.2 0.000000032	Move the decimal point. • If the exponent is positive, move the decimal point to the right. • If the exponent is negative, move the decimal point to the left. Write zeros as placeholders if needed.

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26 Lesson 5 • Scientific Notation

Problem 5

To express 0.000809 in scientific notation, move the decimal point to the right to form a number greater than or equal to 1 and less than 10. The decimal point must be moved four places to the right to form the number 8.09. Drop the zeros to the left of the 8. Notice that the zero between 8 and 9 is not dropped. This number, 8.09, is the first factor in the scientific notation expression. Because the decimal point was moved four places to the right, the exponent of the power of 10 is $^{-}4$. Written in scientific notation, 0.000809 is 8.09×10^{-4}.

Problem 6

This number is written in scientific notation. To write 3.4×10^{-5} in standard form, identify the exponent in the power of 10. What is the exponent? ($^{-}5$) **The exponent is negative, so move the decimal point in the first number to the left the same number of places as the exponent number. Use zeros as placeholders. How many places does the decimal point need to be moved?** (5) **Write four zeros as placeholders in the number. What is 3.4×10^{-5} written in standard form?** (0.000034)

Problem 7

To write 5.02×10^4 in standard form, identify the exponent in the power of 10. What is the exponent? (4) **Because the exponent is positive, move the decimal point in the first number to the right four places. How many digits are after the decimal point in 5.02?** (2) **Because the decimal point needs to be moved four places and there are only two digits after the decimal point in 5.02, two additional zeros are needed as placeholders. What is 5.02×10^4 written in standard form?** (50,200)

BUILD THE CONCEPT

Model how to write a number using scientific notation.

What is the exponent in the power of 10? (8) **How many places will the decimal point move?** (8) **Is the exponent positive or negative?** (positive) **Which direction will the decimal point move?** (right) **Jumps on a number line can be used to move the decimal point the correct number of places. Start at the decimal point and jump 8 times to the right. Place a decimal point where the last jump lands. Number each jump to make sure the correct number is made. Use the digits above the number line and the new decimal point to write the number in standard form. What is the standard form of 1.5×10^8?** (150,000,000) **What is the approximate distance from Earth to the sun?** (150,000,000 km)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 8

What is this number? (85,000,000) **What is the first step in expressing this number in scientific notation?** (Move the decimal point from the right of the ones place to the right of the first digit.) **What is the first factor of the scientific notation?** (8.5) **How many places was the decimal point moved?** (7) **Is the exponent positive or negative?** (positive) **Why?** (The decimal point moved to the left and the scientific notation represents a large number.) **What is the second factor of the scientific notation?** (10^7) **What is 85,000,000 written in scientific notation?** (8.5×10^7)

Problem 9

What is the first factor in the scientific notation expression for 0.0092? (9.2) **How many places was the decimal point moved?** (3) **Is the exponent for the second factor positive or negative?** (negative) **What is the second factor?** (10^{-3}) **What is 0.0092 written in scientific notation?** (9.2×10^{-3})

Problem 10

What is the first step in writing 1.4×10^{-4} in standard form? (Identify the exponent in the power of 10.) **What is the exponent?** (-4) **Should the decimal point be moved to the left or right?** (left) **How many places should the decimal point be moved?** (4) **How many zeros need to be written as placeholders?** (3) **What is the standard form for 1.4×10^{-4}?** (0.00014)

Problem 11

How many places does the decimal point need to be moved? (6) **Which direction does the decimal point need to be moved?** (right) **How many zeros need to be written as placeholders?** (4) **What is the standard form for 2.07×10^{6}?** (2,070,000)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using Guess and Check** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 21.

Have students read the problem.

Ask students what they are to find.

Students are to find 5.5×10^{-7} written in standard form.

Guess 1: Moving the decimal point right yields: 55,000,000

Guess 2: Moving the decimal point left yields: 0.00000055

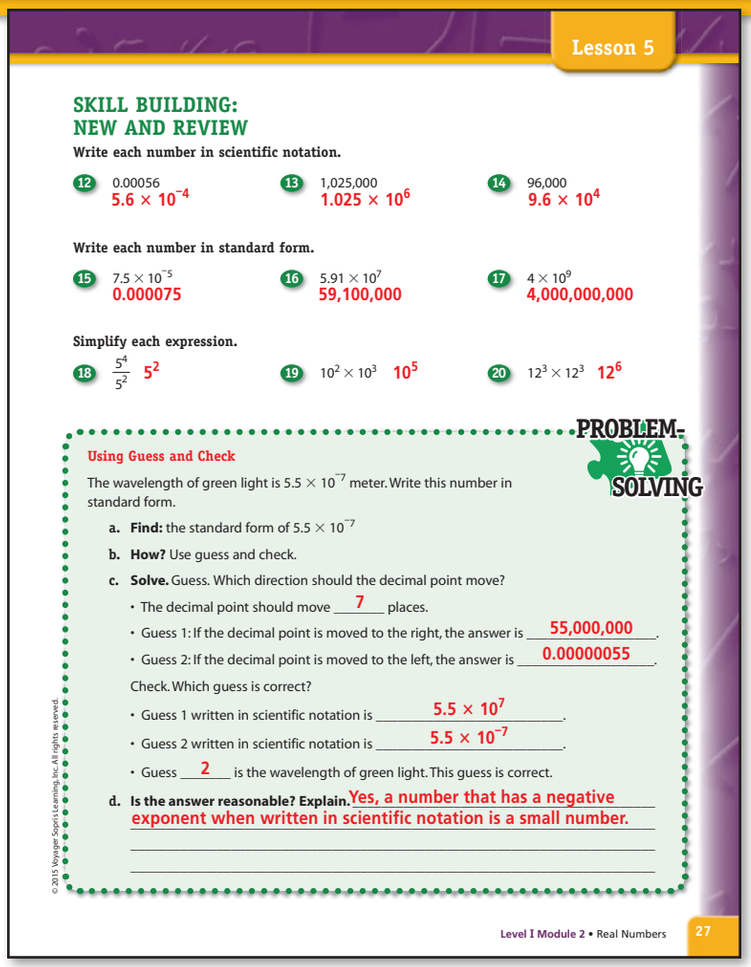
Have students check the guesses by writing the numbers in scientific notation:

Guess 1: 5.5×10^{7}

Guess 2: 5.5×10^{-7}

Guess 2 matches the number in the problem.

The answer is reasonable because a number with a negative exponent when written in scientific notation is a small number.



Lesson 5

SKILL BUILDING: NEW AND REVIEW

Write each number in scientific notation.

12. 0.00056 — 5.6×10^{-4}
13. 1,025,000 — 1.025×10^{6}
14. 96,000 — 9.6×10^{4}

Write each number in standard form.

15. 7.5×10^{-5} — 0.000075
16. 5.91×10^{7} — 59,100,000
17. 4×10^{9} — 4,000,000,000

Simplify each expression.

18. $\frac{5^4}{5^2}$ — 5^2
19. $10^2 \times 10^3$ — 10^5
20. $12^3 \times 12^3$ — 12^6

PROBLEM-SOLVING

Using Guess and Check

The wavelength of green light is 5.5×10^{-7} meter. Write this number in standard form.

a. **Find:** the standard form of 5.5×10^{-7}

b. **How?** Use guess and check.

c. **Solve.** Guess. Which direction should the decimal point move?

- The decimal point should move 7 places.
- Guess 1: If the decimal point is moved to the right, the answer is 55,000,000.
- Guess 2: If the decimal point is moved to the left, the answer is 0.00000055.

Check. Which guess is correct?

- Guess 1 written in scientific notation is 5.5×10^{7}.
- Guess 2 written in scientific notation is 5.5×10^{-7}.
- Guess 2 is the wavelength of green light. This guess is correct.

d. **Is the answer reasonable? Explain.** Yes, a number that has a negative exponent when written in scientific notation is a small number.

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Problems 12–24

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 24 Reminder

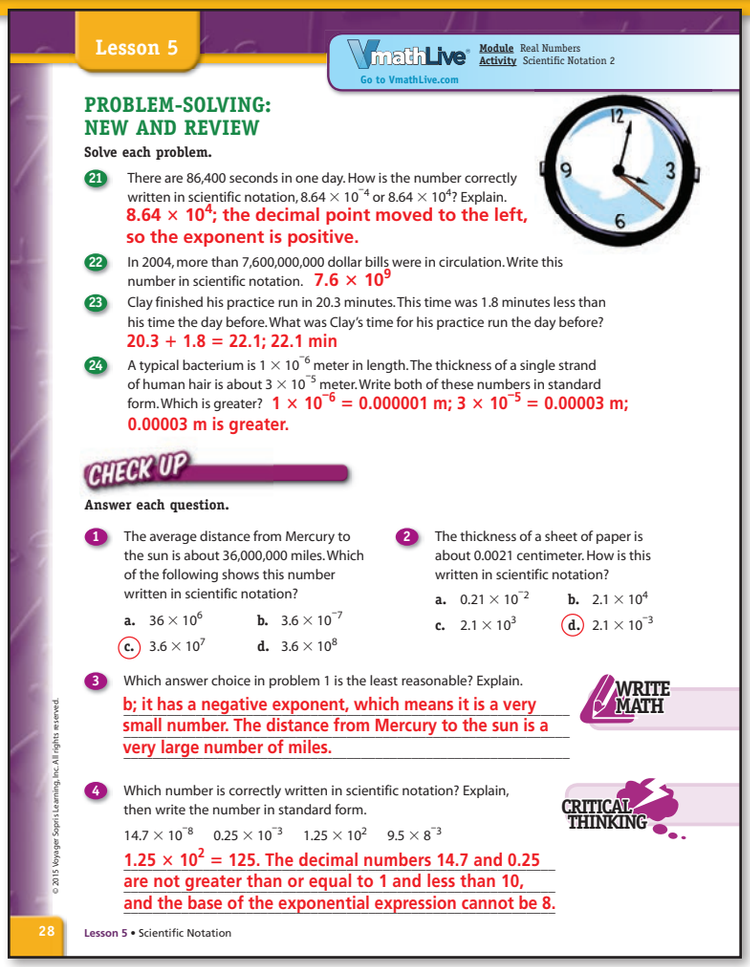
Make sure students know that they have to first change each measurement to standard form, then compare the decimal numbers. Review with students how to compare decimals.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.



Lesson 5

VmathLive Module Real Numbers Activity Scientific Notation 2
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

21 There are 86,400 seconds in one day. How is the number correctly written in scientific notation, 8.64×10^{-4} or 8.64×10^{4}? Explain.
8.64×10^{4}; the decimal point moved to the left, so the exponent is positive.

22 In 2004, more than 7,600,000,000 dollar bills were in circulation. Write this number in scientific notation. **7.6×10^{9}**

23 Clay finished his practice run in 20.3 minutes. This time was 1.8 minutes less than his time the day before. What was Clay's time for his practice run the day before?
20.3 + 1.8 = 22.1; 22.1 min

24 A typical bacterium is 1×10^{-6} meter in length. The thickness of a single strand of human hair is about 3×10^{-5} meter. Write both of these numbers in standard form. Which is greater? **$1 \times 10^{-6} = 0.000001$ m; $3 \times 10^{-5} = 0.00003$ m; 0.00003 m is greater.**

CHECK UP

Answer each question.

1 The average distance from Mercury to the sun is about 36,000,000 miles. Which of the following shows this number written in scientific notation?
a. 36×10^{6} b. 3.6×10^{-7}
(c.) 3.6×10^{7} d. 3.6×10^{8}

2 The thickness of a sheet of paper is about 0.0021 centimeter. How is this written in scientific notation?
a. 0.21×10^{-2} b. 2.1×10^{4}
c. 2.1×10^{3} (d.) 2.1×10^{-3}

3 Which answer choice in problem 1 is the least reasonable? Explain.
WRITE MATH
b; it has a negative exponent, which means it is a very small number. The distance from Mercury to the sun is a very large number of miles.

4 Which number is correctly written in scientific notation? Explain, then write the number in standard form.
CRITICAL THINKING
14.7×10^{-8} 0.25×10^{-3} 1.25×10^{2} 9.5×8^{-3}
$1.25 \times 10^{2} = 125$. The decimal numbers 14.7 and 0.25 are not greater than or equal to 1 and less than 10, and the base of the exponential expression cannot be 8.

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28 Lesson 5 • Scientific Notation

If student answered 1a or 2a: The first factor is not greater than or equal to 1 and less than 10. Remind the student that when writing a number in scientific notation, the first factor must be greater than or equal to 1 and less than 10.

If student answered 1b or 2c: The student confused positive and negative exponents. Use Additional Resources in I.2.5 to reteach scientific notation.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that a negative exponent means the decimal point will be moved to the left, forming a small decimal number.

Technology



Using an online search engine, have students or partners research careers that would need the integer and exponent skills learned in this module. Encourage them to describe how and why these concepts would be needed in specific careers. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Real Numbers
Activity: Scientific Notation 2

Vmath Reteach
Reteach Student Book Module 2 Lesson 5
Reteach Teacher Edition Module 2 Lesson 5

Extra Practice
Student Book page 46

English Language Learners

Use the VmathLive Animated Glossary to review the term *scientific notation*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Students might be confused to see a mathematics vocabulary term that looks like the word *science*. Explain to students that very large and very small numbers are commonly found in various areas of science, so this shorthand form is also used a lot in science. Create a chart on the board to show the students some examples of very large and very small numbers found in science.

Students with Special Needs

Students may need help determining when the decimal point should be moved right and when it should be moved left. Review the steps in the **HOW TO** box and have students help fill in a chart. Students should copy the chart to use as a reference throughout the lesson.

MATH Flash

Lesson 6

Objective

To compare and order real numbers using a number line

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **real number**
  a number that represents a quantity along a continuous line

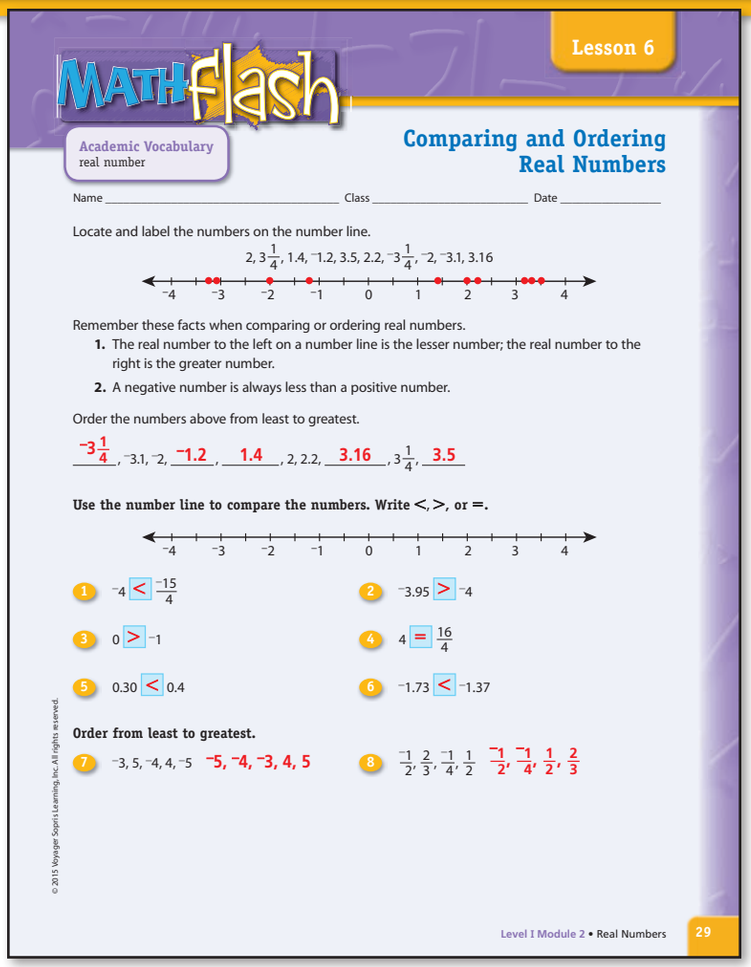
Model the following skills for students.

Look at the number line. How many units are between each tick mark? ($\frac{1}{2}$, or 0.5, unit) **Graph 2 on the number line. Because $3\frac{1}{4}$ is halfway between 3 and $3\frac{1}{2}$, graph $3\frac{1}{4}$ halfway between the tick marks for 3 and $3\frac{1}{2}$. Between what two tick marks should the point for 1.4 be graphed?** (1 and 1.5) **Graph the point 1.4 close to the tick mark for 1.5.**

What is the absolute value of $^-1.2$? (1.2) **What is the absolute value of $^-1$?** (1) **The absolute value of which number is greater?** ($^-1.2$) **This means $^-1.2$ is farther from 0, so graph the point for $^-1.2$ to the left of $^-1$ on the number line. Graph 3.5 and 2.2.**

Which is greater, the absolute value of $^-3$ or the absolute value of $^-3\frac{1}{4}$? (absolute value of $^-3\frac{1}{4}$) **Should the point for $^-3\frac{1}{4}$ be to the left or right of the tick mark for $^-3$?** (left) **Why?** (Its absolute value is greater.) **Graph the point. Now graph the points for $^-2$ and $^-3.1$. The number 3.16 is close to the whole number 3. Graph 3.16 close to the tick mark for 3.**

What are the numbers written in order from least to greatest? ($^-3\frac{1}{4}$, $^-3.1$, $^-2$, $^-1.2$, 1.4, 2, 2.2, 3.16, $3\frac{1}{4}$, 3.5)



MATH Flash — Lesson 6

Academic Vocabulary
real number

Comparing and Ordering Real Numbers

Name _____ Class _____ Date _____

Locate and label the numbers on the number line.

$2, 3\frac{1}{4}, 1.4, ^-1.2, 3.5, 2.2, ^-3\frac{1}{4}, ^-2, ^-3.1, 3.16$

Remember these facts when comparing or ordering real numbers.

1. The real number to the left on a number line is the lesser number; the real number to the right is the greater number.
2. A negative number is always less than a positive number.

Order the numbers above from least to greatest.

$^-3\frac{1}{4}$, $^-3.1$, $^-2$, $^-1.2$, 1.4, 2, 2.2, 3.16, $3\frac{1}{4}$, 3.5

Use the number line to compare the numbers. Write <, >, or =.

1. $^-4 < \frac{^-15}{4}$
2. $^-3.95 > ^-4$
3. $0 > ^-1$
4. $4 = \frac{16}{4}$
5. $0.30 < 0.4$
6. $^-1.73 < ^-1.37$

Order from least to greatest.

7. $^-3, 5, ^-4, 4, ^-5$ $^-5, ^-4, ^-3, 4, 5$
8. $\frac{^-1}{2}, \frac{2}{3}, \frac{^-1}{4}, \frac{1}{2}$ $\frac{^-1}{2}, \frac{^-1}{4}, \frac{1}{2}, \frac{2}{3}$

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Technology

Find and preview short online videos that show real numbers being used in a career. Show the videos to students, then discuss other related careers that could use the same skill. Additional digital content is available through this feature in the eBook.

Gizmos Guided Discovery

Lesson 7

Objective

To use an area model to explore squares and square roots of whole numbers

WITH CONCEPTUAL DEVELOPMENT TECHNOLOGY

Materials

- Gizmo: Square Roots

Gizmos Log In Instructions

- Log in to vmath.voyagersopris.com using your Username and Password provided in VPORT®.
- Select the Gizmos poster.
- Click on the Square Roots Gizmo link.

Lesson Notes

Before beginning the lesson, be sure students have their Student Books and are ready to work at the computers.

In this lesson, students will find square roots and squares of numbers using an area model. They will also distinguish between the square of a number and the square root of a number using the model.

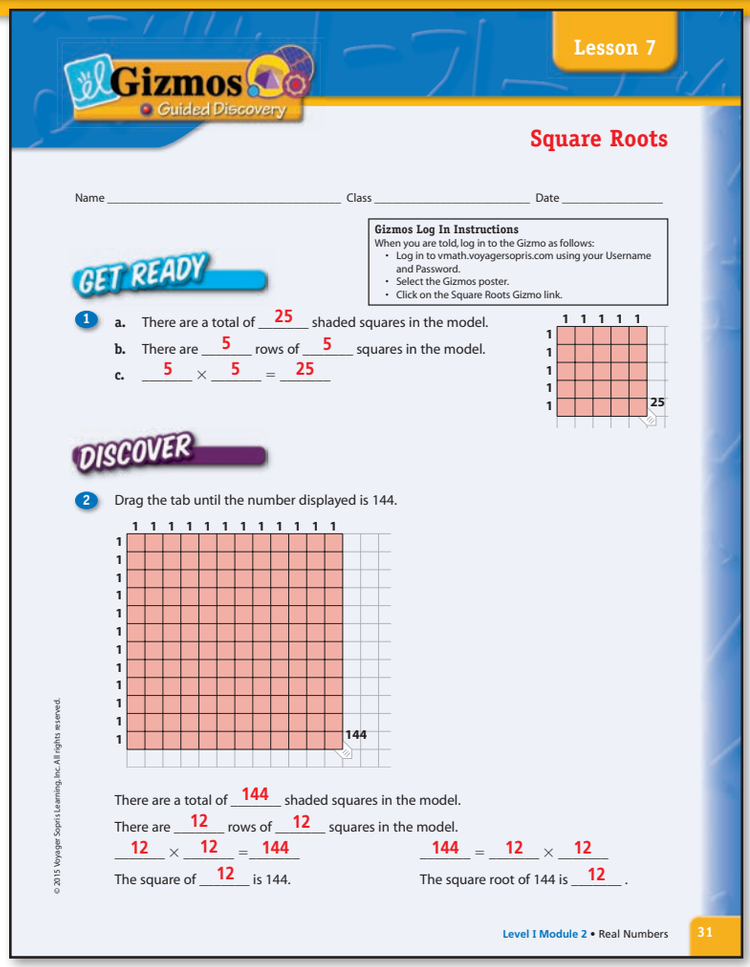
Complete problem 1 before students log in to the Square Roots Gizmo.

If students have additional time, have them answer Assessment Questions 3 and 4 in the Gizmo. They can click on the Check Your Answers button to see how well they did on the assessment.

GET READY

Problem 1

In the model in problem 1, a large square is formed from the smaller shaded squares. How many smaller squares are shaded? (25) **How many rows of shaded squares are there?** (5) **How many shaded squares are in each row?** (5) **Five shaded squares are in each of the 5 rows in the model. So, 5 times 5 equals 25.**



Gizmos Guided Discovery — Lesson 7

Square Roots

Name _____ Class _____ Date _____

Gizmos Log In Instructions
When you are told, log in to the Gizmo as follows:
- Log in to vmath.voyagersopris.com using your Username and Password.
- Select the Gizmos poster.
- Click on the Square Roots Gizmo link.

GET READY

1 a. There are a total of 25 shaded squares in the model.
b. There are 5 rows of 5 squares in the model.
c. 5 × 5 = 25

DISCOVER

2 Drag the tab until the number displayed is 144.

There are a total of 144 shaded squares in the model.
There are 12 rows of 12 squares in the model.
12 × 12 = 144
144 = 12 × 12
The square of 12 is 144.
The square root of 144 is 12.

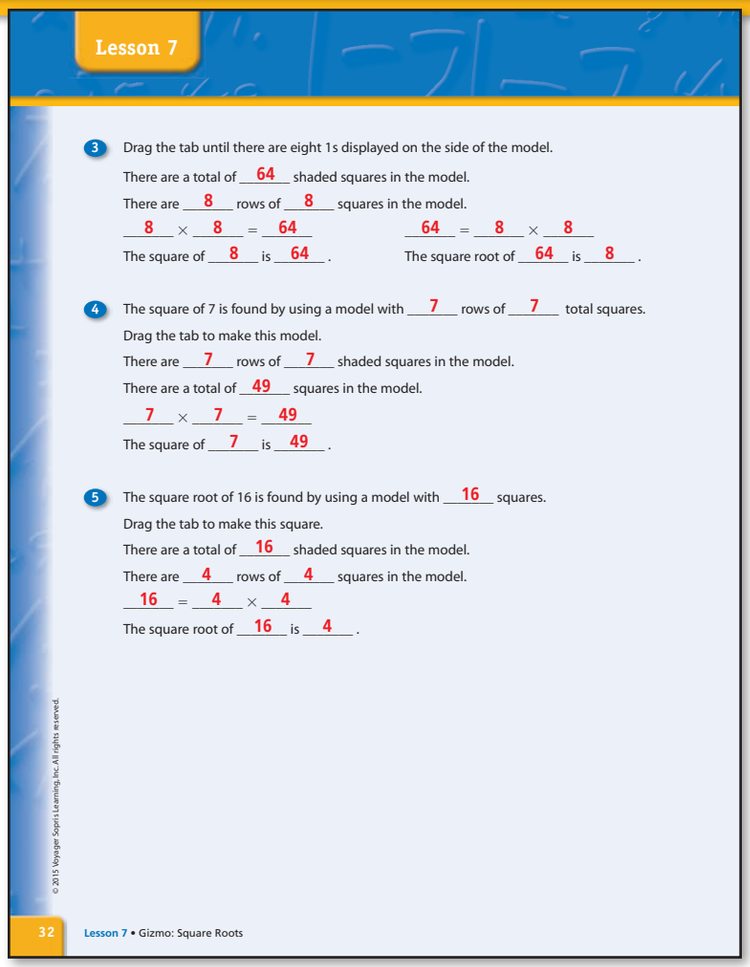
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Level I Module 2 • Real Numbers 31

DISCOVER

Problem 2

Have students log in to the Square Roots Gizmo using the instructions in the box. **The model shown on the screen is like the model in problem 1. Drag the tab at the lower right corner of the model until the number 144 appears next to the tab. How many shaded squares make up the model?** (144) **How many rows of squares are there?** (12) **How many squares are in each row?** (12) **Twelve squares are in each of the 12 rows in the model. So, 12 times 12 equals 144. Another way to say 12 times 12 equals 144 is to say that the square of 12 is 144. There is also another relationship between 12 and 144. One hundred forty-four can be written as the product of 12 and what number?** (12) **When a number can be written as the product of two numbers that are the same, then the number used in the product is called the square root of the original number. So, because 144 is the product of 12 and 12, 12 is the square root of 144.**



Lesson 7

3 Drag the tab until there are eight 1s displayed on the side of the model.

There are a total of 64 shaded squares in the model.

There are 8 rows of 8 squares in the model.

8 × 8 = 64 64 = 8 × 8

The square of 8 is 64. The square root of 64 is 8.

4 The square of 7 is found by using a model with 7 rows of 7 total squares.

Drag the tab to make this model.

There are 7 rows of 7 shaded squares in the model.

There are a total of 49 squares in the model.

7 × 7 = 49

The square of 7 is 49.

5 The square root of 16 is found by using a model with 16 squares.

Drag the tab to make this square.

There are a total of 16 shaded squares in the model.

There are 4 rows of 4 squares in the model.

16 = 4 × 4

The square root of 16 is 4.

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Problem 3

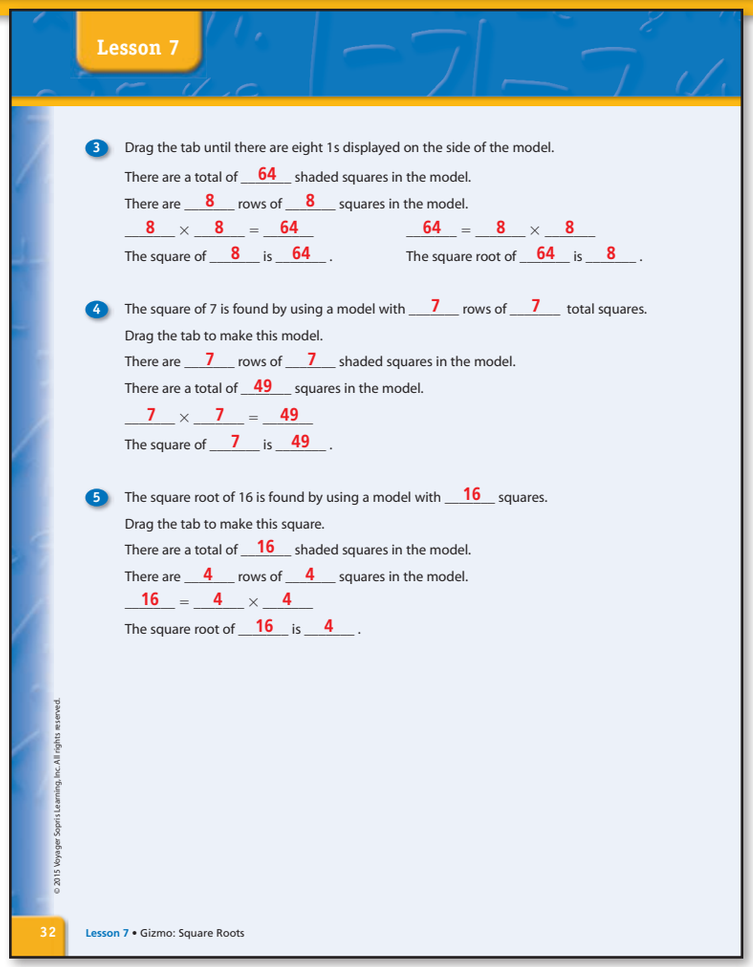
Drag the tab at the lower right corner of the model until there are eight 1s displayed on the side of the model. How many shaded squares make up the model? (64) **How many rows of squares are there?** (8) **How many squares are in each row?** (8) **Eight squares are in each of the 8 rows in the model. So, 8 times 8 equals 64. What is the square of 8?** (64) **Sixty-four can be written as the product of what two identical numbers?** (8 and 8) **Because 64 is the product of 8 and 8, 8 is the square root of 64.**

Problem 4

The square of 7 can be found by using a model. How many rows should be in the model? (7) **How many squares should be in each row?** (7) **Drag the tab at the lower right corner of the model until there are 7 rows of 7 squares in the model. How many squares are in the model?** (49) **So, the product of 7 and itself is 49. So, what is the square of 7?** (49)

Problem 5

To find the square root of 16, the number whose product with itself equals 16 must be found. A model can be used to find the square root of 16. How many shaded squares must be in the model to find the square root of 16? (16) **Drag the tab at the lower right corner of the model until there are 16 squares in the model. How many rows of squares are there?** (4) **How many squares are in each row?** (4) **Four squares are in each of the 4 rows of the model. Sixteen can be written as the product of 4 and 4. What is the square root of 16?** (4)



Lesson 7

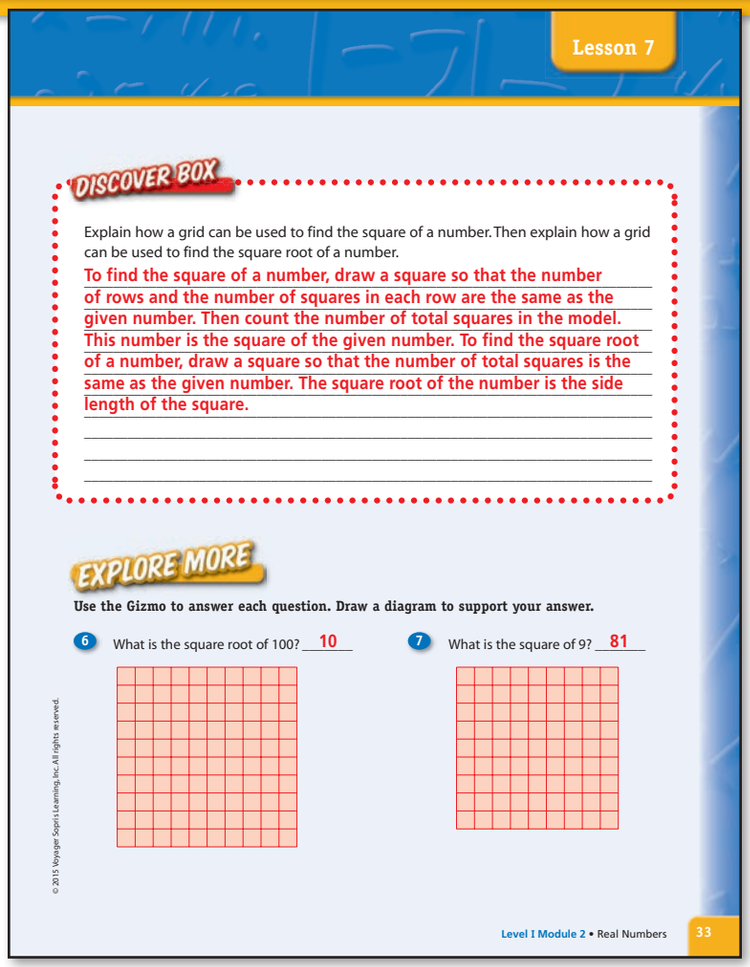
3. Drag the tab until there are eight 1s displayed on the side of the model.
There are a total of 64 shaded squares in the model.
There are 8 rows of 8 squares in the model.
8 × 8 = 64
The square of 8 is 64.
64 = 8 × 8
The square root of 64 is 8.

4. The square of 7 is found by using a model with 7 rows of 7 total squares.
Drag the tab to make this model.
There are 7 rows of 7 shaded squares in the model.
There are a total of 49 squares in the model.
7 × 7 = 49
The square of 7 is 49.

5. The square root of 16 is found by using a model with 16 squares.
Drag the tab to make this square.
There are a total of 16 shaded squares in the model.
There are 4 rows of 4 squares in the model.
16 = 4 × 4
The square root of 16 is 4.

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32 Lesson 7 • Gizmo: Square Roots



Lesson 7

DISCOVER BOX

Explain how a grid can be used to find the square of a number. Then explain how a grid can be used to find the square root of a number.

To find the square of a number, draw a square so that the number of rows and the number of squares in each row are the same as the given number. Then count the number of total squares in the model. This number is the square of the given number. To find the square root of a number, draw a square so that the number of total squares is the same as the given number. The square root of the number is the side length of the square.

EXPLORE MORE

Use the Gizmo to answer each question. Draw a diagram to support your answer.

6 What is the square root of 100? 10

7 What is the square of 9? 81

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DISCOVER BOX

In problem 4, the square of 7 was found. What parts of the model represent 7? (the number of rows in the model and the number of squares in each row) **What part of the model represents the square of 7?** (all the shaded squares in the model) **How can a grid be used to find the square of a number?** (Draw a square so that the number of rows and the number of squares in each row are the same as the given number. Then count the number of shaded squares in the model. This number is the square of the given number.) **In problem 5, the square root of 16 was found. What part of the model represents 16?** (all the shaded squares in the model) **What parts of the model represent the square root of 16?** (the number of rows in the model and the number of squares in each row) **What is true about the number of rows in the model and the number of squares in one row in the model?** (The numbers are the same.) **So, how can a grid be used to find the square root of a number?** (Draw a square so that the number of shaded squares is the same as the given number. Then count the number of rows and the number of squares in each row to make sure that they are the same. This number is the square root of the given number.)

EXPLORE MORE

Problems 6 and 7

Have students work individually. If students need extra guidance, have them use problems 4 and 5 as a guide.

Extension: Have students or partners explore more square and square root problems using area models.

Lesson 8

Objective

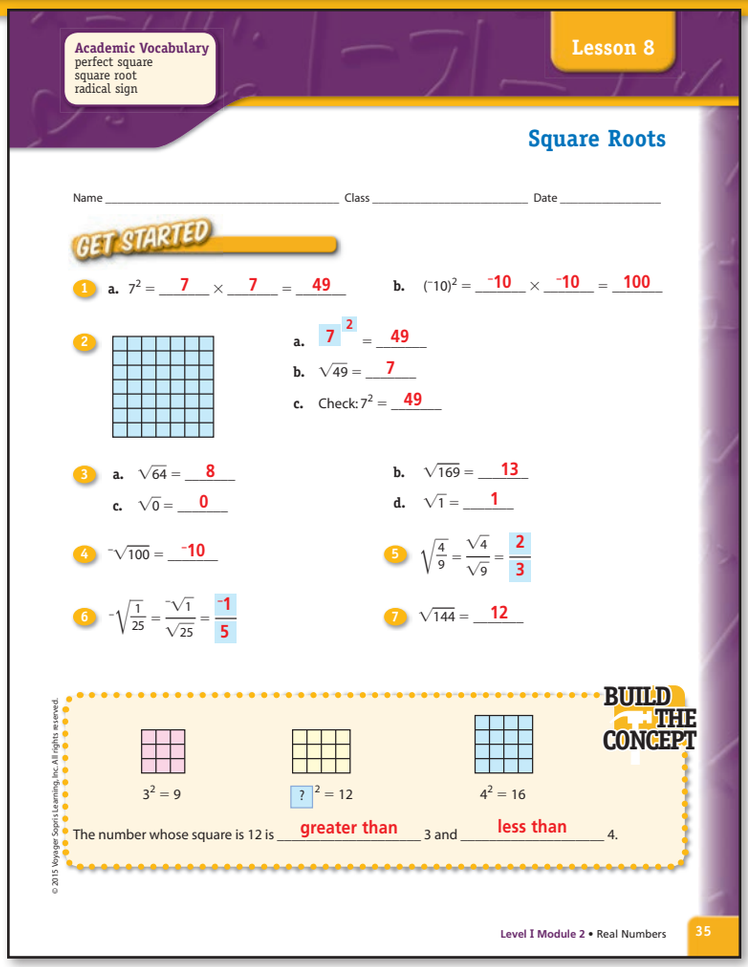
To find square roots of perfect squares

Preskills	Lesson
Exponents	I.2.1

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **perfect square**
  a whole number that is the product of an integer with itself
- **square root**
  a number that when multiplied by itself is equal to a given number
- **radical sign**
  another name for the square root symbol $\sqrt{\ }$



Academic Vocabulary
perfect square
square root
radical sign

Lesson 8

Square Roots

Name _____ Class _____ Date _____

GET STARTED

1 a. $7^2 = 7 \times 7 = 49$ b. $(^-10)^2 = ^-10 \times ^-10 = 100$

2 a. $7^2 = 49$
b. $\sqrt{49} = 7$
c. Check: $7^2 = 49$

3 a. $\sqrt{64} = 8$ b. $\sqrt{169} = 13$
c. $\sqrt{0} = 0$ d. $\sqrt{1} = 1$

4 $^-\sqrt{100} = ^-10$

5 $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

6 $^-\sqrt{\frac{1}{25}} = \frac{^-\sqrt{1}}{\sqrt{25}} = \frac{^-1}{5}$

7 $\sqrt{144} = 12$

BUILD THE CONCEPT

$3^2 = 9$ $?^2 = 12$ $4^2 = 16$

The number whose square is 12 is greater than 3 and less than 4.

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The expression 7^2 is an exponential expression. The base is 7. What is the exponent? (2) **What is the product expressed as repeated multiplication?** (7×7) **What is 7 times 7?** (49)

Look at problem 1b. What is the base of this expression? ($^-10$) **Notice that the negative sign is inside the parentheses. This is done to make sure that the negative sign is included in the base. What is the exponent?** (2) **To evaluate this expression, remember the sign rules for multiplying integers. If the signs of the factors are the same, what is the sign of the product?** (positive) **What is $^-10$ times $^-10$?** (100) **Notice that in problems 1a and 1b, the square of a number was found.**

MODEL NEW SKILLS

Problem 2

In problem 2a, find a number whose square equals 49. Use the model of 49 squares for help. Seven squares are in each of the 7 rows in the model. So, 7^2 equals 49.

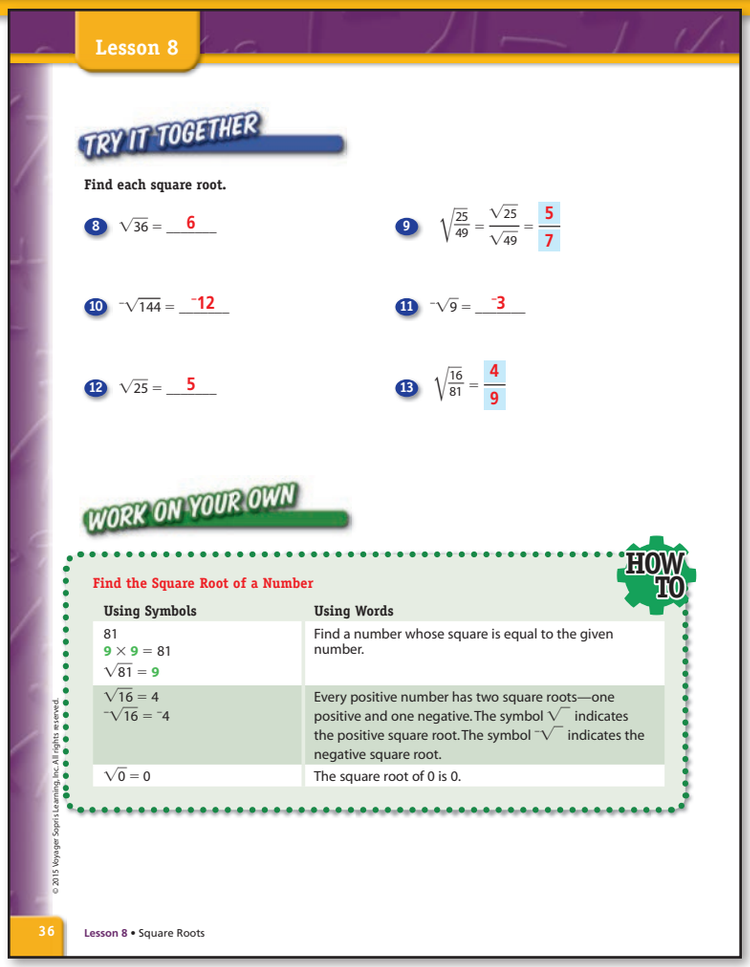
In problem 2b, the expression on the left side of the equal sign is read as *the square root of 49*. Notice the radical sign, or square root symbol, is used to show the square root of a number. The square root of a number is a number that when multiplied by itself equals the original number under the radical sign. Look again at the diagram. What is the square root of 49? (7)

The sentence is now read as *the square root of 49 is seven*. To check, square the answer. What is 7^2? (49) **The answer is correct because 49 is the number under the radical sign.**

Problem 3

A whole number that is the product of an integer with itself is a perfect square. To find the square root of a given number, use guess and check to find a number that when squared is the given number. What number multiplied by itself equals 64? (8) **Sixty-four is a perfect square because 8^2 equals 64. So, the square root of 64 equals 8.**

Another perfect square is 169 because 13^2 equals 169. What is the square root of 169? (13) **What number squared**



Lesson 8

TRY IT TOGETHER

Find each square root.

8. $\sqrt{36} =$ 6

9. $\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} =$ $\frac{5}{7}$

10. $-\sqrt{144} =$ -12

11. $-\sqrt{9} =$ -3

12. $\sqrt{25} =$ 5

13. $\sqrt{\frac{16}{81}} =$ $\frac{4}{9}$

WORK ON YOUR OWN

HOW TO

Find the Square Root of a Number

Using Symbols	Using Words
81 $9 \times 9 = 81$ $\sqrt{81} = 9$	Find a number whose square is equal to the given number.
$\sqrt{16} = 4$ $-\sqrt{16} = -4$	Every positive number has two square roots—one positive and one negative. The symbol $\sqrt{}$ indicates the positive square root. The symbol $-\sqrt{}$ indicates the negative square root.
$\sqrt{0} = 0$	The square root of 0 is 0.

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equals zero? (0) **The square root of zero is zero. One times 1 equals 1. What is the square root of 1?** (1)

Problem 4

When a negative sign is in front of a radical sign, it indicates the negative square root of a number. What number squared is equal to 100? (10) **The negative square root of 100 is -10.**

Problem 5

To find the square root of a fraction, find the square root of the numerator and the square root of the denominator of the fraction. What is the square root of 4? (2) **What is the square root of 9?** (3) **The square root of $\frac{4}{9}$ is $\frac{2}{3}$.**

Problem 6

In this problem, a negative sign is in front of the radical sign. To find the negative square root of a fraction, first find the square root of the numerator and the square root of the denominator, then write a negative sign in front of the numerator. What is the square root of 1? (1) **What is the square root of 25?** (5) **The square root of $\frac{1}{25}$ is $\frac{1}{5}$, so the negative square root of $\frac{1}{25}$ is $-\frac{1}{5}$.**

Problem 7

What is the square root of 144? Guess by starting with 10^2. What is 10^2? (100) **That is too small. What is 11^2?** (121) **That is too small. What is 12^2?** (144) **What is the square root of 144?** (12)

BUILD THE CONCEPT

Model how to find square roots of numbers that are not perfect squares.

Sometimes, square roots are not perfect squares. Look at the diagrams below. The first model represents the square of 3. The diagram makes a square that has 3 on each side. How many individual blocks are there for 3 squared? (9)

Look at the last box. This model represents the square of 4. The diagram makes a square that has 4 on each side. How many individual blocks are there for 4 squared? (16)

Now, look at the middle diagram. Notice that there are 12 individual blocks, but the diagram is not a square. There is no way to make a square with 12 blocks that has the same number on each side. Because of this, it can be said that the square root of 12 will have to be between 3 and 4.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 8

Is 36 a perfect square? (yes) **Why?** (It is the product of 6 times 6.) **What is the square root of 36?** (6)

Problem 9

How is the square root of a fraction found? (by finding the square root of the numerator and the square root of the denominator) **What is the square root of 25?** (5) **Why?** (It is the product of 5 and 5.) **What is the square root of 49?** (7) **Why?** (It is the product of 7 and 7.) **What is the square root of $\frac{25}{49}$?** ($\frac{5}{7}$)

Problem 10

What does the negative sign mean in this problem? (Find the negative square root.) **What is the square root of 144?** (12) **What is the negative square root of 144?** (-12)

Problem 11

Will the answer be positive or negative? (negative) **Why?** (There is a negative sign in front of the radical sign.) **What is the square root of 9?** (3) **What is the negative square root of 9?** (-3)

Problem 12

What is 5 times 5? (25) **What is the square root of 25?** (5)

Problem 13

What is the square root of the numerator? (4)
What is the square root of the denominator? (9)
What is the square root of $\frac{16}{81}$? ($\frac{4}{9}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using Guess and Check** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 23.

Have students read the problem.

From the table, students can see that a square with a side length of 10 feet has an area that is too small, 100 square feet.

A square with a side length of 15 feet has an area that is too large, 225 square feet.

So, the square's side length must be between 10 and 15. Try whole number side lengths:

$11 \times 11 = 121$ – too small

$12 \times 12 = 144$ – too small

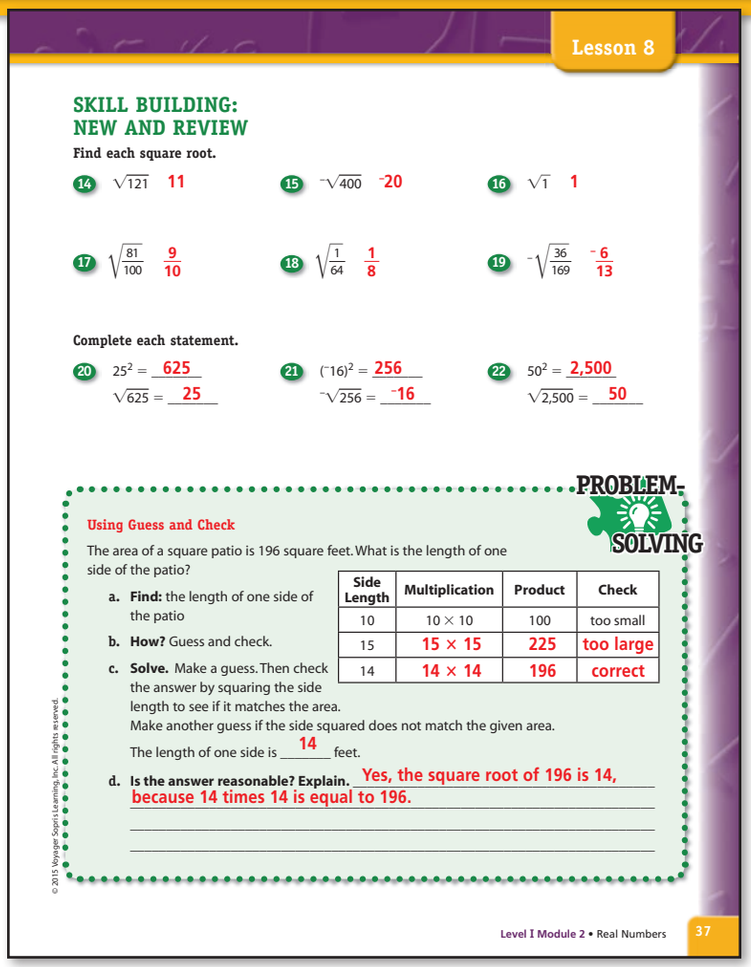
$13 \times 13 = 169$ – too small

$14 \times 14 = 196$ – correct

The answer is reasonable because the square root of 196 is 14; $14 \times 14 = 196$.

Problems 14–26

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 8

SKILL BUILDING: NEW AND REVIEW

Find each square root.

14. $\sqrt{121}$ 11
15. $-\sqrt{400}$ -20
16. $\sqrt{1}$ 1
17. $\sqrt{\frac{81}{100}}$ $\frac{9}{10}$
18. $\sqrt{\frac{1}{64}}$ $\frac{1}{8}$
19. $-\sqrt{\frac{36}{169}}$ $-\frac{6}{13}$

Complete each statement.

20. $25^2 =$ 625; $\sqrt{625} =$ 25
21. $(-16)^2 =$ 256; $-\sqrt{256} =$ -16
22. $50^2 =$ 2,500; $\sqrt{2{,}500} =$ 50

PROBLEM-SOLVING

Using Guess and Check

The area of a square patio is 196 square feet. What is the length of one side of the patio?

a. **Find:** the length of one side of the patio

b. **How?** Guess and check.

c. **Solve.** Make a guess. Then check the answer by squaring the side length to see if it matches the area. Make another guess if the side squared does not match the given area.

Side Length	Multiplication	Product	Check
10	10×10	100	too small
15	15×15	225	too large
14	14×14	196	correct

The length of one side is 14 feet.

d. **Is the answer reasonable? Explain.** Yes, the square root of 196 is 14, because 14 times 14 is equal to 196.

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Problems 15, 19, and 21 **Reminder**

Students must find the negative square root for these problems. Be sure students write a negative sign in the answer.

Problems 24 and 26 **Reminder**

Students are asked to find the length of the side of a square, given the area of the square. Tell students that the formula for the area of a square is $Area = side^2$. To find the square's side length, students must take the square root of the area.

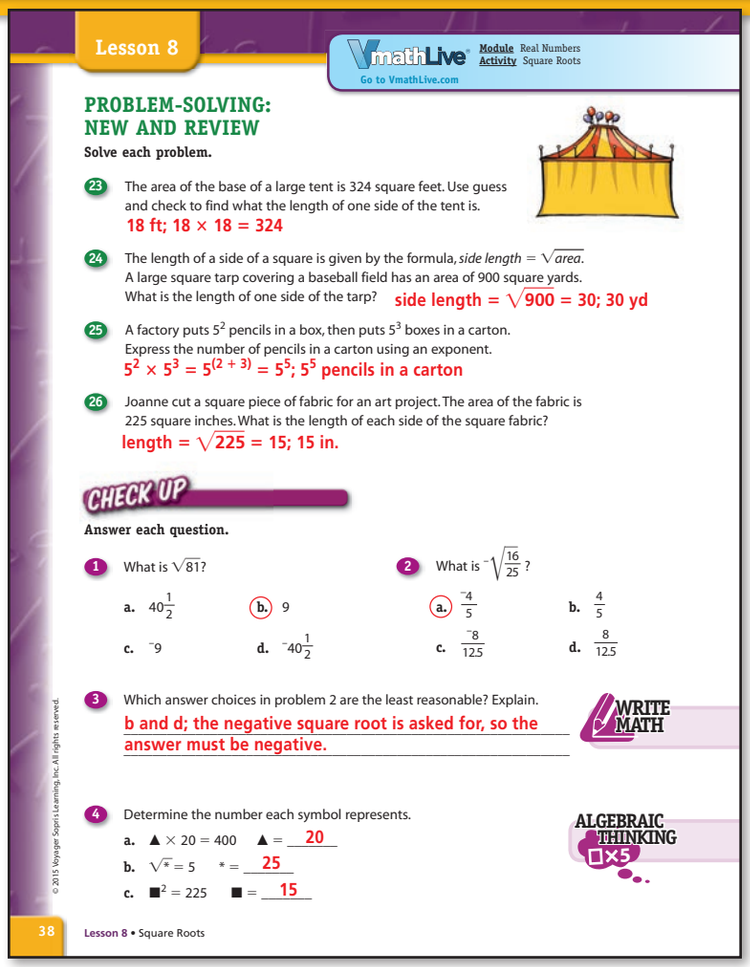
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a, 1d, 2c, or 2d: The student divided each number under the radical sign by 2 instead of finding the square root. Use Additional Resources in I.2.8 to reteach how to find square roots.



Lesson 8

VmathLive Module Real Numbers Activity Square Roots Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

23 The area of the base of a large tent is 324 square feet. Use guess and check to find what the length of one side of the tent is.
18 ft; 18 × 18 = 324

24 The length of a side of a square is given by the formula, *side length* = $\sqrt{area}$. A large square tarp covering a baseball field has an area of 900 square yards. What is the length of one side of the tarp? side length = $\sqrt{900}$ = 30; 30 yd

25 A factory puts 5^2 pencils in a box, then puts 5^3 boxes in a carton. Express the number of pencils in a carton using an exponent.
$5^2 \times 5^3 = 5^{(2+3)} = 5^5$; 5^5 pencils in a carton

26 Joanne cut a square piece of fabric for an art project. The area of the fabric is 225 square inches. What is the length of each side of the square fabric?
length = $\sqrt{225}$ = 15; 15 in.

CHECK UP

Answer each question.

1 What is $\sqrt{81}$?
a. $40\frac{1}{2}$ (b.) 9 c. $^{-}9$ d. $^{-}40\frac{1}{2}$

2 What is $^{-}\sqrt{\frac{16}{25}}$?
(a.) $\frac{^{-}4}{5}$ b. $\frac{4}{5}$ c. $\frac{^{-}8}{125}$ d. $\frac{8}{125}$

3 Which answer choices in problem 2 are the least reasonable? Explain.
b and d; the negative square root is asked for, so the answer must be negative.

WRITE MATH

4 Determine the number each symbol represents.
a. ▲ × 20 = 400 ▲ = 20
b. $\sqrt{*} = 5$ * = 25
c. $■^2 = 225$ ■ = 15

ALGEBRAIC THINKING □×5

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38 Lesson 8 • Square Roots

If student answered 1c or 2b: The student confused negative and positive square roots. Use Additional Resources in I.2.8 to reteach how to find square roots.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the negative sign before the radical sign indicates a negative square root. A negative sign must be included in the answer.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Real Numbers
Activity: Square Roots

Vmath Reteach
Reteach Student Book Module 2 Lesson 8
Reteach Teacher Edition Module 2 Lesson 8

Extra Practice
Student Book page 46

English Language Learners

Use the VmathLive Animated Glossary to review the term *square root*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Give each student a sheet of grid paper. Have students outline squares with 1, 2, 3, and 4 squares on each side, up to 10. Make a table with two columns. The first column will be the number of squares on a side. Label this column *square root*. The second column will be the total number of squares. Label this column *perfect square*. Discuss with students that the number of squares on a side times itself gives the total number of squares outlined.

Students with Special Needs

Have students use a multiplication chart (1 to 10) to locate pairs of numbers with identical factors along the top row and side row of the chart. Have students circle or shade the product of the numbers on the grid. Then have students write the multiplication sentence and the related square root sentence. For example, students should write $2 \times 2 = 4$; $\sqrt{4} = 2$.

HANDS-ON GUIDED DISCOVERY

Lesson 9

Objective

To use grid paper to estimate square roots

CONCEPTUAL DEVELOPMENT

Materials

- Copy Master: Grid Paper
- straightedges

Lesson Notes

Before beginning the lesson, be sure students have their Student Books, copies of Copy Master: Grid Paper, and straightedges.

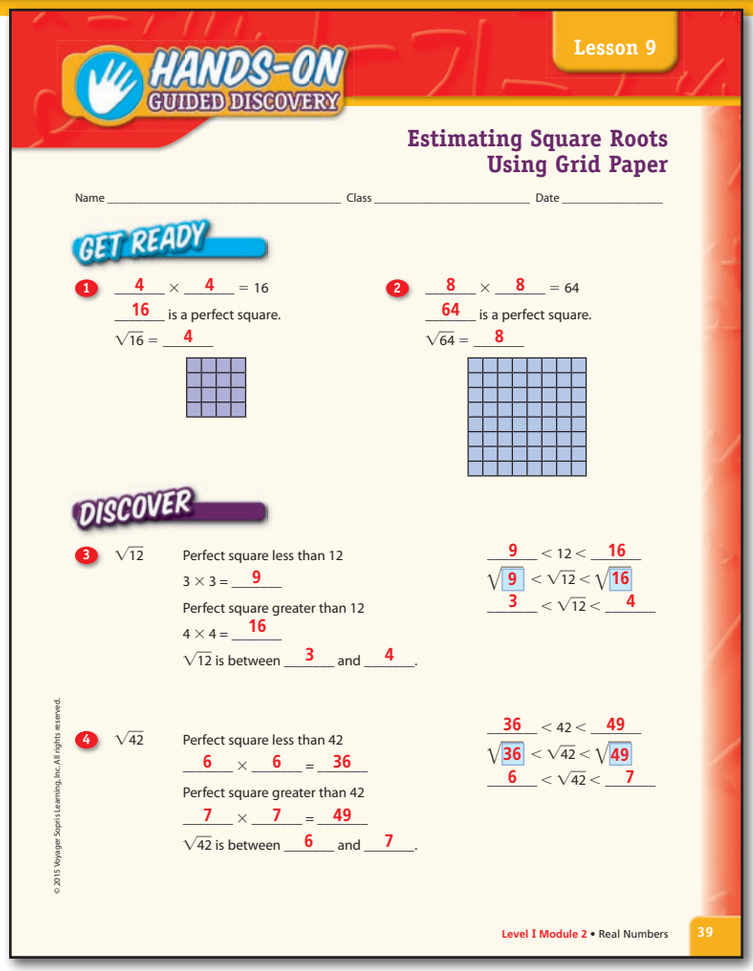
Students will draw squares on the grid paper with side lengths of 3, 4, 5, 6, 7, 8, 9, and 10 grid squares. Suggest that they start at the top left corner of the grid paper and work their way down so that their work is organized and can be used for reference later. Also instruct them to label the side length and total number of grid squares neatly in the center of each square.

Another way to present this lesson is through the use of tiles, Cuisenaire rods, or geoboards. Students can arrange the squares using any one of these manipulatives.

GET READY

Problem 1

Look at problem 1. First, find the number whose product with itself is 16. Use the model of 16 squares to help you. How many squares are in each row of the model? (4) **How many rows are in the model?** (4) **So, 4 times 4 equals 16. A perfect square is a whole number that is the product of an integer with itself. So, 16 is a perfect square. Remember, the square root of a number is a number that when multiplied by itself equals the original number under the radical sign. Look again at the model. What is the square root of 16?** (4)



Lesson 9

HANDS-ON GUIDED DISCOVERY

Estimating Square Roots Using Grid Paper

Name _____ Class _____ Date _____

GET READY

1. 4 × 4 = 16
16 is a perfect square.
$\sqrt{16}$ = 4

2. 8 × 8 = 64
64 is a perfect square.
$\sqrt{64}$ = 8

DISCOVER

3. $\sqrt{12}$
Perfect square less than 12
3 × 3 = 9
Perfect square greater than 12
4 × 4 = 16
$\sqrt{12}$ is between 3 and 4.
9 < 12 < 16
$\sqrt{9} < \sqrt{12} < \sqrt{16}$
3 < $\sqrt{12}$ < 4

4. $\sqrt{42}$
Perfect square less than 42
6 × 6 = 36
Perfect square greater than 42
7 × 7 = 49
$\sqrt{42}$ is between 6 and 7.
36 < 42 < 49
$\sqrt{36} < \sqrt{42} < \sqrt{49}$
6 < $\sqrt{42}$ < 7

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Level I Module 2 • Real Numbers 39

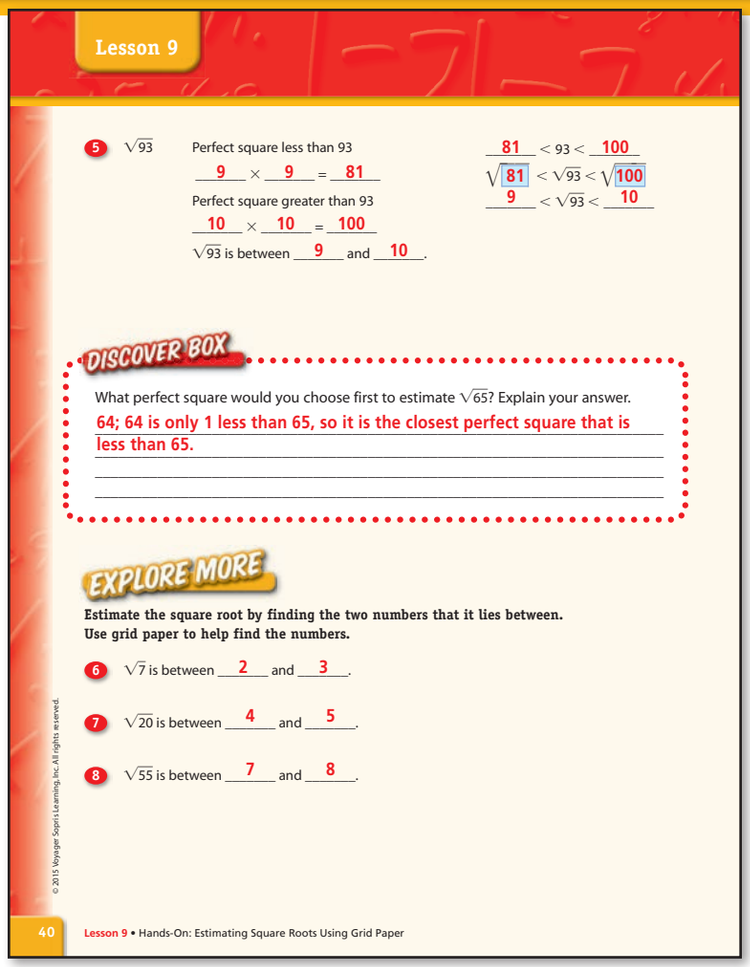
Problem 2

For problem 2, find the number whose product with itself is 64. Use the model of 64 squares to help you. How many squares are in each row of the model? (8) **How many rows are in the model?** (8) **So, 8 times 8 equals 64. A perfect square is a whole number that is the product of an integer with itself. So, 64 is a perfect square. Remember, the square root of a number is a number that when multiplied by itself equals the original number under the radical sign. Look again at the model. What is the square root of 64?** (8)

DISCOVER

Problem 3

In problem 3, there is no whole number that when multiplied by itself is equal to 12. So, the square root of 12 is not a whole number. A model can be used to estimate the square root of 12. On a sheet of grid paper, draw a square that has a side length of 3 units. How many square units are in this figure? (9) **Beside this square, draw a square that has a side length of 4 units. How many square units are in this figure?** (16) **Could a square that covers a total of 12 square units fit inside the square made of 9 square units?** (no) **Could a square that covers a total of 12 square units fit**



Lesson 9

5 $\sqrt{93}$

Perfect square less than 93

9 × 9 = 81

Perfect square greater than 93

10 × 10 = 100

$\sqrt{93}$ is between 9 and 10.

81 < 93 < 100

$\sqrt{81} < \sqrt{93} < \sqrt{100}$

9 < $\sqrt{93}$ < 10

DISCOVER BOX

What perfect square would you choose first to estimate $\sqrt{65}$? Explain your answer.

64; 64 is only 1 less than 65, so it is the closest perfect square that is less than 65.

EXPLORE MORE

Estimate the square root by finding the two numbers that it lies between. Use grid paper to help find the numbers.

6 $\sqrt{7}$ is between 2 and 3.

7 $\sqrt{20}$ is between 4 and 5.

8 $\sqrt{55}$ is between 7 and 8.

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40 Lesson 9 • Hands-On: Estimating Square Roots Using Grid Paper

inside the square made of 16 square units? (yes) **So, 12 lies between the perfect squares 9 and 16. This means that the square root of 12 lies between the square root of 9 and the square root of 16. What is the square root of 9?** (3) **What is the square root of 16?** (4) **So, the square root of 12 lies between 3 and 4. In other words, a square that covers 12 square units has a side length between 3 and 4 units.**

Problem 4

In problem 4, there is no whole number that when multiplied by itself is equal to 42. What does this indicate about the square root of 42? (It is not a whole number.) **On the grid paper, draw a square that has a side length of 5 units. How many square units are in this figure?** (25) **Draw another square on the grid paper that has a side length of 6 units. How many square units make up this figure?** (36) **Draw another square on the grid paper that has a side length of 7 units. How many square units make up this figure?** (49) **A square made of 42 square units is slightly larger than which of the squares?** (the square with 36 square units) **A square made of 42 square units is slightly smaller than which of the squares?** (the square with 49 square units) **According to the figures, 42 lies between which two consecutive perfect squares?** (36 and 49) **So, the square root of 42 lies between the square root of 36 and the square root of 49. What is the square root of 36?** (6) **What is the square root of 49?** (7) **So, the square root of 42 lies between what two numbers?** (6 and 7)

Problem 5

In problem 5, there is no whole number that when multiplied by itself is equal to 93. What does this indicate about the square root of 93? (It is not a whole number.) **On grid paper, draw a square that has a side length of 8 units. How many square units are in this figure?** (64) **Draw another square on the grid paper that has a side length of 9 units. How many square units make up this figure?** (81) **Draw another square on the grid paper that has a side length of 10 units. How many square units make up this figure?** (100) **According to the figures, 93 lies between which two consecutive perfect squares?** (81 and 100) **So, the square root of 93 lies between the square root of 81 and the square root of 100. What is the square root of 81?** (9) **What is the square root of 100?** (10) **So, the square root of 93 lies between what two numbers?** (9 and 10)



Look at your grid paper. What perfect square would you choose first to estimate the square root of 65? (64) **Why?** (Sixty-four is only 1 less than 65, so it is the closest perfect square that is less than 65.)



Problems 6–8

Have students work independently. For each problem, students should refer to the figures that they drew on the grid paper for problems 3–5.

Lesson 10

Objective

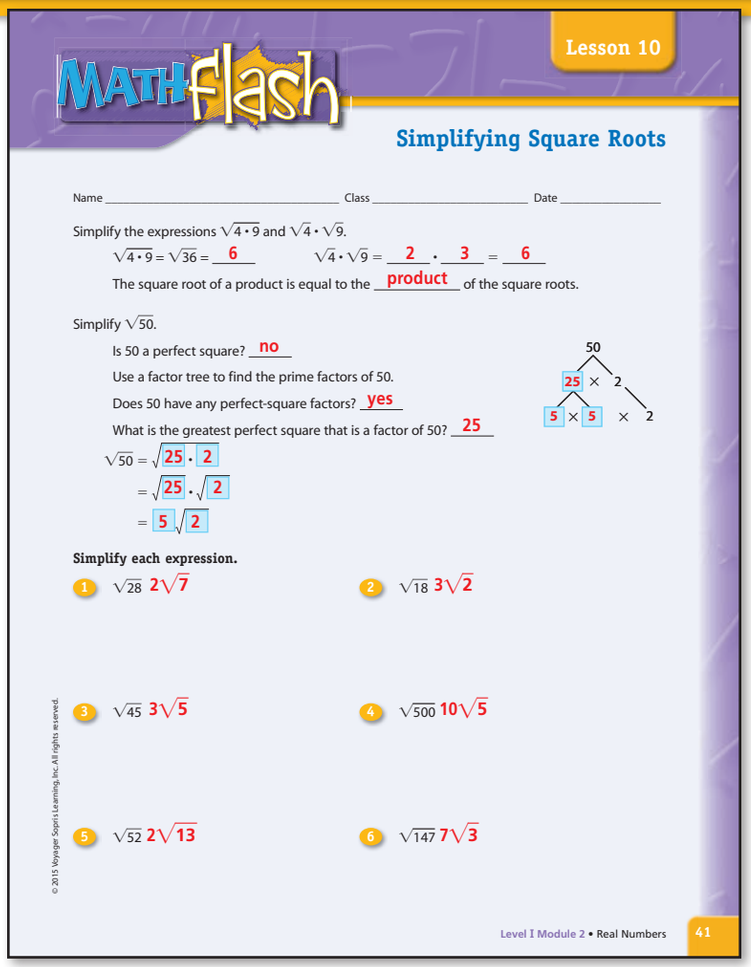
To simplify a square root

Model the following skill for students.

Look at the expression on the left. What is 4 times 9? (36) **What is the square root of 36?** (6) **Look at the expression on the right. What is the square root of 4?** (2) **What is the square root of 9?** (3) **What is 2 times 3?** (6) **This example shows that the square root of a product is equal to the product of the square roots. This property is useful when simplifying expressions involving square roots.**

Work with students to complete the following skills.

Is 50 a perfect square? (no) **Why not?** (It is not the square of a whole number.) **To simplify the square root of 50, you need to find its greatest perfect-square factor. First, use a factor tree to factor 50 completely. The product of 2 and what number equals 50?** (25) **Is 25 factored completely?** (no) **What two prime numbers have a product of 25?** (5 and 5) **What are the prime factors of 50?** (2, 5, and 5) **So, what is the greatest perfect-square factor of 50?** (25) **So, under the square root symbol, write 50 as 25 times 2. The square root of 25 times 2 is equal to the product of what two square roots?** (the square root of 25 and the square root of 2) **What is the square root of 25?** (5) **Because 2 does not have a whole number square root, the expression is in simplest form. What is the square root of 50 written in simplest form?** ($5\sqrt{2}$)



MATH Flash

Lesson 10

Simplifying Square Roots

Name _____ Class _____ Date _____

Simplify the expressions $\sqrt{4 \cdot 9}$ and $\sqrt{4} \cdot \sqrt{9}$.

$\sqrt{4 \cdot 9} = \sqrt{36} =$ 6 $\sqrt{4} \cdot \sqrt{9} =$ 2 · 3 = 6

The square root of a product is equal to the product of the square roots.

Simplify $\sqrt{50}$.

Is 50 a perfect square? no

Use a factor tree to find the prime factors of 50.

Does 50 have any perfect-square factors? yes

What is the greatest perfect square that is a factor of 50? 25

50 → 25 × 2; 25 → 5 × 5

$\sqrt{50} = \sqrt{25 \cdot 2}$

$= \sqrt{25} \cdot \sqrt{2}$

$= 5\sqrt{2}$

Simplify each expression.

1. $\sqrt{28}$ $2\sqrt{7}$
2. $\sqrt{18}$ $3\sqrt{2}$
3. $\sqrt{45}$ $3\sqrt{5}$
4. $\sqrt{500}$ $10\sqrt{5}$
5. $\sqrt{52}$ $2\sqrt{13}$
6. $\sqrt{147}$ $7\sqrt{3}$

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Extra Practice Answer Key

Extra Practice

Name _____ Class _____ Date _____

Lesson PL1 Comparing and Ordering Decimals

Compare. Write >, <, or =.

1. 5.048 < 5.48
2. 0.04693 > 0.04639
3. 16.439 > 12.25
4. 64.0038 < 64.038
5. 0.8093 < 0.8903
6. 14.006 > 12.986

Order from least to greatest.

7. 56.28, 56.82, 56.02
   56.02, 56.28, 56.82
8. 305.45, 304.55, 305.55
   304.55, 305.45, 305.55
9. 0.09, 0.04, 0.004
   0.004, 0.04, 0.09

Order from greatest to least.

10. 34.587, 35.487, 34.857
    35.487, 34.857, 34.587
11. 1.797, 1.795, 1.891
    1.891, 1.797, 1.795
12. 36.400, 36.004, 36.040
    36.400, 36.040, 36.004

Lesson PL2 Using a Problem-Solving Plan

Solve each problem.

1. Each serving of cereal is $\frac{4}{5}$ cup. How many servings are in a box that contains 20 cups of cereal?
   a. Read and understand: How many servings are in a box that has 20 c?
   b. Make a plan: Answer is located on TE page 60.
   c. Solve the problem: $20 \div \frac{4}{5} = 25$; 25 servings
   d. Check the answer: $25 \times \frac{4}{5} = 20$; the answer is correct.
2. Lorraine walked 1.4 miles on Saturday, 0.8 mile on Sunday, and 1.7 miles on Monday. How many total miles did she walk?
   a. Read and understand: How many miles did Lorraine walk?
   b. Make a plan: Add the amounts she walked each day.
   c. Solve the problem: 1.4 + 0.8 + 1.7 = 3.9; 3.9 mi
   d. Check the answer: 3.9 − 1.7 − 0.8 − 1.4 = 0; the answer is correct.

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Extra Practice

Name _____ Class _____ Date _____

Lesson 1 Exponents

Write each product as an exponential expression.

1. $2 \times 2 \times 2 \times 2$ 2^4
2. $^-5 \times ^-5 \times ^-5$ $(^-5)^3$
3. $^-8 \times ^-8$ $(^-8)^2$
4. $3 \times 3 \times 3 \times 3 \times 3$ 3^5
5. $^-6 \times ^-6 \times ^-6 \times ^-6$ $(^-6)^4$
6. $9 \times 9 \times 9$ 9^3

Find the value of each exponential expression.

7. $(^-3)^2$ 9
8. $(^-8)^4$ 4,096
9. $^-(1)^3$ $^-1$
10. $^-(10)^5$ $^-100{,}000$
11. 2^5 32
12. 4^2 16
13. $(^-4)^4$ 256
14. 10^6 1,000,000
15. $(^-12)^2$ 144
16. 2^{10} 1,024
17. $^-(7)^2$ $^-49$
18. $(^-1)^{10}$ 1

Lesson 2 Evaluating Expressions Using Order of Operations

Evaluate each expression.

1. $12 + ^-3 \times 6$ $^-6$
2. $^-6 + 3^2 \div 9$ $^-5$
3. $^-64 \div 2^3 - ^-10$ 2
4. $8 \times (^-7 + 3) \div 4$ $^-8$
5. $^-56 \div (8 \div ^-2)$ 14
6. $24 + (5 - 7)^3 + ^-15$ 1
7. $5^2 + 5 \times (2 - 7)$ 0
8. $(6 - 9)^3 - (9 - 6)^3$ $^-54$
9. $^-60 \div (12 + ^-8) \times 3^2$ $^-135$
10. $^-6 + 10 \div ^-2$ $^-11$
11. $^-7 \times ^-5 - 25$ 10
12. $^-40 + 8^2 \div ^-4$ $^-56$
13. $^-6 \times (8 - 2^2) + ^-20$ $^-44$
14. $(^-4 + 3^2) \times ^-10$ $^-50$
15. $^-13 + (^-44 + ^-64) \div ^-4$ 14
16. $(^-12 \times 5) - 4 \times ^-25$ 40
17. $3^3 + (8 - 2)^2 \div ^-9$ 23
18. $(10 - 2)^2 \div 4^2 + ^-9$ $^-5$

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Extra Practice

Name _____ Class _____ Date _____

Lesson 3 Integer Exponents

Rewrite each expression with a nonnegative exponent.

1. 4^{-3} $\frac{1}{4^3}$
2. 10^{-4} $\frac{1}{10^4}$
3. 7^{-2} $\frac{1}{7^2}$

Evaluate each exponential expression.

4. 5^{-2} $\frac{1}{25}$
5. 3^{-4} $\frac{1}{81}$
6. 1^{-7} 1
7. 2^{-3} $\frac{1}{8}$
8. 4^{-4} $\frac{1}{256}$
9. 6^{-3} $\frac{1}{216}$
10. 3^{-3} $\frac{1}{27}$
11. 5^{-3} $\frac{1}{125}$
12. 9^{-2} $\frac{1}{81}$
13. 8^{-2} $\frac{1}{64}$
14. 2^{-5} $\frac{1}{32}$
15. 10^{-3} $\frac{1}{1{,}000}$
16. 7^{-2} $\frac{1}{49}$
17. 4^{-3} $\frac{1}{64}$
18. 11^{-2} $\frac{1}{121}$

Lesson 4 Properties of Exponents

Simplify each expression.

1. $2^3 \times 2^5$ 2^8
2. $\frac{7^5}{7^3}$ 7^2
3. $4^5 \times 4^2$ 4^7
4. $5^2 \times 5^5$ 5^7
5. $8^4 \times 8^0$ 8^4
6. $\frac{3^7}{3^3}$ 3^4
7. $\frac{10^5}{10}$ 10^4
8. $6^5 \times 6^5$ 6^{10}
9. $\frac{8^4}{8^2}$ 8^2
10. $\frac{5^8}{5^5}$ 5^3
11. $9^2 \times 9^4$ 9^6
12. $\frac{2^{10}}{2^6}$ 2^4
13. $3^6 \times 3^3$ 3^9
14. $\frac{4^5}{4^3}$ 4^2
15. $10^5 \times 10^0$ 10^5

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Extra Practice

Name _____ Class _____ Date _____

Lesson 5 Scientific Notation

Write each number in scientific notation.

1. 0.671 6.71×10^{-1}
2. 584,000 5.84×10^5
3. 0.00004 4×10^{-5}
4. 6,730 6.73×10^3
5. 3,560,000 3.56×10^6
6. 0.0003 3×10^{-4}
7. 75,000 7.5×10^4
8. 0.00021 2.1×10^{-4}
9. 8,750,000 8.75×10^6

Write each number in standard form.

10. 8×10^{-9} 0.000000008
11. 5.05×10^3 5,050
12. 2.12×10^{-5} 0.0000212
13. 6.18×10^5 618,000
14. 7.6×10^{-3} 0.0076
15. 4×10^7 40,000,000
16. 9.35×10^{-6} 0.00000935
17. 2.008×10^3 2,008
18. 3.14159×10^{-10} 0.000000000314159

Lesson 8 Square Roots

Find each square root.

1. $^-\sqrt{25}$ $^-5$
2. $\sqrt{225}$ 15
3. $\sqrt{196}$ 14
4. $\sqrt{9}$ 3
5. $\sqrt{4}$ 2
6. $^-\sqrt{361}$ $^-19$
7. $\sqrt{441}$ 21
8. $\sqrt{1}$ 1
9. $\sqrt{64}$ 8
10. $\sqrt{289}$ 17
11. $\sqrt{121}$ 11
12. $\sqrt{36}$ 6
13. $^-\sqrt{169}$ $^-13$
14. $\sqrt{100}$ 10
15. $^-\sqrt{400}$ $^-20$

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Glossary

base
the number used as a factor in repeated multiplication

exponent
a number that indicates how many times a base is used as a factor in repeated multiplication

inequality
a number sentence that contains the symbol > (greater than) or < (less than)

perfect square
a number that is the product of an integer with itself

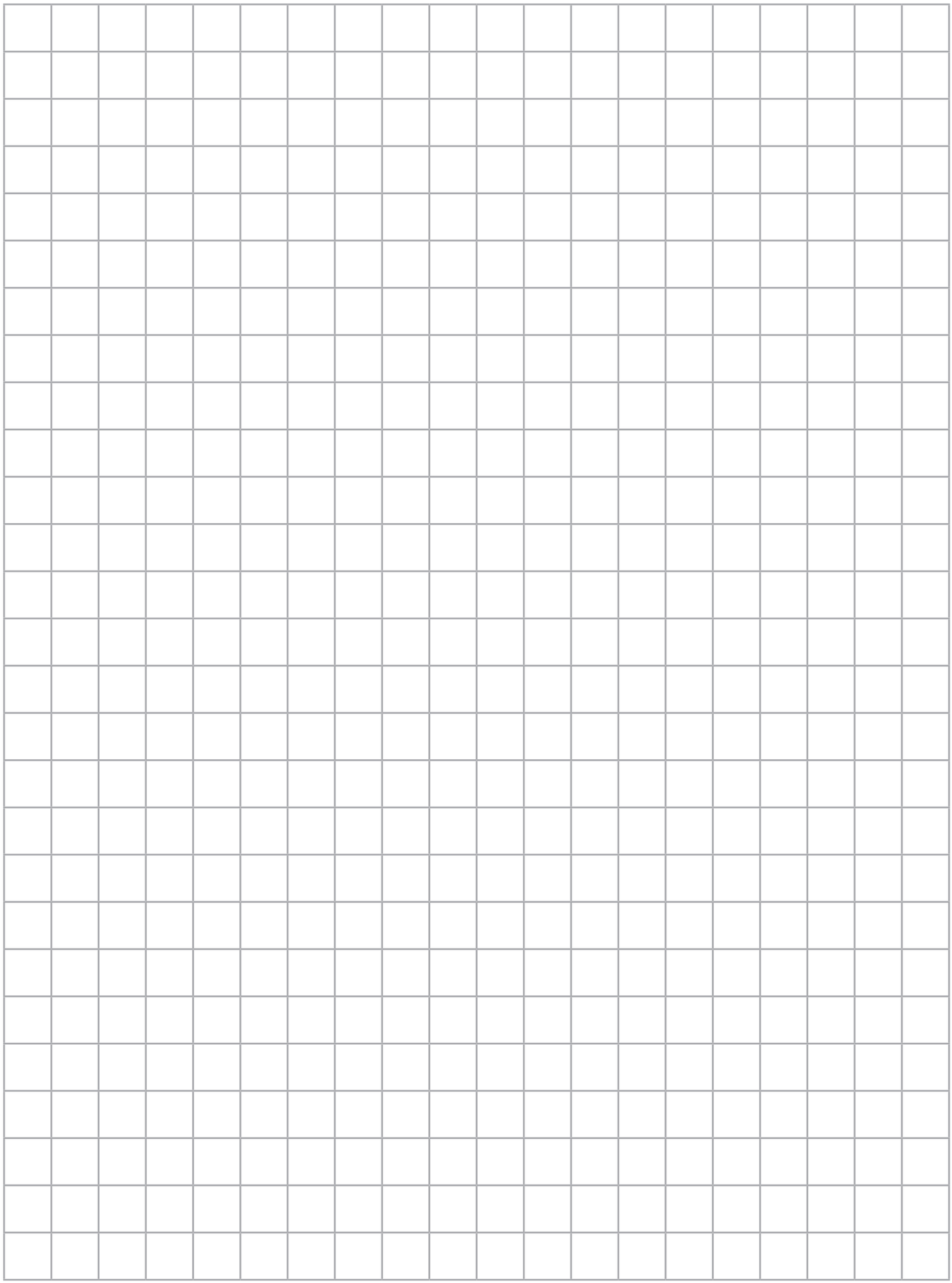
radical sign
another name for the square root symbol $\sqrt{\ }$

real number
a number that represents a quantity along a continuous line

scientific notation
a number expressed as the product of two factors; the first factor is a number greater than or equal to 1 and less than 10, and the second is a power of 10

square root
a number that when multiplied by itself is equal to a given number

Copy Master: Grid Paper



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Equations

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Lesson PL1

Objective

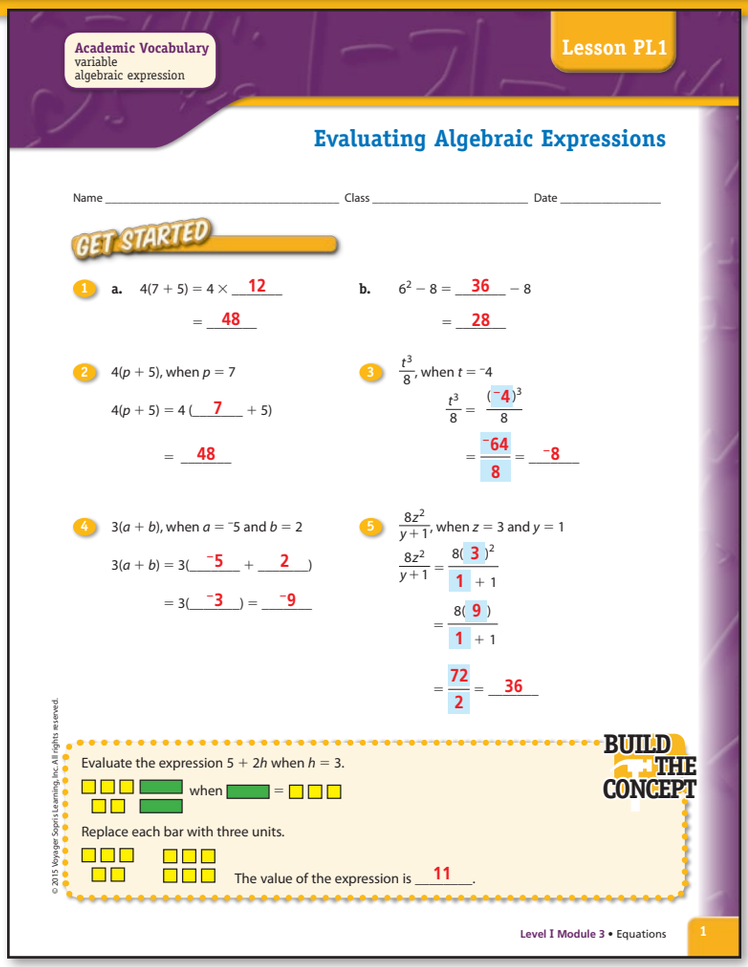
To evaluate an algebraic expression with one or more variables

Preskills	Lesson
Exponents	I.2.1
Evaluating Expressions Using Order of Operations	I.2.2

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **variable**
  a letter or symbol that represents a number
- **algebraic expression**
  an expression that can contain one or more numerals, one or more variables, and one or more operations



Academic Vocabulary
variable
algebraic expression

Lesson PL1

Evaluating Algebraic Expressions

Name _____ Class _____ Date _____

GET STARTED

1 a. $4(7 + 5) = 4 \times$ 12 $= 48$

b. $6^2 - 8 =$ 36 $- 8 = 28$

2 $4(p + 5)$, when $p = 7$

$4(p + 5) = 4($ 7 $+ 5)$

$= 48$

3 $\frac{t^3}{8}$, when $t = -4$

$\frac{t^3}{8} = \frac{(-4)^3}{8}$

$= \frac{-64}{8} = -8$

4 $3(a + b)$, when $a = -5$ and $b = 2$

$3(a + b) = 3($ −5 $+$ 2 $)$

$= 3($ −3 $) =$ −9

5 $\frac{8z^2}{y+1}$, when $z = 3$ and $y = 1$

$\frac{8z^2}{y+1} = \frac{8(3)^2}{1 + 1}$

$= \frac{8(9)}{1 + 1}$

$= \frac{72}{2} = 36$

BUILD THE CONCEPT

Evaluate the expression $5 + 2h$ when $h = 3$.

when ▭ = □□□

Replace each bar with three units.

The value of the expression is 11.

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Level I Module 3 • Equations 1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The order of operations is used to evaluate numeric expressions with more than one operation. In problem 1a, what is the first step in evaluating 4(7 + 5)? (Add the numbers in parentheses.) **What is 7 plus 5?** (12) **What is the next step?** (Multiply 4 and 12.) **What is 4 times 12?** (48)

Look at problem 1b. What is the first step in evaluating 6^2 minus 8? (Evaluate the exponential expression.) **What is 6^2?** (36) **What is the next step?** (Subtract 8 from 36.) **What is 36 minus 8?** (28)

MODEL NEW SKILLS

Problem 2

The expression in this problem is an algebraic expression. Unlike the numeric expressions in problem 1, which contain only numerals and operation symbols, an algebraic expression also contains at least one variable. A variable is a letter or symbol that represents a number. The algebraic expression in this problem is read as *4 times the sum of p and 5*. The variable in the algebraic expression is *p*.

To evaluate an algebraic expression, substitute a number for the variable, then evaluate the numeric expression following the order of operations. What value should be substituted for *p*? (7) **What is the numeric expression when 7 is substituted for the variable?** (4(7 + 5)) **This numeric expression was evaluated in problem 1a. The value of $4(p + 5)$ when *p* equals 7 is 48.**

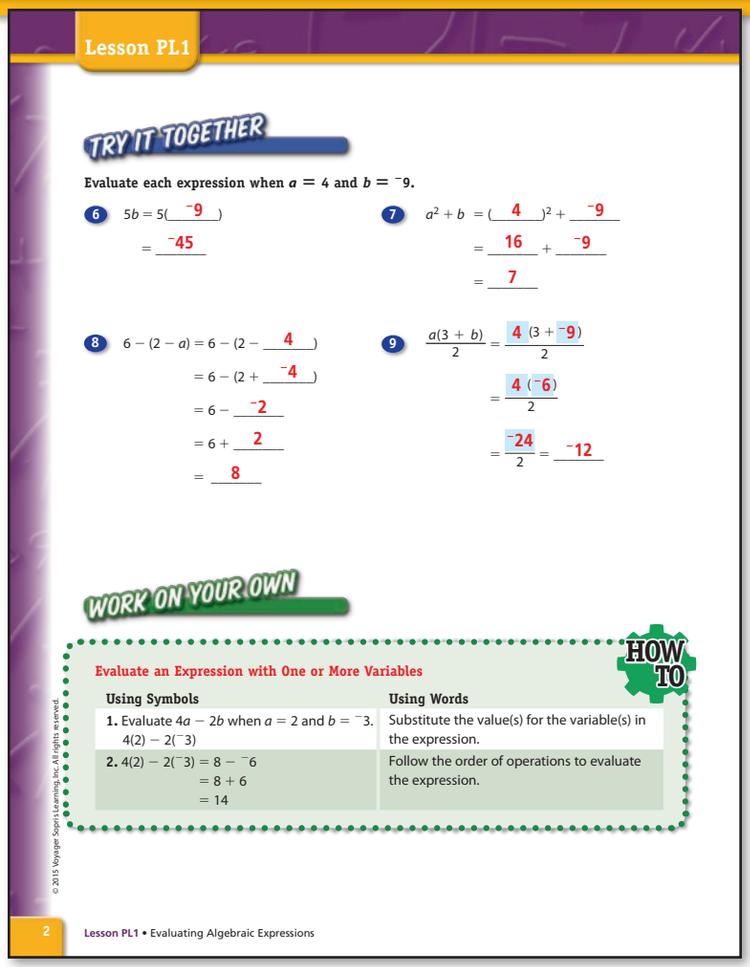
Problem 3

A fraction bar in an expression represents division. This expression is read as *t cubed divided by 8*. What value should be substituted for *t*? ($^-4$) **The numeric expression is $\frac{(-4)^3}{8}$.**

Evaluate the exponential expression first. What is $^-4$ cubed? ($^-64$) **Next, divide $^-64$ by 8. What is 64 divided by 8?** ($^-8$) **The value of $\frac{t^3}{8}$ when *t* equals $^-4$ is $^-8$.**

Problem 4

To evaluate expressions with more than one variable, replace each variable with the indicated value. What are the variables in this expression? (*a* and *b*) **What is the value of *a*?** ($^-5$) **What is the value of *b*?** (2) **What is the numeric**



Lesson PL1

TRY IT TOGETHER

Evaluate each expression when $a = 4$ and $b = -9$.

6 $5b = 5(\underline{-9})$
$= \underline{-45}$

7 $a^2 + b = (\underline{4})^2 + \underline{-9}$
$= \underline{16} + \underline{-9}$
$= \underline{7}$

8 $6 - (2 - a) = 6 - (2 - \underline{4})$
$= 6 - (2 + \underline{-4})$
$= 6 - \underline{-2}$
$= 6 + \underline{2}$
$= \underline{8}$

9 $\frac{a(3 + b)}{2} = \frac{4(3 + \underline{-9})}{2}$
$= \frac{4(\underline{-6})}{2}$
$= \frac{\underline{-24}}{2} = \underline{-12}$

WORK ON YOUR OWN

HOW TO

Evaluate an Expression with One or More Variables

Using Symbols	Using Words
1. Evaluate $4a - 2b$ when $a = 2$ and $b = -3$. $4(2) - 2(-3)$	Substitute the value(s) for the variable(s) in the expression.
2. $4(2) - 2(-3) = 8 - -6$ $= 8 + 6$ $= 14$	Follow the order of operations to evaluate the expression.

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expression with $^-5$ substituted for a and 2 substituted for b? (3($^-5$ + 2))

What is the first step in evaluating 3($^-5$ + 2)? (Add the numbers in parentheses.) **What is $^-5$ plus 2?** ($^-3$) **What is 3 times $^-3$?** ($^-9$) **What is the value of 3(a + b) when a equals $^-5$ and b equals 2?** ($^-9$)

Problem 5

To evaluate $\frac{8z^2}{y + 1}$, replace each variable with its indicated value. What is the numeric expression when z equals 3 and y equals 1? ($\frac{8(3)^2}{1 + 1}$)

Use the order of operations to evaluate the numeric expression. Evaluate the numerator first. Which operation is performed first? (3^2) **What is 3^2?** (9) **What is 8 times 9?** (72)

Next, evaluate the denominator. What is 1 plus 1? (2) **The denominator is 2. Last, divide the numerator by the denominator. What is 72 divided by 2?** (36) **What is the value of $\frac{8z^2}{y + 1}$ when z equals 3 and y equals 1?** (36)

BUILD THE CONCEPT

Model how to evaluate an algebraic expression using algebra tiles.

What is the algebraic expression to be evaluated? (5 + 2h) **The model shows 5 yellow ones tiles and 2 green rectangles. What do the 5 yellow ones tiles represent?** (5) **What does each green rectangle represent?** (the variable h) **What value of h should be substituted into the expression?** (3) **The substitution is shown by replacing each green rectangle with 3 ones tiles. How many ones tiles are there in all?** (11) **What is the value of the algebraic expression?** (11)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

What operation is indicated in the expression 5b? (multiplication) **What value should be substituted for b in the expression?** ($^-9$) **What is 5 times $^-9$?** ($^-45$) **What is the value of 5b when b equals $^-9$?** ($^-45$)

Problem 7

What is the first step in evaluating $a^2 + b$? (Substitute 4 for a and $^-9$ for b in the expression.) **What is the numeric expression with the values of the variables substituted in it?** ($4^2 + -9$) **What should be evaluated first?** (4^2) **What is 4^2?** (16) **What is the next step?** (Add 16 and $^-9$.) **What is 16 plus $^-9$?** (7) **What is the value of $a^2 + b$ when a equals 4 and b equals $^-9$?** (7)

Problem 8

What is the first step in evaluating 6 − (2 − a) when a equals 4? (Substitute 4 for a in the expression.) **What is the first step in evaluating 6 − (2 − 4)?** (Evaluate the expression in parentheses.) **What is 2 minus 4 written as an addition problem?** (2 + $^-4$) **What is 2 plus $^-4$?** ($^-2$)

What is the next step? (Subtract $^-2$ from 6.) **How is $^-2$ subtracted from 6?** (Change subtraction to addition.) **What is the new problem?** (6 + 2) **What is 6 plus 2?** (8) **What is the value of 6 − (2 − a) when a equals 4?** (8)

Problem 9

What is the numeric expression when *a* equals 4 and *b* equals $^-9$? $\left(\frac{4(3 + {}^-9)}{2}\right)$ What is 3 plus $^-9$? $(^-6)$ What is 4 times $^-6$? $(^-24)$ What is $^-24$ divided by 2? $(^-12)$ What is the value of $\frac{a(3 + b)}{2}$ when *a* equals 4 and *b* equals $^-9$? $(^-12)$

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 10–25

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 15, 18, and 21 Reminder

Remind students to evaluate the numerator, then the denominator. Then divide the numerator by the denominator to find the value of the expression.

CHECK UP

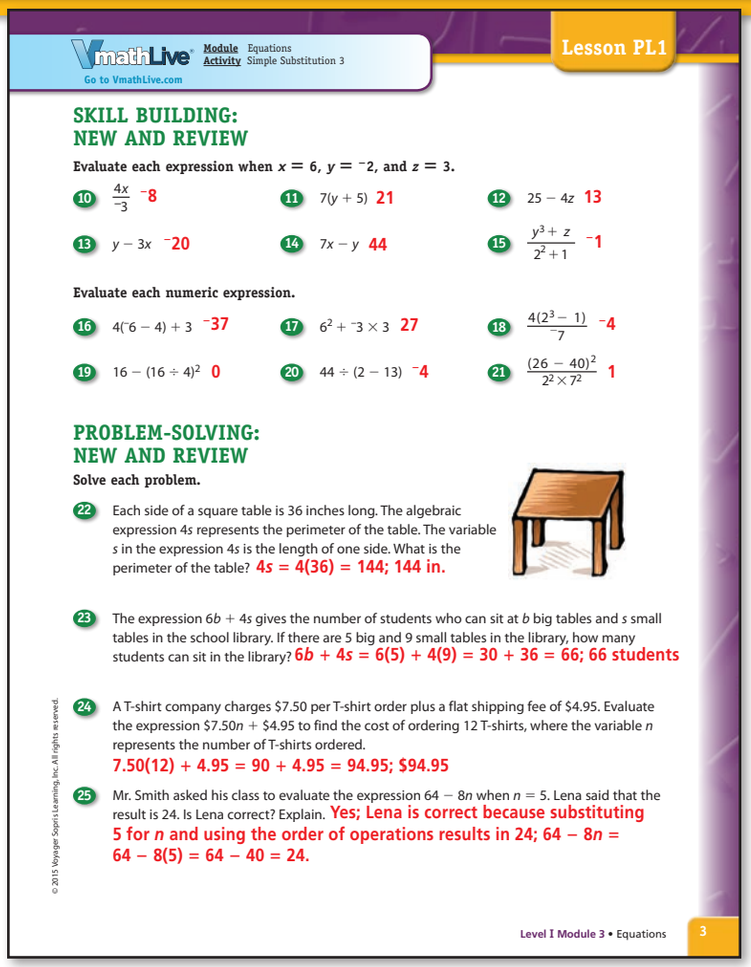
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1d: The student added the values in the parentheses incorrectly. Review with the student how to add integers.

If student answered 1b: The student did not evaluate the expression in the parentheses first. Use Additional Resources in I.2.2 to reteach how to evaluate expressions using the order of operations.



VmathLive Module Activity Equations Simple Substitution 3
Go to VmathLive.com

Lesson PL1

SKILL BUILDING: NEW AND REVIEW

Evaluate each expression when $x = 6$, $y = {}^-2$, and $z = 3$.

10. $\frac{4x}{^-3}$ $^-8$
11. $7(y + 5)$ 21
12. $25 - 4z$ 13
13. $y - 3x$ $^-20$
14. $7x - y$ 44
15. $\frac{y^3 + z}{2^2 + 1}$ $^-1$

Evaluate each numeric expression.

16. $4(^-6 - 4) + 3$ $^-37$
17. $6^2 + {}^-3 \times 3$ 27
18. $\frac{4(2^3 - 1)}{^-7}$ $^-4$
19. $16 - (16 \div 4)^2$ 0
20. $44 \div (2 - 13)$ $^-4$
21. $\frac{(26 - 40)^2}{2^2 \times 7^2}$ 1

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

22. Each side of a square table is 36 inches long. The algebraic expression $4s$ represents the perimeter of the table. The variable s in the expression $4s$ is the length of one side. What is the perimeter of the table? $4s = 4(36) = 144$; 144 in.

23. The expression $6b + 4s$ gives the number of students who can sit at b big tables and s small tables in the school library. If there are 5 big and 9 small tables in the library, how many students can sit in the library? $6b + 4s = 6(5) + 4(9) = 30 + 36 = 66$; 66 students

24. A T-shirt company charges \$7.50 per T-shirt order plus a flat shipping fee of \$4.95. Evaluate the expression $\$7.50n + \$4.95$ to find the cost of ordering 12 T-shirts, where the variable n represents the number of T-shirts ordered. $7.50(12) + 4.95 = 90 + 4.95 = 94.95$; \$94.95

25. Mr. Smith asked his class to evaluate the expression $64 - 8n$ when $n = 5$. Lena said that the result is 24. Is Lena correct? Explain. Yes; Lena is correct because substituting 5 for n and using the order of operations results in 24; $64 - 8n = 64 - 8(5) = 64 - 40 = 24$.

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Level I Module 3 • Equations 3

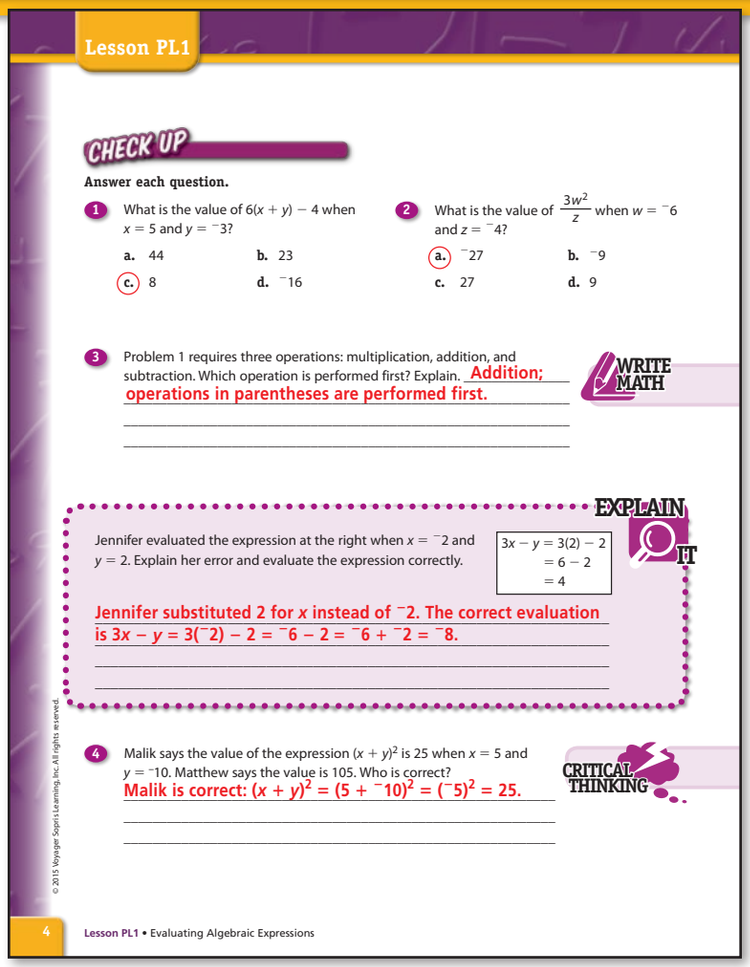
If student answered 2b or 2d: The student did not evaluate $(^-6)^2$, but used the 2 as a factor instead. Use Additional Resources in I.2.1 to reteach how to evaluate exponential expressions.

If student answered 2c: The student used the wrong sign for the quotient. Review the rules for dividing integers with the student.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to follow the order of operations. Use Additional Resources in I.2.2 to reteach how to evaluate expressions using the order of operations.



Lesson PL1

CHECK UP

Answer each question.

1. What is the value of $6(x + y) - 4$ when $x = 5$ and $y = {}^-3$?

   a. 44 **b.** 23

   (c.) 8 **d.** $^-16$

2. What is the value of $\frac{3w^2}{z}$ when $w = {}^-6$ and $z = {}^-4$?

   (a.) $^-27$ **b.** $^-9$

   c. 27 **d.** 9

3. Problem 1 requires three operations: multiplication, addition, and subtraction. Which operation is performed first? Explain. Addition; operations in parentheses are performed first.

WRITE MATH

EXPLAIN IT

Jennifer evaluated the expression at the right when $x = {}^-2$ and $y = 2$. Explain her error and evaluate the expression correctly.

$3x - y = 3(2) - 2$
$= 6 - 2$
$= 4$

Jennifer substituted 2 for x instead of $^-2$. The correct evaluation is $3x - y = 3(^-2) - 2 = {}^-6 - 2 = {}^-6 + {}^-2 = {}^-8$.

4. Malik says the value of the expression $(x + y)^2$ is 25 when $x = 5$ and $y = {}^-10$. Matthew says the value is 105. Who is correct?

   Malik is correct: $(x + y)^2 = (5 + {}^-10)^2 = ({}^-5)^2 = 25$.

CRITICAL THINKING

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EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Equations
Activity: Simple Substitution 3

Vmath Reteach
Reteach Student Book Module 3 Lesson PL1
Reteach Teacher Edition Module 3 Lesson PL1

Extra Practice
Student Book page 45

English Language Learners

Use the VmathLive Animated Glossary to review the terms *variable* and *algebraic expression*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Draw a two-column table on the board; label the first column *variable*, p, and label the second column *algebraic expression*, $p + 2$. Tell students the variable represents the number of pencils a student has. Ask for volunteers to state the number of pencils they have and the value of the expression. Discuss the meaning of a variable, a letter whose value may change, or vary.

Students with Special Needs

Have students use a highlighter to highlight both the variable and the number to be substituted for the variable when evaluating an algebraic expression. For example, in the expression $4x - 2$, when $x = 3$, have students highlight the x after the 4 and the 3.

Lesson PL2

Objective

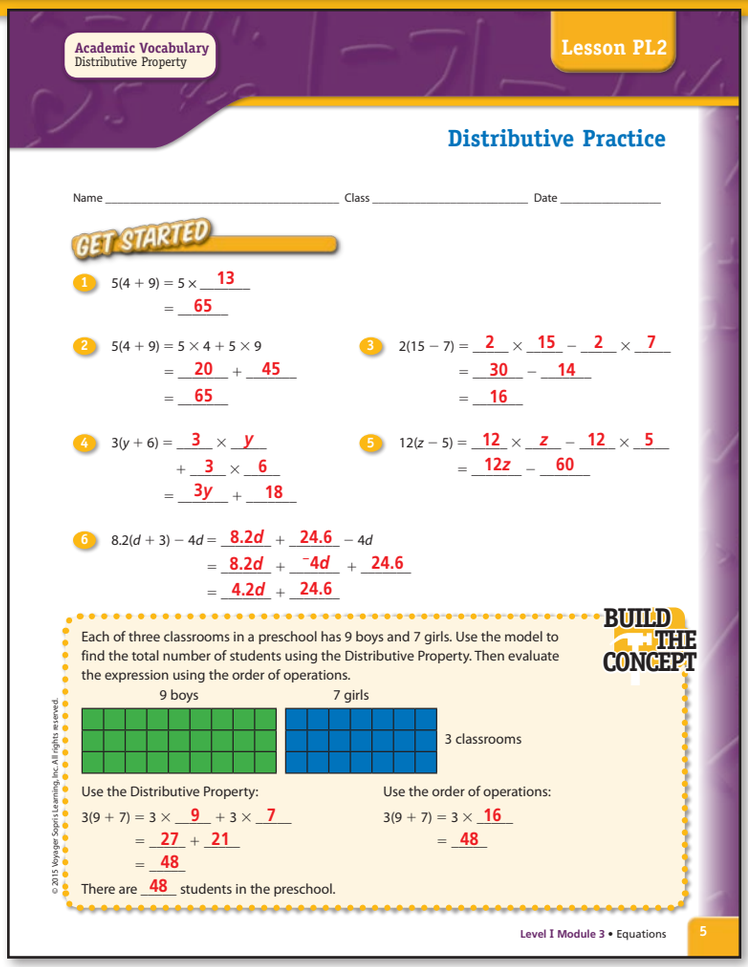
To use the Distributive Property to simplify an algebraic expression

Preskills	Lesson
Evaluating Expressions Using Order of Operations	I.2.2

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **Distributive Property**
  the property of distributing one operation over another, and the answer is the same
  Examples: $a(b + c) = a \times b + a \times c$ and $a(b - c) = a \times b - a \times c$



Academic Vocabulary
Distributive Property

Lesson PL2

Distributive Practice

Name _____ Class _____ Date _____

GET STARTED

1. $5(4 + 9) = 5 \times$ 13 $=$ 65

2. $5(4 + 9) = 5 \times 4 + 5 \times 9$
$=$ 20 $+$ 45
$=$ 65

3. $2(15 - 7) =$ 2 $\times$ 15 $-$ 2 $\times$ 7
$=$ 30 $-$ 14
$=$ 16

4. $3(y + 6) =$ 3 $\times$ y
$+$ 3 $\times$ 6
$=$ 3y $+$ 18

5. $12(z - 5) =$ 12 $\times$ z $-$ 12 $\times$ 5
$=$ 12z $-$ 60

6. $8.2(d + 3) - 4d =$ 8.2d $+$ 24.6 $-$ 4d
$=$ 8.2d $+$ $^-$4d $+$ 24.6
$=$ 4.2d $+$ 24.6

BUILD THE CONCEPT

Each of three classrooms in a preschool has 9 boys and 7 girls. Use the model to find the total number of students using the Distributive Property. Then evaluate the expression using the order of operations.

9 boys 7 girls

3 classrooms

Use the Distributive Property:
$3(9 + 7) = 3 \times$ 9 $+ 3 \times$ 7
$=$ 27 $+$ 21
$=$ 48

Use the order of operations:
$3(9 + 7) = 3 \times$ 16
$=$ 48

There are 48 students in the preschool.

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Level I Module 3 • Equations 5

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Use the order of operations to evaluate the expression in problem 1. What operation should be performed first? (addition in parentheses) **What is 4 plus 9?** (13) **What operation should be performed next?** (multiplication) **What is 5 times 13?** (65) **The value of the expression is 65.**

MODEL NEW SKILLS

Problem 2

What operation is indicated when an operation is written before parentheses? (multiplication) **The Distributive Property can be used to evaluate this expression. To use the Distributive Property in this problem, multiply each number in parentheses by 5.**

What is 5 times 4? (20) **What is 5 times 9?** (45) **Because the operation in parentheses is addition, add these two products. What is 20 plus 45?** (65) **Notice that the expression in problem 1 is the same as the expression in problem 2. The answer in problem 1 was found using the order of operations, and the answer in problem 2 was found using the Distributive Property. Both answers are the same.**

Multiplying a number and the sum of two numbers is the same as multiplying each number in parentheses by the number outside the parentheses and adding the products.

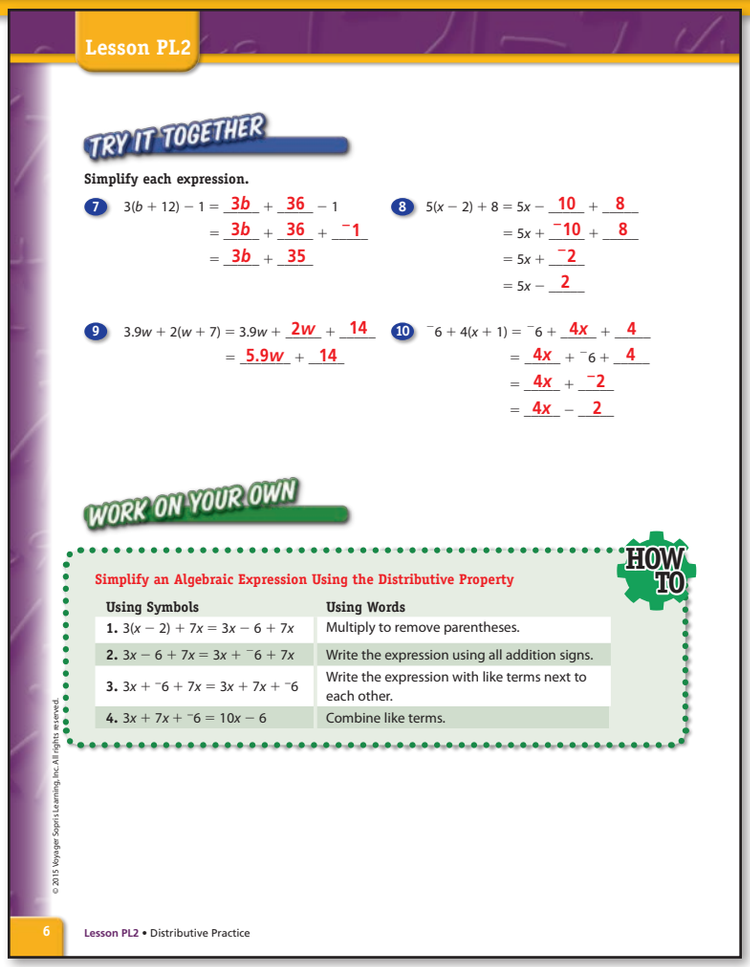
Problem 3

To use the Distributive Property in this problem, multiply each number in parentheses by 2. What is 2 times 15? (30) **What is 2 times 7?** (14) **Notice that a subtraction sign is between the two products because a subtraction sign was in the original expression inside the parentheses. What is 30 minus 14?** (16)

Multiplying a number and the difference of two numbers is the same as multiplying each number in parentheses by the number outside the parentheses and subtracting the products.

Problem 4

The Distributive Property is ordinarily used to simplify expressions with variables. To use the Distributive Property in this problem, first multiply both terms in parentheses by 3. Write the product 3 times y first. Then


Lesson PL2

TRY IT TOGETHER

Simplify each expression.

7. $3(b + 12) - 1 = \underline{3b} + \underline{36} - 1$
$= \underline{3b} + \underline{36} + \underline{^{-}1}$
$= \underline{3b} + \underline{35}$

8. $5(x - 2) + 8 = 5x - \underline{10} + \underline{8}$
$= 5x + \underline{^{-}10} + \underline{8}$
$= 5x + \underline{^{-}2}$
$= 5x - \underline{2}$

9. $3.9w + 2(w + 7) = 3.9w + \underline{2w} + \underline{14}$
$= \underline{5.9w} + \underline{14}$

10. $^{-}6 + 4(x + 1) = \underline{^{-}6} + \underline{4x} + \underline{4}$
$= \underline{4x} + \underline{^{-}6} + \underline{4}$
$= \underline{4x} + \underline{^{-}2}$
$= \underline{4x} - \underline{2}$

WORK ON YOUR OWN

HOW TO

Simplify an Algebraic Expression Using the Distributive Property

Using Symbols	Using Words
1. $3(x - 2) + 7x = 3x - 6 + 7x$	Multiply to remove parentheses.
2. $3x - 6 + 7x = 3x + {}^{-}6 + 7x$	Write the expression using all addition signs.
3. $3x + {}^{-}6 + 7x = 3x + 7x + {}^{-}6$	Write the expression with like terms next to each other.
4. $3x + 7x + {}^{-}6 = 10x - 6$	Combine like terms.

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6 Lesson PL2 • Distributive Practice

write the product 3 times 6. How is 3 times *y* written as a term? (3*y*) **What is 3 times 6?** (18) **These are not like terms, so they cannot be combined. What is the simplified expression?** (3*y* + 18)

Problem 5

To simplify the expression using the Distributive Property, first multiply to remove the parentheses. What is the first product? (12 × *z*) **What is the second product?** (12 × 5) **Notice that a subtraction sign is between the two products because a subtraction sign was in the original expression in parentheses.**

How is 12 times *z* written as a term? (12*z*) **What is 12 times 5?** (60) **Can these terms be combined?** (no) **Why not?** (They are not like terms.) **What is the simplified expression?** (12*z* − 60)

Problem 6

To simplify this expression, first multiply each term in parentheses by 8.2. What terms are in parentheses? (*d* and 3) **What term is not in parentheses?** (4*d*) **What terms should be multiplied by 8.2?** (*d* and 3) **How is 8.2 times *d* written?** (8.2*d*) **What is 8.2 times 3?** (24.6)

How is the entire expression written? (8.2*d* + 24.6 − 4*d*) **Write the expression using all addition signs with the like terms next to each other. How is the entire expression written now?** (8.2*d* + $^{-}$4*d* + 24.6) **What are the like terms in the expression now?** (8.2*d* and $^{-}$4*d*) **What is 8.2*d* plus $^{-}$4*d*?** (4.2*d*) **What is the simplified expression?** (4.2*d* + 24.6)

BUILD THE CONCEPT

Model how to use the Distributive Property to simplify an algebraic expression.

How is the number of boys shown in the model? (3 rows of 9 green tiles) **How is the number of girls shown in the model?** (3 rows of 7 blue tiles) **The expression 3(9 + 7) represents the total number of students. How can the Distributive Property be used to find the total number of students?** (Multiply both numbers in parentheses by 3.) **What is the first product?** (3 × 9) **What is the second product?** (3 × 7) **What is 3 times 9?** (27) **What is 3 times 7?** (21) **How many students are in the preschool?** (48) **What is the first step in evaluating the expression using the order of operations?** (add the numbers in parentheses) **What is 9 plus 7?** (16) **What is the next step?** (multiplication) **What is 3 times 16?** (48) **Are the answers the same?** (yes)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 7

What should be done first to simplify the expression in this problem? (Multiply to remove the parentheses.) **What is the expression after multiplying to remove the parentheses?** (3*b* + 36 − 1) **What is the expression written using all addition signs?** (3*b* + 36 + $^{-}$1) **What terms can be combined?** (36 and $^{-}$1) **What is 36 plus $^{-}$1?** (35) **What is the simplified expression?** (3*b* + 35)

Problem 8

What is the expression after multiplication is used to remove the parentheses? (5*x* − 10 + 8) **What is the expression written using all addition signs?** (5*x* + $^{-}$10 + 8) **What terms can be combined?** ($^{-}$10 and 8) **What is $^{-}$10 plus 8?** ($^{-}$2) **What is the simplified expression?** (5*x* − 2)

Problem 9

What is the expression after multiplying to remove parentheses? $(3.9w + 2w + 14)$ **What terms can be combined?** $(3.9w$ and $2w)$ **What is the simplified expression?** $(5.9w + 14)$

Problem 10

What is the expression after multiplying to remove parentheses? $(^{-}6 + 4x + 4)$ **What is the expression when the numeric terms are moved next to each other?** $(4x + {}^{-}6 + 4)$ **What is the simplified expression using subtraction?** $(4x - 2)$

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using a 4-Step Plan** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving the Distributive Property. The instruction is immediately followed by application of the strategy in problem 26.

Have students read the problem.

Review the 4-step plan with students:

Step 1 is to determine what needs to be found.

Step 2 is to determine how the problem will be solved.

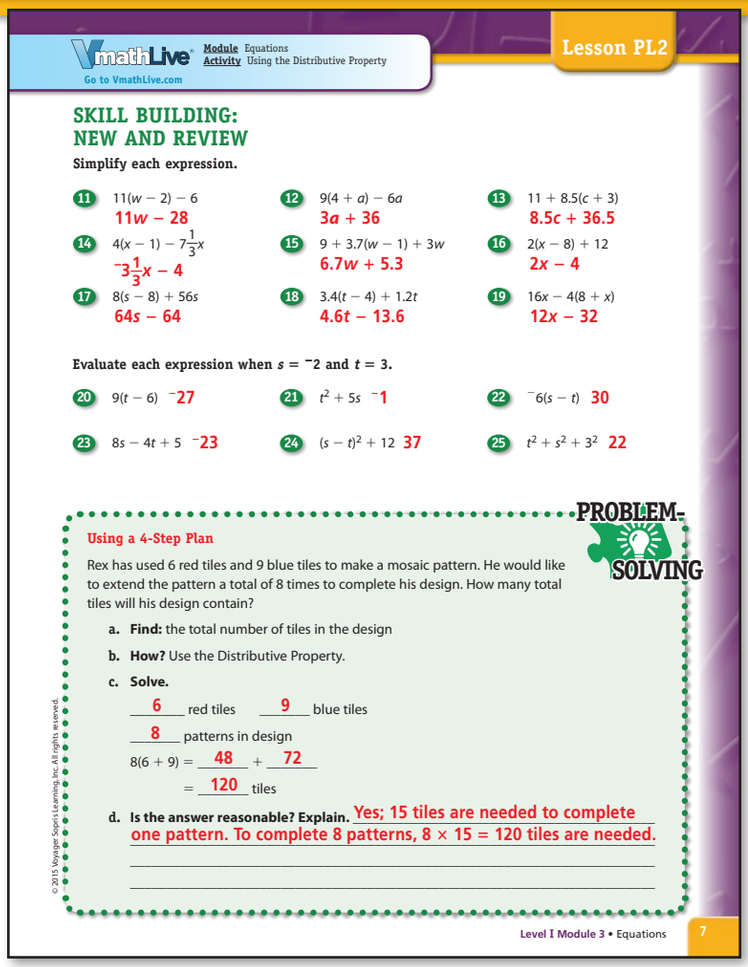
Step 3 is to solve the problem.

Step 4 is to check the answer for reasonableness.

Students should identify how many red and blue tiles are necessary for one pattern, then use the Distributive Property to determine the total number of tiles used in the completed design:

$$8(6 + 9) = 48 + 72 = 120 \text{ tiles}$$

The answer is reasonable because 15 tiles are needed to complete 1 pattern, so to complete 8 patterns, $8 \times 15 = 120$ tiles are needed.



VmathLive Module Activity Equations Using the Distributive Property
Go to VmathLive.com

Lesson PL2

SKILL BUILDING: NEW AND REVIEW

Simplify each expression.

11. $11(w - 2) - 6$ — $11w - 28$
12. $9(4 + a) - 6a$ — $3a + 36$
13. $11 + 8.5(c + 3)$ — $8.5c + 36.5$
14. $4(x - 1) - 7\frac{1}{3}x$ — $^{-}3\frac{1}{3}x - 4$
15. $9 + 3.7(w - 1) + 3w$ — $6.7w + 5.3$
16. $2(x - 8) + 12$ — $2x - 4$
17. $8(s - 8) + 56s$ — $64s - 64$
18. $3.4(t - 4) + 1.2t$ — $4.6t - 13.6$
19. $16x - 4(8 + x)$ — $12x - 32$

Evaluate each expression when $s = {}^{-}2$ and $t = 3$.

20. $9(t - 6)$ — $^{-}27$
21. $t^2 + 5s$ — $^{-}1$
22. $^{-}6(s - t)$ — 30
23. $8s - 4t + 5$ — $^{-}23$
24. $(s - t)^2 + 12$ — 37
25. $t^2 + s^2 + 3^2$ — 22

PROBLEM-SOLVING

Using a 4-Step Plan

Rex has used 6 red tiles and 9 blue tiles to make a mosaic pattern. He would like to extend the pattern a total of 8 times to complete his design. How many total tiles will his design contain?

a. **Find:** the total number of tiles in the design

b. **How?** Use the Distributive Property.

c. **Solve.**

6 red tiles 9 blue tiles

8 patterns in design

$8(6 + 9) = 48 + 72$

$= 120$ tiles

d. **Is the answer reasonable? Explain.** Yes; 15 tiles are needed to complete one pattern. To complete 8 patterns, $8 \times 15 = 120$ tiles are needed.

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Level I Module 3 • Equations 7

Problems 11–29

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 11, 14, 16–19, and 28 Reminder

The result should be written with a subtraction sign rather than the addition of a negative term.

Problem 29 Reminder

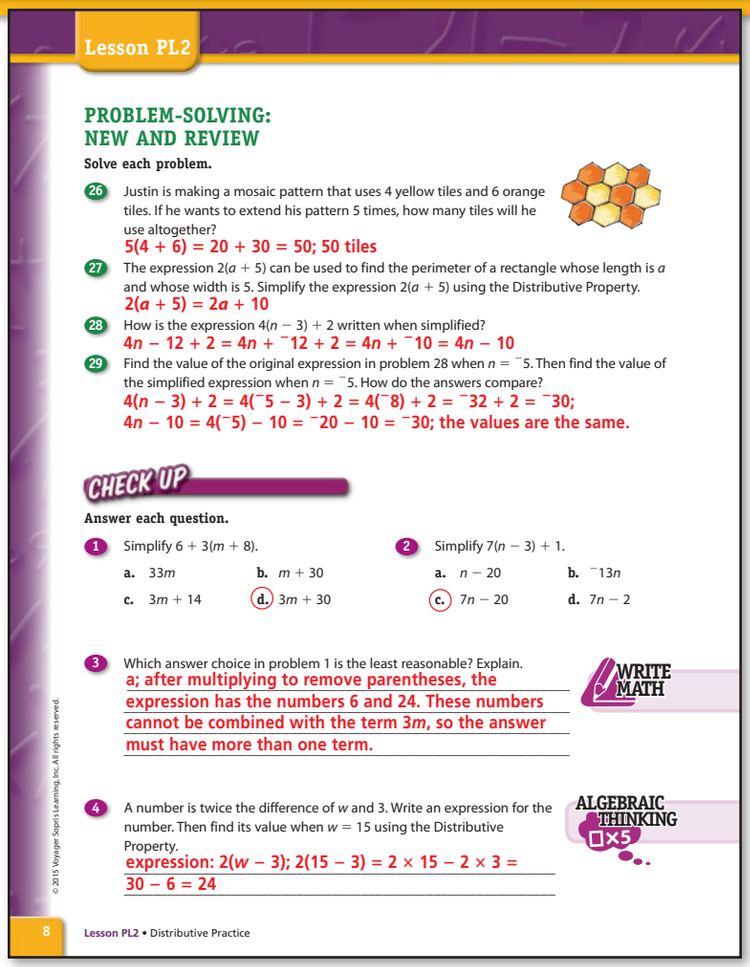
The students will need to substitute $^{-}5$ for n in both the original expression and the result of the preceding problem.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.



Lesson PL2

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

26 Justin is making a mosaic pattern that uses 4 yellow tiles and 6 orange tiles. If he wants to extend his pattern 5 times, how many tiles will he use altogether?
5(4 + 6) = 20 + 30 = 50; 50 tiles

27 The expression 2(*a* + 5) can be used to find the perimeter of a rectangle whose length is *a* and whose width is 5. Simplify the expression 2(*a* + 5) using the Distributive Property.
2(*a* + 5) = 2*a* + 10

28 How is the expression 4(*n* − 3) + 2 written when simplified?
4*n* − 12 + 2 = 4*n* + ⁻12 + 2 = 4*n* + ⁻10 = 4*n* − 10

29 Find the value of the original expression in problem 28 when *n* = ⁻5. Then find the value of the simplified expression when *n* = ⁻5. How do the answers compare?
4(*n* − 3) + 2 = 4(⁻5 − 3) + 2 = 4(⁻8) + 2 = ⁻32 + 2 = ⁻30;
4*n* − 10 = 4(⁻5) − 10 = ⁻20 − 10 = ⁻30; the values are the same.

CHECK UP

Answer each question.

1 Simplify 6 + 3(*m* + 8).
a. 33*m* b. *m* + 30 c. 3*m* + 14 (d.) 3*m* + 30

2 Simplify 7(*n* − 3) + 1.
a. *n* − 20 b. ⁻13*n* (c.) 7*n* − 20 d. 7*n* − 2

3 Which answer choice in problem 1 is the least reasonable? Explain.
a; after multiplying to remove parentheses, the expression has the numbers 6 and 24. These numbers cannot be combined with the term 3*m*, so the answer must have more than one term.

WRITE MATH

4 A number is twice the difference of *w* and 3. Write an expression for the number. Then find its value when *w* = 15 using the Distributive Property.
expression: 2(*w* − 3); 2(15 − 3) = 2 × 15 − 2 × 3 = 30 − 6 = 24

ALGEBRAIC THINKING □×5

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8 Lesson PL2 • Distributive Practice

If student answered 1a or 2b: The student treated all terms as variable terms. Remind the student that when simplifying expressions, the numeric terms and variable terms must be added separately.

If student answered 1b or 2a: The student multiplied the number outside parentheses by only the second term in parentheses, rather than by both terms. Use Additional Resources in I.3.PL2 to reteach how to use the Distributive Property to simplify expressions.

If student answered 1c or 2d: The student multiplied the number outside parentheses by only the first term in parentheses, rather than by both terms. Use Additional Resources in I.3.PL2 to reteach how to use the Distributive Property to simplify expressions.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that a numeric term must be part of the answer to problem 1. Remind the student that only like terms may be combined.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Equations
Activity: Using the Distributive Property

Vmath Reteach
Reteach Student Book Module 3 Lesson PL2
Reteach Teacher Edition Module 3 Lesson PL2

Extra Practice
Student Book page 45

English Language Learners

Use the VmathLive Animated Glossary to review the term *Distributive Property*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Have students work together to simplify expressions. Have the students say aloud the steps in simplifying the expression, making sure they distributed over every term in the parentheses. Tell students that the Distributive Property is named as such because one operation, multiplication, is distributed over another, addition or subtraction.

Students with Special Needs

Students sometimes fail to distribute a number over the entire expression in parentheses. Remind students to multiply each number in the expression by drawing arrows from the term outside parentheses to each term in parentheses.

Lesson 1

Objective

To solve an equation requiring more than two steps

Preskills	Lesson
Distributive Practice	I.3.PL2

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To simplify this expression, first change the subtraction sign to an addition sign. How is the expression written using all addition signs? ($^-2x + ^-3 + 8x$) **Now look for like terms. What are the like terms in ^-2x plus $^-3$ plus $8x$?** (^-2x and $8x$)

Combine like terms by adding the coefficients of the like terms. What is $^-2$ plus 8? (6) **Then ^-2x plus $8x$ is $6x$. The expression now is $6x$ plus $^-3$. What is the customary way to write the final answer?** ($6x - 3$)

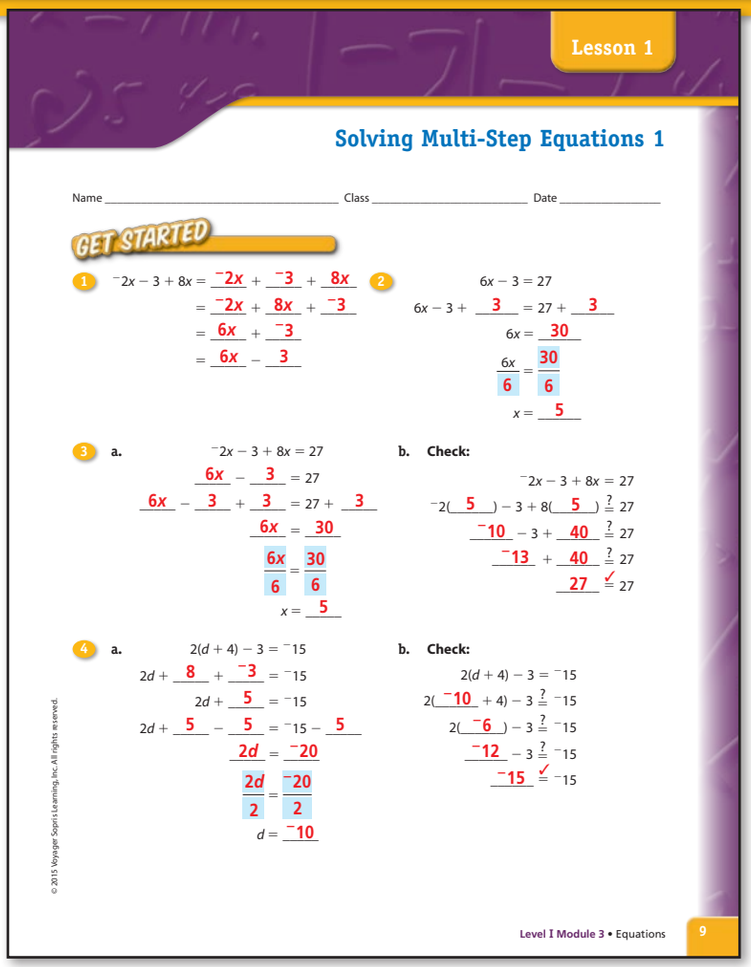
Problem 2

This is a two-step equation; it will take two operations to solve the equation. What is the first step in solving this equation? (Add 3 to both sides of the equation.) **What is $6x$ minus 3 plus 3?** ($6x$) **What is 27 plus 3?** (30) **What is the equation now?** ($6x = 30$) **Divide both sides of the equation by 6. What is $6x$ divided by 6?** (x) **What is 30 divided by 6?** (5) **The solution of the equation is x equals 5.**

MODEL NEW SKILLS

Problem 3

Notice that the expression on the left side of the equation is not simplified. It has two terms with the variable x. Before solving the equation, simplify the expression on the left side of the equation by combining like terms. Notice that the expression on the left side of the equation is the same as the expression in problem 1.



Lesson 1

Solving Multi-Step Equations 1

Name _____ Class _____ Date _____

GET STARTED

1 $^-2x - 3 + 8x = {}^-2x + {}^-3 + 8x$
$= {}^-2x + 8x + {}^-3$
$= 6x + {}^-3$
$= 6x - 3$

2 $6x - 3 = 27$
$6x - 3 + 3 = 27 + 3$
$6x = 30$
$\frac{6x}{6} = \frac{30}{6}$
$x = 5$

3 a. $^-2x - 3 + 8x = 27$
$6x - 3 = 27$
$6x - 3 + 3 = 27 + 3$
$6x = 30$
$\frac{6x}{6} = \frac{30}{6}$
$x = 5$

b. Check:
$^-2x - 3 + 8x = 27$
$^-2(5) - 3 + 8(5) \stackrel{?}{=} 27$
$^-10 - 3 + 40 \stackrel{?}{=} 27$
$^-13 + 40 \stackrel{?}{=} 27$
$27 \stackrel{✓}{=} 27$

4 a. $2(d + 4) - 3 = {}^-15$
$2d + 8 + {}^-3 = {}^-15$
$2d + 5 = {}^-15$
$2d + 5 - 5 = {}^-15 - 5$
$2d = {}^-20$
$\frac{2d}{2} = \frac{^-20}{2}$
$d = {}^-10$

b. Check:
$2(d + 4) - 3 = {}^-15$
$2(^-10 + 4) - 3 \stackrel{?}{=} {}^-15$
$2(^-6) - 3 \stackrel{?}{=} {}^-15$
$^-12 - 3 \stackrel{?}{=} {}^-15$
$^-15 \stackrel{✓}{=} {}^-15$

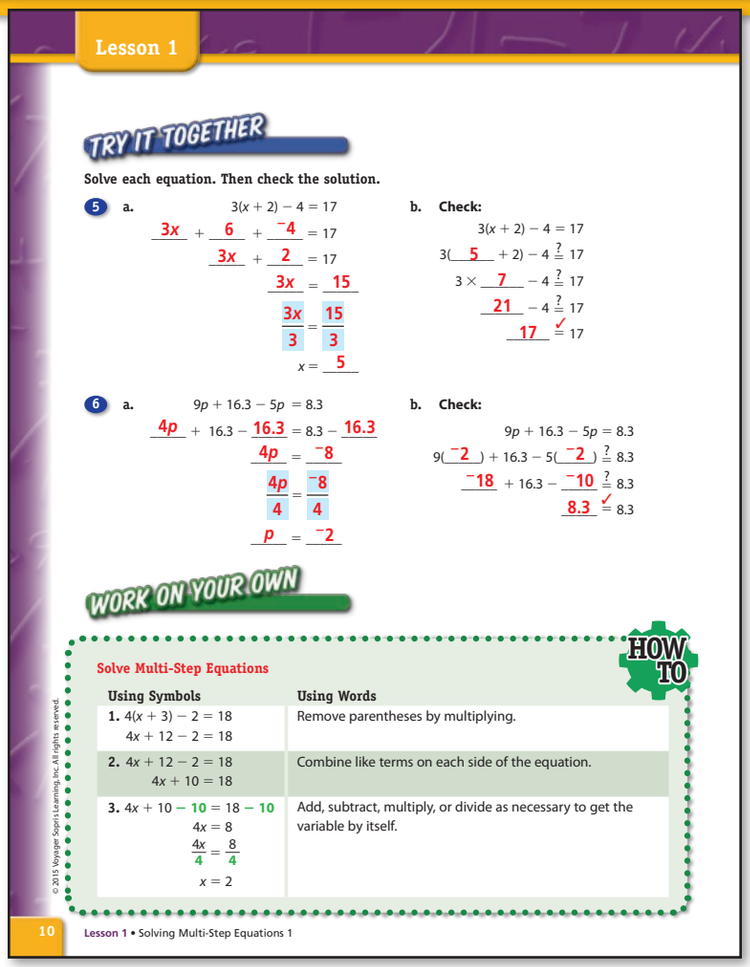
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Level I Module 3 • Equations 9

Use the results of problem 1. The expression can be written as $6x$ minus 3. The equation is now $6x$ minus 3 equals 27, which is a two-step equation. This equation was solved in problem 2. When 3 is added to both sides of the equation, the result is $6x$ equals 30. Then both sides of the equation are divided by 6. What is the solution of the equation? ($x = 5$)

To check the solution of an equation, substitute the solution for the variable in the original equation. Substitute the solution, 5, for both x's in the original equation. Follow the order of operations when solving the equation. The equation is $^-2(5) - 3 + 8(5) = 27$. If both sides of the equation equal the same value, then the solution is correct.

What is $^-2$ times 5? ($^-10$) **What is 8 times 5?** (40) **What is $^-10$ minus 3?** ($^-13$) **What is $^-13$ plus 40?** (27) **Twenty-seven equals 27, so the solution is correct. Write a check mark above the equal sign to show that the answer is correct.**


Lesson 1

TRY IT TOGETHER

Solve each equation. Then check the solution.

5 a. $3(x + 2) - 4 = 17$

$3x + 6 + {}^{-}4 = 17$

$3x + 2 = 17$

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

b. Check:

$3(x + 2) - 4 = 17$

$3(5 + 2) - 4 \stackrel{?}{=} 17$

$3 \times 7 - 4 \stackrel{?}{=} 17$

$21 - 4 \stackrel{?}{=} 17$

$17 \stackrel{\checkmark}{=} 17$

6 a. $9p + 16.3 - 5p = 8.3$

$4p + 16.3 - 16.3 = 8.3 - 16.3$

$4p = {}^{-}8$

$\frac{4p}{4} = \frac{{}^{-}8}{4}$

$p = {}^{-}2$

b. Check:

$9p + 16.3 - 5p = 8.3$

$9({}^{-}2) + 16.3 - 5({}^{-}2) \stackrel{?}{=} 8.3$

$-18 + 16.3 - {}^{-}10 \stackrel{?}{=} 8.3$

$8.3 \stackrel{\checkmark}{=} 8.3$

WORK ON YOUR OWN

HOW TO

Solve Multi-Step Equations

Using Symbols	Using Words
1. $4(x + 3) - 2 = 18$ $4x + 12 - 2 = 18$	Remove parentheses by multiplying.
2. $4x + 12 - 2 = 18$ $4x + 10 = 18$	Combine like terms on each side of the equation.
3. $4x + 10 - 10 = 18 - 10$ $4x = 8$ $\frac{4x}{4} = \frac{8}{4}$ $x = 2$	Add, subtract, multiply, or divide as necessary to get the variable by itself.

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10 Lesson 1 • Solving Multi-Step Equations 1

Problem 4

In this problem, the expression on the left side of the equation is not simplified. The expression includes parentheses. Before this equation can be solved, this expression must be simplified.

What should be done first? (Multiply to remove the parentheses.) **What is the result of multiplying to remove the parentheses and changing all subtraction signs to addition signs?** ($2d + 8 + {}^{-}3$)

What is the next step in simplifying this expression? (Add like terms.) **What is 8 plus ⁻3?** (5) **Now the two-step equation can be solved. What should be done first to solve the equation?** (Subtract 5 from both sides of the equation.) **What is 2*d* plus 5 minus 5?** ($2d$) **What is ⁻15 minus 5?** ($^{-}20$) **Now solve the equation 2*d* equals ⁻20. Divide both sides of the equation by 2. What is 2*d* divided by 2?** (d) **What is ⁻20 divided by 2?** ($^{-}10$) **The solution is *d* equals ⁻10.**

Check the solution in problem 4b. Substitute the solution, ⁻10, for *d* in the original equation. If both sides of the equation equal the same value, then the solution is correct. Use the order of operations to evaluate the expression on the left side of the equation and check the solution.

Simplify the parentheses first. What is ⁻10 plus 4? ($^{-}6$) **What is 2 times ⁻6?** ($^{-}12$) **What is ⁻12 minus 3?** ($^{-}15$) **Negative 15 equals ⁻15, so the solution is correct. Write a check mark above the equal sign to show that the answer is correct.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What should be done first to simplify the expression on the left side of the equation? (Multiply to remove the parentheses.) **What is 3 times *x* written as a term?** ($3x$) **What is 3 times 2?** (6) **What is the new equation written with all addition signs?** ($3x + 6 + {}^{-}4 = 17$)

Is the expression on the left side of the equation simplified? (no) **What is the next step in simplifying the expression on the left side?** (Combine the numeric terms 6 and ⁻4.) **What is the resulting equation?** ($3x + 2 = 17$)

Now solve the equation. What should be done first? (Subtract 2 from both sides of the equation.) **What is the next step?** (Divide both sides of the equation by 3.) **What is the solution?** ($x = 5$)

Check the solution by substituting 5 for *x* in the original equation. What is the equation when 5 is substituted for *x*? ($3(5 + 2) - 4 = 17$) **What is 5 plus 2?** (7) **What is 3 times 7?** (21) **What is 21 minus 4?** (17) **Both sides of the equation equal 17, so the solution is correct. Write a check mark above the equal sign to show that the answer is correct.**

Problem 6

What is the first step in solving this equation? (Combine like terms.) **What is the resulting equation?** ($4p + 16.3 = 8.3$) **Can the equation be solved now?** (yes) **Why?** (The expressions on both sides of the equation cannot be simplified further.) **What is subtracted from both sides of the equation?** (16.3) **What are both sides of the equation divided by?** (4) **What is the solution?** ($p = {}^{-}2$) **How can the solution be checked?** (Substitute ⁻2 for p in the original equation.) **Is the solution correct?** (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Writing an Equation** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving writing and solving an equation from a given problem. The instruction is immediately followed by application of the strategy in problem 13.

Have students read the problem.

Ask students what they are to find.

Students should solve the two-step equation for x, the original cost of one shirt.

$$5(x - 3) = 100$$

$$x = 23$$

The answer is reasonable because the equation checks when 23 is substituted for x in the original equation.

Problems 7–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 8 and 11 Reminder

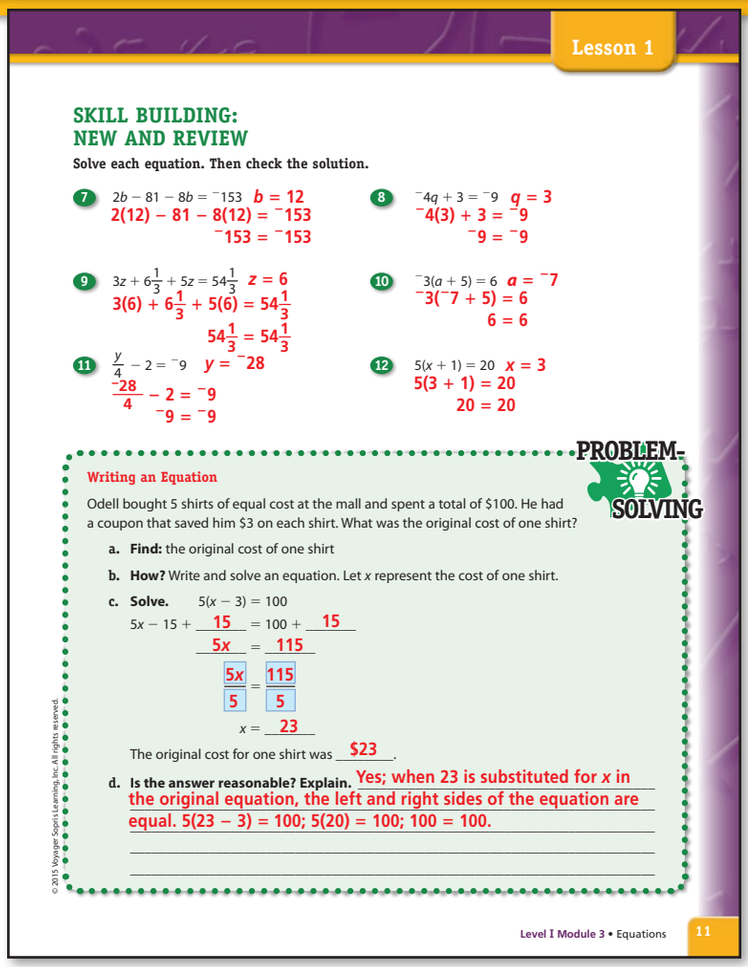
The expressions in these problems do not need to be simplified. These problems review solving two-step equations.

Problem 14 Reminder

Students must substitute 14 for n in the expression $n - 4$ to find the width of the rectangle.

Problem 15 Reminder

Suggest to students that they write the ratio of the number of free throws made to the number of free throws attempted as a fraction, then convert the fraction to a percent.



Lesson 1

SKILL BUILDING: NEW AND REVIEW

Solve each equation. Then check the solution.

7. $2b - 81 - 8b = -153$ $b = 12$
$2(12) - 81 - 8(12) = -153$
$-153 = -153$

8. $-4q + 3 = -9$ $q = 3$
$-4(3) + 3 = -9$
$-9 = -9$

9. $3z + 6\frac{1}{3} + 5z = 54\frac{1}{3}$ $z = 6$
$3(6) + 6\frac{1}{3} + 5(6) = 54\frac{1}{3}$
$54\frac{1}{3} = 54\frac{1}{3}$

10. $-3(a + 5) = 6$ $a = -7$
$-3(-7 + 5) = 6$
$6 = 6$

11. $\frac{y}{4} - 2 = -9$ $y = -28$
$\frac{-28}{4} - 2 = -9$
$-9 = -9$

12. $5(x + 1) = 20$ $x = 3$
$5(3 + 1) = 20$
$20 = 20$

PROBLEM-SOLVING

Writing an Equation

Odell bought 5 shirts of equal cost at the mall and spent a total of $100. He had a coupon that saved him $3 on each shirt. What was the original cost of one shirt?

a. **Find:** the original cost of one shirt

b. **How?** Write and solve an equation. Let x represent the cost of one shirt.

c. **Solve.** $5(x - 3) = 100$
$5x - 15 + 15 = 100 + 15$
$5x = 115$
$\frac{5x}{5} = \frac{115}{5}$
$x = 23$
The original cost for one shirt was $23.

d. **Is the answer reasonable? Explain.** Yes; when 23 is substituted for x in the original equation, the left and right sides of the equation are equal. $5(23 - 3) = 100$; $5(20) = 100$; $100 = 100$.

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CHECK UP

ASSESS INFORMALLY

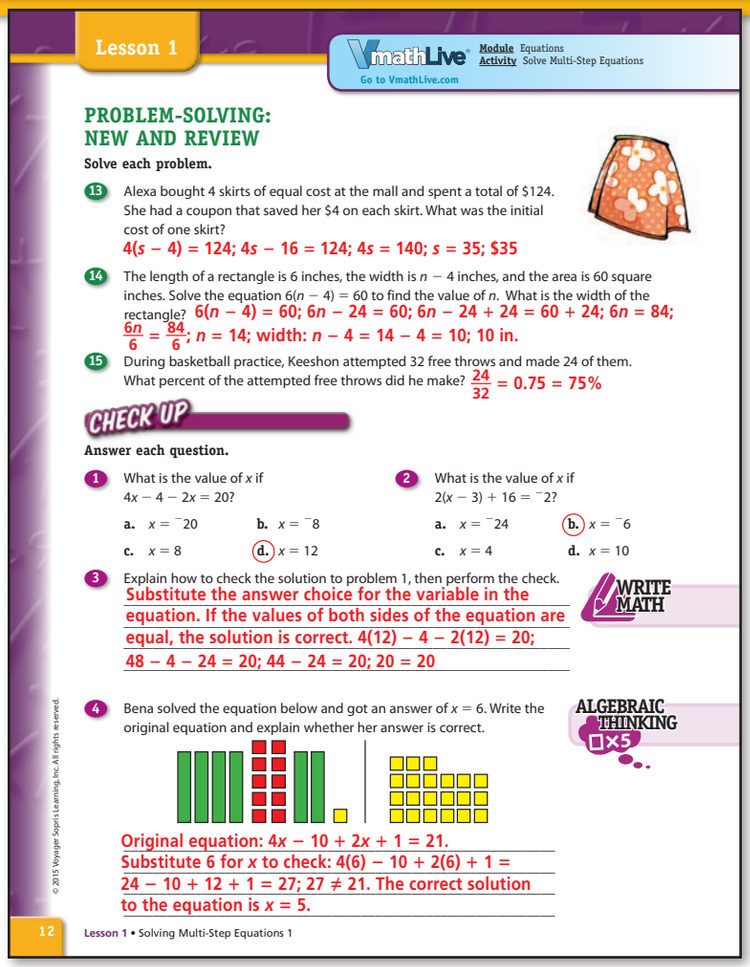
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student made a sign error in the division step of the solution. Remind the student to check the sign of each result when working with integers.

If student answered 1b: The student subtracted 4 from both sides of the equation instead of adding 4 and made a sign error. Remind the student that the opposite operation is used when solving an equation.

If student answered 1c: The student subtracted 4 from both sides of the equation instead of adding 4. Remind the student that the opposite operation is used when solving an equation.



Lesson 1

VmathLive Module Equations Activity Solve Multi-Step Equations Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

13 Alexa bought 4 skirts of equal cost at the mall and spent a total of $124. She had a coupon that saved her $4 on each skirt. What was the initial cost of one skirt?
$4(s - 4) = 124$; $4s - 16 = 124$; $4s = 140$; $s = 35$; $35

14 The length of a rectangle is 6 inches, the width is $n - 4$ inches, and the area is 60 square inches. Solve the equation $6(n - 4) = 60$ to find the value of n. What is the width of the rectangle? $6(n - 4) = 60$; $6n - 24 = 60$; $6n - 24 + 24 = 60 + 24$; $6n = 84$; $\frac{6n}{6} = \frac{84}{6}$; $n = 14$; width: $n - 4 = 14 - 4 = 10$; 10 in.

15 During basketball practice, Keeshon attempted 32 free throws and made 24 of them. What percent of the attempted free throws did he make? $\frac{24}{32} = 0.75 = 75\%$

CHECK UP

Answer each question.

1 What is the value of x if $4x - 4 - 2x = 20$?
a. $x = -20$ b. $x = -8$ c. $x = 8$ (d.) $x = 12$

2 What is the value of x if $2(x - 3) + 16 = -2$?
a. $x = -24$ (b.) $x = -6$ c. $x = 4$ d. $x = 10$

3 Explain how to check the solution to problem 1, then perform the check.
Substitute the answer choice for the variable in the equation. If the values of both sides of the equation are equal, the solution is correct. $4(12) - 4 - 2(12) = 20$; $48 - 4 - 24 = 20$; $44 - 24 = 20$; $20 = 20$

WRITE MATH

4 Bena solved the equation below and got an answer of $x = 6$. Write the original equation and explain whether her answer is correct.

ALGEBRAIC THINKING

Original equation: $4x - 10 + 2x + 1 = 21$.
Substitute 6 for x to check: $4(6) - 10 + 2(6) + 1 = 24 - 10 + 12 + 1 = 27$; $27 \neq 21$. The correct solution to the equation is $x = 5$.

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If student answered 2a: The student multiplied in the final step instead of dividing. Remind the student that the opposite operation is used when solving an equation.

If student answered 2c: The student made an error combining the numeric terms -6 and 16. Remind the student to follow the rules when working with integers to solve equations.

If student answered 2d: The student added 16 to both sides of the equation instead of subtracting. Remind the student that the opposite operation is used when solving an equation.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to check a solution by substituting the solution for the variable in the original equation.

Technology



Have students practice math fluency while competing against one another online in VmathLive activities. Online videos in VmathLive reinforce math concepts. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Equations
Activity: Solve Multi-Step Equations

Vmath Reteach
Reteach Student Book Module 3 Lesson 1
Reteach Teacher Edition Module 3 Lesson 1

Extra Practice
Student Book page 46

English Language Learners

Use the VmathLive Animated Glossary to review the terms *algebraic equation* and *substitute*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write a multi-step equation on the board, such as $-13 = 3(x - 4) + 2$, and solve it. Then have students write in words what was done to the equation during each step. This activity will help students practice the vocabulary associated with algebra.

Students with Special Needs

Students should know that whatever is done to one side of an equation must be done to the other side as well. Therefore, they might be confused when they simplify one side of an equation but not the other. Show that simplifying one side of an equation does not change the value; it just changes the appearance. Use a simple problem to demonstrate: $9 + 9 + 9 + 9 + 9 = 5(9) = 45$. The value of the original $9 + 9 + 9 + 9 + 9$ was never changed; it was just simplified.

Lesson 2

Objective

To solve multi-step equations that contain fractions and decimals

Preskills	Lesson
Reciprocals	I.1.9
Operations with Positive and Negative Fractions	I.1.13–I.1.15
Solving Multi-Step Equations 1	I.3.1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What should be done to simplify the expression in this problem? (Multiply to remove the parentheses.) **What is 0.4 times 5?** (2) **What is the simplified expression?** ($0.4w - 2$)

Problem 2

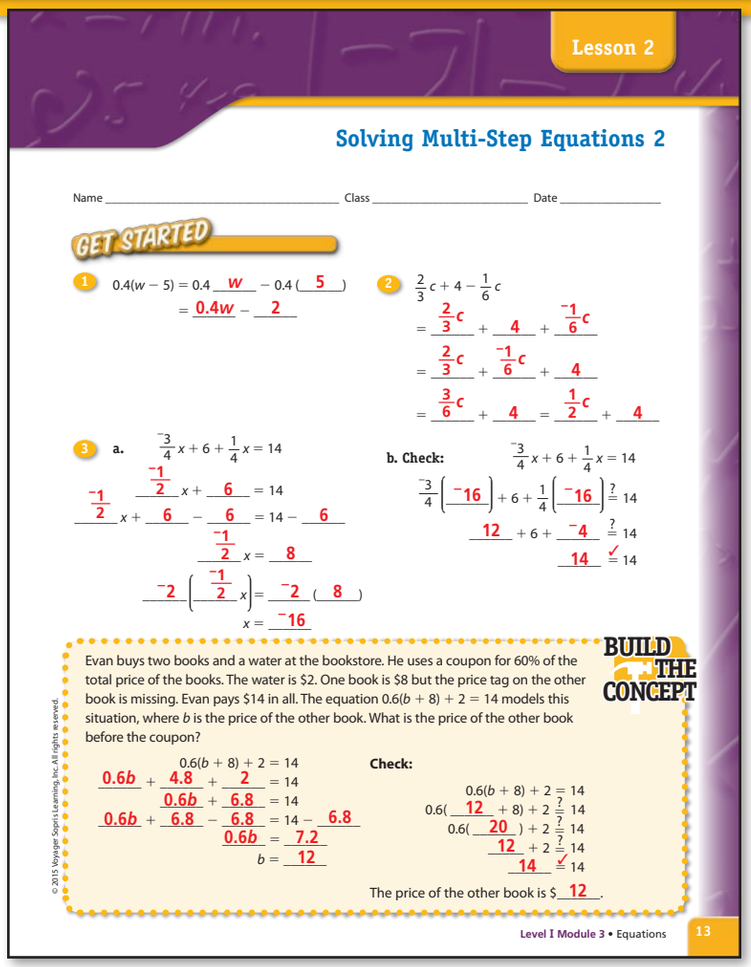
To simplify this expression, first change the subtraction sign to an addition sign. What is the expression written using all addition signs? ($\frac{2}{3}c + 4 + \frac{-1}{6}c$) **Now look for like terms. What are the like terms in $\frac{2}{3}c + 4 + \frac{-1}{6}c$?** ($\frac{2}{3}c$ and $\frac{-1}{6}c$)

Combine like terms by adding the coefficients. What is $\frac{2}{3}$ plus $\frac{-1}{6}$? ($\frac{3}{6}$) **Then $\frac{2}{3}c$ plus $\frac{-1}{6}c$ is $\frac{3}{6}c$. The expression now is $\frac{3}{6}c + 4$. What is this expression written in simplest form?** ($\frac{1}{2}c + 4$)

MODEL NEW SKILLS

Problem 3

Are the expressions on both sides of the equation simplified? (no) **What needs to be done to simplify the expressions?** (Combine the like terms on the left side of the equation.) **What are the like terms?** ($\frac{-3}{4}x$ and $\frac{1}{4}x$) **What is $\frac{-3}{4}x$ plus $\frac{1}{4}x$ in simplest form?** ($\frac{-1}{2}x$)



Lesson 2

Solving Multi-Step Equations 2

Name _____ Class _____ Date _____

GET STARTED

1. $0.4(w - 5) = 0.4(\underline{w}) - 0.4(\underline{5})$
$= \underline{0.4w} - \underline{2}$

2. $\frac{2}{3}c + 4 - \frac{1}{6}c$
$= \underline{\frac{2}{3}c} + \underline{4} + \underline{\frac{-1}{6}c}$
$= \underline{\frac{2}{3}c} + \underline{\frac{-1}{6}c} + \underline{4}$
$= \underline{\frac{3}{6}c} + \underline{4} = \underline{\frac{1}{2}c} + \underline{4}$

3. a. $\frac{-3}{4}x + 6 + \frac{1}{4}x = 14$
$\underline{\frac{-1}{2}}x + \underline{6} - \underline{6} = 14 - \underline{6}$
$\underline{\frac{-1}{2}}x = \underline{8}$
$\underline{-2}\left(\underline{\frac{-1}{2}}x\right) = \underline{-2}(\underline{8})$
$x = \underline{-16}$

b. Check: $\frac{-3}{4}x + 6 + \frac{1}{4}x = 14$
$\frac{-3}{4}(\underline{-16}) + 6 + \frac{1}{4}(\underline{-16}) \stackrel{?}{=} 14$
$\underline{12} + 6 + \underline{-4} \stackrel{?}{=} 14$
$\underline{14} \stackrel{\checkmark}{=} 14$

BUILD THE CONCEPT

Evan buys two books and a water at the bookstore. He uses a coupon for 60% of the total price of the books. The water is $2. One book is $8 but the price tag on the other book is missing. Evan pays $14 in all. The equation $0.6(b + 8) + 2 = 14$ models this situation, where b is the price of the other book. What is the price of the other book before the coupon?

$0.6(b + 8) + 2 = 14$
$\underline{0.6b} + \underline{4.8} + \underline{2} = 14$
$\underline{0.6b} + \underline{6.8} = 14$
$\underline{0.6b} + \underline{6.8} - \underline{6.8} = 14 - \underline{6.8}$
$\underline{0.6b} = \underline{7.2}$
$b = \underline{12}$

Check:
$0.6(b + 8) + 2 = 14$
$0.6(\underline{12} + 8) + 2 \stackrel{?}{=} 14$
$0.6(\underline{20}) + 2 \stackrel{?}{=} 14$
$\underline{12} + 2 \stackrel{?}{=} 14$
$\underline{14} \stackrel{\checkmark}{=} 14$

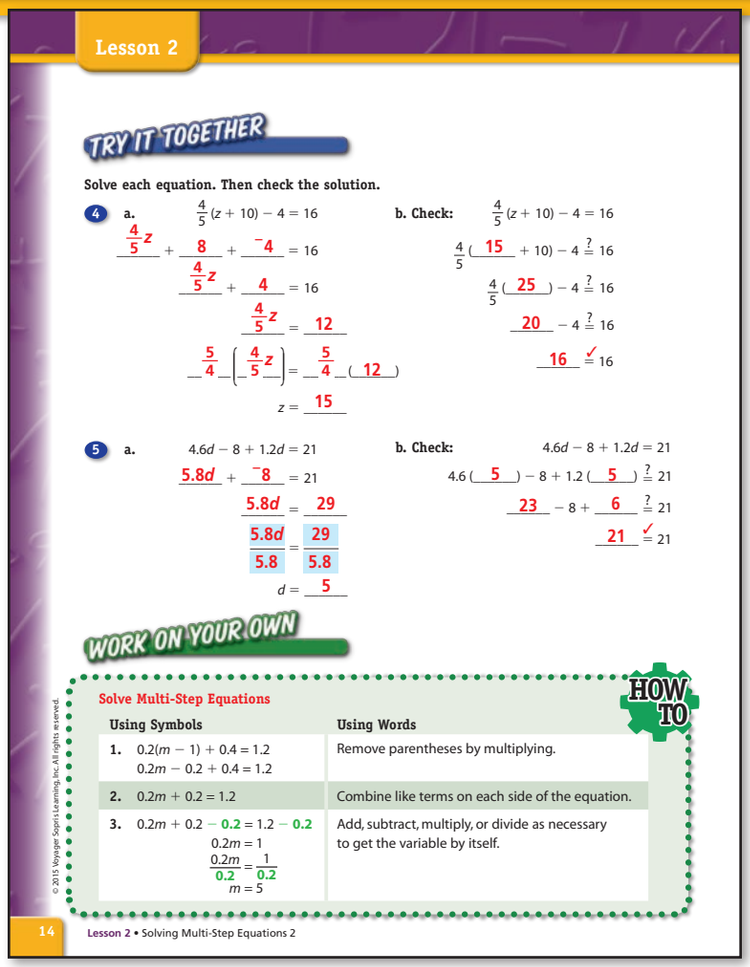
The price of the other book is $\$\underline{12}$.

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Level I Module 3 • Equations 13

What is the equation now? ($\frac{-1}{2}x + 6 = 14$) **What should be done next to isolate the variable in this two-step equation?** (Subtract 6 from each side.) **When 6 is subtracted from both sides, what is the result?** ($\frac{-1}{2}x = 8$) **Finally, multiply both sides of the equation by the reciprocal of $\frac{-1}{2}$. What is the reciprocal of $\frac{-1}{2}$?** ($^-2$) **So, multiply each side of the equation by $^-2$. What is $^-2$ times $\frac{-1}{2}x$?** (x) **What is $^-2$ times 8?** ($^-16$) **What is the solution of the equation?** ($x = {}^-16$)

How is the solution of an equation checked? (Substitute the solution for the variable in the original equation.) **What is the original equation when the solution is substituted for both x's?** ($\frac{-3}{4}(^-16) + 6 + \frac{1}{4}(^-16) = 14$) **What is $\frac{-3}{4}$ times $^-16$?** (12) **What is $\frac{1}{4}$ times $^-16$?** ($^-4$) **What is 12 plus 6?** (18) **What is 18 plus $^-4$?** (14) **Is the solution correct?** (yes) **Why?** (Both sides of the equation are the same number.) **Write a check mark above the equal sign to show that the answer is correct.**



Lesson 2

TRY IT TOGETHER

Solve each equation. Then check the solution.

4 a. $\frac{4}{5}(z + 10) - 4 = 16$

$\frac{4}{5}z + 8 + {}^{-}4 = 16$

$\frac{4}{5}z + 4 = 16$

$\frac{4}{5}z = 12$

$\frac{5}{4}\left(\frac{4}{5}z\right) = \frac{5}{4}(12)$

$z = 15$

b. Check: $\frac{4}{5}(z + 10) - 4 = 16$

$\frac{4}{5}(15 + 10) - 4 \stackrel{?}{=} 16$

$\frac{4}{5}(25) - 4 \stackrel{?}{=} 16$

$20 - 4 \stackrel{?}{=} 16$

$16 \stackrel{\checkmark}{=} 16$

5 a. $4.6d - 8 + 1.2d = 21$

$5.8d + {}^{-}8 = 21$

$5.8d = 29$

$\frac{5.8d}{5.8} = \frac{29}{5.8}$

$d = 5$

b. Check: $4.6d - 8 + 1.2d = 21$

$4.6(5) - 8 + 1.2(5) \stackrel{?}{=} 21$

$23 - 8 + 6 \stackrel{?}{=} 21$

$21 \stackrel{\checkmark}{=} 21$

WORK ON YOUR OWN

HOW TO

Solve Multi-Step Equations

Using Symbols	Using Words
1. $0.2(m - 1) + 0.4 = 1.2$ $0.2m - 0.2 + 0.4 = 1.2$	Remove parentheses by multiplying.
2. $0.2m + 0.2 = 1.2$	Combine like terms on each side of the equation.
3. $0.2m + 0.2 - 0.2 = 1.2 - 0.2$ $0.2m = 1$ $\frac{0.2m}{0.2} = \frac{1}{0.2}$ $m = 5$	Add, subtract, multiply, or divide as necessary to get the variable by itself.

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BUILD THE CONCEPT

Model how to use an equation to solve a real-world problem.

Read the problem. What is to be found? (the price of the other book) **How is the price of the other book represented in the problem?** (as the variable *b*)

What is the equation? ($0.6(b + 8) + 2 = 14$) **Are the expressions on both sides of the equation simplified?** (no) **What needs to be done to simplify the expressions?** (Multiply to remove parentheses, then combine the like terms on the left side.) **What is the result of multiplying to remove parentheses?** ($0.6b + 4.8 + 2$) **What like terms need to be combined?** (4.8 and 2) **What is 4.8 plus 2?** (6.8)

What should be done first to isolate the variable in this two-step equation? (Subtract 6.8 from each side.) **When 6.8 is subtracted from both sides, what is the result?** ($0.6b = 7.2$) **What should be done next?** (Divide both sides by 0.6.) **What is 7.2 divided by 0.6?** (12) **What is the solution of the equation?** ($b = 12$)

Now check the solution. What is the original equation after the solution is substituted for *b*? ($0.6(12 + 8) + 2 = 14$) **What is 12 plus 8?** (20) **What is 0.6 times 20?** (12) **What is 12 plus 2?** (14) **Is the solution correct?** (yes) **What is the price of the other book?** ($12)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What should be done first to simplify the expression on the left side of the equation? (Multiply to remove the parentheses.) **What is $\frac{4}{5}$ times 10?** (8) **What is the new equation written with all addition signs?** ($\frac{4}{5}z + 8 + {}^{-}4 = 16$)

Is the expression on the left side of the equation simplified? (no) **What is the next step in simplifying the expression on the left side?** (Combine the numeric terms 8 and $^{-}4$.) **What is the resulting equation?** ($\frac{4}{5}z + 4$)

Now solve the equation. What should be done first? (Subtract 4 from both sides.) **What is the resulting equation** ($\frac{4}{5}z = 12$) **What is the next step?** (Multiply both sides of the equation by the reciprocal of $\frac{4}{5}$.) **What is the reciprocal of $\frac{4}{5}$?** ($\frac{5}{4}$) **What is the solution?** ($z = 15$)

Check the solution by substituting 15 for *z* in the original equation. What is the equation when 15 is substituted for *z*? ($\frac{4}{5}(15 + 10) - 4 = 16$) **What is 15 plus 10?** (25) **What is $\frac{4}{5}$ times 25?** (20) **What is 20 minus 4?** (16) **Do both sides equal the same number?** (yes) **The solution is correct.**

Problem 5

What is the first step in solving this equation? (Combine like terms.) **What is the resulting equation?** ($5.8d + {}^{-}8 = 21$) **Can the equation be solved now?** (yes) **Why?** (The expressions on both sides of the equation cannot be simplified further.) **What is subtracted from both sides of the equation?** ($^{-}8$) **By what number are both sides of the equation divided?** (5.8) **What is the solution?** ($d = 5$) **How can the solution be checked?** (Substitute 5 for *d* in the original equation.) **Is the solution correct?** (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

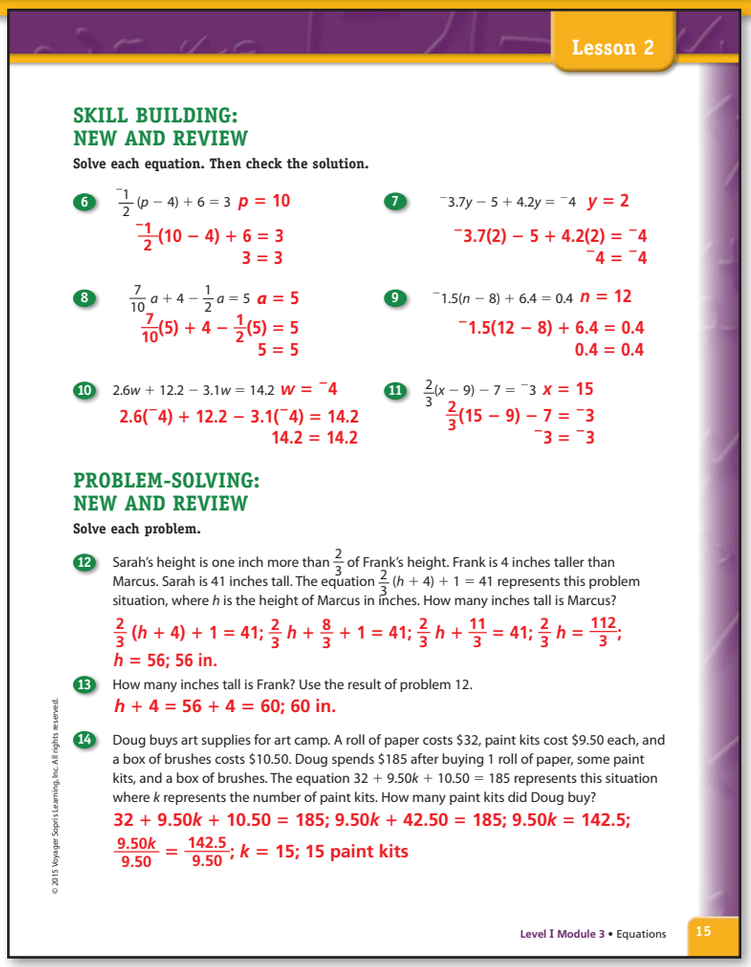
Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 6–14

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 13 Reminder

Students should identify the expression from problem 12 that represents Frank's height. Then they should use the solution from problem 12 to evaluate the expression to answer the question.



Lesson 2

SKILL BUILDING: NEW AND REVIEW

Solve each equation. Then check the solution.

6. $\frac{-1}{2}(p - 4) + 6 = 3$ $p = 10$
$\frac{-1}{2}(10 - 4) + 6 = 3$
$3 = 3$

7. $-3.7y - 5 + 4.2y = -4$ $y = 2$
$-3.7(2) - 5 + 4.2(2) = -4$
$-4 = -4$

8. $\frac{7}{10}a + 4 - \frac{1}{2}a = 5$ $a = 5$
$\frac{7}{10}(5) + 4 - \frac{1}{2}(5) = 5$
$5 = 5$

9. $-1.5(n - 8) + 6.4 = 0.4$ $n = 12$
$-1.5(12 - 8) + 6.4 = 0.4$
$0.4 = 0.4$

10. $2.6w + 12.2 - 3.1w = 14.2$ $w = -4$
$2.6(-4) + 12.2 - 3.1(-4) = 14.2$
$14.2 = 14.2$

11. $\frac{2}{3}(x - 9) - 7 = -3$ $x = 15$
$\frac{2}{3}(15 - 9) - 7 = -3$
$-3 = -3$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

12. Sarah's height is one inch more than $\frac{2}{3}$ of Frank's height. Frank is 4 inches taller than Marcus. Sarah is 41 inches tall. The equation $\frac{2}{3}(h + 4) + 1 = 41$ represents this problem situation, where h is the height of Marcus in inches. How many inches tall is Marcus?
$\frac{2}{3}(h + 4) + 1 = 41$; $\frac{2}{3}h + \frac{8}{3} + 1 = 41$; $\frac{2}{3}h + \frac{11}{3} = 41$; $\frac{2}{3}h = \frac{112}{3}$; $h = 56$; 56 in.

13. How many inches tall is Frank? Use the result of problem 12.
$h + 4 = 56 + 4 = 60$; 60 in.

14. Doug buys art supplies for art camp. A roll of paper costs $32, paint kits cost $9.50 each, and a box of brushes costs $10.50. Doug spends $185 after buying 1 roll of paper, some paint kits, and a box of brushes. The equation $32 + 9.50k + 10.50 = 185$ represents this situation where k represents the number of paint kits. How many paint kits did Doug buy?
$32 + 9.50k + 10.50 = 185$; $9.50k + 42.50 = 185$; $9.50k = 142.5$; $\frac{9.50k}{9.50} = \frac{142.5}{9.50}$; $k = 15$; 15 paint kits

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Level I Module 3 • Equations 15

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student began by dividing both sides improperly by -0.5. Use Additional Resources in I.3.2 to review the process for solving an equation that contains parentheses.

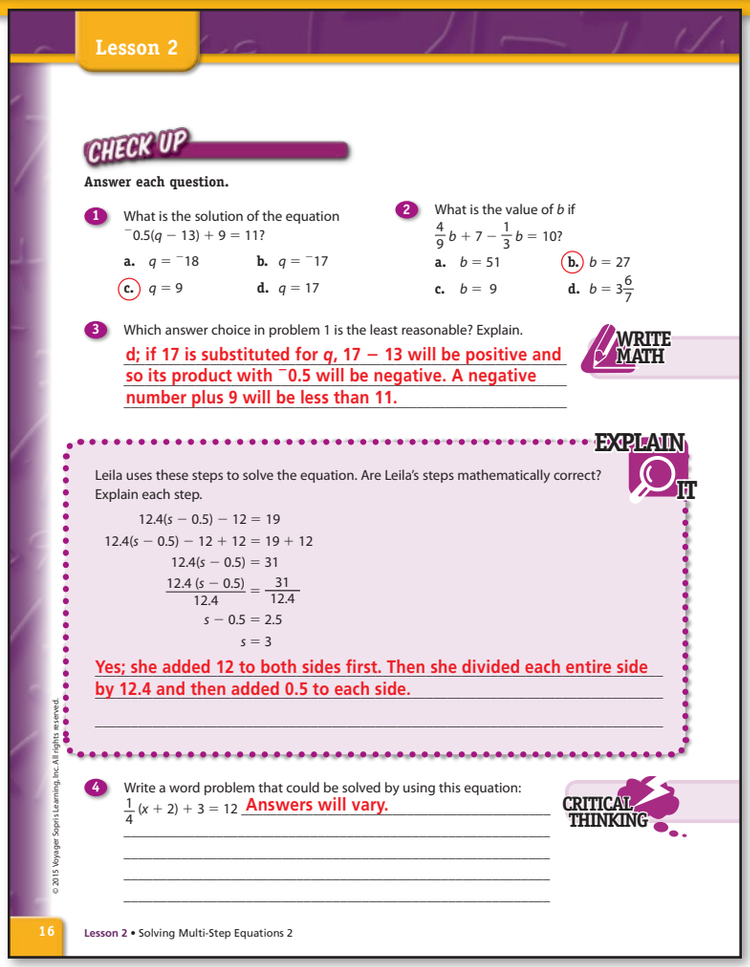
If student answered 1b or 1d: The student did not multiply correctly when removing parentheses. Use Additional Resources in I.3.2 to review the process for solving an equation that contains parentheses.

If student answered 2a or 2d: The student used the wrong operation when isolating the variable. Use Additional Resources in I.3.2 to review the process for solving a multi-step equation.

If student answered 2c: The student subtracted fractions incorrectly. Use Additional Resources in I.1.14 to review how to subtract fractions.



If student answered problem 3 incorrectly: While students work together, review how to predict the signs of sums, differences, and products based on the signs and absolute values of the numbers in the given problem.



Lesson 2

CHECK UP

Answer each question.

1. What is the solution of the equation $^{-}0.5(q - 13) + 9 = 11$?

   a. $q = {}^{-}18$ b. $q = {}^{-}17$ (c.) $q = 9$ d. $q = 17$

2. What is the value of b if $\frac{4}{9}b + 7 - \frac{1}{3}b = 10$?

   a. $b = 51$ (b.) $b = 27$ c. $b = 9$ d. $b = 3\frac{6}{7}$

3. Which answer choice in problem 1 is the least reasonable? Explain.

   WRITE MATH

   d; if 17 is substituted for q, 17 − 13 will be positive and so its product with $^{-}0.5$ will be negative. A negative number plus 9 will be less than 11.

EXPLAIN IT

Leila uses these steps to solve the equation. Are Leila's steps mathematically correct? Explain each step.

$$12.4(s - 0.5) - 12 = 19$$
$$12.4(s - 0.5) - 12 + 12 = 19 + 12$$
$$12.4(s - 0.5) = 31$$
$$\frac{12.4\,(s - 0.5)}{12.4} = \frac{31}{12.4}$$
$$s - 0.5 = 2.5$$
$$s = 3$$

Yes; she added 12 to both sides first. Then she divided each entire side by 12.4 and then added 0.5 to each side.

4. Write a word problem that could be solved by using this equation: $\frac{1}{4}(x + 2) + 3 = 12$ Answers will vary.

   CRITICAL THINKING

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16 Lesson 2 • Solving Multi-Step Equations 2

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 3 Lesson 2

Reteach Teacher Edition Module 3 Lesson 2

Extra Practice

Student Book page 46

English Language Learners

Use the VmathLive Animated Glossary to review the terms *like terms* and *reciprocal*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Because this lesson contains fractions, be careful to specify whether students are to be finding the simplest form of a fraction or the simplest form of an expression.

Students with Special Needs

It may be helpful to break apart the process of solving these equations into three parts for students: simplifying each side of the equation, solving the resulting two-step equation, and checking the solution.

Lesson 3

Objective

To solve an equation with variables on both sides

Preskills	Lesson
Distributive Practice	I.3.PL2
Solving Multi-Step Equations 1	I.3.1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

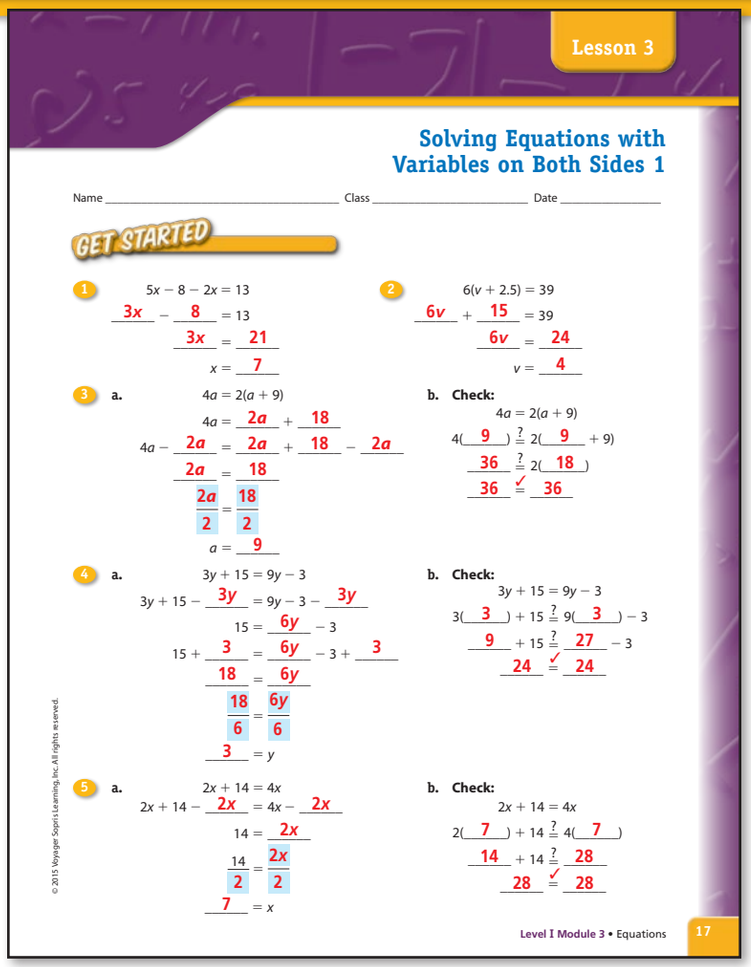
The expression on the left side of this equation must be simplified before the equation can be solved. What needs to be done to simplify the expression? (Combine the like terms.) **What are the like terms?** ($5x$ and $^{-}2x$) **What is the equation when the like terms are combined?** ($3x - 8 = 13$)

To solve the equation, first add 8 to each side. What is the equation now? ($3x = 21$) **Divide both sides of the equation by 3. What is the solution of the equation?** ($x = 7$)

Problem 2

The expression on the left side of this equation must be simplified before the equation can be solved. Use the Distributive Property to remove the parentheses. What is 6 times *v*? ($6v$) **What is 6 times 2.5?** (15)

The expression on the left side of the equation simplifies to $6v$ plus 15. Now the equation can be solved. Subtract 15 from both sides of the equation. Then divide each side by 6. What is the solution of the equation? ($v = 4$)



Lesson 3

Solving Equations with Variables on Both Sides 1

Name _____ Class _____ Date _____

GET STARTED

1. $5x - 8 - 2x = 13$
   $3x - 8 = 13$
   $3x = 21$
   $x = 7$

2. $6(v + 2.5) = 39$
   $6v + 15 = 39$
   $6v = 24$
   $v = 4$

3. a. $4a = 2(a + 9)$
   $4a = 2a + 18$
   $4a - 2a = 2a + 18 - 2a$
   $2a = 18$
   $\frac{2a}{2} = \frac{18}{2}$
   $a = 9$

   b. Check:
   $4a = 2(a + 9)$
   $4(9) \stackrel{?}{=} 2(9 + 9)$
   $36 \stackrel{?}{=} 2(18)$
   $36 \stackrel{\checkmark}{=} 36$

4. a. $3y + 15 = 9y - 3$
   $3y + 15 - 3y = 9y - 3 - 3y$
   $15 = 6y - 3$
   $15 + 3 = 6y - 3 + 3$
   $18 = 6y$
   $\frac{18}{6} = \frac{6y}{6}$
   $3 = y$

   b. Check:
   $3y + 15 = 9y - 3$
   $3(3) + 15 \stackrel{?}{=} 9(3) - 3$
   $9 + 15 \stackrel{?}{=} 27 - 3$
   $24 \stackrel{\checkmark}{=} 24$

5. a. $2x + 14 = 4x$
   $2x + 14 - 2x = 4x - 2x$
   $14 = 2x$
   $\frac{14}{2} = \frac{2x}{2}$
   $7 = x$

   b. Check:
   $2x + 14 = 4x$
   $2(7) + 14 \stackrel{?}{=} 4(7)$
   $14 + 14 \stackrel{?}{=} 28$
   $28 \stackrel{\checkmark}{=} 28$

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MODEL NEW SKILLS

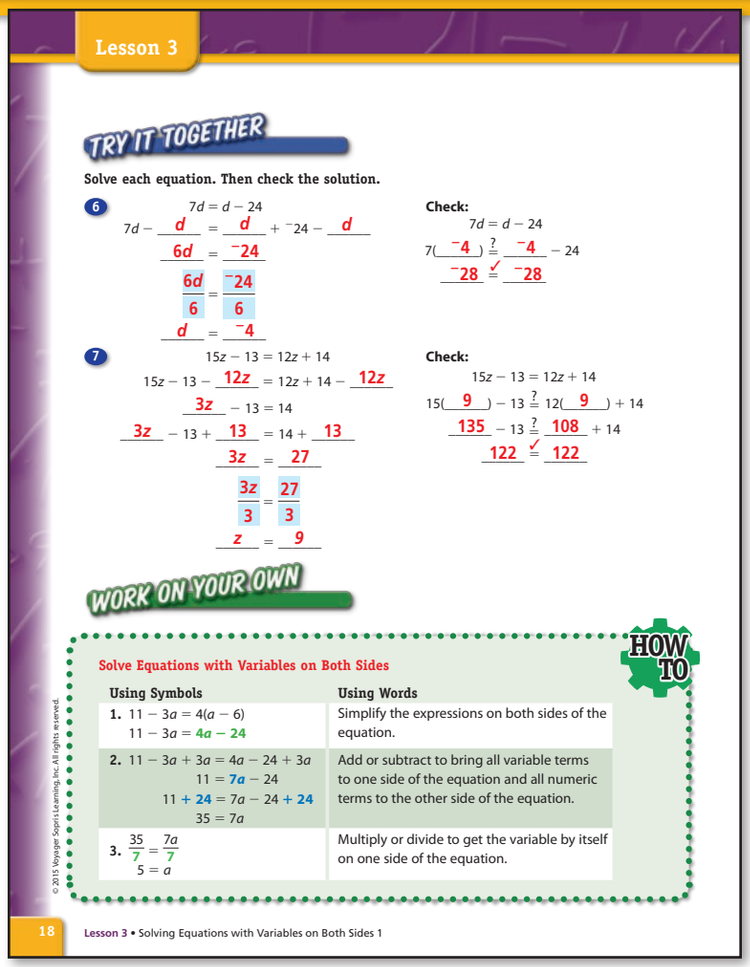
Problem 3

Equations can have variable terms and numeric terms on both sides of the equal sign. To solve these equations, first simplify the expressions on both sides of the equal sign. Next, use addition and subtraction to bring all variable terms to one side of the equation. Then solve the equation.

Look at the equation in problem 3a. The expression on the left side of the equation is $4a$. It is already simplified. The expression on the right side is $2(a + 9)$. It must be simplified. The Distributive Property can be used to remove the parentheses. The expression $2(a + 9)$ is equal to $2a$ plus 18.

The equation is now $4a$ equals $2a$ plus 18. Notice that variable terms are on both sides of the equation. To get all the variable terms on the left side, subtract $2a$ from both sides of the equation. What is $4a$ minus $2a$? ($2a$) **Notice that $2a$ plus 18 minus $2a$ written with like terms next to each other is $2a$ minus $2a$ plus 18. Because $2a$ minus $2a$ is zero, $2a$ plus 18 minus $2a$ is 18. The resulting equation is $2a$ equals 18. Divide both sides by 2. What is the solution?** ($a = 9$)

The next step is to check the answer. Substitute the solution, 9, for the *a*'s on both sides in the original


Lesson 3

TRY IT TOGETHER

Solve each equation. Then check the solution.

6 $7d = d - 24$

$7d - \underline{d} = \underline{d} + {}^{-}24 - \underline{d}$

$\underline{6d} = \underline{{}^{-}24}$

$\frac{\underline{6d}}{\underline{6}} = \frac{\underline{{}^{-}24}}{\underline{6}}$

$\underline{d} = \underline{{}^{-}4}$

Check:

$7d = d - 24$

$7(\underline{{}^{-}4}) \stackrel{?}{=} \underline{{}^{-}4} - 24$

$\underline{{}^{-}28} \stackrel{\checkmark}{=} \underline{{}^{-}28}$

7 $15z - 13 = 12z + 14$

$15z - 13 - \underline{12z} = 12z + 14 - \underline{12z}$

$\underline{3z} - 13 = 14$

$\underline{3z} - 13 + \underline{13} = 14 + \underline{13}$

$\underline{3z} = \underline{27}$

$\frac{\underline{3z}}{\underline{3}} = \frac{\underline{27}}{\underline{3}}$

$\underline{z} = \underline{9}$

Check:

$15z - 13 = 12z + 14$

$15(\underline{9}) - 13 \stackrel{?}{=} 12(\underline{9}) + 14$

$\underline{135} - 13 \stackrel{?}{=} \underline{108} + 14$

$\underline{122} \stackrel{\checkmark}{=} \underline{122}$

WORK ON YOUR OWN

HOW TO

Solve Equations with Variables on Both Sides

Using Symbols	Using Words
1. $11 - 3a = 4(a - 6)$ $11 - 3a = 4a - 24$	Simplify the expressions on both sides of the equation.
2. $11 - 3a + 3a = 4a - 24 + 3a$ $11 = 7a - 24$ $11 + 24 = 7a - 24 + 24$ $35 = 7a$	Add or subtract to bring all variable terms to one side of the equation and all numeric terms to the other side of the equation.
3. $\frac{35}{7} = \frac{7a}{7}$ $5 = a$	Multiply or divide to get the variable by itself on one side of the equation.

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equation. The equation is now $4(9) = 2(9 + 9)$. Because 4 times 9 equals 36, and 2 times 18 equals 36, the solution is correct. Write a check mark above the equal sign to show the solution is correct.

Problem 4

Variable terms and numeric terms are on both sides of this equation. The expressions are simplified, so add or subtract to bring all the variable terms to one side of the equal sign. It is customary to keep the coefficient of the variable term positive. To get a variable term with a positive coefficient, subtract $3y$ from both sides of the equation. What is $3y$ plus 15 minus $3y$? (15) What is $9y$ minus 3 minus $3y$? ($6y - 3$)

The equation is now 15 equals $6y$ minus 3, and the variable term has a positive coefficient. This equation is a two-step equation that can be solved. Add 3 to both sides to bring the numeric terms to the same side. What is 15 plus 3? (18) What is $6y$ minus 3 plus 3? ($6y$) What is the resulting equation? ($18 = 6y$) To finish solving the equation, divide both sides by 6. The solution is y equals 3.

The next step is to check the answer. Substitute the solution, 3, for the y's on both sides of the original equation. The equation is now $3(3) + 15 = 9(3) - 3$. What is 3 times 3? (9) What is 9 times 3? (27) What is 9 plus 15? (24) What is 27 minus 3? (24) Is the solution correct? (yes) Why? (The values of both sides of the equation are the same.) Write a check mark above the equal sign to show the solution is correct.

Problem 5

What are the variable terms in this problem? ($2x$ and $4x$) What is $2x$ minus $4x$? (${}^{-}2x$) What is $4x$ minus $2x$? ($2x$) It is easier to work with positive coefficients of variables. What must be done to get the variable x on one side of the equation with a positive coefficient? (Subtract $2x$ from both sides.) What is the resulting equation? ($14 = 2x$) By what number should both sides of the equation be divided to get x by itself? (2) What is x? ($x = 7$)

The next step is to check the answer. Substitute the solution, 7, into the original equation. What is 2 times 7? (14) What is 4 times 7? (28) What is 14 plus 14? (28) Twenty-eight equals 28, so the solution is correct. Write a check mark above the equal sign to show the solution is correct.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

Do the expressions on either side of the equation need to be simplified? (no) Are variable terms on both sides of the equation? (yes)

What should be done to bring the variable terms to the left side of the equation to keep the coefficient of the variable term positive? (Subtract d from both sides of the equation.) To subtract d from the right side of the equation, it is helpful to write minus 24 as plus negative 24. What is $7d$ minus d? ($6d$) What is d plus ${}^{-}24$ minus d? (${}^{-}24$) What is the equation now? ($6d = {}^{-}24$)

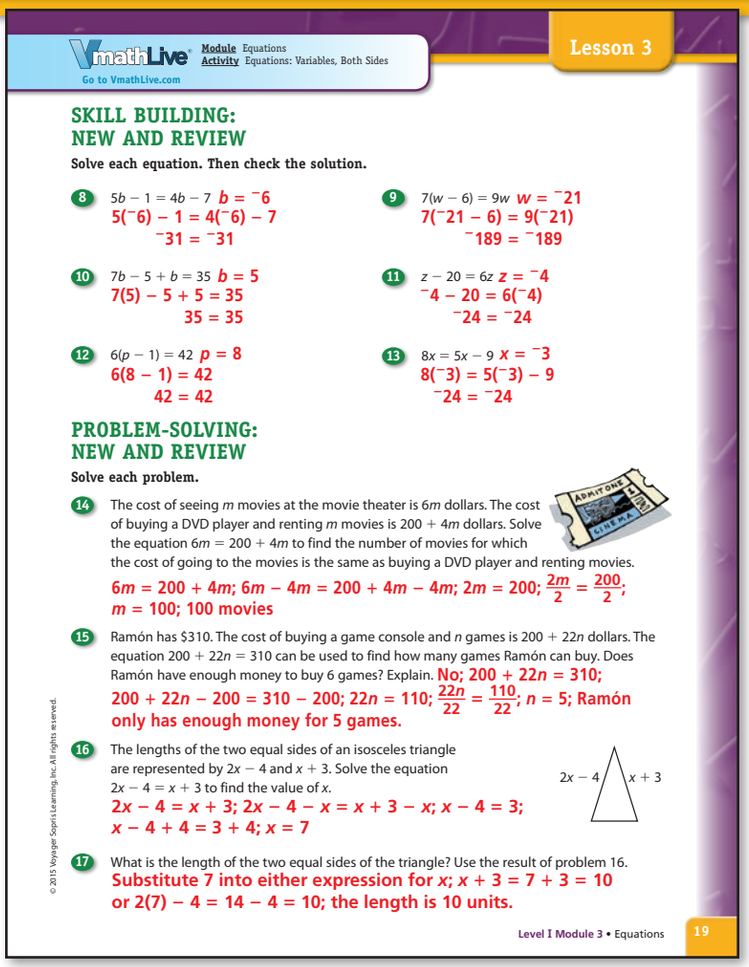
Divide both sides of the equation by 6. What is $6d$ divided by 6? (d) What is ${}^{-}24$ divided by 6? (${}^{-}4$) What is the solution? ($d = {}^{-}4$)

Substitute the solution, ${}^{-}4$, for both d's. What is 7 times ${}^{-}4$? (${}^{-}28$) What is ${}^{-}4$ minus 24? (${}^{-}28$) Because ${}^{-}28$ equals ${}^{-}28$, the solution is correct.

Problem 7

Are variable terms on both sides of the equation? (yes) **Are numeric terms on both sides of the equation?** (yes) **What should be done to both sides of the equation to keep the variable term positive?** (Subtract 12*z*.) **Why?** (Twelve is less than 15.) **What is the resulting equation?** ($3z - 13 = 14$) **What should be done next?** (Add 13 to both sides.) **What is the resulting equation?** ($3z = 27$) **What is the final step?** (Divide both sides of the equation by 3.) **What is the solution?** ($z = 9$)

Substitute the solution, 9, for both *z*'s. What is the equation when 9 is substituted for the variable? ($15(9) - 13 = 12(9) + 14$) **What is 15 times 9?** (135) **What is 12 times 9?** (108) **What is 135 minus 13?** (122) **What is 108 plus 14?** (122) **Why is the solution correct?** (Both sides equal the same value.)



VmathLive Module Equations Activity Equations: Variables, Both Sides
Go to VmathLive.com

Lesson 3

SKILL BUILDING: NEW AND REVIEW

Solve each equation. Then check the solution.

8. $5b - 1 = 4b - 7$ $b = -6$
$5(-6) - 1 = 4(-6) - 7$
$-31 = -31$

9. $7(w - 6) = 9w$ $w = -21$
$7(-21 - 6) = 9(-21)$
$-189 = -189$

10. $7b - 5 + b = 35$ $b = 5$
$7(5) - 5 + 5 = 35$
$35 = 35$

11. $z - 20 = 6z$ $z = -4$
$-4 - 20 = 6(-4)$
$-24 = -24$

12. $6(p - 1) = 42$ $p = 8$
$6(8 - 1) = 42$
$42 = 42$

13. $8x = 5x - 9$ $x = -3$
$8(-3) = 5(-3) - 9$
$-24 = -24$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

14. The cost of seeing *m* movies at the movie theater is $6m$ dollars. The cost of buying a DVD player and renting *m* movies is $200 + 4m$ dollars. Solve the equation $6m = 200 + 4m$ to find the number of movies for which the cost of going to the movies is the same as buying a DVD player and renting movies.
$6m = 200 + 4m$; $6m - 4m = 200 + 4m - 4m$; $2m = 200$; $\frac{2m}{2} = \frac{200}{2}$; $m = 100$; 100 movies

15. Ramón has $310. The cost of buying a game console and *n* games is $200 + 22n$ dollars. The equation $200 + 22n = 310$ can be used to find how many games Ramón can buy. Does Ramón have enough money to buy 6 games? Explain. No; $200 + 22n = 310$; $200 + 22n - 200 = 310 - 200$; $22n = 110$; $\frac{22n}{22} = \frac{110}{22}$; $n = 5$; Ramón only has enough money for 5 games.

16. The lengths of the two equal sides of an isosceles triangle are represented by $2x - 4$ and $x + 3$. Solve the equation $2x - 4 = x + 3$ to find the value of *x*.
$2x - 4 = x + 3$; $2x - 4 - x = x + 3 - x$; $x - 4 = 3$; $x - 4 + 4 = 3 + 4$; $x = 7$

17. What is the length of the two equal sides of the triangle? Use the result of problem 16.
Substitute 7 into either expression for *x*; $x + 3 = 7 + 3 = 10$ or $2(7) - 4 = 14 - 4 = 10$; the length is 10 units.

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Level I Module 3 • Equations 19

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 8–17

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

CHECK UP

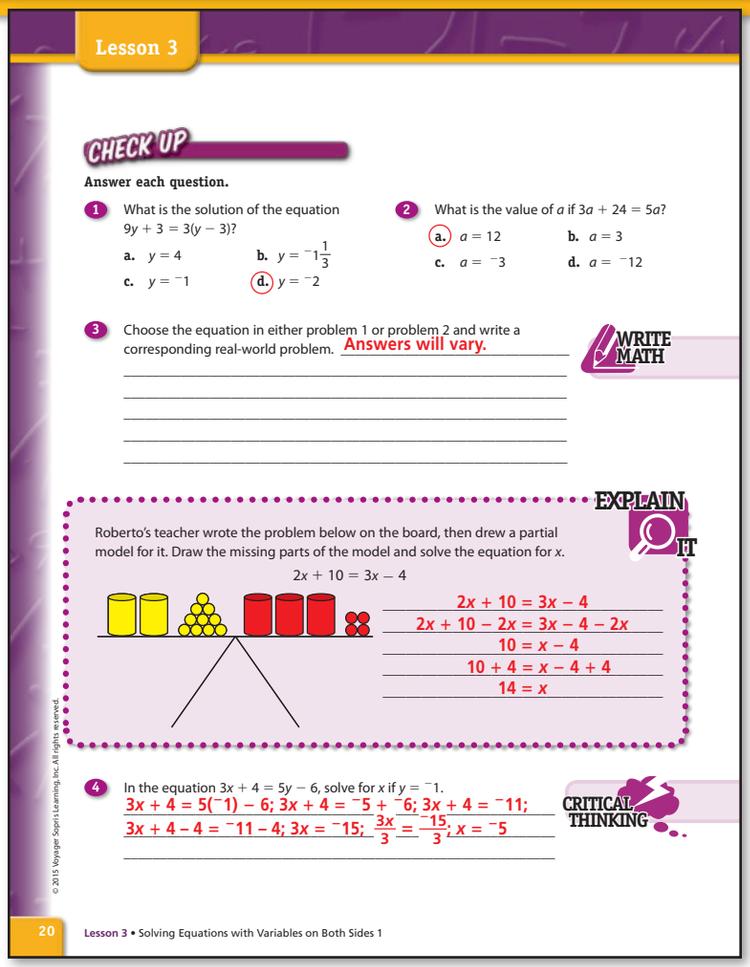
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1b: The student did not bring the variable terms to one side of the equation but instead ignored one of the variable terms. Use Additional Resources in I.3.3 to reteach how to solve equations with variables on both sides.

If student answered 1c: The student did not multiply to remove the parentheses correctly. Use Additional Resources in I.3.PL2 to reteach how to simplify algebraic expressions using the Distributive Property.



Lesson 3

CHECK UP

Answer each question.

1 What is the solution of the equation $9y + 3 = 3(y - 3)$?

a. $y = 4$ b. $y = -1\frac{1}{3}$

c. $y = -1$ d. (d.) $y = -2$

2 What is the value of a if $3a + 24 = 5a$?

(a.) $a = 12$ b. $a = 3$

c. $a = -3$ d. $a = -12$

3 Choose the equation in either problem 1 or problem 2 and write a corresponding real-world problem. Answers will vary.

WRITE MATH

EXPLAIN IT

Roberto's teacher wrote the problem below on the board, then drew a partial model for it. Draw the missing parts of the model and solve the equation for x.

$2x + 10 = 3x - 4$

$2x + 10 = 3x - 4$
$2x + 10 - 2x = 3x - 4 - 2x$
$10 = x - 4$
$10 + 4 = x - 4 + 4$
$14 = x$

4 In the equation $3x + 4 = 5y - 6$, solve for x if $y = -1$.

$3x + 4 = 5(-1) - 6$; $3x + 4 = -5 + -6$; $3x + 4 = -11$;
$3x + 4 - 4 = -11 - 4$; $3x = -15$; $\frac{3x}{3} = \frac{-15}{3}$; $x = -5$

CRITICAL THINKING

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If student answered 2b or 2c: The student added the coefficients of the variable terms instead of subtracting when bringing the variable terms to one side of the equation. Use Additional Resources in I.3.3 to reteach how to solve equations with variables on both sides.

If student answered 2d: The student used the wrong sign in the answer. Use Additional Resources in I.3.PL2 to reteach how to simplify algebraic expressions.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, encourage the student to look back through the lesson for a real-world problem that can be used as model and adapted to the chosen equation.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Equations
Activity: Equations: Variables, Both Sides

Vmath Reteach
Reteach Student Book Module 3 Lesson 3
Reteach Teacher Edition Module 3 Lesson 3

Extra Practice
Student Book page 47

English Language Learners

Use the VmathLive Animated Glossary to review the terms *simplify* and *algebraic equation*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Be sure students know the operations and their opposites. Review with them that addition and subtraction are opposites and multiplication and division are opposites.

Students with Special Needs

Have students use algebra tiles or another manipulative to solve equations. Have students work with a partner to compare their answers and to help each other solve an equation. Work various simple examples with students using algebra tiles.

Lesson 4

Objective

To solve equations that contain fractions, decimals, and variables on both sides

Preskills	Lesson
Solving Multi-Step Equations 2	I.3.2
Solving Equations with Variables on Both Sides 1	I.3.3

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

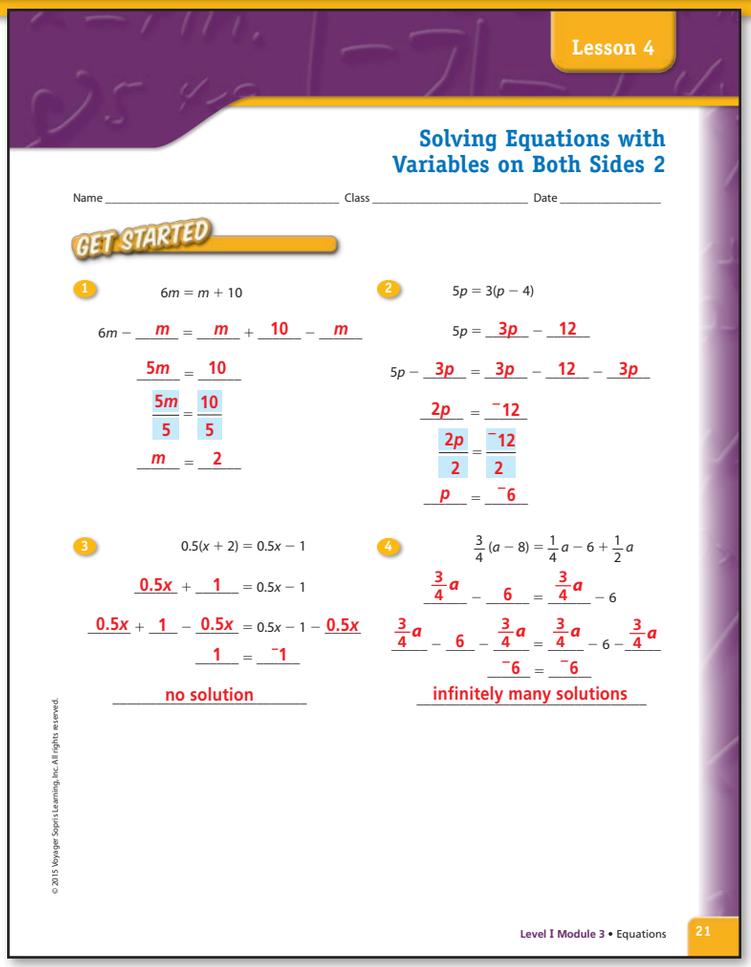
Do the expressions on either side of the equation need to be simplified? (no) **Are variable terms on both sides of the equation?** (yes)

What should be done to bring the variable term to the left side of the equation to keep the coefficient of the variable term positive? (Subtract *m* from both sides.) **What is 6*m* minus *m*?** (5*m*) **What is *m* plus 10 minus *m*?** (10) **What is the equation now?** (5*m* = 10)

Divide both sides of the equation by 5. What is 5*m* divided by 5? (*m*) **What is 10 divided by 5?** (2) **What is the solution?** (*m* = 2)

Problem 2

The expression on the right side of this equation must be simplified before the equation can be solved. Multiply to remove the parentheses. What is 3 times *p*? (3*p*) **What is 3 times 4** (12) **The expression on the right side of the equation simplifies to 3*p* – 12. Now the equation can be solved. Subtract 3*p* from both sides of the equation. Then divide each side by 2. What is the solution of the equation?** ($p = {}^{-}6$)



Lesson 4

Solving Equations with Variables on Both Sides 2

Name _____ Class _____ Date _____

GET STARTED

1. $6m = m + 10$

$6m - m = m + 10 - m$

$5m = 10$

$\frac{5m}{5} = \frac{10}{5}$

$m = 2$

2. $5p = 3(p - 4)$

$5p = 3p - 12$

$5p - 3p = 3p - 12 - 3p$

$2p = {}^{-}12$

$\frac{2p}{2} = \frac{{}^{-}12}{2}$

$p = {}^{-}6$

3. $0.5(x + 2) = 0.5x - 1$

$0.5x + 1 = 0.5x - 1$

$0.5x + 1 - 0.5x = 0.5x - 1 - 0.5x$

$1 = {}^{-}1$

no solution

4. $\frac{3}{4}(a - 8) = \frac{1}{4}a - 6 + \frac{1}{2}a$

$\frac{3}{4}a - 6 = \frac{3}{4}a - 6$

$\frac{3}{4}a - 6 - \frac{3}{4}a = \frac{3}{4}a - 6 - \frac{3}{4}a$

${}^{-}6 = {}^{-}6$

infinitely many solutions

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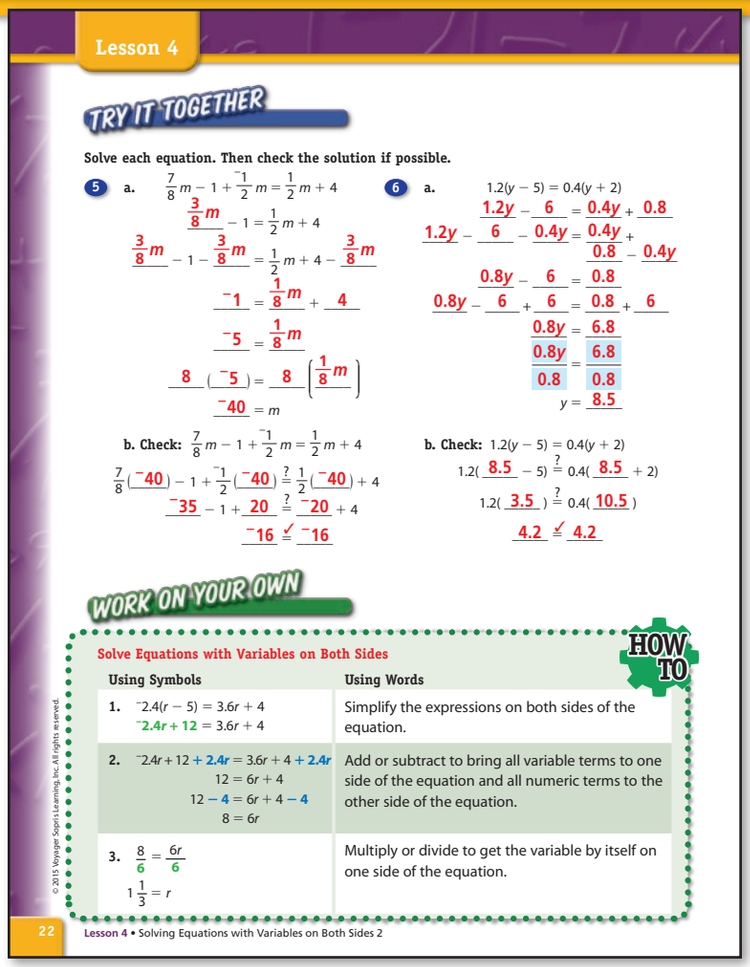
Level I Module 3 • Equations 21

MODEL NEW SKILLS

Problem 3

What is the equation? ($0.5(x + 2) = 0.5x - 1$) **Are the expressions on both sides of the equation simplified?** (no) **What needs to be done to simplify the expressions?** (Multiply to remove parentheses on the left side of the equation.) **What is the result of multiplying to remove parentheses?** ($0.5x + 1$)

Are variable terms on both sides of the equation? (yes) **Are numeric terms on both sides of the equation?** (yes) **What are the variable terms?** (They are both 0.5*x*.) **So, the variable terms are the same. Try subtracting 0.5*x* from both sides. What is 0.5*x* plus 1 minus 0.5*x*?** (1) **What is 0.5*x* minus 1 minus 0.5*x*?** ($^{-}1$) **What is the resulting equation?** ($1 = {}^{-}1$) **Is this equation ever true?** (no) **Some equations have no solution. When an equation is solved and the resulting equation is never true, the equation has no solution.**


Lesson 4

TRY IT TOGETHER

Solve each equation. Then check the solution if possible.

5 a. $\frac{7}{8}m - 1 + \frac{-1}{2}m = \frac{1}{2}m + 4$

$\frac{3}{8}m - 1 = \frac{1}{2}m + 4$

$\frac{3}{8}m - 1 - \frac{3}{8}m = \frac{1}{2}m + 4 - \frac{3}{8}m$

$-1 = \frac{1}{8}m + 4$

$-5 = \frac{1}{8}m$

$8(-5) = 8\left(\frac{1}{8}m\right)$

$-40 = m$

b. Check: $\frac{7}{8}m - 1 + \frac{-1}{2}m = \frac{1}{2}m + 4$

$\frac{7}{8}(-40) - 1 + \frac{-1}{2}(-40) \stackrel{?}{=} \frac{1}{2}(-40) + 4$

$-35 - 1 + 20 \stackrel{?}{=} -20 + 4$

$-16 \stackrel{\checkmark}{=} -16$

6 a. $1.2(y - 5) = 0.4(y + 2)$

$1.2y - 6 = 0.4y + 0.8$

$1.2y - 6 - 0.4y = 0.4y + 0.8 - 0.4y$

$0.8y - 6 = 0.8$

$0.8y - 6 + 6 = 0.8 + 6$

$0.8y = 6.8$

$\frac{0.8y}{0.8} = \frac{6.8}{0.8}$

$y = 8.5$

b. Check: $1.2(y - 5) = 0.4(y + 2)$

$1.2(8.5 - 5) \stackrel{?}{=} 0.4(8.5 + 2)$

$1.2(3.5) \stackrel{?}{=} 0.4(10.5)$

$4.2 \stackrel{\checkmark}{=} 4.2$

WORK ON YOUR OWN

Solve Equations with Variables on Both Sides

HOW TO

Using Symbols	Using Words
1. $-2.4(r - 5) = 3.6r + 4$ $-2.4r + 12 = 3.6r + 4$	Simplify the expressions on both sides of the equation.
2. $-2.4r + 12 + 2.4r = 3.6r + 4 + 2.4r$ $12 = 6r + 4$ $12 - 4 = 6r + 4 - 4$ $8 = 6r$	Add or subtract to bring all variable terms to one side of the equation and all numeric terms to the other side of the equation.
3. $\frac{8}{6} = \frac{6r}{6}$ $1\frac{1}{3} = r$	Multiply or divide to get the variable by itself on one side of the equation.

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Problem 4

What is the equation? ($\frac{3}{4}(a - 8) = \frac{1}{4}a - 6 + \frac{1}{2}a$) **Are the expressions on both sides of the equation simplified?** (no) **What needs to be done to simplify the expressions?** (On the left side, multiply to remove parentheses. On the right side, combine the like terms.) **On the left side of the equation, what is the result of multiplying to remove parentheses?** ($\frac{3}{4}a - 6$) **On the right side of the equation, what is the result of combining like terms?** ($\frac{3}{4}a - 6$)

Are variable terms on both sides of the equation? (yes) **Are numeric terms on both sides of the equation?** (yes) **What are the variable terms?** (They are both $\frac{3}{4}a$.) **So, the variable terms are the same. Try subtracting $\frac{3}{4}a$ from both sides. On the left, what is $\frac{3}{4}a$ minus 6 minus $\frac{3}{4}a$?** ($^-6$) **On the right, what is $\frac{3}{4}a$ minus 6 minus $\frac{3}{4}a$?** ($^-6$) **What is the resulting equation?** ($^-6 = {}^-6$) **Is this equation always true?** (yes) **Some equations have an infinite number of solutions. That is, no matter what value is substituted for the variable, the resulting equation is true. When an equation is solved and the resulting equation is always true, the equation has infinitely many solutions.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What is the first step in solving this equation? (Combine like terms.) **What is the resulting equation?** ($\frac{3}{8}m - 1 = \frac{1}{2}m + 4$) **Can the equation be solved now?** (yes) **Why?** (The expressions on both sides of the equation cannot be simplified further.)

Are variable terms on both sides of the equation? (yes) **Are numeric terms on both sides of the equation?** (yes) **What should be done to both sides of the equation to keep the variable term positive?** (Subtract $\frac{3}{8}m$.) **Why?** ($\frac{3}{8}$ is less than $\frac{1}{2}$.) **What is the resulting equation?** ($^-1 = \frac{1}{8}m + 4$) **What should be done next?** (Subtract 4 from both sides.) **What is the resulting equation?** ($^-5 = \frac{1}{8}m$) **What is the final step?** (Multiply both sides of the equation by 8.) **What is the solution?** ($^-40 = m$)

Substitute the solution, $^-40$, for all three m's. How is the equation written when $^-40$ is substituted for the variable? ($\frac{7}{8}(-40) - 1 + \frac{-1}{2}(-40) = \frac{1}{2}(-40) + 4$) **What is $\frac{7}{8}$ times $^-40$?** ($^-35$) **What is $\frac{-1}{2}$ times $^-40$?** (20) **What is $^-35$ minus 1 plus 20?** ($^-16$) **What is $\frac{1}{2}$ times $^-40$?** ($^-20$) **What is $^-20$ plus 4?** ($^-16$) **Why is the solution correct?** (Both sides equal the same value.)

Problem 6

What is the first step in solving this equation? (Multiply to remove parentheses on both sides.) **What is the resulting equation?** ($1.2y - 6 = 0.4y + 0.8$) **Are variable terms on both sides of the equation?** (yes) **Are numeric terms on both sides of the equation?** (yes) **What should be done to both sides of the equation to keep the variable term positive?** (Subtract $0.4y$.) **What is the resulting equation?** ($0.8y - 6 = 0.8$) **What should be done next?** (Add 6 to both sides.) **What is the resulting equation?** ($0.8y = 6.8$) **What is the final step?** (Divide both sides of the equation by 0.8.) **What is the solution?** ($y = 8.5$)

How can the solution be checked? (Substitute 8.5 for y in the original equation.) **Is the solution correct?** (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problem 7–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

PROBLEM-SOLVING This problem illustrates the **Writing an Equation** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving writing and solving an equation from a given problem. The instruction is immediately followed by application of the strategy in problem 13.

Have students read the problem. Ask students what they are to find. Suggest that students first write an expression using a variable for each company's cost. If necessary, have students write a word model for the total cost before writing each expression. Once the expressions are written, have students explain why they can be set equal to each other. Then have students write and solve the equation.

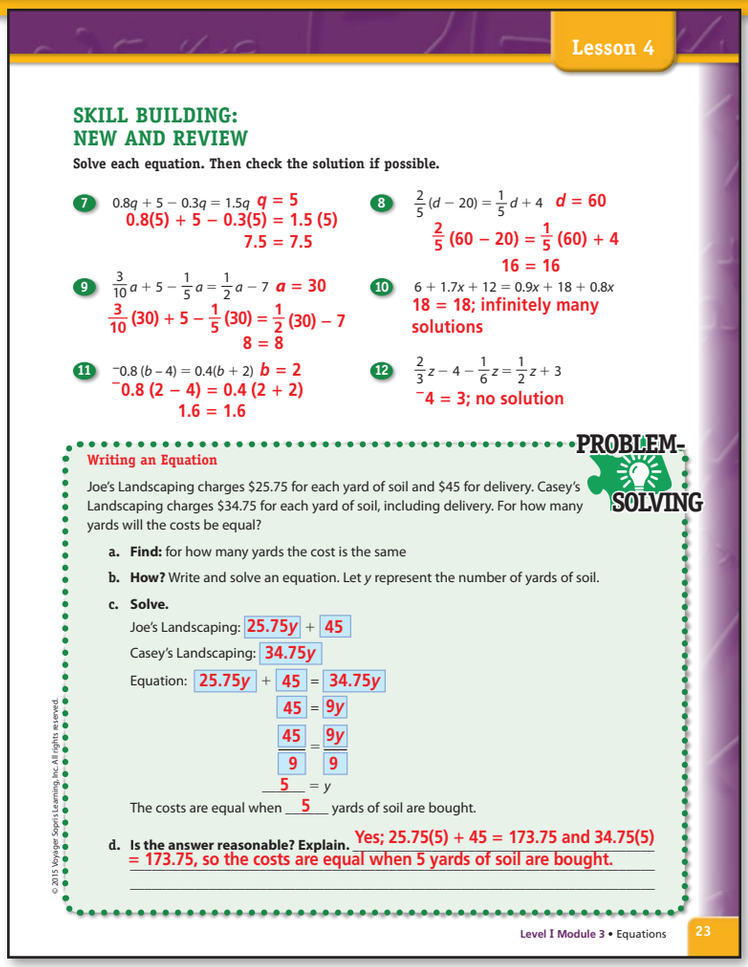
Ask students to explain what the solution of the equation represents in the problem. Then have students check their solution by substituting the value of the variable in each of the original expressions and showing that the total costs are equal.

Problem 10 Reminder

Once students collect like terms, they will have the equation $1.7x + 18 = 1.7x + 18$. At this point, they should recognize that the equation will have infinitely many solutions.

Problem 12 Reminder

Once students collect like terms, they will have the equation $(\frac{1}{2}z - 4 = \frac{1}{2}z + 3)$. At this point, they should recognize that the equation will have no solution.



Lesson 4

SKILL BUILDING: NEW AND REVIEW

Solve each equation. Then check the solution if possible.

7. $0.8q + 5 - 0.3q = 1.5q$ $q = 5$
$0.8(5) + 5 - 0.3(5) = 1.5\ (5)$
$7.5 = 7.5$

8. $\frac{2}{5}(d - 20) = \frac{1}{5}d + 4$ $d = 60$
$\frac{2}{5}(60 - 20) = \frac{1}{5}(60) + 4$
$16 = 16$

9. $\frac{3}{10}a + 5 - \frac{1}{5}a = \frac{1}{2}a - 7$ $a = 30$
$\frac{3}{10}(30) + 5 - \frac{1}{5}(30) = \frac{1}{2}(30) - 7$
$8 = 8$

10. $6 + 1.7x + 12 = 0.9x + 18 + 0.8x$
$18 = 18$; infinitely many solutions

11. $^-0.8\ (b - 4) = 0.4(b + 2)$ $b = 2$
$^-0.8\ (2 - 4) = 0.4\ (2 + 2)$
$1.6 = 1.6$

12. $\frac{2}{3}z - 4 - \frac{1}{6}z = \frac{1}{2}z + 3$
$^-4 = 3$; no solution

PROBLEM-SOLVING

Writing an Equation

Joe's Landscaping charges $25.75 for each yard of soil and $45 for delivery. Casey's Landscaping charges $34.75 for each yard of soil, including delivery. For how many yards will the costs be equal?

a. **Find:** for how many yards the cost is the same

b. **How?** Write and solve an equation. Let y represent the number of yards of soil.

c. **Solve.**
Joe's Landscaping: $25.75y + 45$
Casey's Landscaping: $34.75y$
Equation: $25.75y + 45 = 34.75y$
$45 = 9y$
$\frac{45}{9} = \frac{9y}{9}$
$5 = y$
The costs are equal when 5 yards of soil are bought.

d. **Is the answer reasonable? Explain.** Yes; 25.75(5) + 45 = 173.75 and 34.75(5) = 173.75, so the costs are equal when 5 yards of soil are bought.

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CHECK UP

ASSESS INFORMALLY

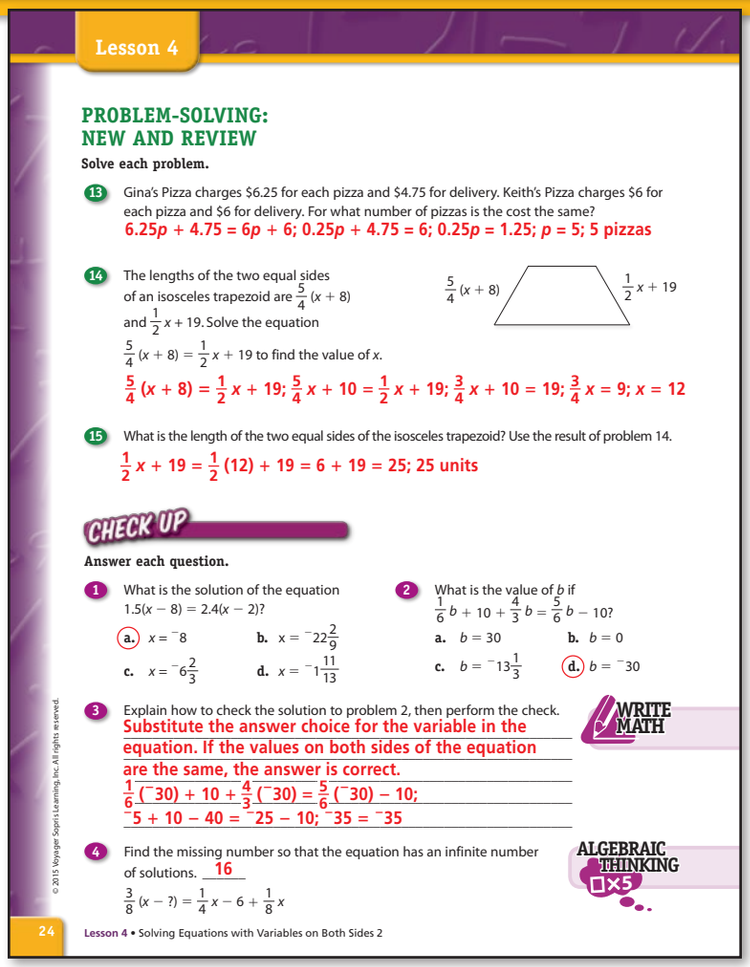
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 or problem 2 incorrectly:
The student made an arithmetic error. Remind the student to write each step completely and go back and check his or her work.



WRITE MATH **If student answered problem 3 incorrectly:** While students work together, remind the student that the answer is always checked by substituting the value of the variable into the original equation he or she started with.



Lesson 4

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

13 Gina's Pizza charges $6.25 for each pizza and $4.75 for delivery. Keith's Pizza charges $6 for each pizza and $6 for delivery. For what number of pizzas is the cost the same?

$6.25p + 4.75 = 6p + 6$; $0.25p + 4.75 = 6$; $0.25p = 1.25$; $p = 5$; 5 pizzas

14 The lengths of the two equal sides of an isosceles trapezoid are $\frac{5}{4}(x + 8)$ and $\frac{1}{2}x + 19$. Solve the equation $\frac{5}{4}(x + 8) = \frac{1}{2}x + 19$ to find the value of x.

$\frac{5}{4}(x + 8)$ $\frac{1}{2}x + 19$

$\frac{5}{4}(x + 8) = \frac{1}{2}x + 19$; $\frac{5}{4}x + 10 = \frac{1}{2}x + 19$; $\frac{3}{4}x + 10 = 19$; $\frac{3}{4}x = 9$; $x = 12$

15 What is the length of the two equal sides of the isosceles trapezoid? Use the result of problem 14.

$\frac{1}{2}x + 19 = \frac{1}{2}(12) + 19 = 6 + 19 = 25$; 25 units

CHECK UP

Answer each question.

1 What is the solution of the equation $1.5(x - 8) = 2.4(x - 2)$?

(a.) $x = -8$ b. $x = -22\frac{2}{9}$ c. $x = -6\frac{2}{3}$ d. $x = -1\frac{11}{13}$

2 What is the value of b if $\frac{1}{6}b + 10 + \frac{4}{3}b = \frac{5}{6}b - 10$?

a. $b = 30$ b. $b = 0$ c. $b = -13\frac{1}{3}$ (d.) $b = -30$

3 Explain how to check the solution to problem 2, then perform the check. WRITE MATH

Substitute the answer choice for the variable in the equation. If the values on both sides of the equation are the same, the answer is correct.

$\frac{1}{6}(-30) + 10 + \frac{4}{3}(-30) = \frac{5}{6}(-30) - 10$;

$-5 + 10 - 40 = -25 - 10$; $-35 = -35$

4 Find the missing number so that the equation has an infinite number of solutions. 16 ALGEBRAIC THINKING

$\frac{3}{8}(x - ?) = \frac{1}{4}x - 6 + \frac{1}{8}x$

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Technology

Have students think about math in their world, using an online search engine to find references describing different methods for modeling and solving equations with variables on both sides of the equal sign. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 3 Lesson 4

Reteach Teacher Edition Module 3 Lesson 4

Extra Practice

Student Book page 47

English Language Learners

Review the meaning of a solution of an equation with students. Show that a solution is a value that, when substituted for the variable in the equation, results in an equation that is always true. Then discuss the different kinds of solutions that a linear equation in one variable can have: one solution, no solution, infinitely many solutions. A table of values can be used to demonstrate how an equation with infinitely many solutions always results in a true equation no matter what value is chosen for the variable.

Students with Special Needs

In this lesson there are multiple terms on each side of the equation, which can cause students to have difficulty keeping track of the like terms. For these students, suggest that they use a colored pencil to lightly shade the variable terms in each equation.

Gizmos Guided Discovery

Lesson 5

Objective

To use word, number, and symbol tiles to write an equation in words and in symbols

WITH CONCEPTUAL DEVELOPMENT TECHNOLOGY

Materials

- Gizmo: Using Algebraic Equations

Gizmos Log In Instructions

- Log in to vmath.voyagersopris.com using your Username and Password provided in VPORT®.
- Select the Gizmos poster.
- Click on the Using Algebraic Equations Gizmo link.

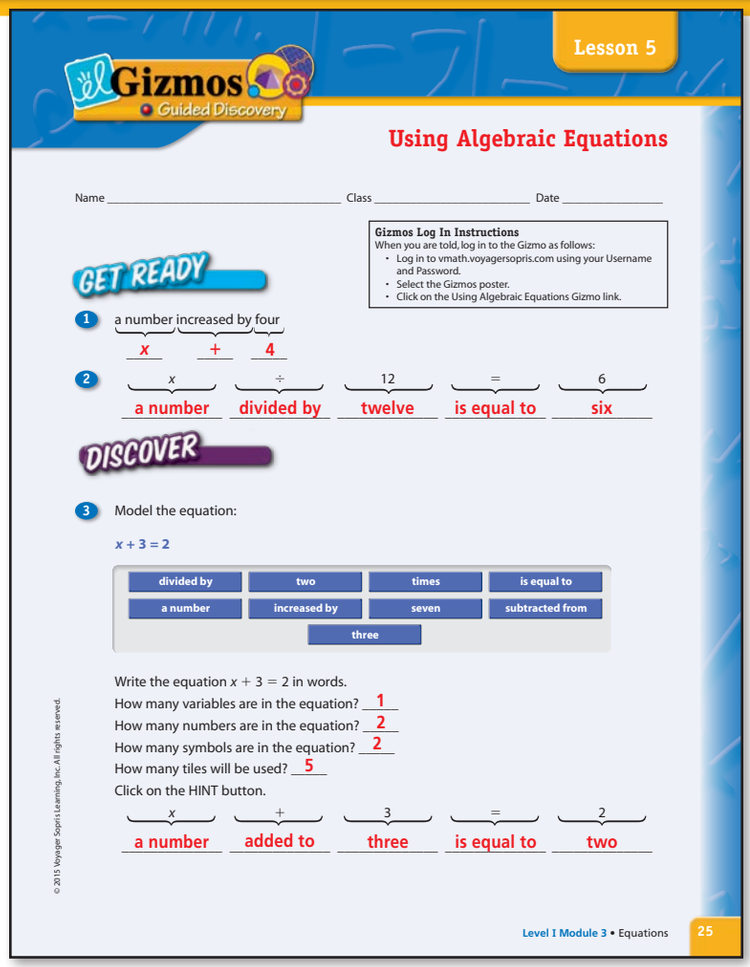
Lesson Notes

Before beginning the lesson, be sure students have their Student Books and are ready to work at the computers.

Complete problems 1 and 2 before students log in to the Using Algebraic Equations Gizmo.

Note that the Gizmo uses a dot for multiplication. Introduce students to this notation. Also note that the Gizmo recognizes commutativity of addition and multiplication so that more than one answer is considered correct.

If students have additional time, have them answer Assessment Questions 1, 2, 4, and 5 in the Gizmo. They can click on the Check Your Answers button to see how well they did on the assessment.



Gizmos Guided Discovery — Lesson 5

Using Algebraic Equations

Name _____ Class _____ Date _____

Gizmos Log In Instructions
When you are told, log in to the Gizmo as follows:
- Log in to vmath.voyagersopris.com using your Username and Password.
- Select the Gizmos poster.
- Click on the Using Algebraic Equations Gizmo link.

GET READY

1. a number increased by four: x + 4

2. x (a number) ÷ (divided by) 12 (twelve) = (is equal to) 6 (six)

DISCOVER

3. Model the equation: x + 3 = 2

divided by | two | times | is equal to
a number | increased by | seven | subtracted from
three

Write the equation x + 3 = 2 in words.
How many variables are in the equation? 1
How many numbers are in the equation? 2
How many symbols are in the equation? 2
How many tiles will be used? 5
Click on the HINT button.

x (a number) + (added to) 3 (three) = (is equal to) 2 (two)

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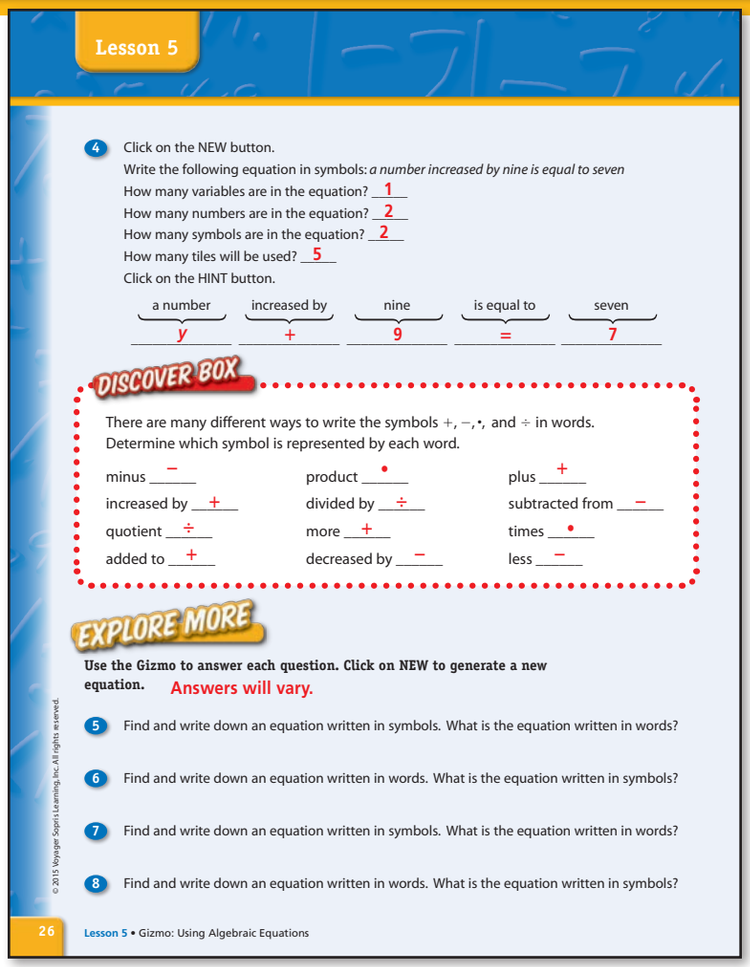
GET READY

Problem 1

To write a word phrase as an algebraic expression, first choose a variable to represent the unknown quantity. Any variable can be used. In this problem, use *x*. Then look for the words that indicate the operation. What operation is indicated by the words *increased by*? (addition) **In this expression, 4 is being added to a number. Therefore, the variable is written first, then an addition sign, then 4. The algebraic expression for *a number increased by four* is *x* + 4.**

Problem 2

To write an algebraic equation as a word phrase, first identify the operation. What operation is indicated? (division) **One way to write the division sign in words is to write *divided by*. The words *a number* are usually used to indicate a variable. One way to write the equal sign in words is to write *is equal to*. The expression in words is read as it appears from left to right: *a number divided by twelve is equal to six*.**


Lesson 5

4 Click on the NEW button.
Write the following equation in symbols: *a number increased by nine is equal to seven*
How many variables are in the equation? 1
How many numbers are in the equation? 2
How many symbols are in the equation? 2
How many tiles will be used? 5
Click on the HINT button.

a number	increased by	nine	is equal to	seven
y	+	9	=	7

DISCOVER BOX

There are many different ways to write the symbols +, −, •, and ÷ in words. Determine which symbol is represented by each word.

minus −	product •	plus +
increased by +	divided by ÷	subtracted from −
quotient ÷	more +	times •
added to +	decreased by −	less −

EXPLORE MORE

Use the Gizmo to answer each question. Click on NEW to generate a new equation. Answers will vary.

5 Find and write down an equation written in symbols. What is the equation written in words?

6 Find and write down an equation written in words. What is the equation written in symbols?

7 Find and write down an equation written in symbols. What is the equation written in words?

8 Find and write down an equation written in words. What is the equation written in symbols?

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26 Lesson 5 • Gizmo: Using Algebraic Equations

DISCOVER

Problem 3

Have students log in to the Using Algebraic Equations Gizmo using the instructions in the box. **The equation *x* plus 3 equals 2 is shown. How many variables are in the equation?** (1) **What is it?** (*x*) **How many numbers are in the equation?** (2) **What are they?** (3 and 2) **How many symbols are in the equation?** (2) **What are they?** (+ and =)

So, five tiles will be needed—one tile for the variable, one tile for each number, and one tile for each symbol. Click on the HINT button. How many tiles are left? (5) **Which tile represents *x*?** (a number) **Which tile represents the addition sign?** (added to) Note: The tile that appears for addition could be *plus, added to,* or *increased by*. **Which tile represents 3?** (three) **Which tile represents the equal sign?** (is equal to) **Which tile represents 2?** (two) **What is the equation written in words?** (a number added to [plus or increased by] three is equal to two)

Problem 4

Click on the NEW button until an equation given in words appears on your screen. The equation *a number increased by nine is equal to seven* is shown. What words indicate the variable? (a number) **How many variables are in the equation?** (1) **Which words indicate numbers?** (nine and seven) **How many numbers are in the equation?** (2) **Which words indicate symbols?** (*increased by* and *is equal to*) **How many symbols are in the equation?** (2) **So, five tiles will be needed—one tile for the variable, one tile for each number, and one tile for each symbol. Click on the HINT button. How many tiles are left?** (5) **Which tile represents *a number*?** (*y*) Note: The tile that appears for the variable could be any letter. **Which tile represents *increased by*?** (+) **Which tile represents *nine*?** (9) **Which tile represents *is equal to*?** (=) **Which tile represents *seven*?** (7) **What is the equation written in symbols?** ($y + 9 = 7$)

DISCOVER BOX

Many different words are used for the symbols +, −, •, and ÷. Read each word and determine which symbol is indicated.

EXPLORE MORE

Problems 5–8

Have students work individually. In these problems, students will write equations in words and in symbols. Some equations may use a dot for multiplication. Students should be introduced to this notation before beginning the lesson.

Extension: Have students or partners write algebraic equations in words, then trade papers and write the equations in numbers and symbols.

Lesson 6

Objective

To solve a word problem by using an equation

Preskills	Lesson
Solving Multi-Step Equations 1	I.3.1
Solving Multi-Step Equations 2	I.3.2

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What operation is indicated in the word statement *15 increased by a number is 22*? (addition) **Use *n* for the variable. How is the statement written as an equation?** ($15 + n = 22$)

Problem 2

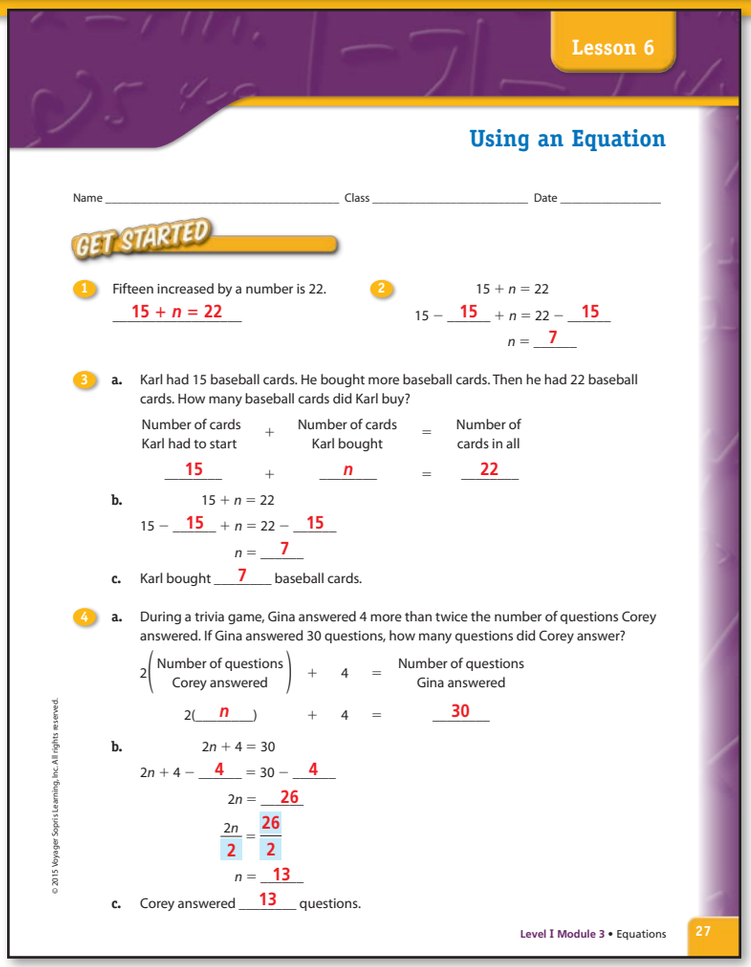
What operation is needed to solve this equation? (subtraction) **What number is subtracted?** (15) **What is 15 minus 15 plus *n*?** (*n*) **What is 22 minus 15?** (7) **What is the solution of the equation 15 plus *n* equals 22?** ($n = 7$)

MODEL NEW SKILLS

Problem 3

Often a word problem can be solved by writing and solving an equation. In problem 3, the number of cards Karl bought is unknown. Let the variable *n* represent the number of cards Karl bought.

Karl started with 15 cards, then bought *n* more. This means that Karl had 15 plus *n* cards. The problem states that after he bought more, Karl had 22 cards. The equation 15 plus *n* equals 22 represents Karl starting with 15 cards, buying *n* more, and ending with 22 cards. The solution of this equation is the number of cards Karl bought.



Lesson 6

Using an Equation

Name _____ Class _____ Date _____

GET STARTED

1 Fifteen increased by a number is 22.

15 + *n* = 22

2

15 + *n* = 22

15 − 15 + *n* = 22 − 15

n = 7

3 a. Karl had 15 baseball cards. He bought more baseball cards. Then he had 22 baseball cards. How many baseball cards did Karl buy?

Number of cards Karl had to start	+	Number of cards Karl bought	=	Number of cards in all
15	+	*n*	=	22

b.

15 + *n* = 22

15 − 15 + *n* = 22 − 15

n = 7

c. Karl bought 7 baseball cards.

4 a. During a trivia game, Gina answered 4 more than twice the number of questions Corey answered. If Gina answered 30 questions, how many questions did Corey answer?

2(Number of questions Corey answered) + 4 = Number of questions Gina answered

2(*n*) + 4 = 30

b.

2*n* + 4 = 30

2*n* + 4 − 4 = 30 − 4

2*n* = 26

$\frac{2n}{2} = \frac{26}{2}$

n = 13

c. Corey answered 13 questions.

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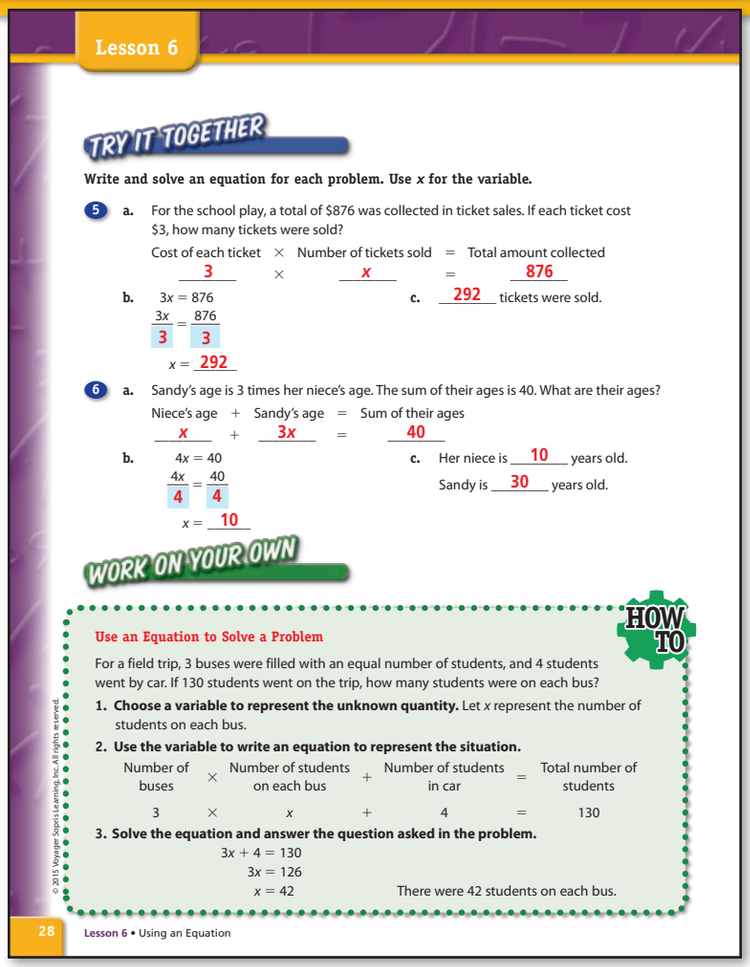
Look at problem 3b. To solve the equation, subtract 15 from both sides of the equation. The solution is *n* equals 7. Look at problem 3c. Because *n* equals 7, Karl bought 7 baseball cards.

Problem 4

In problem 4, what is the unknown quantity? (the number of questions Corey answered) **Let the variable *n* represent the number of questions Corey answered. The problem indicates that Gina answered 4 more than twice the number of questions Corey answered. If *n* represents the number of questions Corey answered, how can the number of questions Gina answered be represented?** ($2n + 4$)

The algebraic expression 2*n* plus 4 represents the number of questions Gina answered. How many questions did Gina answer? (30) **The equation 2*n* plus 4 equals 30 represents the problem. The solution of this equation is the number of questions Corey answered.**

Look at problem 4b. What is the first step in solving the equation? (Subtract 4 from both sides of the equation.) **What is the new equation?** ($2n = 26$) **What is the next step?**



Lesson 6

TRY IT TOGETHER

Write and solve an equation for each problem. Use *x* for the variable.

5 **a.** For the school play, a total of $876 was collected in ticket sales. If each ticket cost $3, how many tickets were sold?

Cost of each ticket	×	Number of tickets sold	=	Total amount collected
3	×	*x*	=	876

b. $3x = 876$

$\frac{3x}{3} = \frac{876}{3}$

$x =$ 292

c. 292 tickets were sold.

6 **a.** Sandy's age is 3 times her niece's age. The sum of their ages is 40. What are their ages?

Niece's age	+	Sandy's age	=	Sum of their ages
x	+	3*x*	=	40

b. $4x = 40$

$\frac{4x}{4} = \frac{40}{4}$

$x =$ 10

c. Her niece is 10 years old.

Sandy is 30 years old.

WORK ON YOUR OWN

HOW TO

Use an Equation to Solve a Problem

For a field trip, 3 buses were filled with an equal number of students, and 4 students went by car. If 130 students went on the trip, how many students were on each bus?

1. **Choose a variable to represent the unknown quantity.** Let *x* represent the number of students on each bus.
2. **Use the variable to write an equation to represent the situation.**

Number of buses	×	Number of students on each bus	+	Number of students in car	=	Total number of students
3	×	*x*	+	4	=	130

3. **Solve the equation and answer the question asked in the problem.**

$3x + 4 = 130$

$3x = 126$

$x = 42$

There were 42 students on each bus.

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(Divide both sides of the equation by 2.) **What is 2*n* divided by 2?** (*n*) **What is 26 divided by 2?** (13) **What is the solution of the equation?** ($n = 13$) **Look at problem 4c. Because *n* equals 13, Corey answered 13 questions.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

How can this word problem be solved? (Write an equation to represent the situation, then solve the equation.) **What is unknown in this problem?** (the number of tickets sold) **How can this unknown amount be represented?** (by a variable)

Let *x* represent the number of tickets sold. What was the cost of each ticket? ($3) **What was the total amount collected?** ($876) **Given the cost of each ticket and the number of tickets sold, how is the total amount collected found?** (Multiply the cost of each ticket by the number of tickets sold.)

What expression represents the cost of each ticket multiplied by the number of tickets sold? ($3x$) **What equation can be written to represent the situation?** ($3x = 876$)

To solve the equation, what needs to be done to both sides of the equation? (Divide both sides by 3.) **What is the solution of the equation?** ($x = 292$) **How many tickets were sold?** (292 tickets)

Problem 6

What are the unknowns in this problem? (the niece's age and Sandy's age) **Let *x* represent the niece's age. What is known about Sandy's age?** (Sandy's age is 3 times her niece's age.) **What expression can be used to represent Sandy's age?** ($3x$)

What other information is given in the problem about their ages? (The sum of their ages is 40.) **What equation can be written to represent the situation?** ($x + 3x = 40$) **What needs to be done to solve the equation?** (Combine like terms.) **What is *x* plus 3*x*?** ($4x$)

If $4x$ equals 40, what is *x*? ($x = 10$) **So, Sandy's niece is 10 years old because the variable *x* represents the niece's age. How old is Sandy?** (30 years old) **Why?** (Three times 10 equals 30.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 7–17

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 11 Reminder

This problem asks for Chris's age and Julie's age. Be sure students understand they can use one person's age to find the other person's age.

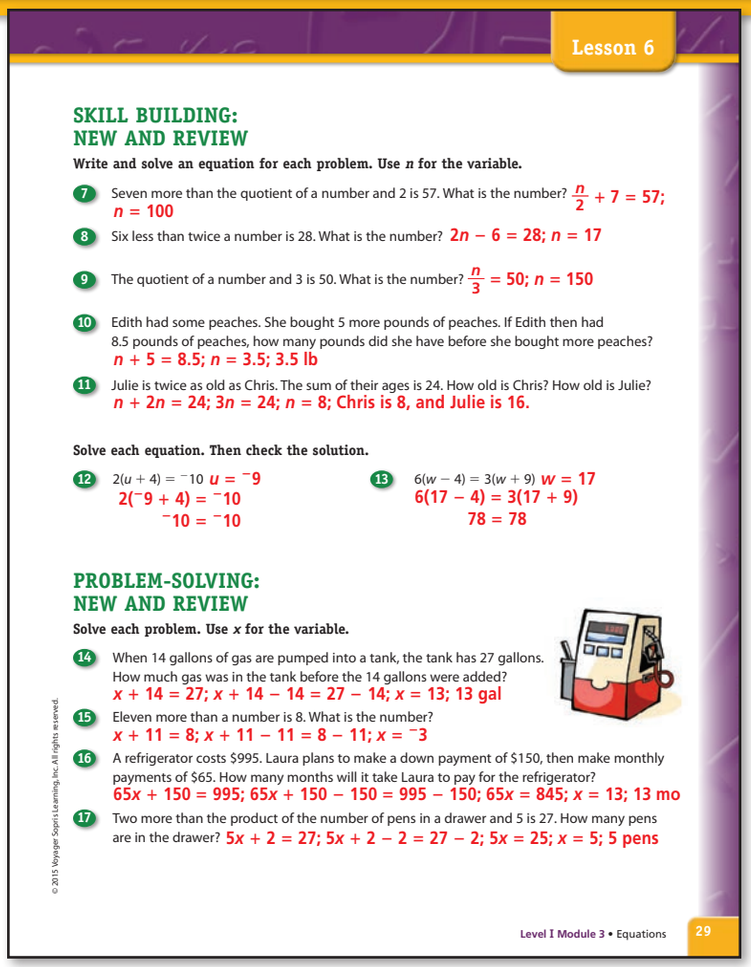
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student did not recognize the words *less than* indicated subtraction instead of addition. Use Additional Resources in I.3.6 to reteach how to set up and solve an equation.



Lesson 6

SKILL BUILDING: NEW AND REVIEW

Write and solve an equation for each problem. Use *n* for the variable.

7. Seven more than the quotient of a number and 2 is 57. What is the number? $\frac{n}{2} + 7 = 57$; $n = 100$
8. Six less than twice a number is 28. What is the number? $2n - 6 = 28$; $n = 17$
9. The quotient of a number and 3 is 50. What is the number? $\frac{n}{3} = 50$; $n = 150$
10. Edith had some peaches. She bought 5 more pounds of peaches. If Edith then had 8.5 pounds of peaches, how many pounds did she have before she bought more peaches? $n + 5 = 8.5$; $n = 3.5$; 3.5 lb
11. Julie is twice as old as Chris. The sum of their ages is 24. How old is Chris? How old is Julie? $n + 2n = 24$; $3n = 24$; $n = 8$; Chris is 8, and Julie is 16.

Solve each equation. Then check the solution.

12. $2(u + 4) = {}^-10$ $u = {}^-9$
$2({}^-9 + 4) = {}^-10$
${}^-10 = {}^-10$
13. $6(w - 4) = 3(w + 9)$ $w = 17$
$6(17 - 4) = 3(17 + 9)$
$78 = 78$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem. Use *x* for the variable.

14. When 14 gallons of gas are pumped into a tank, the tank has 27 gallons. How much gas was in the tank before the 14 gallons were added? $x + 14 = 27$; $x + 14 - 14 = 27 - 14$; $x = 13$; 13 gal
15. Eleven more than a number is 8. What is the number? $x + 11 = 8$; $x + 11 - 11 = 8 - 11$; $x = {}^-3$
16. A refrigerator costs $995. Laura plans to make a down payment of $150, then make monthly payments of $65. How many months will it take Laura to pay for the refrigerator? $65x + 150 = 995$; $65x + 150 - 150 = 995 - 150$; $65x = 845$; $x = 13$; 13 mo
17. Two more than the product of the number of pens in a drawer and 5 is 27. How many pens are in the drawer? $5x + 2 = 27$; $5x + 2 - 2 = 27 - 2$; $5x = 25$; $x = 5$; 5 pens

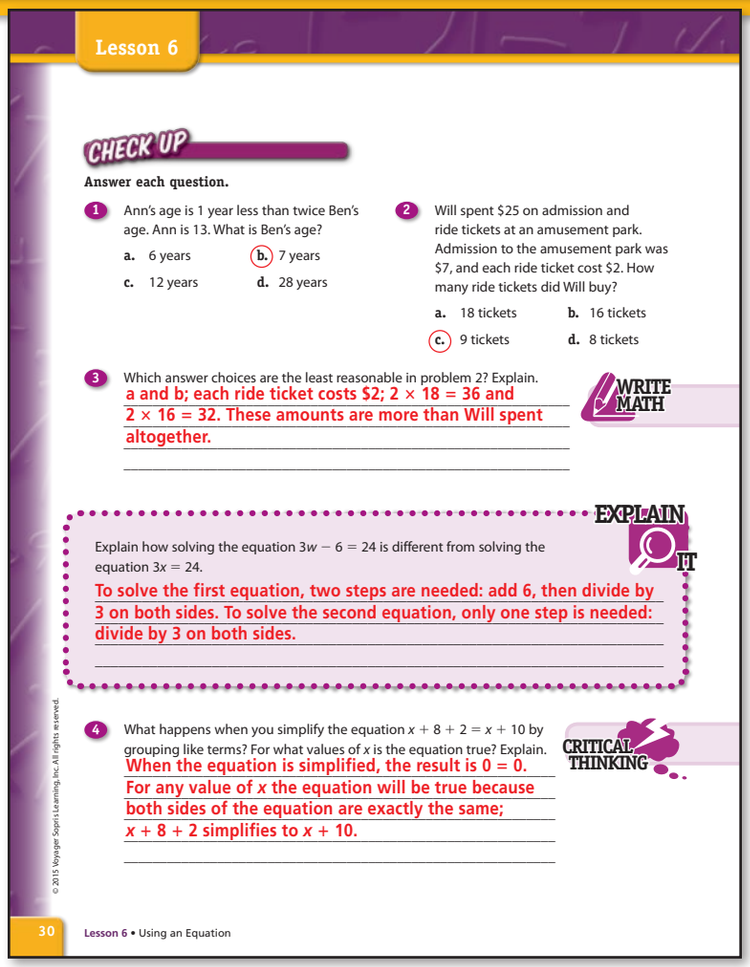
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If student answered 1c or 1d: The student did not recognize that the word *twice* indicated multiplication. Use Additional Resources in I.3.6 to reteach how to set up and solve an equation.

If student answered 2a: The student found the cost of all the tickets, not an individual ticket. Use Additional Resources in I.3.1 to reteach how to solve an equation.

If student answered 2b: The student added 7 instead of subtracting 7. Use Additional Resources in I.3.1 to reteach how to solve an equation.



Lesson 6

CHECK UP

Answer each question.

1. Ann's age is 1 year less than twice Ben's age. Ann is 13. What is Ben's age?
   a. 6 years (b.) 7 years
   c. 12 years d. 28 years

2. Will spent $25 on admission and ride tickets at an amusement park. Admission to the amusement park was $7, and each ride ticket cost $2. How many ride tickets did Will buy?
   a. 18 tickets b. 16 tickets
   (c.) 9 tickets d. 8 tickets

3. Which answer choices are the least reasonable in problem 2? Explain. WRITE MATH
   a and b; each ride ticket costs $2; 2 × 18 = 36 and 2 × 16 = 32. These amounts are more than Will spent altogether.

EXPLAIN IT

Explain how solving the equation $3w - 6 = 24$ is different from solving the equation $3x = 24$.

To solve the first equation, two steps are needed: add 6, then divide by 3 on both sides. To solve the second equation, only one step is needed: divide by 3 on both sides.

4. What happens when you simplify the equation $x + 8 + 2 = x + 10$ by grouping like terms? For what values of x is the equation true? Explain. CRITICAL THINKING
   When the equation is simplified, the result is 0 = 0. For any value of x the equation will be true because both sides of the equation are exactly the same; $x + 8 + 2$ simplifies to $x + 10$.

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If student answered problem 3 incorrectly: While students work together, remind the student to use the cost of one ride ticket, $2, and to see that answer choices a and b are unreasonable. Be sure the student understands that the total amount that Will spent was $25.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 3 Lesson 6
Reteach Teacher Edition Module 3 Lesson 6

Extra Practice
Student Book page 48

English Language Learners

Use the VmathLive Animated Glossary to review the term *algebraic equation*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

If students have trouble writing equations to represent problems, encourage them to draw a picture. For example, show the following problem:

> A school store has 14 unopened packs of pens. The store also has 4 pens on the counter display. If the store has 172 pens in all, how many pens are in each unopened pack?

To model this problem, draw 14 boxes on the board to represent the unopened packages. Label each one with an *x*. Then write a plus sign, and draw 4 lines to represent the pens on the counter. Guide students to use this model to help them write an equation and solve.

Students with Special Needs

Have students write the following steps on an index card to use when writing and solving an equation:

Step 1: Write an equation.
Step 2: Simplify by combining like terms.
Step 3: Undo any addition or subtraction.
Step 4: Undo any multiplication or division.

Lesson 7

Objective

To solve a word problem using a table

Preskills	Lesson
Evaluating Algebraic Expressions	I.3.PL1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What operations are indicated in the word phrase *the sum of 6 times a number and 10*? (addition and multiplication) **What algebraic expression shows *the sum of 6 times a number and 10*?** ($6n + 10$)

In problem 1b, substitute 8 for *n* to find the value of *6n* plus 10 when *n* equals 8. What is 6 times 8? (48) **What is 48 plus 10?** (58) **What is the value of *6n* plus 10 when *n* equals 8?** (58)

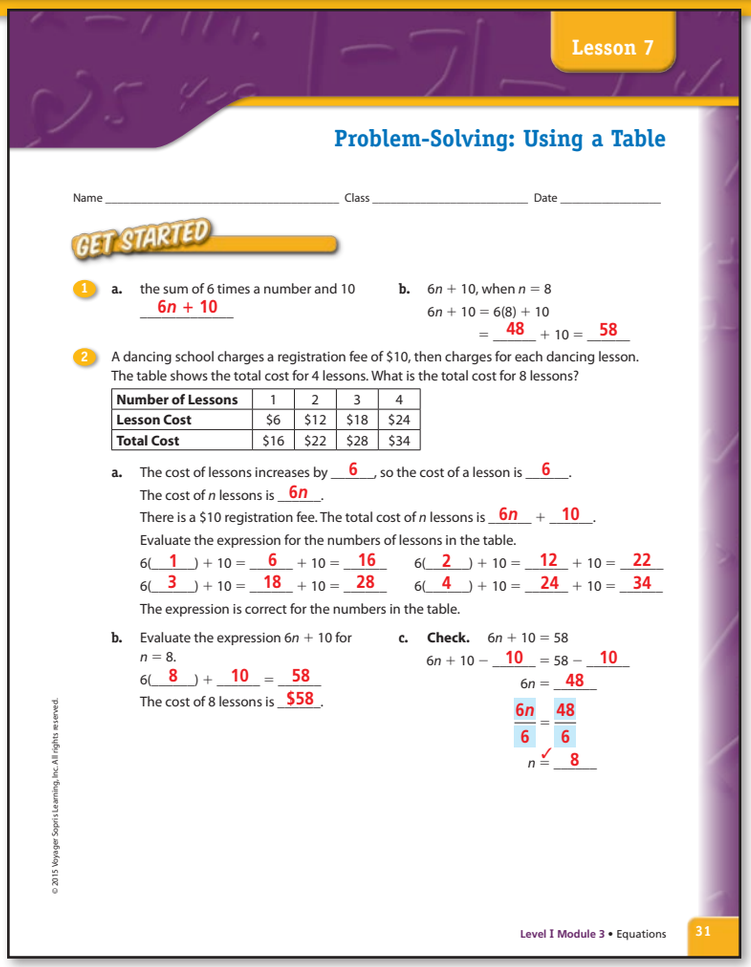
MODEL NEW SKILLS

Problem 2

To solve this problem first ask, "What am I trying to find?" What needs to be found in this problem? (the total cost for 8 dancing lessons) **Now ask, "How can I find the answer?" Use the information in the table to write an expression for the total cost of *n* lessons. The expression can be evaluated for *n* equals 8.**

Look at the table. By what number does the number of lessons increase? (1) **By what number does the total cost for the lessons increase?** (6) **As the number of lessons increases by 1, the cost of the lessons increases by 6. So, each lesson costs $6.**

If *n* represents the number of lessons, 6 times *n*, or *6n*, represents the cost of *n* lessons. Remember that there is also a $10 registration fee. The total cost of *n* lessons is the sum of 6 times *n* and 10. This was written as an algebraic expression in problem 1a. What is the expression? ($6n + 10$)



Lesson 7

Problem-Solving: Using a Table

Name _____ Class _____ Date _____

GET STARTED

1 **a.** the sum of 6 times a number and 10
6n + 10

b. 6n + 10, when n = 8
6n + 10 = 6(8) + 10
= 48 + 10 = 58

2 A dancing school charges a registration fee of $10, then charges for each dancing lesson. The table shows the total cost for 4 lessons. What is the total cost for 8 lessons?

Number of Lessons	1	2	3	4
Lesson Cost	$6	$12	$18	$24
Total Cost	$16	$22	$28	$34

a. The cost of lessons increases by 6, so the cost of a lesson is 6.
The cost of n lessons is 6n.
There is a $10 registration fee. The total cost of n lessons is 6n + 10.
Evaluate the expression for the numbers of lessons in the table.
6(1) + 10 = 6 + 10 = 16
6(2) + 10 = 12 + 10 = 22
6(3) + 10 = 18 + 10 = 28
6(4) + 10 = 24 + 10 = 34
The expression is correct for the numbers in the table.

b. Evaluate the expression 6n + 10 for n = 8.
6(8) + 10 = 58
The cost of 8 lessons is $58.

c. **Check.** 6n + 10 = 58
6n + 10 − 10 = 58 − 10
6n = 48
6n/6 = 48/6
n = 8 ✓

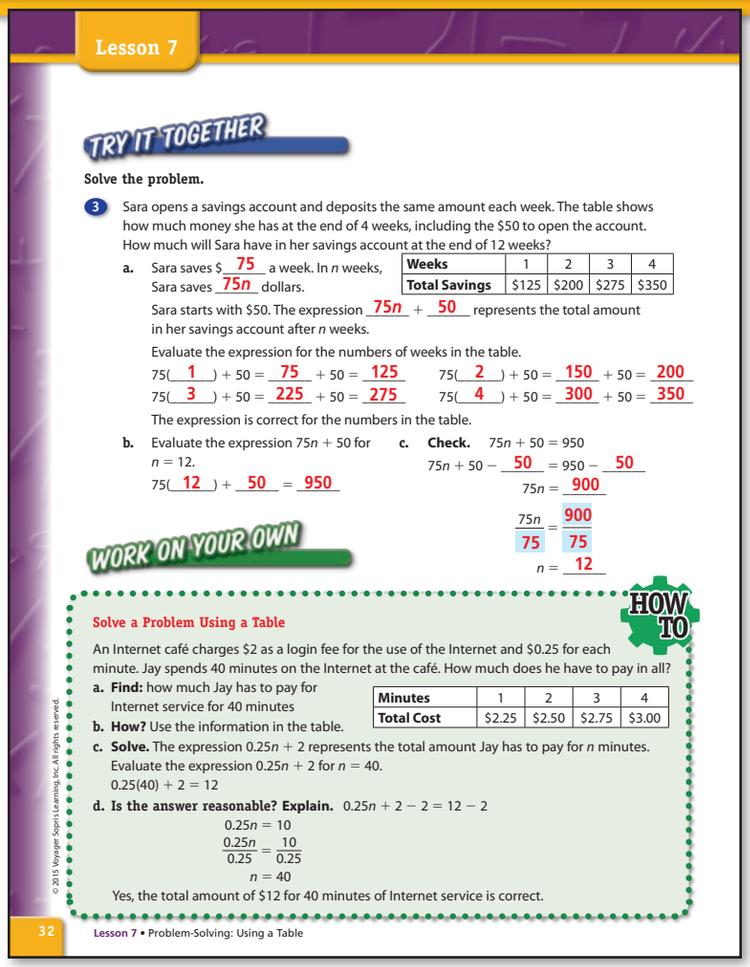
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Evaluate the expression *6n* plus 10 for the numbers of lessons given in the table. For *n* equals 1, how is the expression evaluated? ($6(1) + 10 = 16$) **For *n* equals 2, how is the expression evaluated?** ($6(2) + 10 = 22$) **What is the value of the expression when *n* equals 3?** (28) **What is the value of the expression when *n* equals 4?** (34)

The values found match the values in the table, so the expression is correct. Now solve the problem by evaluating the expression for *n* equals 8. Substitute 8 for *n* in the expression. What is the expression with 8 substituted for *n*? ($6(8) + 10$) **This expression was evaluated in problem 1b. What is the value of *6n* plus 10 when *n* equals 8?** (58) **The total cost of 8 lessons is $58.**

Finally ask, "Is the answer correct?" To answer this question, solve the equation *6n* plus 10 equals 58. To solve the equation, subtract 10 from both sides. What is 58 minus 10? (48) **If *6n* equals 48, what is *n*?** ($n = 8$) **Therefore, the cost of $58 for 8 lessons is correct.**



Lesson 7

TRY IT TOGETHER

Solve the problem.

3 Sara opens a savings account and deposits the same amount each week. The table shows how much money she has at the end of 4 weeks, including the $50 to open the account. How much will Sara have in her savings account at the end of 12 weeks?

Weeks	1	2	3	4
Total Savings	$125	$200	$275	$350

a. Sara saves $ 75 a week. In n weeks, Sara saves 75n dollars.

Sara starts with $50. The expression 75n + 50 represents the total amount in her savings account after n weeks.

Evaluate the expression for the numbers of weeks in the table.

75(1) + 50 = 75 + 50 = 125
75(2) + 50 = 150 + 50 = 200
75(3) + 50 = 225 + 50 = 275
75(4) + 50 = 300 + 50 = 350

The expression is correct for the numbers in the table.

b. Evaluate the expression $75n + 50$ for $n = 12$.

75(12) + 50 = 950

c. **Check.** $75n + 50 = 950$

$75n + 50 - 50 = 950 - 50$

$75n = 900$

$\frac{75n}{75} = \frac{900}{75}$

$n = 12$

WORK ON YOUR OWN

HOW TO

Solve a Problem Using a Table

An Internet café charges $2 as a login fee for the use of the Internet and $0.25 for each minute. Jay spends 40 minutes on the Internet at the café. How much does he have to pay in all?

Minutes	1	2	3	4
Total Cost	$2.25	$2.50	$2.75	$3.00

a. **Find:** how much Jay has to pay for Internet service for 40 minutes

b. **How?** Use the information in the table.

c. **Solve.** The expression $0.25n + 2$ represents the total amount Jay has to pay for n minutes. Evaluate the expression $0.25n + 2$ for $n = 40$.

$0.25(40) + 2 = 12$

d. **Is the answer reasonable? Explain.** $0.25n + 2 - 2 = 12 - 2$

$0.25n = 10$

$\frac{0.25n}{0.25} = \frac{10}{0.25}$

$n = 40$

Yes, the total amount of $12 for 40 minutes of Internet service is correct.

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TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 3

First ask, "What am I trying to find?" What needs to be found in this problem? (how much money Sara will have in her savings account at the end of 12 weeks) **Now ask, "How can I find the answer?" How can the answer be found?** (Use the information in the table to write an expression for her savings after n weeks. Evaluate the expression for n equals 12.)

By how much does the balance in Sara's account increase each week? ($75) **How much does Sara deposit in her account each week?** ($75) **Using the variable *n*, what expression can be used to show Sara's savings after *n* weeks?** ($75n$) **How much did Sara start with?** ($50) **What expression can be used to show the total amount in Sara's account?** ($75n + 50$)

Evaluate the expression 75*n* plus 50 for the numbers of weeks given in the table to show that it is the correct expression. What is the value of the expression when *n* equals 1? (125) **What is the value of the expression when *n* equals 2?** (200) **What is the value of the expression when *n* equals 3?** (275) **What is the value of the expression when *n* equals 4?** (350) **Do the values match the values given in the table?** (yes) **Then the expression may be used to solve the problem.**

Substitute 12 for *n* in the expression and evaluate. What is the expression with 12 substituted for *n*? ($75(12) + 50$) **Multiply, then add. What is the value of the expression when *n* equals 12?** (950) **What will be the balance in Sara's account after 12 weeks?** ($950)

To check the answer, solve the given equation 75*n* plus 50 equals 950. What is the equation after 50 is subtracted from both sides? ($75n = 900$) **What is *n*?** ($n = 12$) **How much will Sara have in her savings account at the end of 12 weeks?** ($950) **Is the answer correct?** (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 4–11

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 9 Reminder

Students should set up a percent equation to solve this problem.

Problem 10 Reminder

Students must first write an expression for the total cost of buying *n* tickets, then substitute to see if he has enough money.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student added the number of letters and the initial amount instead of using the table to solve the problem. Use Additional Resources in I.3.7 to reteach how to use a table to solve a problem.

If student answered 1b or 2b: The student multiplied the amount of the increment by the number of items and did not consider the initial amount. Use Additional Resources in I.3.7 to reteach how to use a table to solve a problem.

SKILL BUILDING: NEW AND REVIEW

Solve each problem.

4. As hikers went up a mountain, the temperature decreased at a constant rate. When they started hiking, the temperature was 62°F. The table shows how the temperature changed during 3 hours. What was the temperature after 6 hours? $62 - 3n$; $62 - 3(6) = 44$; 44°F

Hours	0	1	2	3
Temperature (°F)	62°	59°	56°	53°

5. Mrs. Black wants to buy a pillow sewn with her daughter's name. The pillow costs $25 plus a charge for each letter in the name. How much will the pillow cost if there are 8 letters in her daughter's name? $2n + 25$; $2(8) + 25 = 41$; $41

Letters	1	2	3	4
Pillow Cost	$25	$25	$25	$25
Total Cost	$27	$29	$31	$33

Solve each equation. Then check the solution.

6. $8a - 4 + a = 32$ $a = 4$
$8(4) - 4 + 4 = 32$
$32 = 32$

7. $7b - 1 = b - 1$ $b = 0$
$7(0) - 1 = 0 - 1$
$^-1 = ^-1$

8. $3c - 15 = ^-60$ $c = ^-15$
$3(^-15) - 15 = ^-60$
$^-60 = ^-60$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.



9. Keiko paid $21 for a jacket. The jacket is on sale for 70% off of the original price. What was the original price of the jacket?
$100\% - 70\% = 30\%$; $21 = 0.3 \times b$; $b = 70$; $70

10. Carlos is ordering concert tickets by mail. There is a $4 handling fee for the order. The table shows the cost of buying tickets by mail. Carlos has $570. Does he have enough to buy 16 tickets? Explain.

Tickets	1	2	3	4
Handling Fee	$4	$4	$4	$4
Total Cost	$39	$74	$109	$144

Yes; the cost of 16 tickets is $35n + 4 = 35(16) + 4 = 560 + 4 = 564$; $564 < $570

11. The entrance fee to a state fair is $8. The cost for rides is shown in the table. Jen went on 6 rides at the fair. How much did she spend in all?

Rides	1	2	3	4
Entrance Fee	$8	$8	$8	$8
Total Cost	$12	$16	$20	$24

$8 + 4n$; $= 8 + 4(6) = 8 + 24 = 32$; $32

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If student answered 1d, 2a, or 2d: The student multiplied numbers given in the problem but did not write and evaluate an expression to represent the information given in the table. Use Additional Resources in I.3.7 to reteach how to use a table to solve a problem.

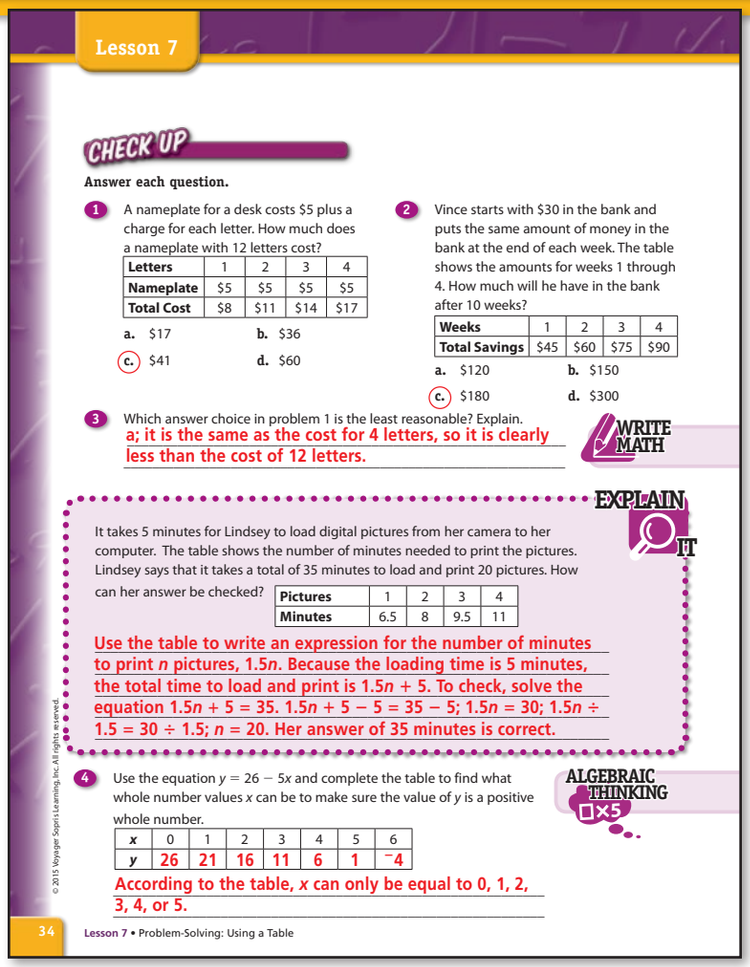


WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student to look for the amount of increase in each cost, then add 5 to find the expression for problem 1. To check the expression, substitute each value found in the table (1–4) in the expression. If the expression is correct, the value of the expression will equal the value of the corresponding price in the table.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 7

CHECK UP

Answer each question.

1. A nameplate for a desk costs $5 plus a charge for each letter. How much does a nameplate with 12 letters cost?

Letters	1	2	3	4
Nameplate	$5	$5	$5	$5
Total Cost	$8	$11	$14	$17

a. $17 b. $36 (c.) $41 d. $60

2. Vince starts with $30 in the bank and puts the same amount of money in the bank at the end of each week. The table shows the amounts for weeks 1 through 4. How much will he have in the bank after 10 weeks?

Weeks	1	2	3	4
Total Savings	$45	$60	$75	$90

a. $120 b. $150 (c.) $180 d. $300

3. Which answer choice in problem 1 is the least reasonable? Explain.
a; it is the same as the cost for 4 letters, so it is clearly less than the cost of 12 letters.

WRITE MATH

EXPLAIN IT

It takes 5 minutes for Lindsey to load digital pictures from her camera to her computer. The table shows the number of minutes needed to print the pictures. Lindsey says that it takes a total of 35 minutes to load and print 20 pictures. How can her answer be checked?

Pictures	1	2	3	4
Minutes	6.5	8	9.5	11

Use the table to write an expression for the number of minutes to print n pictures, $1.5n$. Because the loading time is 5 minutes, the total time to load and print is $1.5n + 5$. To check, solve the equation $1.5n + 5 = 35$. $1.5n + 5 - 5 = 35 - 5$; $1.5n = 30$; $1.5n \div 1.5 = 30 \div 1.5$; $n = 20$. Her answer of 35 minutes is correct.

4. Use the equation $y = 26 - 5x$ and complete the table to find what whole number values x can be to make sure the value of y is a positive whole number.

ALGEBRAIC THINKING

x	0	1	2	3	4	5	6
y	26	21	16	11	6	1	$^{-}4$

According to the table, x can only be equal to 0, 1, 2, 3, 4, or 5.

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34 Lesson 7 • Problem-Solving: Using a Table

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 3 Lesson 7
Reteach Teacher Edition Module 3 Lesson 7

Extra Practice
Student Book page 48

English Language Learners

Talk about the meaning of the word *deposit*, which is mentioned in problem 3 of the Try It Together section. Show the students a bag of 10 counters. Tell them that every day you will deposit 5 counters into the bag. Explain that depositing means adding. After one day, there will be 10 + 5, or 15, counters in the bag. Have students work together to create a table that compares the number of days with the number of counters in the bag.

Students with Special Needs

Help students who have difficulty understanding and reading a table by focusing on each row or column individually. While showing a table on the board or screen, cover information that is not being considered at that time to prevent students from being distracted. Focus on how to write an expression that relates the information in each particular column.

For example, create a table to show one CD costs $10, two CDs cost $20, three CDs cost $30, and so on. When showing the first column, write, Cost = $10 × 1 = $10. Shift the masking to show the next column, then write, Cost = $10 × 2 = $20, and so on.

Ask the students to find what is changing in the number sentences (the number of CDs and the cost, which is affected by the number of CDs). Point out that because the number of CDs changes, a variable is used to represent the number of CDs. So, the expression for this example is 10n.

Have students work in pairs. Repeat this exercise by creating different tables and categories.

Lesson 8

Objective

To solve equations and practice an open-ended response item

Model the following skill for students.

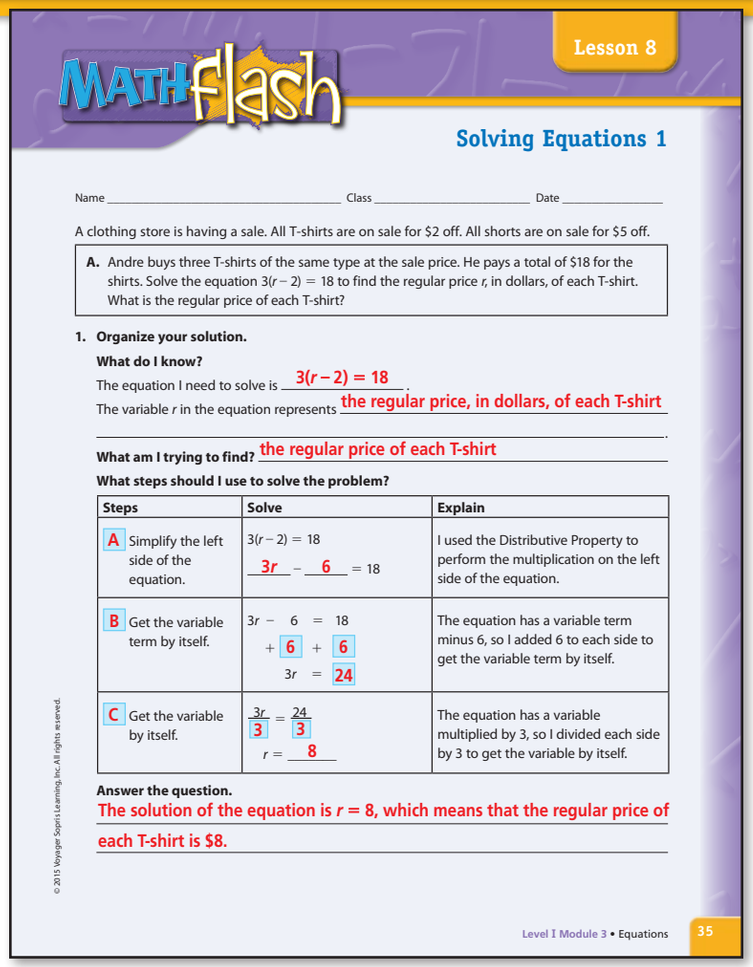
Part 1: Organize your solution: What do I know? **The first part of answering an open-ended question is to organize your solution. Read the problem. Find the given information from the problem. What equation are you given to solve?** (3(*r* – 2) = 18) **What does the variable in this equation represent?** (the regular price, in dollars, of each T-shirt)

What am I trying to find? **Read the problem again. What does the problem ask for?** (the regular price of each T-shirt)

What steps can I use to solve the problem? **Now that all the given information is found, you can break down the problem into steps to find the solution of the equation.**

Direct students' attention to the table. **When solving a problem, you can use a table like this one to organize your work. Each row represents a step you take to solve the problem. Write the letter *A* in the box in the first row. According to the table, what is Step A?** (Simplify the left side of the equation.) **What is the expression on the left side of the equation?** (3(*r* – 2)) **You can use the Distributive Property to simplify this expression. First, multiply 3 and *r*. What is the product?** (3*r*) **Then multiply 3 and 2. What is the product?** (6) **Next, subtract the two products. What simplified expression did you write for the left side of the equation?** (3*r* – 6) **Now look at the Explain column. The text in this column explains in words the math you did in the Solve column. What words explain what you did in the Solve column?** (I used the Distributive Property to perform the multiplication on the left side of the equation.)

Direct students' attention to the second row in the table. **Write the letter *B* in the box in the second row. What is Step B?** (Get the variable term by itself.) **What is the variable term in the equation?** (3*r*) **What number is subtracted from 3*r*?** (6) **What operation is the inverse of subtraction?** (addition) **Add 6 to each side of the equation. What equation did you write in the Solve column after adding 6 to each side?** (3*r* = 24) **Look at the sentence in the Explain column. Does this sentence correctly describe the work you did?** (yes)



MATHFlash

Lesson 8

Solving Equations 1

Name _____ Class _____ Date _____

A clothing store is having a sale. All T-shirts are on sale for $2 off. All shorts are on sale for $5 off.

A. Andre buys three T-shirts of the same type at the sale price. He pays a total of $18 for the shirts. Solve the equation 3(*r* – 2) = 18 to find the regular price *r*, in dollars, of each T-shirt. What is the regular price of each T-shirt?

1. Organize your solution.

What do I know?

The equation I need to solve is 3(*r* – 2) = 18.

The variable *r* in the equation represents the regular price, in dollars, of each T-shirt.

What am I trying to find? the regular price of each T-shirt

What steps should I use to solve the problem?

Steps	Solve	Explain
A Simplify the left side of the equation.	3(*r* – 2) = 18 3*r* – 6 = 18	I used the Distributive Property to perform the multiplication on the left side of the equation.
B Get the variable term by itself.	3*r* – 6 = 18 + 6 + 6 3*r* = 24	The equation has a variable term minus 6, so I added 6 to each side to get the variable term by itself.
C Get the variable by itself.	$\frac{3r}{3} = \frac{24}{3}$ *r* = 8	The equation has a variable multiplied by 3, so I divided each side by 3 to get the variable by itself.

Answer the question.

The solution of the equation is *r* = 8, which means that the regular price of each T-shirt is $8.

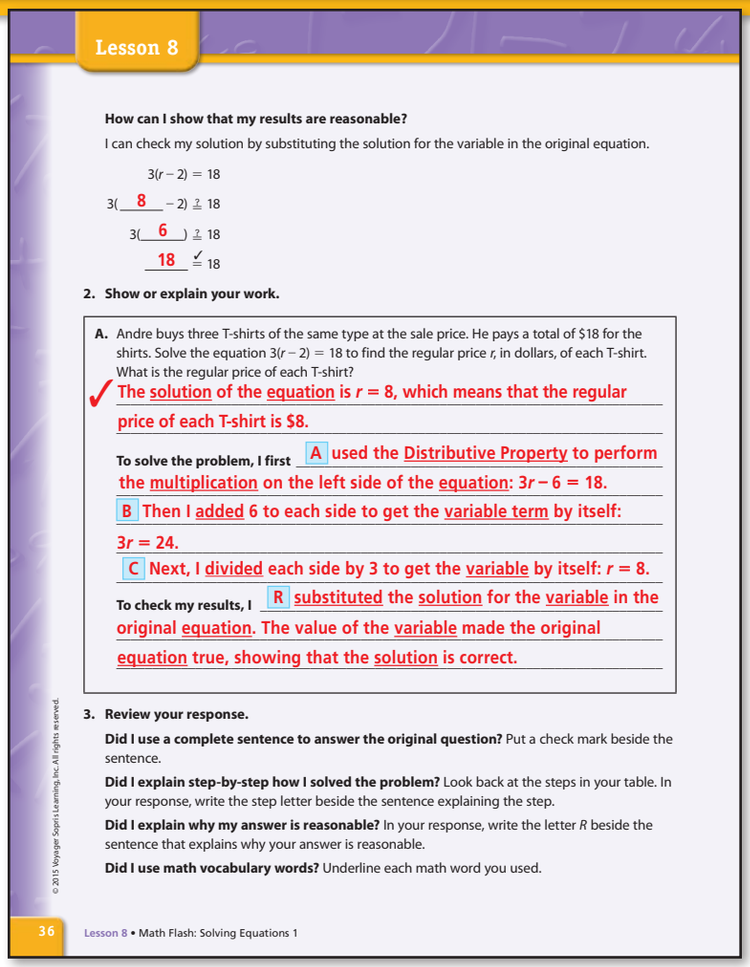
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Level I Module 3 • Equations 35

Direct students' attention to the third row in the table. **Write the letter *C* in the box in the third row. What is Step C?** (Get the variable by itself.) **What is the variable in the equation?** (*r*) **What number is the variable multiplied by?** (3) **What operation is the inverse of multiplication?** (division) **Divide each side of the equation by 3. What equation did you write in the Solve column after dividing each side by 3?** (*r* = 8) **What is the solution of the original equation?** (*r* = 8) **Look at the sentence in the Explain column. Does this sentence correctly describe the work you did?** (yes)

Answer the question. **Answer the original question using a complete sentence. What is the regular price of each T-shirt?** (The solution of the equation is *r* = 8, which means that the regular price of each T-shirt is $8.)

How can I show that my results are reasonable? **Decide whether the results of the problem are reasonable. In this**



Lesson 8

How can I show that my results are reasonable?

I can check my solution by substituting the solution for the variable in the original equation.

$3(r - 2) = 18$

$3(\underline{8} - 2) \stackrel{?}{=} 18$

$3(\underline{6}) \stackrel{?}{=} 18$

$\underline{18} \stackrel{\checkmark}{=} 18$

2. Show or explain your work.

A. Andre buys three T-shirts of the same type at the sale price. He pays a total of $18 for the shirts. Solve the equation $3(r - 2) = 18$ to find the regular price r, in dollars, of each T-shirt. What is the regular price of each T-shirt?

✓ The solution of the equation is $r = 8$, which means that the regular price of each T-shirt is $8.

To solve the problem, I first [A] used the Distributive Property to perform the multiplication on the left side of the equation: $3r - 6 = 18$.

[B] Then I added 6 to each side to get the variable term by itself: $3r = 24$.

[C] Next, I divided each side by 3 to get the variable by itself: $r = 8$.

To check my results, I [R] substituted the solution for the variable in the original equation. The value of the variable made the original equation true, showing that the solution is correct.

3. Review your response.

Did I use a complete sentence to answer the original question? Put a check mark beside the sentence.

Did I explain step-by-step how I solved the problem? Look back at the steps in your table. In your response, write the step letter beside the sentence explaining the step.

Did I explain why my answer is reasonable? In your response, write the letter *R* beside the sentence that explains why your answer is reasonable.

Did I use math vocabulary words? Underline each math word you used.

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problem, you solved an equation. You can check the solution by substituting the solution for the variable in the original equation. What is the original equation? ($3(r - 2) = 18$) **What value should you substitute for *r* in the original equation?** (8) **If both sides of the equation simplify to the same number, your solution is correct. How do you know that your solution is correct?** (Both sides of the equation simplify to 18.)

Work with students to complete these skills.

Part 2: Show or explain your work. **Now that you are organized, the next part is to show or explain your work. First, answer the original question using a complete sentence. Your response should next describe in words how you solved the problem. To write these steps in words, use your answers from the table and finish the statement, "To solve the problem, I first …." Be sure to use complete sentences. Next, use complete sentences to describe how you checked the results for reasonableness. To do this, complete the sentence, "To check my results, I …."**

Part 3: Review your response. **Finally, review your written response by reading through the questions in Part 3. After you have read the questions and followed the instructions, you should have a check mark; the letters *A*, *B*, *C*, and *R*; and underlined vocabulary words in your response.**

Technology

Using an online search engine, have students or partners research careers that would need to solve equations with variables. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

Lesson 9

Objective

To solve equations and practice an open-ended response item

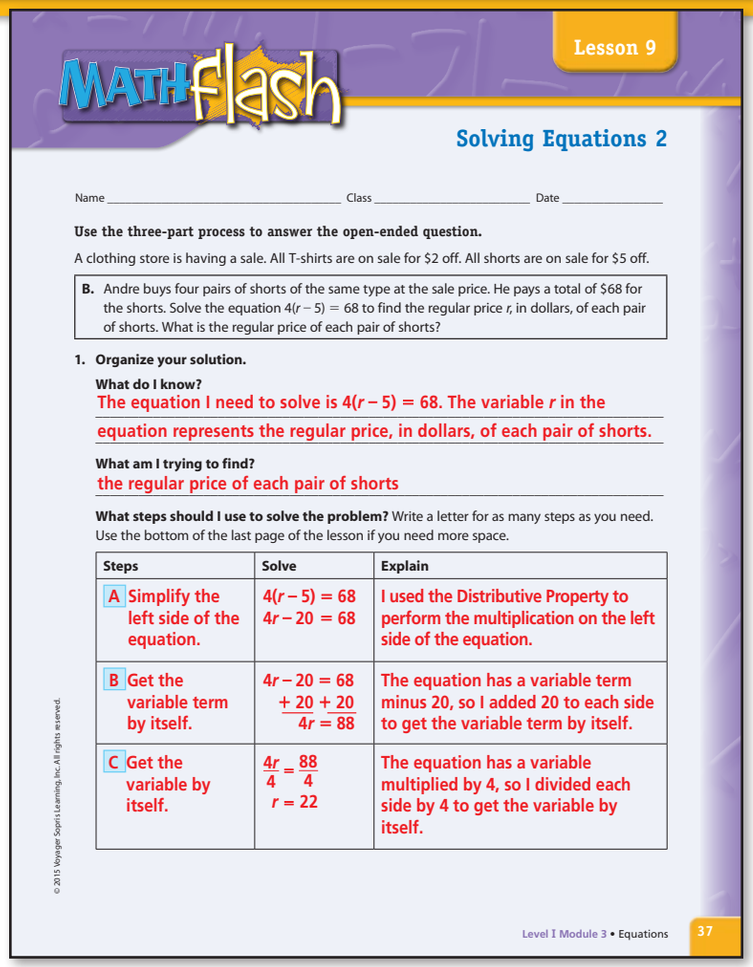
Model the following skills for students.

Part 1: Organize your solution: What do I know? **The first part of answering an open-ended question is to organize your solution. Read the problem. Find the given information from the problem. What equation are you given to solve?** ($4(r - 5) = 68$) **What does the variable in this equation represent?** (the regular price, in dollars, of each pair of shorts)

What am I trying to find? **Read the problem again. What does the problem ask for?** (the regular price of each pair of shorts)

What steps can I use to solve the problem? **Now that all the given information is found, you can break down the problem into steps to find the solution of the equation.**

Direct students' attention to the table. **When solving a problem, you can use a table like this one to organize your work. Each row represents a step you take to solve the problem. Write the letter *A* in the box in the first row. What is the first step you need to do?** (Simplify the left side of the equation.) **Write the words in the Steps column of the first row. What is the equation that you need to solve?** ($4(r - 5) = 68$) **Write the equation in the first row of the Solve column. You can use the Distributive Property to simplify this expression. First, multiply 4 and *r*. What is the product?** ($4r$) **Then multiply 4 and 5. What is the product?** (20) **Next, subtract the two products. Write the simplified expression under the equation in the first row of the table in the Solve column. What simplified expression did you write for the left side of the equation?** ($4r - 20$) **Now complete the Explain column for the first row. What sentence could you write in the Explain column to describe the work you did in this step?** (I used the Distributive Property to perform the multiplication on the left side of the equation.) **Write those words in the Explain column.**



MATH Flash

Lesson 9

Solving Equations 2

Name _____ Class _____ Date _____

Use the three-part process to answer the open-ended question.

A clothing store is having a sale. All T-shirts are on sale for $2 off. All shorts are on sale for $5 off.

B. Andre buys four pairs of shorts of the same type at the sale price. He pays a total of $68 for the shorts. Solve the equation $4(r - 5) = 68$ to find the regular price r, in dollars, of each pair of shorts. What is the regular price of each pair of shorts?

1. Organize your solution.

What do I know?
The equation I need to solve is $4(r - 5) = 68$. The variable r in the equation represents the regular price, in dollars, of each pair of shorts.

What am I trying to find?
the regular price of each pair of shorts

What steps should I use to solve the problem? Write a letter for as many steps as you need. Use the bottom of the last page of the lesson if you need more space.

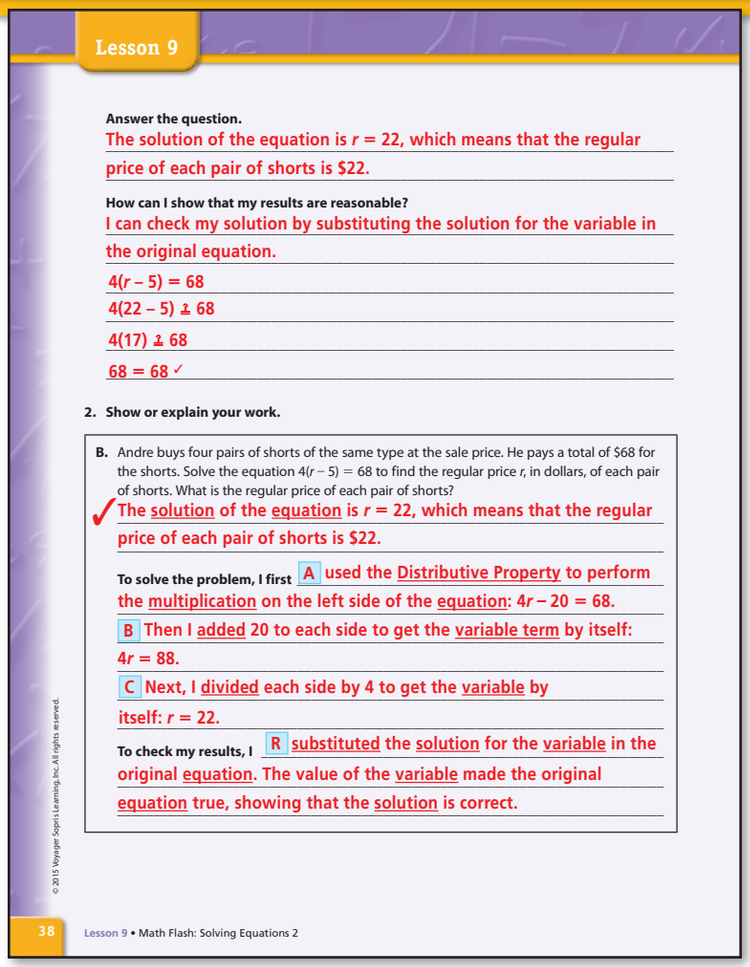
Steps	Solve	Explain
A Simplify the left side of the equation.	$4(r - 5) = 68$ $4r - 20 = 68$	I used the Distributive Property to perform the multiplication on the left side of the equation.
B Get the variable term by itself.	$4r - 20 = 68$ $+20 \quad +20$ $4r = 88$	The equation has a variable term minus 20, so I added 20 to each side to get the variable term by itself.
C Get the variable by itself.	$\frac{4r}{4} = \frac{88}{4}$ $r = 22$	The equation has a variable multiplied by 4, so I divided each side by 4 to get the variable by itself.

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Direct students' attention to the second row in the table. **Write the letter *B* in the box in the second row. What is the next step you need to take to solve the equation?** (Get the variable term by itself.) **Write those words in the Steps column of the second row. What is the variable term in the equation?** ($4r$) **What number is subtracted from $4r$?** (20) **What operation is the inverse of subtraction?** (addition) **Add 20 to each side of the equation. Record your work in the second row of the Solve column. What equation did you write in the Solve column after adding 20 to each side?** ($4r = 88$) **What sentence could you write in the Explain column to describe the work you did in this step?** (The equation has a variable term minus 20, so I added 20 to each side to get the variable term by itself.)

Direct students' attention to the third row in the table. **Write the letter *C* in the box in the third row. What is the next step you need to take?** (Get the variable by itself.) **Write those words in the Steps column of the third row. What number is the variable multiplied by?** (4) **What operation is the inverse of multiplication?** (division) **Divide each side of the equation by 4. Record your work in the third row of the Solve column. What equation did you write in the Solve column after dividing each side by 4?** ($r = 22$) **What is the solution of the original equation?** ($r = 22$) **What sentence could you write in the Explain column to describe the work you did in this step?**



Lesson 9

Answer the question.
The solution of the equation is $r = 22$, which means that the regular price of each pair of shorts is $22.

How can I show that my results are reasonable?
I can check my solution by substituting the solution for the variable in the original equation.

$4(r - 5) = 68$
$4(22 - 5) \stackrel{?}{=} 68$
$4(17) \stackrel{?}{=} 68$
$68 = 68$ ✓

2. **Show or explain your work.**

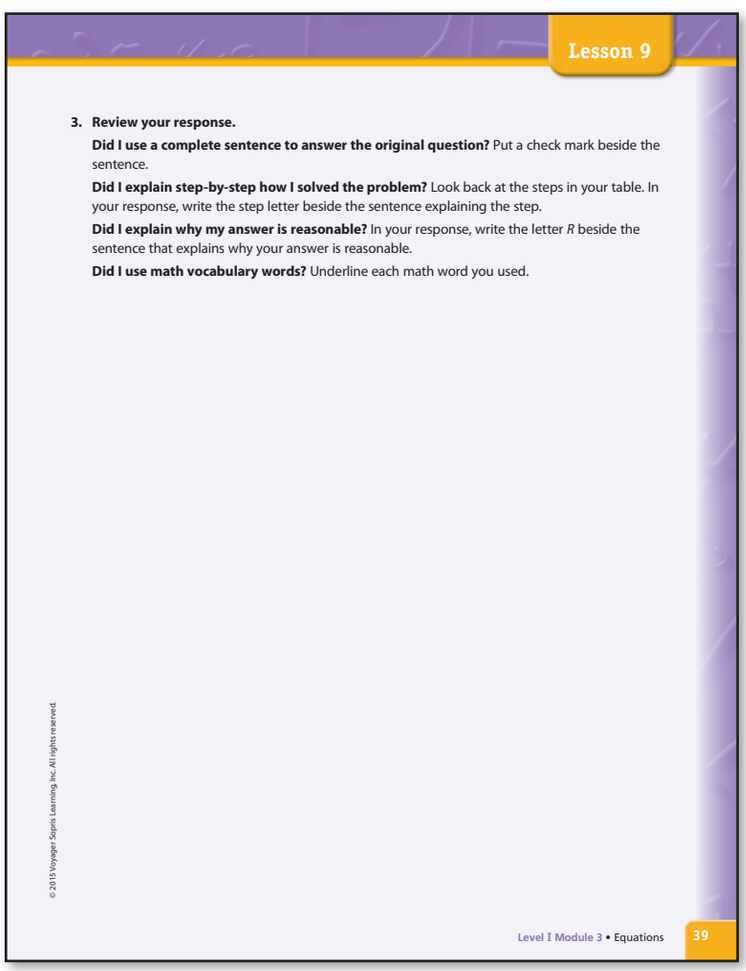
B. Andre buys four pairs of shorts of the same type at the sale price. He pays a total of $68 for the shorts. Solve the equation $4(r - 5) = 68$ to find the regular price r, in dollars, of each pair of shorts. What is the regular price of each pair of shorts?

✓ The solution of the equation is $r = 22$, which means that the regular price of each pair of shorts is $22.

To solve the problem, I first [A] used the Distributive Property to perform the multiplication on the left side of the equation: $4r - 20 = 68$. [B] Then I added 20 to each side to get the variable term by itself: $4r = 88$. [C] Next, I divided each side by 4 to get the variable by itself: $r = 22$.

To check my results, I [R] substituted the solution for the variable in the original equation. The value of the variable made the original equation true, showing that the solution is correct.

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Lesson 9

3. **Review your response.**

Did I use a complete sentence to answer the original question? Put a check mark beside the sentence.

Did I explain step-by-step how I solved the problem? Look back at the steps in your table. In your response, write the step letter beside the sentence explaining the step.

Did I explain why my answer is reasonable? In your response, write the letter *R* beside the sentence that explains why your answer is reasonable.

Did I use math vocabulary words? Underline each math word you used.

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(The equation has a variable multiplied by 4, so I divided each side by 4 to get the variable by itself.)

Answer the question. Answer the original question using a complete sentence. What is the regular price of each pair of shorts? (The solution of the equation is $r = 22$, which means that the regular price of each pair of shorts is $8.)

How can I show that my results are reasonable? Decide whether the results of the problem are reasonable. In this problem, you solved an equation. You can check the solution by substituting the solution for the variable in the original equation. What is the original equation? ($4(r - 5) = 68$) **What value should you substitute for *r* in the original equation?** (22) **If both sides of the equation simplify to the same number, your solution is correct. How do you know that your solution is correct?** (Both sides of the equation simplify to 68.)

Work with students to complete these skills.

Part 2: Show or explain your work. Now that you are organized, the next part is to show or explain your work. First, answer the original question using a complete sentence. Your response should next describe in words how you solved the problem. To write these steps in words, use your answers from the table and finish the statement, "To solve the problem, I first …." Be sure to use complete sentences. Next, use complete sentences to describe how you checked the results for reasonableness. To do this, complete the sentence, "To check my results, I …."

Part 3: Review your response. Finally, review your written response by reading through the questions in Part 3. After you have read the questions and followed the instructions, you should have a check mark; the letters *A*, *B*, *C*, and *R*; and underlined vocabulary words in your response.

Lesson 10

Objective

To solve real-world problems by writing and solving equations with variables on both sides

Preskills	Lesson
Solving Multi-Step Equations 2	I.3.2
Solving Equations with Variables on Both Sides 2	I.3.4

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Do the expressions on either side of the equation need to be simplified? (no) **Are variable terms on both sides of the equation?** (yes)

What should be done to bring the variable term to the left side of the equation to keep the coefficient of the variable term positive? (Subtract 4.45*n* from both sides of the equation.) **What is 6.85*n* minus 4.45*n*?** (2.4*n*) **What is 4.45*n* minus 8 minus 4.45*n*?** ($^-8$) **What is the equation now?** ($2.4n + 4 = {}^-8$)

What should be done next? (Subtract 4 from each side.) **What is the resulting equation?** ($2.4n = {}^-12$)

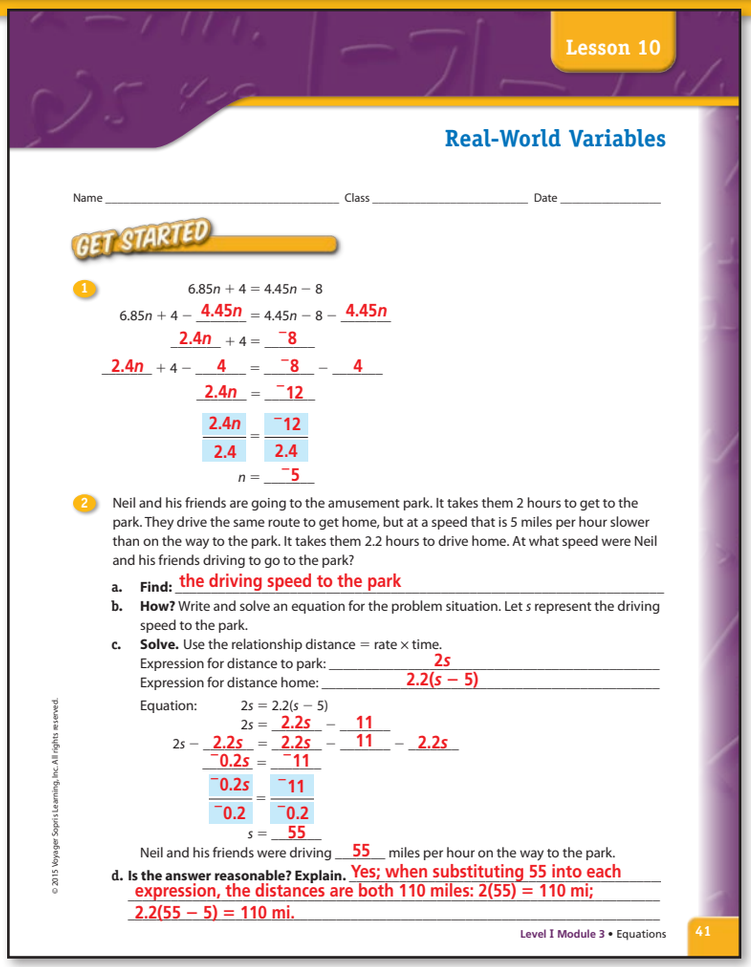
Divide both sides of the equation by 2.4. What is 2.4*n* divided by 2.4? (*n*) **What is $^-12$ divided by 2.4?** ($^-5$) **What is the solution?** ($n = {}^-5$)

MODEL NEW SKILLS

Problem 2

To solve this problem first ask, "What am I trying to find?" What needs to be found in this problem? (the driving speed to the park) **Now ask, "How can I find the answer?" Use the information in the problem to write an equation.**

What will the variable *s* represent? (the driving speed to the park) **Remember that speed is a rate. The problem indicates that speed is a rate that compares miles and hours, or distance and time.**



Lesson 10

Real-World Variables

Name _____ Class _____ Date _____

GET STARTED

1. $6.85n + 4 = 4.45n - 8$

$6.85n + 4 - 4.45n = 4.45n - 8 - 4.45n$

$2.4n + 4 = {}^-8$

$2.4n + 4 - 4 = {}^-8 - 4$

$2.4n = {}^-12$

$\frac{2.4n}{2.4} = \frac{{}^-12}{2.4}$

$n = {}^-5$

2. Neil and his friends are going to the amusement park. It takes them 2 hours to get to the park. They drive the same route to get home, but at a speed that is 5 miles per hour slower than on the way to the park. It takes them 2.2 hours to drive home. At what speed were Neil and his friends driving to go to the park?

a. **Find:** the driving speed to the park

b. **How?** Write and solve an equation for the problem situation. Let *s* represent the driving speed to the park.

c. **Solve.** Use the relationship distance = rate × time.

Expression for distance to park: $2s$

Expression for distance home: $2.2(s - 5)$

Equation: $2s = 2.2(s - 5)$

$2s = 2.2s - 11$

$2s - 2.2s = 2.2s - 11 - 2.2s$

$^-0.2s = {}^-11$

$\frac{^-0.2s}{^-0.2} = \frac{{}^-11}{{}^-0.2}$

$s = 55$

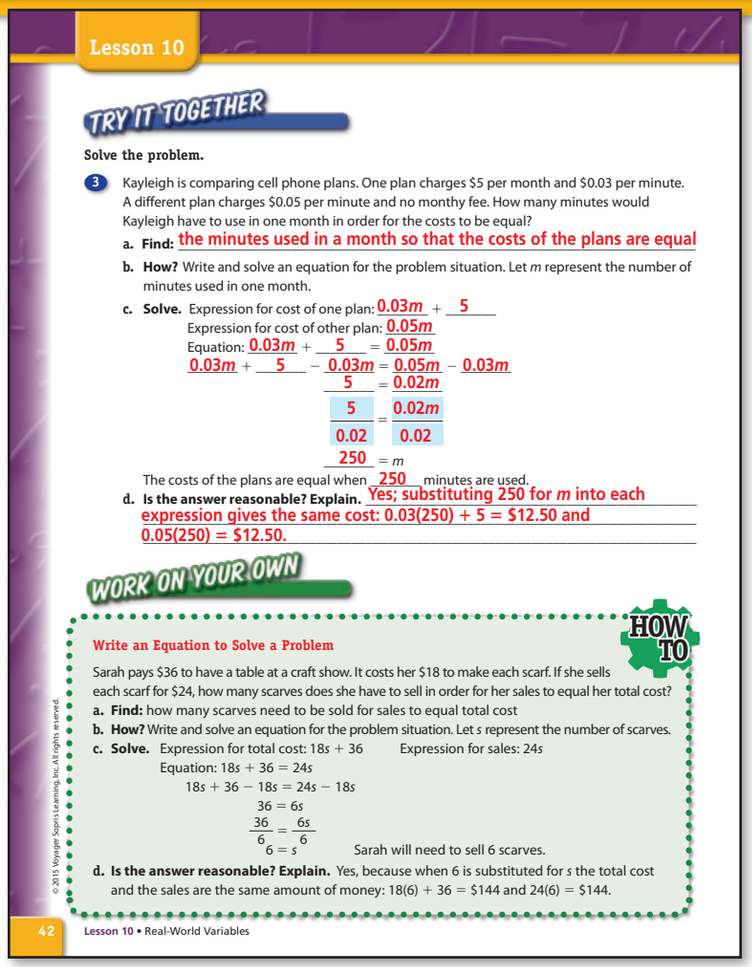
Neil and his friends were driving 55 miles per hour on the way to the park.

d. **Is the answer reasonable? Explain.** Yes; when substituting 55 into each expression, the distances are both 110 miles: $2(55) = 110$ mi; $2.2(55 - 5) = 110$ mi.

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The relationship between distance, rate, and time can be used to write an equation. Neil and his friends drive twice: from home to the park and from the park to home. How do the two distances compare? (The distances are equal.) **So this distance can be represented in two ways: in terms of the drive to the park and in terms of the drive home.**

Distance can be written as a product of the rate, or speed in this case, and time. On the way to the park, what is the driving speed? (*s*) **How long did it take to get to the park?** (2 hours) **So an expression for the distance from home to the park is the rate times time, or *s* times 2, or 2*s*. Now write an expression for the distance from the park to home using the information in the problem. What is known about the speed on the way home?** (It was 5 miles per hour slower than the speed on the way to the park.) **Because *s* represents the speed to the park and the speed home was 5 miles per hour slower, the expression *s* – 5 represents the driving speed on the way home. How long did it take to drive home?** (2.2 hours) **So an expression for the distance from the park to home is *s* – 5 times 2.2, or 2.2 times the quantity of *s* – 5. Because the distances are equal, form the equation by setting the expressions equal to each other. What is the equation?** ($2s = 2.2(s - 5)$)



Lesson 10

TRY IT TOGETHER

Solve the problem.

3 Kayleigh is comparing cell phone plans. One plan charges $5 per month and $0.03 per minute. A different plan charges $0.05 per minute and no monthly fee. How many minutes would Kayleigh have to use in one month in order for the costs to be equal?

a. **Find:** the minutes used in a month so that the costs of the plans are equal

b. **How?** Write and solve an equation for the problem situation. Let m represent the number of minutes used in one month.

c. **Solve.** Expression for cost of one plan: $0.03m + 5$

Expression for cost of other plan: $0.05m$

Equation: $0.03m + 5 = 0.05m$

$0.03m + 5 - 0.03m = 0.05m - 0.03m$

$5 = 0.02m$

$\frac{5}{0.02} = \frac{0.02m}{0.02}$

$250 = m$

The costs of the plans are equal when 250 minutes are used.

d. **Is the answer reasonable? Explain.** Yes; substituting 250 for m into each expression gives the same cost: 0.03(250) + 5 = $12.50 and 0.05(250) = $12.50.

WORK ON YOUR OWN

HOW TO

Write an Equation to Solve a Problem

Sarah pays $36 to have a table at a craft show. It costs her $18 to make each scarf. If she sells each scarf for $24, how many scarves does she have to sell in order for her sales to equal her total cost?

a. **Find:** how many scarves need to be sold for sales to equal total cost

b. **How?** Write and solve an equation for the problem situation. Let s represent the number of scarves.

c. **Solve.** Expression for total cost: $18s + 36$ Expression for sales: $24s$

Equation: $18s + 36 = 24s$

$18s + 36 - 18s = 24s - 18s$

$36 = 6s$

$\frac{36}{6} = \frac{6s}{6}$

$6 = s$

Sarah will need to sell 6 scarves.

d. **Is the answer reasonable? Explain.** Yes, because when 6 is substituted for s the total cost and the sales are the same amount of money: 18(6) + 36 = $144 and 24(6) = $144.

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What is the first step in solving this equation? (multiplying to remove parentheses) **What is the resulting equation?** ($2s = 2.2s - 11$) **What is the next step?** (Subtract $2.2s$ from both sides to get the variable terms on one side of the equation.) **What is the resulting equation?** ($^-0.2s = ^-11$) **What is the last step?** (Divide each side by $^-0.2$.) **What is the solution of the equation?** ($s = 55$) **What does s represent?** (the driving speed to the park) **What was the driving speed of Neil and his friends on the way to the park?** (55 mph)

To check the answer, substitute the value of the variable into each expression and verify that the distances are the same. What is the expression for the distance to the park after 55 is substituted for s? (2(55)) **What is 2 times 55?** (110) **What is the expression for the distance home after 55 is substituted for s?** (2.2(55 – 5)) **What is 55 minus 5?** (50) **What is 2.2 times 50?** (110) **Because the distances are the same, the answer checks.**

TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 3

To solve this problem first ask, "What am I trying to find?" What needs to be found in this problem? (the number of minutes for which the cost of the two plans is equal) **Now ask, "How can I find the answer?" Use the information in the problem to write an equation.**

What will the variable m represent? (the number of minutes) **Because the problem asks when the costs of the plans will be the same, write an expression for the cost of each plan. Then the equation will be written by setting these expressions equal to each other.**

How much does the first plan charge for each minute? ($0.03) **If m is the number of minutes used, what expression represents the cost of the first plan if m minutes are used in one month?** ($0.03m$) **Does the first plan have any other charges?** (yes) **What is the charge?** ($5 per month) **So add 5 to the expression $0.03m$ to get the total cost of the first plan for one month. How much does the second plan charge for each minute?** ($0.05) **If m is the number of minutes used, what expression represents the total cost of the second plan if m minutes are used in one month?** ($0.05m$) **What equation represents the situation in which the costs are equal?** ($0.03m + 5 = 0.05m$)

What is the first step in solving this equation? (Subtract $0.03m$ from both sides to get the variable terms on one side of the equation.) **What is the resulting equation?** ($5 = 0.02m$) **What is the next step?** (Divide each side by 0.02.) **What is the solution of the equation?** ($m = 250$) **What does m represent?** (the number of minutes that makes the costs of the plans equal) **How many minutes make the costs of the plans equal?** (250 min)

To check the answer, substitute the value of the variable into each expression and verify that the costs are the same. What is the expression for the cost of the first plan after 250 is substituted for *m*? (0.03(250) + 5) **What is 0.03 times 250?** (7.50) **What is 7.50 plus 5?** (12.50) **What is the expression for the cost of the second plan after 250 is substituted for *m*?** (0.05(250)) **What is 0.05 times 250?** (12.50) **Because the costs are the same, the answer checks.**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

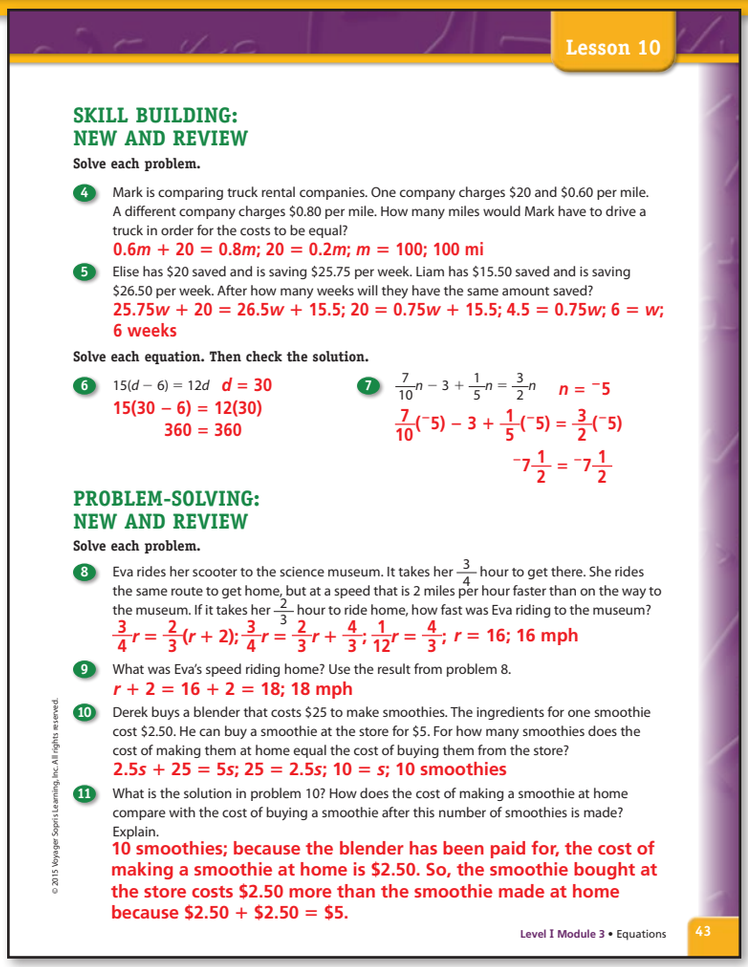
Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 4–11

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 11 **Reminder**

Remind students that after 10 smoothies, the blender has been paid for. The cost of making smoothies at home after making 10 smoothies is only the cost of the ingredients.



Lesson 10

SKILL BUILDING: NEW AND REVIEW

Solve each problem.

4. Mark is comparing truck rental companies. One company charges $20 and $0.60 per mile. A different company charges $0.80 per mile. How many miles would Mark have to drive a truck in order for the costs to be equal?
   $0.6m + 20 = 0.8m$; $20 = 0.2m$; $m = 100$; 100 mi
5. Elise has $20 saved and is saving $25.75 per week. Liam has $15.50 saved and is saving $26.50 per week. After how many weeks will they have the same amount saved?
   $25.75w + 20 = 26.5w + 15.5$; $20 = 0.75w + 15.5$; $4.5 = 0.75w$; $6 = w$; 6 weeks

Solve each equation. Then check the solution.

6. $15(d - 6) = 12d$ $d = 30$
   $15(30 - 6) = 12(30)$
   $360 = 360$
7. $\frac{7}{10}n - 3 + \frac{1}{5}n = \frac{3}{2}n$ $n = -5$
   $\frac{7}{10}(-5) - 3 + \frac{1}{5}(-5) = \frac{3}{2}(-5)$
   $-7\frac{1}{2} = -7\frac{1}{2}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

8. Eva rides her scooter to the science museum. It takes her $\frac{3}{4}$ hour to get there. She rides the same route to get home, but at a speed that is 2 miles per hour faster than on the way to the museum. If it takes her $\frac{2}{3}$ hour to ride home, how fast was Eva riding to the museum?
   $\frac{3}{4}r = \frac{2}{3}(r + 2)$; $\frac{3}{4}r = \frac{2}{3}r + \frac{4}{3}$; $\frac{1}{12}r = \frac{4}{3}$; $r = 16$; 16 mph
9. What was Eva's speed riding home? Use the result from problem 8.
   $r + 2 = 16 + 2 = 18$; 18 mph
10. Derek buys a blender that costs $25 to make smoothies. The ingredients for one smoothie cost $2.50. He can buy a smoothie at the store for $5. For how many smoothies does the cost of making them at home equal the cost of buying them from the store?
    $2.5s + 25 = 5s$; $25 = 2.5s$; $10 = s$; 10 smoothies
11. What is the solution in problem 10? How does the cost of making a smoothie at home compare with the cost of buying a smoothie after this number of smoothies is made? Explain.
    10 smoothies; because the blender has been paid for, the cost of making a smoothie at home is $2.50. So, the smoothie bought at the store costs $2.50 more than the smoothie made at home because $2.50 + $2.50 = $5.

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CHECK UP

ASSESS INFORMALLY

Error Analysis

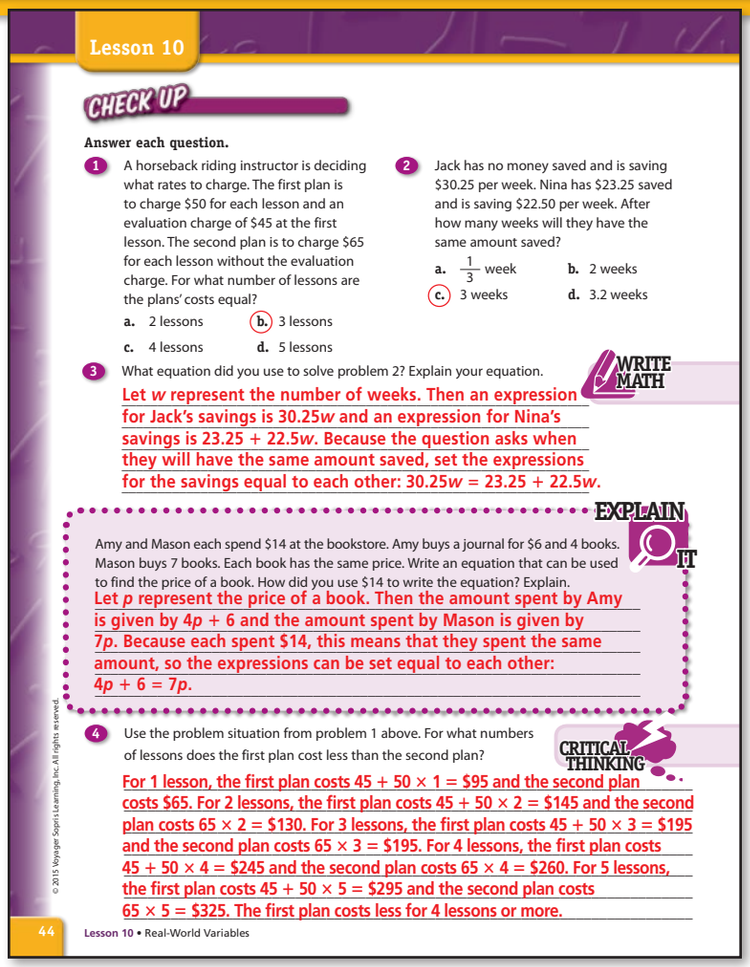
Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 or problem 2 incorrectly:
The student made an error when writing or solving the equation. The student may benefit from making a table of values before writing the expressions for each equation.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to use expressions to describe the two situations that will be set equal to each other.



Lesson 10

CHECK UP

Answer each question.

1. A horseback riding instructor is deciding what rates to charge. The first plan is to charge $50 for each lesson and an evaluation charge of $45 at the first lesson. The second plan is to charge $65 for each lesson without the evaluation charge. For what number of lessons are the plans' costs equal?

   a. 2 lessons (b.) 3 lessons

   c. 4 lessons d. 5 lessons

2. Jack has no money saved and is saving $30.25 per week. Nina has $23.25 saved and is saving $22.50 per week. After how many weeks will they have the same amount saved?

   a. $\frac{1}{3}$ week b. 2 weeks

   (c.) 3 weeks d. 3.2 weeks

3. What equation did you use to solve problem 2? Explain your equation.

   WRITE MATH

   Let *w* represent the number of weeks. Then an expression for Jack's savings is 30.25*w* and an expression for Nina's savings is 23.25 + 22.5*w*. Because the question asks when they will have the same amount saved, set the expressions for the savings equal to each other: 30.25*w* = 23.25 + 22.5*w*.

EXPLAIN IT

Amy and Mason each spend $14 at the bookstore. Amy buys a journal for $6 and 4 books. Mason buys 7 books. Each book has the same price. Write an equation that can be used to find the price of a book. How did you use $14 to write the equation? Explain.

Let *p* represent the price of a book. Then the amount spent by Amy is given by 4*p* + 6 and the amount spent by Mason is given by 7*p*. Because each spent $14, this means that they spent the same amount, so the expressions can be set equal to each other: 4*p* + 6 = 7*p*.

4. Use the problem situation from problem 1 above. For what numbers of lessons does the first plan cost less than the second plan?

   CRITICAL THINKING

   For 1 lesson, the first plan costs 45 + 50 × 1 = $95 and the second plan costs $65. For 2 lessons, the first plan costs 45 + 50 × 2 = $145 and the second plan costs 65 × 2 = $130. For 3 lessons, the first plan costs 45 + 50 × 3 = $195 and the second plan costs 65 × 3 = $195. For 4 lessons, the first plan costs 45 + 50 × 4 = $245 and the second plan costs 65 × 4 = $260. For 5 lessons, the first plan costs 45 + 50 × 5 = $295 and the second plan costs 65 × 5 = $325. The first plan costs less for 4 lessons or more.

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EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 3 Lesson 10

Reteach Teacher Edition Module 3 Lesson 10

Extra Practice

Student Book page 49

English Language Learners

With one of the problems from the lesson, demonstrate how to use unit analysis to check that the expressions on both sides of the equation are reasonable. Have students describe the units for each constant, variable, and entire expression. Remind students that words like $0.03 per minute indicate the rate.

Students with Special Needs

Encourage students to use a word model, such as what was used in Lesson I.3.6, to write each expression before writing the equation. In this lesson, the word models *cost per unit* × *number of units* + *initial cost* and *rate* × *time* will be useful to know when students are writing the expressions for each side of the equation.

Extra Practice Answer Key

Extra Practice

Name _____ Class _____ Date _____

Lesson PL1 Evaluating Algebraic Expressions

Evaluate each expression when $x = 5$, $y = -6$, and $z = 8$.

1. $7x + 2y$ **23**
2. $\frac{10(-3y)}{45}$ **4**
3. $z + 3x - 15$ **8**
4. $\frac{-65}{x}$ **-13**
5. $90 - y + 3z$ **120**
6. $15z - 2x$ **110**
7. $-10x + 5y$ **-80**
8. $100 - 5z + 10y$ **0**
9. $\frac{-36}{y}$ **6**
10. $\frac{5(-4x)}{-10}$ **10**
11. $\frac{x^2 + y^2}{53 + 8}$ **1**
12. $3z - 2x$ **14**
13. $\frac{2z}{10 + y}$ **4**
14. $x - 6 + z$ **7**
15. $\frac{-25}{x}$ **-5**
16. $5z - 26$ **14**
17. $41 - 17y - 18$ **125**
18. $100 - 9z - 6$ **22**

Lesson PL2 Distributive Practice

Simplify each expression.

1. $8a + 3(a - 17)$ **$11a - 51$**
2. $-20 - a + 6(8 + a)$ **$5a + 28$**
3. $\frac{1}{8}(a - 80) + 20a$ **$20\frac{1}{8}a - 10$**
4. $15 + 5(a - \frac{4}{5})$ **$5a + 11$**
5. $9(11 - a) + 20 + 88a$ **$79a + 119$**
6. $10(5 + a)$ **$10a + 50$**
7. $9 + 3(a - 23)$ **$3a - 60$**
8. $4(c + 8) + 12c$ **$16c + 32$**
9. $-15 - t + 4(t + 5)$ **$3t + 5$**
10. $4(w + 15) - 3w$ **$w + 60$**
11. $17 + 10.5(t - 1.5)$ **$10.5t + 1.25$**
12. $17 + \frac{4}{5}(y - 2)$ **$\frac{4}{5}y - 15\frac{2}{5}$**
13. $3 + 15(y - 2)$ **$15y - 27$**
14. $6(x - 2) + x$ **$7x - 12$**
15. $\frac{3}{8}(8 - t) + 4t$ **$3\frac{5}{8}t + 3$**

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Extra Practice

Name _____ Class _____ Date _____

Lesson 1 Solving Multi-Step Equations 1

Solve each equation. Then check the solution.

1. $-4a + 8a = -48$ **$a = -12$**
   $-4(-12) + 8(-12) = -48$
   $-48 = -48$
2. $-2(b - 6) = 32$ **$b = -10$**
   $-2(-10 - 6) = 32$
   $32 = 32$
3. $8c + 5 - 5c + 7 = 36$ **$c = -16$**
   $8(-16) + 5 - 5(-16) + 7 = -36$
   $-36 = -36$
4. $4d + 6 - 6d = 120$ **$d = -57$**
   $4(-57) + 6 - 6(-57) = 120$
   $120 = 120$
5. $4(x - 7) + 5x - 2 = -3$ **$x = 3$**
   $4(3 - 7) + 5(3) - 2 = -3$
   $-3 = -3$
6. $10m + 1 - m = -80$ **$m = -9$**
   $10(-9) + 1 - (-9) = -80$
   $-80 = -80$
7. $13z + 45 - 7z = -15$ **$z = -10$**
   $13(-10) + 45 - 7(-10) = -15$
   $-15 = -15$
8. $-5(c - 8) = 5$ **$m = 7$**
   $-5(7 - 8) = 5$
   $5 = 5$

Lesson 2 Solving Multi-Step Equations 2

Solve each equation. Then check the solution.

1. $\frac{5}{8}x - 17 - \frac{3}{16}x = -3$ **$x = 32$**
   $\frac{5}{8}(32) - 17 - \frac{3}{16}(32) = -3$
   $-3 = -3$
2. $1.25(d - 5) + 7 = 32$ **$d = 25$**
   $1.25(25 - 5) + 7 = 32$
   $32 = 32$
3. $\frac{2}{3}(y + 24) - 15 = -5$ **$y = -9$**
   $\frac{2}{3}(-9 + 24) - 15 = -5$
   $-5 = -5$
4. $-4.75m - 4.8 + 2.25m = -24.8$ **$m = 8$**
   $-4.75(8) - 4.8 + 2.25(8) = -24.8$
   $-24.8 = -24.8$
5. $12 + \frac{2}{5}(t - 10) = 10$ **$t = 5$**
   $12 + \frac{2}{5}(5 - 10) = 10$
   $10 = 10$
6. $14.5 - 6.8z + 10.5 = -9$ **$z = 5$**
   $14.5 - 6.8(5) + 10.5 = -9$
   $-9 = -9$

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Extra Practice

Name _____ Class _____ Date _____

Lesson 3 Solving Equations with Variables on Both Sides 1

Solve each equation. Then check the solution.

1. $11a + 4 = -3a - 24$ **$a = -2$**
   $11(-2) + 4 = -3(-2) - 24$
   $-18 = -18$
2. $5c + 3.5 = 2.5c - 6.5$ **$c = -4$**
   $5(-4) + 3.5 = 2.5(-4) - 6.5$
   $-16.5 = -16.5$
3. $x + 110 = 26x + 10$ **$x = 4$**
   $4 + 110 = 26(4) + 10$
   $114 = 114$
4. $4(-3 + z) = -2z$ **$z = 2$**
   $4(-3 + 2) = -2(2)$
   $-4 = -4$
5. $1.7h = -13 + 1.2h$ **$h = -26$**
   $1.7(-26) = -13 + 1.2(-26)$
   $-44.2 = -44.2$
6. $7 - 2a = 3a - 8$ **$a = 3$**
   $7 - 2(3) = 3(3) - 8$
   $1 = 1$
7. $16 - 7y = 18 - 5y$ **$y = -1$**
   $16 - 7(-1) = 18 - 5(-1)$
   $23 = 23$
8. $4x - 3 = -18 - x$ **$x = -3$**
   $4(-3) - 3 = -18 - (-3)$
   $-15 = -15$

Lesson 4 Solving Equations with Variables on Both Sides 2

Solve each equation. Then check the solution.

1. $\frac{7}{9}b + 8 - \frac{1}{3}b = \frac{1}{9}b + 6$ **$b = -6$**
   $\frac{7}{9}(-6) + 8 - \frac{1}{3}(-6) = \frac{1}{9}(-6) + 6$
   $5\frac{1}{3} = 5\frac{1}{3}$
2. $-4 + 2.1x + 10 = 1.7x + 14 + 0.4x$
   6 = 14; no solution
3. $0.5q + 16 - 0.4q = 8.1q$ **$q = 2$**
   $0.5(2) + 16 - 0.4(2) = 8.1(2)$
   $16.2 = 16.2$
4. $\frac{5}{6}(d - 18) = \frac{1}{6}d + 3$ **$d = 27$**
   $\frac{5}{6}(27 - 18) = \frac{1}{6}(27) + 3$
   $7\frac{1}{2} = 7\frac{1}{2}$
5. $\frac{-3}{4}m - 8 + \frac{5}{8}m = \frac{1}{2}m + 7$ **$m = -24$**
   $\frac{-3}{4}(-24) - 8 + \frac{5}{8}(-24) = \frac{1}{2}(-24) + 7$
   $-5 = -5$
6. $-0.2(x - 5) = 0.3(x - 6)$ **$x = 5.6$**
   $-0.2(5.6 - 5) = 0.3(5.6 - 6)$
   $-0.12 = -0.12$; infinitely many solutions

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Extra Practice

Name _____ Class _____ Date _____

Lesson 6 Using an Equation

Write and solve an equation for each problem. Use n for the variable.

1. Fifty-one less than the product of 7 and a number is 19. What is the number?
   $7n - 51 = 19$; $n = 10$
2. Ten less than the quotient of a number and 3 is 1. What is the number?
   $\frac{n}{3} - 10 = 1$; $n = 33$
3. Five more than the product of 17 and a number is 107. What is the number?
   $17n + 5 = 107$; $n = 6$
4. If the product of 11 and a number is decreased by 22, the result is 99. What is the number?
   $11n - 22 = 99$; $n = 11$
5. Three friends decide to order a large pizza that has 12 slices. Each person has the same number of slices and 3 are left over. How many slices did each person have?
   $3n + 3 = 12$; $n = 3$; $n = 3$ slices
6. Three friends went on a fishing trip. They caught 20 fish. If the first friend caught 8 fish and the second caught 3 times as many as the third, how many fish did the third friend catch?
   $8 + 3n + n = 20$; $n = 3$; the third friend caught 3 fish.

Lesson 7 Problem-Solving: Using a Table

Solve each equation. Then check the solution.

1. Joe has a rare baseball card. The longer he waits to sell the card, the more he can get for it. The card was worth $7 when he bought it. After 1 week, it was worth $9. Using the table below, determine how much Joe can sell the card for if he waits 10 weeks.

Weeks	1	2	3	4
Cost ($)	9	11	13	15

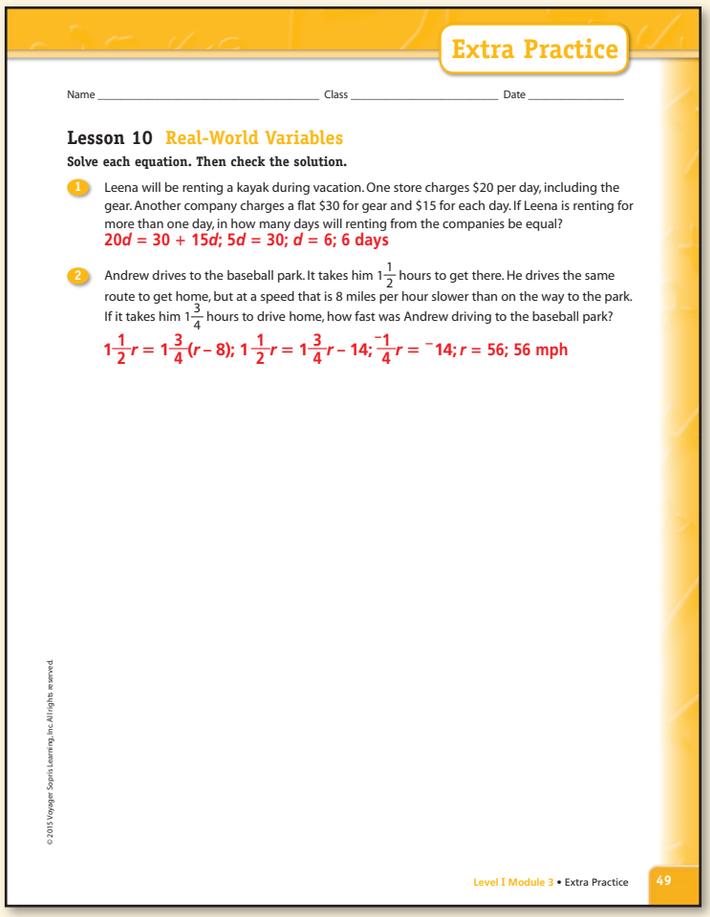
   $2n + 7$; $2(10) + 7 = 27$; $27

2. Yuri owns a restaurant and she wants to decorate it with origami cranes. She already has 12 cranes, but she needs more. The table shows the number of cranes she can fold every hour. How many cranes will she have after 6 hours?

Hours	1	2	3	4
Cranes	34	56	78	100

   $22n + 12 = 22(6) + 12 = 144$; 144 cranes

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Extra Practice

Name _____ Class _____ Date _____

Lesson 10 Real-World Variables

Solve each equation. Then check the solution.

1. Leena will be renting a kayak during vacation. One store charges $20 per day, including the gear. Another company charges a flat $30 for gear and $15 for each day. If Leena is renting for more than one day, in how many days will renting from the companies be equal?
   $20d = 30 + 15d$; $5d = 30$; $d = 6$; 6 days

2. Andrew drives to the baseball park. It takes him $1\frac{1}{2}$ hours to get there. He drives the same route to get home, but at a speed that is 8 miles per hour slower than on the way to the park. If it takes him $1\frac{3}{4}$ hours to drive home, how fast was Andrew driving to the baseball park?
   $1\frac{1}{2}r = 1\frac{3}{4}(r - 8)$; $1\frac{1}{2}r = 1\frac{3}{4}r - 14$; $\frac{-1}{4}r = {}^{-}14$; $r = 56$; 56 mph

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Glossary

algebraic expression
an expression that can contain one or more numerals, one or more variables, and one or more operations

Distributive Property
the property of distributing one operation over another, and the answer is the same

Examples: $a(b + c) = a \times b + a \times c$ and $a(b - c) = a \times b - a \times c$

variable
a letter or symbol that represents a number

Functions Part A

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Lesson PL1

Objective

To identify and graph a point on a coordinate plane

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **coordinate plane**
  a graph formed by two number lines intersecting at a right angle
- ***x*-axis**
  the horizontal number line of a coordinate plane
- ***y*-axis**
  the vertical number line of a coordinate plane
- **origin**
  the point where the *x*-axis and *y*-axis intersect
- **quadrants**
  the four sections formed by the intersection of the *x*-axis and *y*-axis; labeled with Roman numerals
- **ordered pair**
  a pair of numbers used to locate a point on a coordinate plane

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

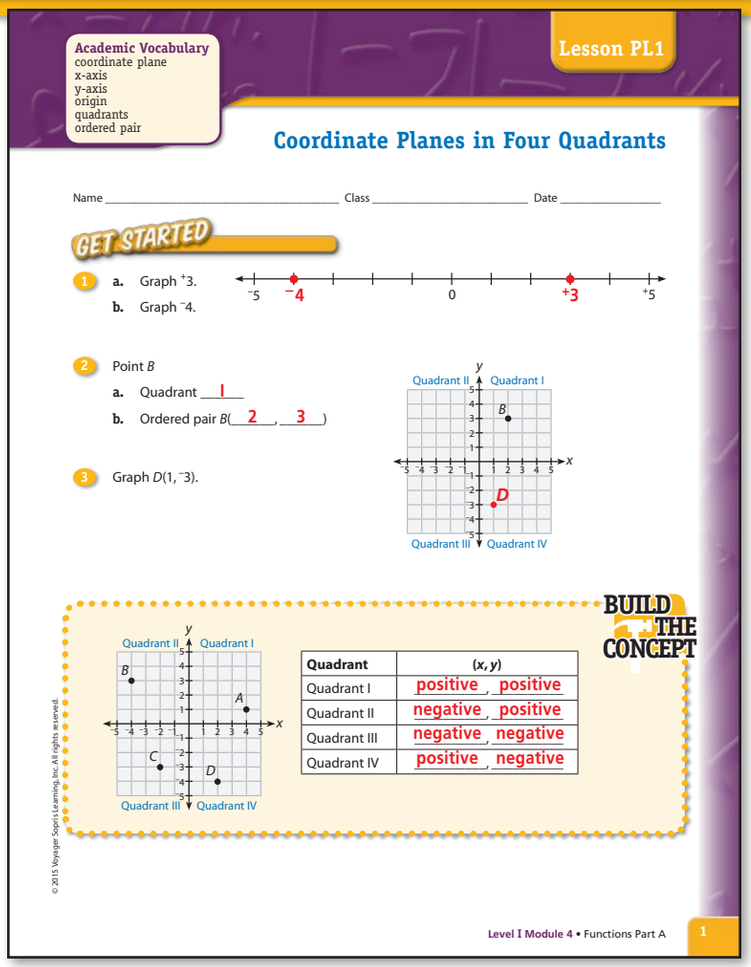
Problem 1

On a number line, positive integers are to the right of 0. How many units to the right of 0 is $^{+}3$? (3) **Graph $^{+}3$. Negative integers are to the left of 0. How many units to the left of 0 is $^{-}4$?** (4) **Graph $^{-}4$.**

MODEL NEW SKILLS

Problem 2

A coordinate plane is formed by two number lines intersecting at a right angle. The horizontal number line is the *x*-axis, and the vertical number line is the *y*-axis. The point where the *x*-axis and *y*-axis intersect is the origin.



Academic Vocabulary
coordinate plane
x-axis
y-axis
origin
quadrants
ordered pair

Lesson PL1

Coordinate Planes in Four Quadrants

Name _____ Class _____ Date _____

GET STARTED

1 a. Graph $^{+}3$.
b. Graph $^{-}4$.

2 Point *B*
a. Quadrant I
b. Ordered pair *B*(2 , 3)

3 Graph *D*(1, $^{-}3$).

BUILD THE CONCEPT

Quadrant	(*x*, *y*)
Quadrant I	positive , positive
Quadrant II	negative , positive
Quadrant III	negative , negative
Quadrant IV	positive , negative

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Level I Module 4 • Functions Part A 1

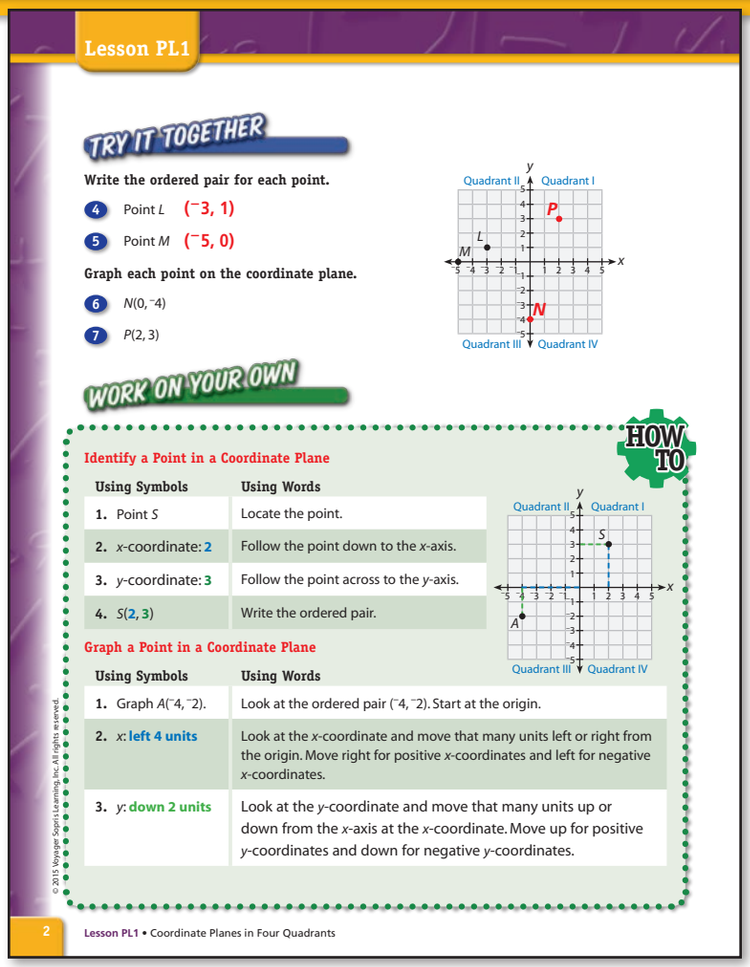
On the *x*-axis, the positive numbers are to the right of the origin, and the negative numbers are to the left of the origin. On the *y*-axis, the positive numbers are above the *x*-axis, and the negative numbers are below the *x*-axis.

Positive numbers do not usually have a positive sign in front of them. The integer $^{+}3$ is the same as 3. Negative numbers will always have a negative sign in front of them.

Notice that when the *x*-axis and *y*-axis intersect, four sections are formed. These sections are called quadrants. Look at the labeled coordinate plane. In which quadrant is *B* located? (Quadrant I)

An ordered pair is a pair of numbers used to locate a point on a coordinate plane. The origin is the ordered pair (0, 0). The first number is the *x*-coordinate, and it indicates how many units left or right of the *y*-axis the point is.

The second number is the *y*-coordinate, and it indicates how many units up or down from the *x*-axis the point is. Ordered pairs are always written with the *x*-coordinate first and the *y*-coordinate second.



To find the *x*-coordinate for *B*, follow *B* straight down to the *x*-axis. The *x*-coordinate of *B* is 2. To find the *y*-coordinate for *B*, follow *B* across to the *y*-axis. The *y*-coordinate of *B* is 3. The ordered pair for *B* is (2, 3).

Problem 3

Graph points on a coordinate plane by using the ordered pair. The first number of the ordered pair is the *x*-coordinate, and it indicates how many units to move left or right from the origin. The second number of the ordered pair is the *y*-coordinate, and it indicates how many units to move up or down from the *x*-axis.

What is the *x*-coordinate for *D*? (1) **On the *x*-axis, numbers to the right of the origin are positive and numbers to the left of the origin are negative. Start at the origin and move one unit to the right.**

What is the *y*-coordinate for *D*? (⁻3) **On the *y*-axis, numbers above the *x*-axis are positive and numbers below the *x*-axis are negative. Start at 1 on the *x*-axis and move three units down. Graph *D* at this location. Point *D* is in Quadrant IV.**

BUILD THE CONCEPT

Model how to interpret positions of points on a coordinate plane.

Look at the coordinate plane. What is the ordered pair for *A*? ((4, 1)) **In which quadrant is *A* located?** (Quadrant I) **The *x*-coordinate and the *y*-coordinate of *A* are both positive. Are all *x*- and *y*-coordinates in Quadrant I positive?** (yes) **What is the ordered pair for *B*?** ((⁻4, 3)) **In which quadrant is *B* located?** (Quadrant II) **The *x*-coordinate of *B* is negative and the *y*-coordinate of *B* is positive. Are all *x*-coordinates negative and all *y*-coordinates positive in Quadrant II?** (yes) **What is the ordered pair for *C*?** ((⁻2, ⁻3)) **In which quadrant is *C* located?** (Quadrant III) **The *x*-coordinate and the *y*-coordinate of *C* are both negative. Are all *x*- and *y*-coordinates in Quadrant III negative?** (yes) **What is the ordered pair for *D*?** ((2, ⁻4)) **In which quadrant is *D* located?** (Quadrant IV) **The *x*-coordinate of *D* is positive and the *y*-coordinate of *D* is negative. Are all *x*-coordinates positive and all *y*-coordinates negative in Quadrant IV?** (yes)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

Locate *L* on the coordinate plane. In which quadrant is *L*? (Quadrant II) **Follow the point down to the *x*-axis. What point on the *x*-axis lines up with *L*?** (⁻3) **What is the *x*-coordinate of *L*?** (⁻3) **Follow the point across to the *y*-axis. What point on the *y*-axis lines up with *L*?** (1) **What is the *y*-coordinate of *L*?** (1) **What is the ordered pair for *L*?** ((⁻3, 1))

Problem 5

Locate *M* on the coordinate plane. On which axis is this point located? (the *x*-axis) **What number on the *x*-axis lines up with *M*?** (⁻5) **Is ⁻5 the *x*-coordinate or the *y*-coordinate?** (*x*-coordinate) **Follow the point across to the *y*-axis. What point on the y-axis lines up with *M*?** (0) **What is the ordered pair for *M*?** ((⁻5, 0)) **Points on the *x*-axis have 0 as the *y*-coordinate.**

Problem 6

What is the *x*-coordinate for *N*? (0) **How many units from the origin is this point on the *x*-axis?** (0) **What is the *y*-coordinate for *N*?** ($^{-}4$) **Is *N* above or below the *x*-axis?** (below) **Graph (0, $^{-}4$). Point *N* is on the *y*-axis. Points that are on the *y*-axis have 0 as the *x*-coordinate.**

Problem 7

In which quadrant will *P* be located? (Quadrant I) **Why?** (Both coordinates are positive.) **Graph (2, 3).**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 8–23

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 8–19 **Reminder**

Errors may occur if students confuse the *x*-coordinate and the *y*-coordinate.

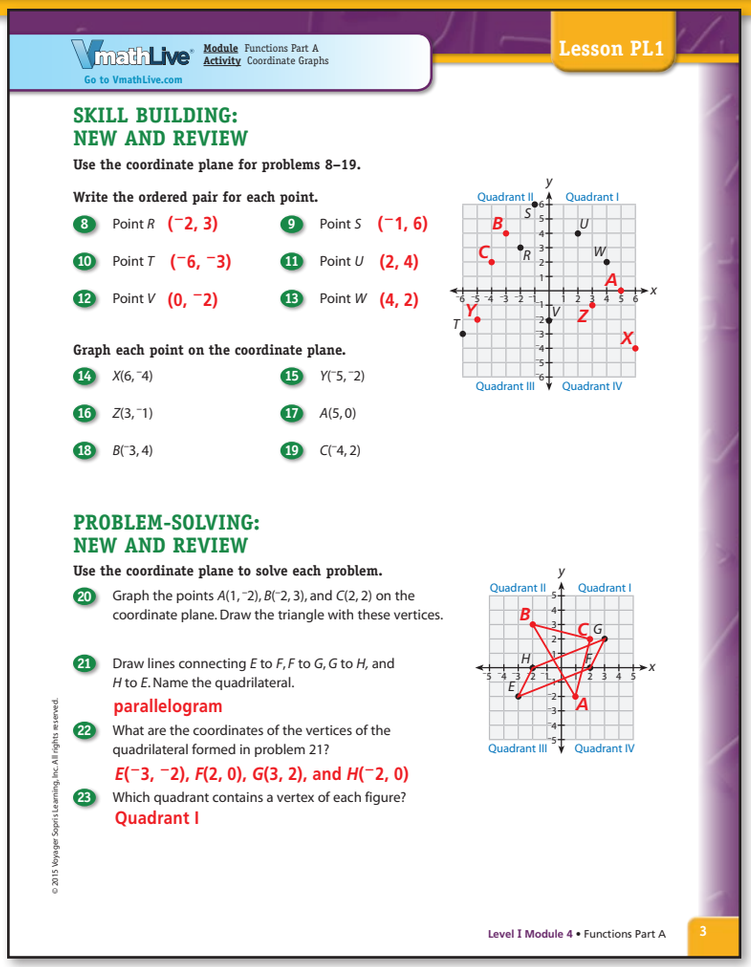
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2d: The student reversed the numbers in the ordered pair. Use Additional Resources in I.4.PL1 to reteach ordered pairs.



VmathLive Module Functions Part A Activity Coordinate Graphs
Go to VmathLive.com

Lesson PL1

SKILL BUILDING: NEW AND REVIEW

Use the coordinate plane for problems 8–19.

Write the ordered pair for each point.

8. Point *R* (−2, 3)
9. Point *S* (−1, 6)
10. Point *T* (−6, −3)
11. Point *U* (2, 4)
12. Point *V* (0, −2)
13. Point *W* (4, 2)

Graph each point on the coordinate plane.

14. *X*(6, −4)
15. *Y*(−5, −2)
16. *Z*(3, −1)
17. *A*(5, 0)
18. *B*(−3, 4)
19. *C*(−4, 2)

Quadrant II Quadrant I Quadrant III Quadrant IV

PROBLEM-SOLVING: NEW AND REVIEW

Use the coordinate plane to solve each problem.

20. Graph the points *A*(1, −2), *B*(−2, 3), and *C*(2, 2) on the coordinate plane. Draw the triangle with these vertices.
21. Draw lines connecting *E* to *F*, *F* to *G*, *G* to *H*, and *H* to *E*. Name the quadrilateral.
    parallelogram
22. What are the coordinates of the vertices of the quadrilateral formed in problem 21?
    E(−3, −2), *F*(2, 0), *G*(3, 2), and *H*(−2, 0)
23. Which quadrant contains a vertex of each figure?
    Quadrant I

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Level I Module 4 • Functions Part A 3

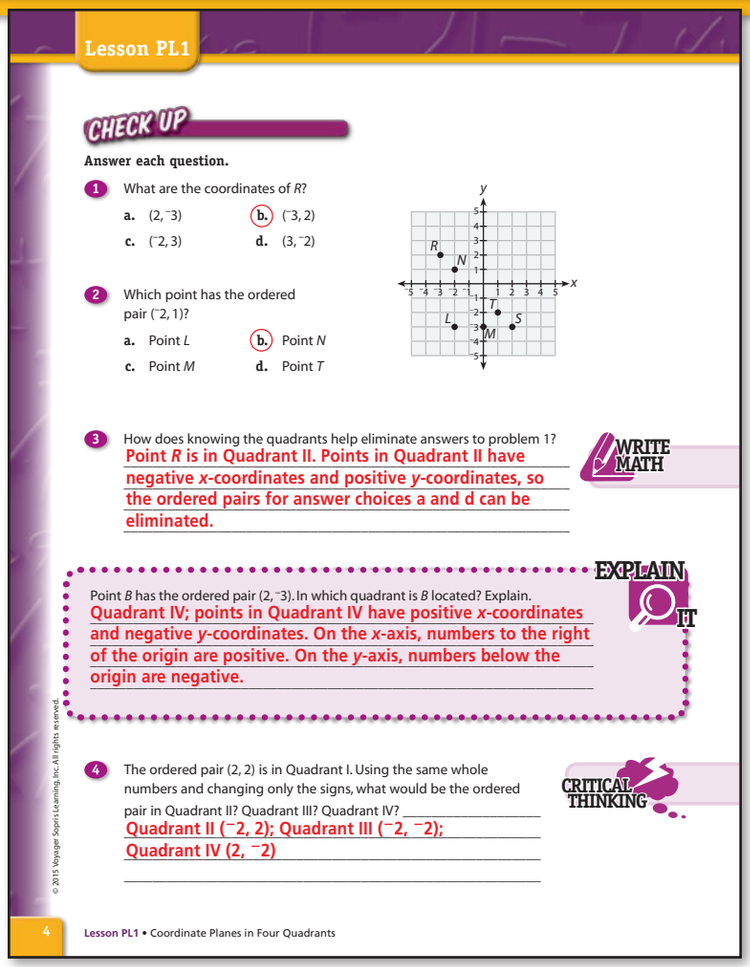
If student answered 1d: The student used the wrong sign for the integers in the ordered pair. Use Additional Resources in I.4.PL1 to reteach ordered pairs.

WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student the four quadrants and the coordinates of points found in each quadrant. Only answer choices that have a negative integer for the *x*-coordinate and a positive integer for the *y*-coordinate are possible answers for problem 1.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson PL1

CHECK UP

Answer each question.

1. What are the coordinates of *R*?
   a. (2, ⁻3) b. (⁻3, 2)
   c. (⁻2, 3) d. (3, ⁻2)

2. Which point has the ordered pair (⁻2, 1)?
   a. Point *L* b. Point *N*
   c. Point *M* d. Point *T*

3. How does knowing the quadrants help eliminate answers to problem 1?
   Point *R* is in Quadrant II. Points in Quadrant II have negative *x*-coordinates and positive *y*-coordinates, so the ordered pairs for answer choices a and d can be eliminated.

WRITE MATH

EXPLAIN IT

Point *B* has the ordered pair (2, ⁻3). In which quadrant is *B* located? Explain.
Quadrant IV; points in Quadrant IV have positive *x*-coordinates and negative *y*-coordinates. On the *x*-axis, numbers to the right of the origin are positive. On the *y*-axis, numbers below the origin are negative.

4. The ordered pair (2, 2) is in Quadrant I. Using the same whole numbers and changing only the signs, what would be the ordered pair in Quadrant II? Quadrant III? Quadrant IV?
   Quadrant II (⁻2, 2); Quadrant III (⁻2, ⁻2); Quadrant IV (2, ⁻2)

CRITICAL THINKING

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4 Lesson PL1 • Coordinate Planes in Four Quadrants

Technology

Have students think about math in their world, using an online search engine to find references describing how they could use a coordinate plane to describe locations. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Functions Part A
Activity: Coordinate Graphs

Vmath Reteach
Reteach Student Book Module 4 Lesson PL1
Reteach Teacher Edition Module 4 Lesson PL1

Extra Practice
Student Book page 57

English Language Learners

Use the VmathLive Animated Glossary to review the terms *coordinate plane, x-axis, y-axis, x-coordinate, y-coordinate,* and *ordered pair.* Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Students may be overwhelmed by the vocabulary in this lesson. Create a reference card and allow students to use it throughout the lesson. Give students a large coordinate plane. Label the following parts of the coordinate plane: Quadrant I, Quadrant II, Quadrant III, Quadrant IV, origin, *x*-axis, and *y*-axis.

Students with Special Needs

Use masking tape to create the *x*- and *y*-axes on the floor. Identify the horizontal line as the *x*-axis and the vertical line as the *y*-axis. Place a round sticker at the intersection to signify the origin.

Give each student an ordered pair. Have the student read the ordered pair and stand in the quadrant where the point would be located. List the four quadrants on the board. Write each student's ordered pair under the quadrant where the point is located.

Lesson PL2

Objective

To write a ratio, find an equivalent ratio including a ratio in simplest form, and compare ratios

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **ratio**
  a comparison of two quantities by division
- **equivalent ratios**
  ratios that can be written as equivalent fractions

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

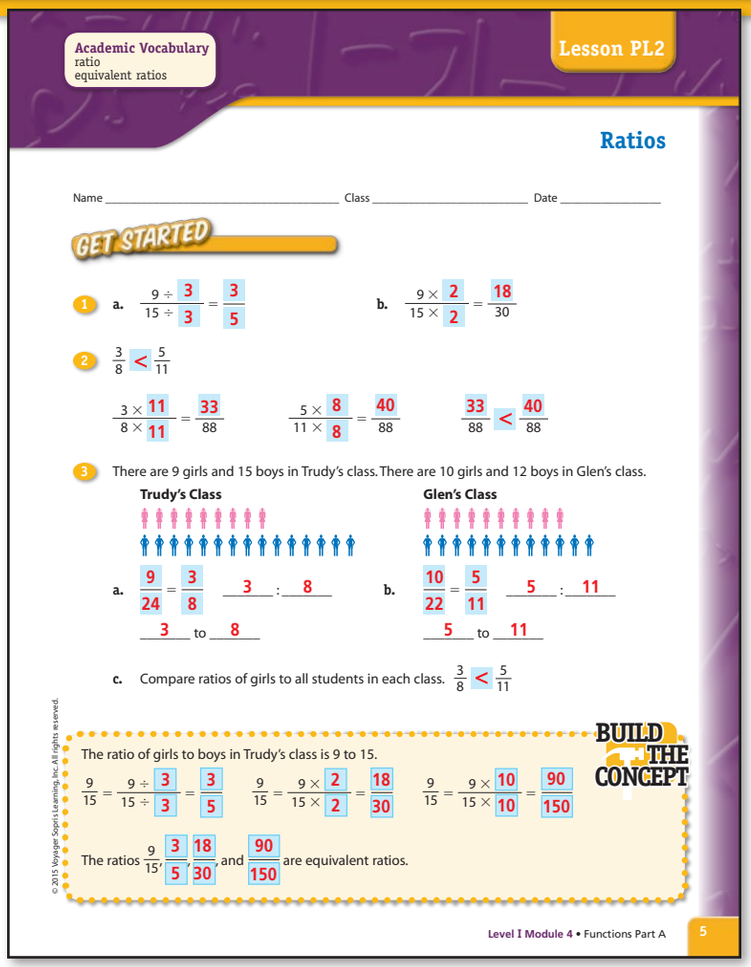
Problem 1

To write a fraction in simplest form, divide the numerator and denominator by their greatest common factor. What is the GCF of 9 and 15? (3) **What is 9 divided by 3?** (3) **What is 15 divided by 3?** (5) **What is $\frac{9}{15}$ written in simplest form?** ($\frac{3}{5}$)

To find equivalent fractions, multiply or divide the numerator and denominator of the fraction by the same number. What is 15 multiplied by to get 30? (2) **Multiply the numerator by 2. What fraction with a denominator of 30 is equivalent to $\frac{9}{15}$?** ($\frac{18}{30}$)

Problem 2

To compare fractions, rewrite the fractions using their least common denominator. What is the least common multiple of 8 and 11? (88) **The LCD of $\frac{3}{8}$ and $\frac{5}{11}$ is 88. What fraction with a denominator of 88 is equivalent to $\frac{3}{8}$?** ($\frac{33}{88}$) **What fraction with a denominator of 88 is equivalent to $\frac{5}{11}$?** ($\frac{40}{88}$) **Now that the denominators of the fractions are the same, compare the numerators. How does 33 compare with 40?** (Thirty-three is less than 40.) **Because 33 is less than 40, $\frac{33}{88}$ is less than $\frac{40}{88}$, and $\frac{3}{8}$ is less than $\frac{5}{11}$. Write the less than symbol between the two fractions.**



Academic Vocabulary
ratio
equivalent ratios

Lesson PL2

Ratios

Name _____ Class _____ Date _____

GET STARTED

1 a. $\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$ b. $\frac{9 \times 2}{15 \times 2} = \frac{18}{30}$

2 $\frac{3}{8} < \frac{5}{11}$

$\frac{3 \times 11}{8 \times 11} = \frac{33}{88}$ $\frac{5 \times 8}{11 \times 8} = \frac{40}{88}$ $\frac{33}{88} < \frac{40}{88}$

3 There are 9 girls and 15 boys in Trudy's class. There are 10 girls and 12 boys in Glen's class.

Trudy's Class

Glen's Class

a. $\frac{9}{24} = \frac{3}{8}$ 3 : 8 3 to 8

b. $\frac{10}{22} = \frac{5}{11}$ 5 : 11 5 to 11

c. Compare ratios of girls to all students in each class. $\frac{3}{8} < \frac{5}{11}$

BUILD THE CONCEPT

The ratio of girls to boys in Trudy's class is 9 to 15.

$\frac{9}{15} = \frac{9 \div 3}{15 \div 3} = \frac{3}{5}$ $\frac{9}{15} = \frac{9 \times 2}{15 \times 2} = \frac{18}{30}$ $\frac{9}{15} = \frac{9 \times 10}{15 \times 10} = \frac{90}{150}$

The ratios $\frac{9}{15}$, $\frac{3}{5}$, $\frac{18}{30}$, and $\frac{90}{150}$ are equivalent ratios.

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Level I Module 4 • Functions Part A 5

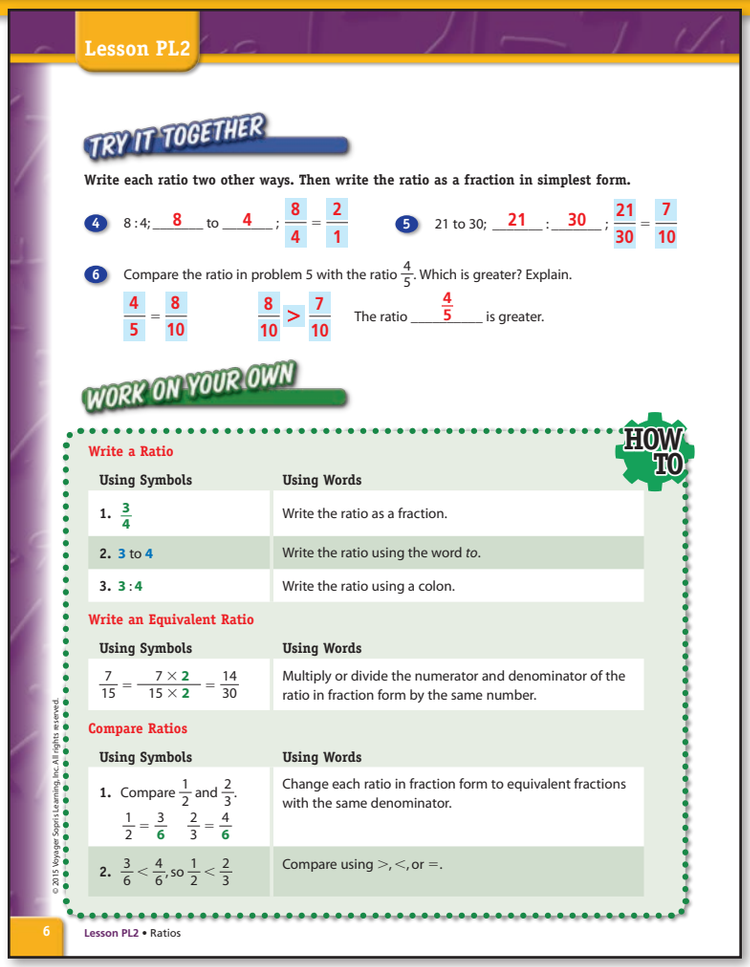
MODEL NEW SKILLS

Problem 3

A ratio is a comparison of two quantities by division. In problem 3, a ratio can be used to compare the number of girls to the number of all students in Trudy's class. How many girls are in Trudy's class? (9) **How many students?** (24)

Write the ratio of girls to all students as a fraction with the number of girls as the numerator and the number of students as the denominator. Then write the ratio in simplest form by writing the fraction in simplest form. What is the simplest form of $\frac{9}{24}$? ($\frac{3}{8}$) **The ratio in simplest form is written as a fraction, but it is read as *three to eight*, not *three-eighths*.**

The ratio can be written two other ways—with a colon or with the word *to*. Write the ratio 3 to 8 with a colon, then write it with the word *to*. Regardless of how the ratio is written, it is always read as *three to eight*. Now, write the ratio of girls to all students in Glen's class. The ratio of girls to all students is 10 to 22. Write the ratio as a fraction. Is this ratio written in simplest form? (no) **Write the ratio in simplest form by writing the fraction in simplest form.**



Lesson PL2

TRY IT TOGETHER

Write each ratio two other ways. Then write the ratio as a fraction in simplest form.

4. 8 : 4; 8 to 4; $\frac{8}{4} = \frac{2}{1}$

5. 21 to 30; 21 : 30; $\frac{21}{30} = \frac{7}{10}$

6. Compare the ratio in problem 5 with the ratio $\frac{4}{5}$. Which is greater? Explain.

$\frac{4}{5} = \frac{8}{10}$ $\frac{8}{10} > \frac{7}{10}$ The ratio $\frac{4}{5}$ is greater.

WORK ON YOUR OWN

HOW TO

Write a Ratio

Using Symbols	Using Words
1. $\frac{3}{4}$	Write the ratio as a fraction.
2. 3 to 4	Write the ratio using the word *to*.
3. 3 : 4	Write the ratio using a colon.

Write an Equivalent Ratio

Using Symbols	Using Words
$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$	Multiply or divide the numerator and denominator of the ratio in fraction form by the same number.

Compare Ratios

Using Symbols	Using Words
1. Compare $\frac{1}{2}$ and $\frac{2}{3}$. $\frac{1}{2} = \frac{3}{6}$ $\frac{2}{3} = \frac{4}{6}$	Change each ratio in fraction form to equivalent fractions with the same denominator.
2. $\frac{3}{6} < \frac{4}{6}$, so $\frac{1}{2} < \frac{2}{3}$	Compare using >, <, or =.

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6 Lesson PL2 • Ratios

What is the ratio of girls to all students written in simplest form? (5 to 11) **Write the ratio 5 to 11 with a colon and with the word *to*. Regardless of how the ratio is written, it is read as *five to eleven*.**

Compare the ratios of girls to all students in each class by comparing $\frac{3}{8}$ with $\frac{5}{11}$. To compare two ratios, rewrite the ratios in fraction form as equivalent fractions using the LCD of the fractions. The fraction $\frac{3}{8}$ was compared with $\frac{5}{11}$ in problem 2. How does $\frac{3}{8}$ compare with $\frac{5}{11}$? ($\frac{3}{8} < \frac{5}{11}$)

Because the ratio 3 to 8 is less than the ratio 5 to 11, the ratio of girls to all students in Trudy's class is less than the ratio of girls to all students in Glen's class.

BUILD THE CONCEPT

Model how to find equivalent ratios.

The ratio of girls to boys in Trudy's class is 9 to 15. Multiplication and division can be used to write ratios that are equivalent to this ratio. Equivalent ratios are ratios that can be written as equivalent fractions.

First, write the ratio as a fraction. Now, use division to write an equivalent ratio. What is the GCF of 9 and 15? (3) **So, write an equivalent fraction by dividing the numerator and denominator by 3. What is 9 divided by 3?** (3) **What is 15 divided by 3?** (5) **What is the equivalent ratio?** ($\frac{3}{5}$)

Next, use multiplication to write an equivalent ratio. Start with the original ratio written as the fraction $\frac{9}{15}$. Multiply the numerator and denominator by 2. What is 9 times 2? (18) **What is 15 times 2?** (30) **What is the equivalent ratio written as a fraction?** ($\frac{18}{30}$)

Finally, write one more equivalent ratio by multiplying the numerator and denominator of $\frac{9}{15}$ by 10. What is the equivalent ratio? ($\frac{90}{150}$) **So, the ratios $\frac{9}{15}$, $\frac{3}{5}$, $\frac{18}{30}$, and $\frac{90}{150}$ are all equivalent.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What are the three ways a ratio can be written? (as a fraction, with a colon, or with the word *to*) **How is the ratio in problem 4 written?** (with a colon) **How is the ratio written using the word *to*?** (8 to 4) **How is the ratio written as a fraction?** ($\frac{8}{4}$) **How is the simplest form of the ratio found?** (by writing the fraction form of the ratio in simplest form)

What is $\frac{8}{4}$ written in simplest form? ($\frac{2}{1}$) **Because this fraction represents a ratio, it is left as an improper fraction instead of changing it to a whole number.**

Problem 5

What is the ratio in problem 5? (21 to 30) **How is the ratio written using a colon?** (21 : 30) **How is the ratio written as a fraction?** ($\frac{21}{30}$) **What is $\frac{21}{30}$ written in simplest form?** ($\frac{7}{10}$)

Problem 6

Which ratios are to be compared? (21 to 30 and $\frac{4}{5}$) **How can two ratios be compared?** (by comparing the fraction forms of the ratios in simplest form) **What two fractions should be compared?** ($\frac{7}{10}$ and $\frac{4}{5}$) **How are the fractions compared?** (by rewriting each fraction using the LCD of the fractions, then comparing the numerators)

What is the LCD of $\frac{7}{10}$ and $\frac{4}{5}$? (10) **What fraction is equivalent to $\frac{4}{5}$ and has a denominator of 10?** ($\frac{8}{10}$) **Which is greater, $\frac{7}{10}$ or $\frac{8}{10}$?** ($\frac{8}{10}$) **Which ratio is greater?** ($\frac{4}{5}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Table** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a table. The instruction is immediately followed by application of the strategy in problem 18.

The problem asks students to find whether Tory or Sara has the greater ratio of airplane models to total models.

Have students start by making a table. Tory has 10 airplane models and 24 total models. Sara has 9 airplane models and 18 total models. Next, have students write ratios of airplane models to total models for each person.

Tory: $\frac{10}{24}$ Sara: $\frac{9}{18}$

Write each ratio in simplest form.

Tory: $\frac{5}{12}$ Sara: $\frac{1}{2}$

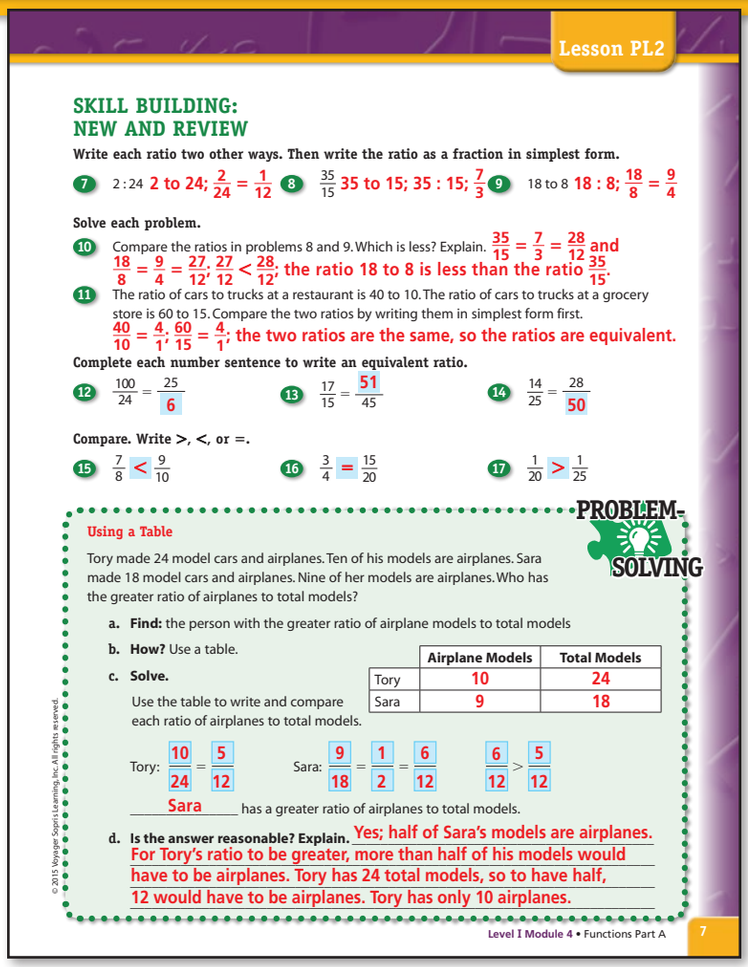
To compare the ratios, write each fraction as an equivalent fraction with the same denominator.

Tory: $\frac{5}{12}$ Sara: $\frac{6}{12}$

Because $\frac{6}{12}$ is greater than $\frac{5}{12}$, Sara has the greater ratio of airplanes to total models.

Problems 7–21

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson PL2

SKILL BUILDING: NEW AND REVIEW

Write each ratio two other ways. Then write the ratio as a fraction in simplest form.

7 2 : 24 2 to 24; $\frac{2}{24} = \frac{1}{12}$ 8 $\frac{35}{15}$ 35 to 15; 35 : 15; $\frac{7}{3}$ 9 18 to 8 18 : 8; $\frac{18}{8} = \frac{9}{4}$

Solve each problem.

10 Compare the ratios in problems 8 and 9. Which is less? Explain. $\frac{35}{15} = \frac{7}{3} = \frac{28}{12}$ and $\frac{18}{8} = \frac{9}{4} = \frac{27}{12}$, $\frac{27}{12} < \frac{28}{12}$; the ratio 18 to 8 is less than the ratio $\frac{35}{15}$.

11 The ratio of cars to trucks at a restaurant is 40 to 10. The ratio of cars to trucks at a grocery store is 60 to 15. Compare the two ratios by writing them in simplest form first.
$\frac{40}{10} = \frac{4}{1}$, $\frac{60}{15} = \frac{4}{1}$; the two ratios are the same, so the ratios are equivalent.

Complete each number sentence to write an equivalent ratio.

12 $\frac{100}{24} = \frac{25}{6}$ 13 $\frac{17}{15} = \frac{51}{45}$ 14 $\frac{14}{25} = \frac{28}{50}$

Compare. Write >, <, or =.

15 $\frac{7}{8} < \frac{9}{10}$ 16 $\frac{3}{4} = \frac{15}{20}$ 17 $\frac{1}{20} > \frac{1}{25}$

PROBLEM-SOLVING

Using a Table

Tory made 24 model cars and airplanes. Ten of his models are airplanes. Sara made 18 model cars and airplanes. Nine of her models are airplanes. Who has the greater ratio of airplanes to total models?

a. **Find:** the person with the greater ratio of airplane models to total models

b. **How?** Use a table.

c. **Solve.**

Use the table to write and compare each ratio of airplanes to total models.

	Airplane Models	Total Models
Tory	10	24
Sara	9	18

Tory: $\frac{10}{24} = \frac{5}{12}$ Sara: $\frac{9}{18} = \frac{1}{2} = \frac{6}{12}$ $\frac{6}{12} > \frac{5}{12}$

Sara has a greater ratio of airplanes to total models.

d. **Is the answer reasonable? Explain.** Yes; half of Sara's models are airplanes. For Tory's ratio to be greater, more than half of his models would have to be airplanes. Tory has 24 total models, so to have half, 12 would have to be airplanes. Tory has only 10 airplanes.

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Problem 18 Reminder

Encourage students to write each ratio as a fraction in simplest form first. Then remind them to write equivalent fractions using the LCD to compare the fractions.

CHECK UP

ASSESS INFORMALLY

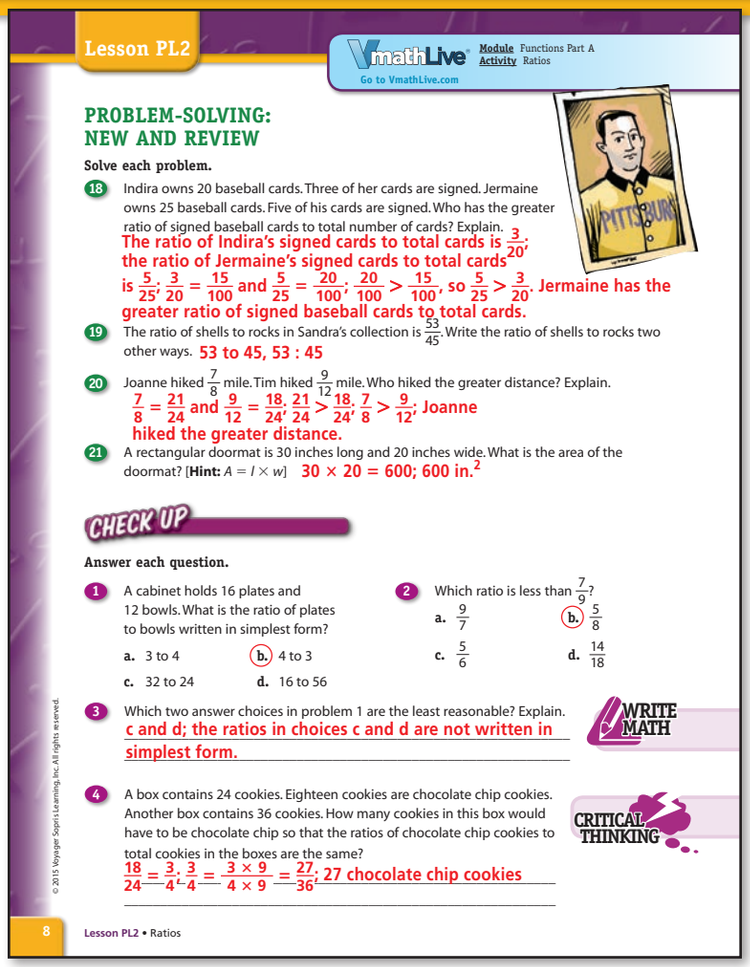
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student did not write the numbers of the ratio in the correct order. Remind the student that the order of the numbers in a ratio is important.

If student answered 1c or 1d: The student did not recognize that the ratios are not written in simplest form.

If student answered problem 2 incorrectly: The student chose a ratio that was greater than or equal to the given ratio. Use Additional Resources in I.4.PL2 to reteach how to compare ratios.



Lesson PL2

VmathLive Module Activity Functions Part A Ratios
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

18 Indira owns 20 baseball cards. Three of her cards are signed. Jermaine owns 25 baseball cards. Five of his cards are signed. Who has the greater ratio of signed baseball cards to total number of cards? Explain.
The ratio of Indira's signed cards to total cards is $\frac{3}{20}$; the ratio of Jermaine's signed cards to total cards is $\frac{5}{25}$. $\frac{3}{20} = \frac{15}{100}$ and $\frac{5}{25} = \frac{20}{100}$; $\frac{20}{100} > \frac{15}{100}$, so $\frac{5}{25} > \frac{3}{20}$. Jermaine has the greater ratio of signed baseball cards to total cards.

19 The ratio of shells to rocks in Sandra's collection is $\frac{53}{45}$. Write the ratio of shells to rocks two other ways. 53 to 45, 53 : 45

20 Joanne hiked $\frac{7}{8}$ mile. Tim hiked $\frac{9}{12}$ mile. Who hiked the greater distance? Explain.
$\frac{7}{8} = \frac{21}{24}$ and $\frac{9}{12} = \frac{18}{24}$; $\frac{21}{24} > \frac{18}{24}$; $\frac{7}{8} > \frac{9}{12}$; Joanne hiked the greater distance.

21 A rectangular doormat is 30 inches long and 20 inches wide. What is the area of the doormat? [**Hint:** $A = l \times w$] $30 \times 20 = 600$; 600 in.^{2}

CHECK UP

Answer each question.

1 A cabinet holds 16 plates and 12 bowls. What is the ratio of plates to bowls written in simplest form?
a. 3 to 4 (b.) 4 to 3 c. 32 to 24 d. 16 to 56

2 Which ratio is less than $\frac{7}{9}$?
a. $\frac{9}{7}$ (b.) $\frac{5}{8}$ c. $\frac{5}{6}$ d. $\frac{14}{18}$

3 Which two answer choices in problem 1 are the least reasonable? Explain. WRITE MATH
c and d; the ratios in choices c and d are not written in simplest form.

4 A box contains 24 cookies. Eighteen cookies are chocolate chip cookies. Another box contains 36 cookies. How many cookies in this box would have to be chocolate chip so that the ratios of chocolate chip cookies to total cookies in the boxes are the same? CRITICAL THINKING
$\frac{18}{24} = \frac{3}{4}$; $\frac{3}{4} = \frac{3 \times 9}{4 \times 9} = \frac{27}{36}$; 27 chocolate chip cookies

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8 Lesson PL2 • Ratios



If student answered problem 3 incorrectly: While students work together, remind the student that the ratios in answer choices c and d are not written in simplest form. Remind the student to read the problem carefully.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Functions Part A
Activity: Ratios

Vmath Reteach
Reteach Student Book Module 4 Lesson PL2
Reteach Teacher Edition Module 4 Lesson PL2

Extra Practice
Student Book page 57

English Language Learners

Use the VmathLive Animated Glossary to review the terms *ratio* and *equivalent fractions*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write the word *equivalent* on the board and underline the first four letters. Ask students to think of other words that they have seen before with this prefix. Possible words include *equal* and *equilateral*. Discuss what it means for ratios to be equivalent.

Write different amounts on the board and ask students to name equivalent amounts. For example, use the following: $3 (12 quarters), 3 bags of 5 marbles each (15 marbles), 1 hour (60 minutes), and so on.

Students with Special Needs

Help students understand that equivalent ratios are related by multiplication or division. Take students through this exercise to see this relationship. Distribute red and yellow counters to students. Explain that a recipe for strawberry lemonade calls for 3 parts of lemonade and 2 parts of strawberry juice. Tell students to use a red counter to represent each part of strawberry juice and a yellow counter to represent each part of lemonade.

Have students complete a table that lists the ingredients for multiple batches of strawberry lemonade. Ask students to discuss patterns that they see in their tables. Guide students to see the multiplication or division patterns. Be sure students understand that as long as the ratios of strawberry juice to lemonade stay the same, the batches of strawberry lemonade will have the same flavor.

Lesson 1

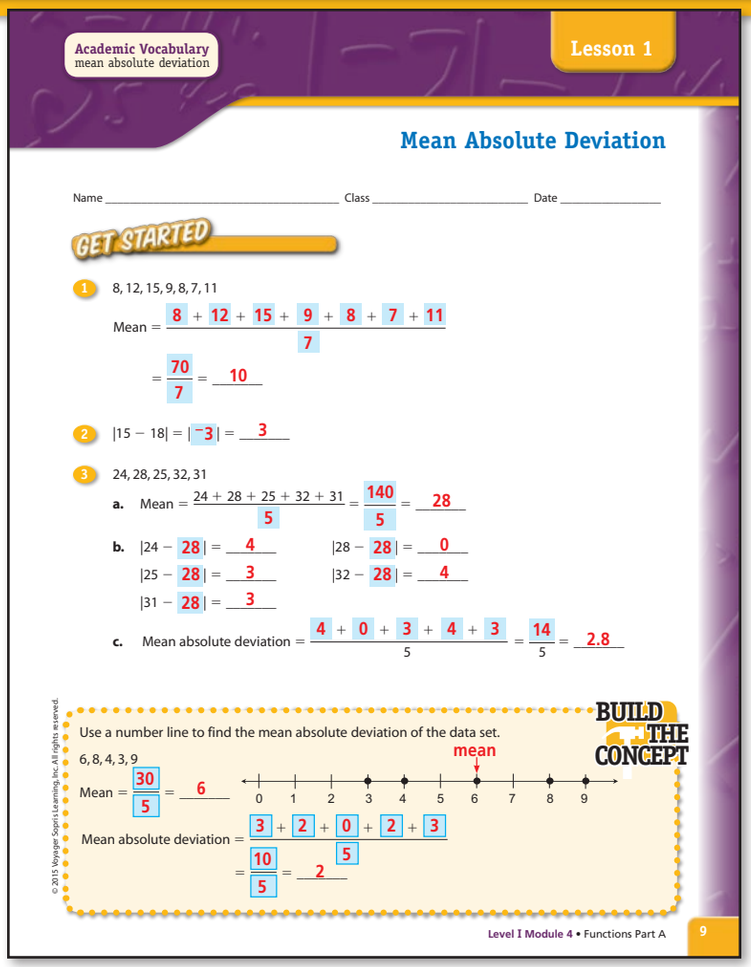
Objective

To find the mean absolute deviation of a data set

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **mean absolute deviation**
  the average distance a data value is from the mean in a data set


Academic Vocabulary
mean absolute deviation

Lesson 1

Mean Absolute Deviation

Name _____ Class _____ Date _____

GET STARTED

1. 8, 12, 15, 9, 8, 7, 11

Mean = $\frac{8 + 12 + 15 + 9 + 8 + 7 + 11}{7}$

$= \frac{70}{7} =$ 10

2. $|15 - 18| = |{}^{-}3| =$ 3

3. 24, 28, 25, 32, 31

a. Mean = $\frac{24 + 28 + 25 + 32 + 31}{5} = \frac{140}{5} =$ 28

b. $|24 - 28| =$ 4 $|28 - 28| =$ 0

$|25 - 28| =$ 3 $|32 - 28| =$ 4

$|31 - 28| =$ 3

c. Mean absolute deviation = $\frac{4 + 0 + 3 + 4 + 3}{5} = \frac{14}{5} =$ 2.8

BUILD THE CONCEPT

Use a number line to find the mean absolute deviation of the data set.

6, 8, 4, 3, 9

Mean = $\frac{30}{5} =$ 6

Mean absolute deviation = $\frac{3 + 2 + 0 + 2 + 3}{5}$

$= \frac{10}{5} =$ 2

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Look at the set of data. The mean of a data set describes the data by identifying the average data value. The mean of a data set is found by dividing the sum of the data values by the number of values. What sum must be found? (8 + 12 + 15 + 9 + 8 + 7 + 11) **How many data values are there?** (7)

What is 8 plus 12 plus 15 plus 9 plus 8 plus 7 plus 11? (70) **What is 70 divided by 7?** (10) **What is the mean, or average value, of this data set?** (10)

Problem 2

To evaluate the expression |15 − 18|, what should be done first? (Subtract 18 from 15.) **What is 15 minus 18?** ($^{-}3$) **What should be done next?** (Find the absolute value of $^{-}3$.) **What is the absolute value of $^{-}3$?** (3)

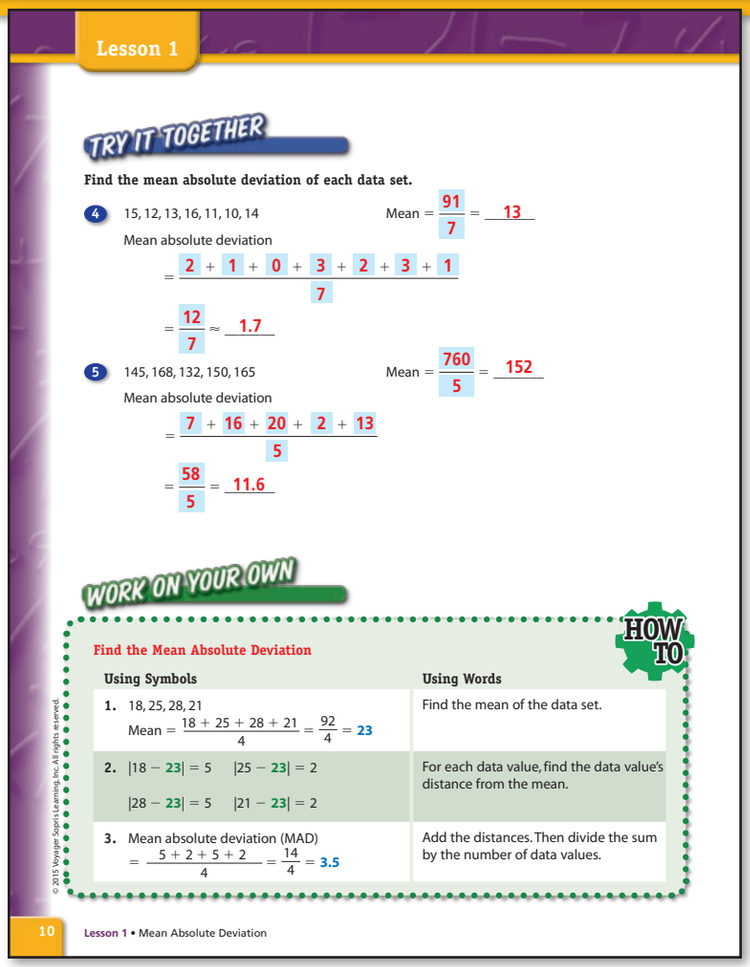
MODEL NEW SKILLS

Problem 3

Another number that describes a data set is the mean absolute deviation. The mean absolute deviation is the average distance a data value is from the mean.

To find the mean absolute deviation of a data set, first find the mean of the data set. How is the mean of a data set found? (by dividing the sum of the data values by the number of data values) **Look at part a. How many data values are there?** (5) **What is 24 plus 28 plus 25 plus 32 plus 31?** (140) **What is 140 divided by 5?** (28) **So, the mean of the data set is 28.**

Look at part b. The next step in finding the mean absolute deviation is to find each data value's distance from the mean. Remember that the distance between two points can be found by calculating the absolute value of the difference of the two points. To find the distance between the first data value, 24, and the mean, 28, subtract 28 from 24 and find the absolute value of the result. What is the absolute value of 24 minus 28? (4) **Now find the distance between the second data value, 28, and the mean, 28. What is the absolute value of 28 minus 28?** (0) **How is the distance between the third value and the mean found?** (by finding the absolute value of 25 minus 28) **What is the absolute value of 25 minus 28?** (3) **How is the distance between the fourth value and the mean found?** (by finding the absolute value of 32 minus 28) **What is the absolute value of 32 minus 28?** (4) **How is the distance between the last value and the mean found?** (by finding the absolute value of 31 minus 28) **What is the absolute value of 31 minus 28?** (3)



Lesson 1

TRY IT TOGETHER

Find the mean absolute deviation of each data set.

4 15, 12, 13, 16, 11, 10, 14 Mean = $\frac{91}{7}$ = 13

Mean absolute deviation

$= \frac{2 + 1 + 0 + 3 + 2 + 3 + 1}{7}$

$= \frac{12}{7} \approx 1.7$

5 145, 168, 132, 150, 165 Mean = $\frac{760}{5}$ = 152

Mean absolute deviation

$= \frac{7 + 16 + 20 + 2 + 13}{5}$

$= \frac{58}{5} = 11.6$

WORK ON YOUR OWN

HOW TO

Find the Mean Absolute Deviation

Using Symbols	Using Words
1. 18, 25, 28, 21 Mean = $\frac{18 + 25 + 28 + 21}{4} = \frac{92}{4} = 23$	Find the mean of the data set.
2. $\lvert 18 - 23 \rvert = 5$ $\lvert 25 - 23 \rvert = 2$ $\lvert 28 - 23 \rvert = 5$ $\lvert 21 - 23 \rvert = 2$	For each data value, find the data value's distance from the mean.
3. Mean absolute deviation (MAD) = $\frac{5 + 2 + 5 + 2}{4} = \frac{14}{4} = 3.5$	Add the distances. Then divide the sum by the number of data values.

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10 Lesson 1 • Mean Absolute Deviation

Look at part c. To find the mean absolute deviation, add the distances from part b and divide the sum by the number of data values. What numbers should be added? (4, 0, 3, 4, 3) **What is the sum of 4, 0, 3, 4, and 3?** (14) **What is 14 divided by 5?** (2.8) **The mean absolute deviation of the data set is 2.8. This means that the average distance between a data value and the mean is 2.8.**

BUILD THE CONCEPT

Model how to use a number line to find the mean absolute deviation of a data set.

Look at the data. How many values are there? (5) **To find the mean absolute deviation of the data, first find the mean. What is the sum of 6, 8, 4, 3, and 9?** (30) **What is 30 divided by 5?** (6) **The data values are shown on the number line. Draw an arrow that points to 6 on the number line, and label it as the mean.**

The mean absolute deviation is found by adding all of the data values' distances from the mean, then dividing the sum by the number of values. Look at the number line. How many units from 6 is 3? (3) **How many units from 6 is 4?** (2) **How many units from 6 is 6?** (0) **How many units from 6 is 8?** (2) **How many units from 6 is 9?** (3) **What distances are to be added?** (3, 2, 0, 2, 3) **What is the sum of 3, 2, 0, 2, and 3?** (10) **What is 10 divided by 5?** (2) **What is the mean absolute deviation of the data set?** (2) **So, the average distance between a data value and the mean is 2. According to the number line, what is the greatest distance between a point and the mean?** (3) **What is the least distance?** (0) **Because the average distance, 2, is between the least and greatest distances, 0 and 3, the answer makes sense.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What is the first step in finding the mean absolute deviation of a data set? (Find the mean of the data set.) **How many data values are there?** (7) **What is the sum of 15, 12, 13, 16, 11, 10, and 14?** (91) **What is 91 divided by 7?** (13) **What is the mean?** (13)

The next step in finding the mean absolute deviation is to find each data value's distance from the mean. What is the distance between the first data value, 15, and the mean, 13? (2) **What is the distance between the second data value, 12, and the mean, 13?** (1) **What is the distance between 13 and the mean?** (0) **What is the distance between 16 and the mean?** (3) **What is the distance between 11 and the mean?** (2) **What is the distance between 10 and the mean?** (3) **What is the distance between 14 and the mean?** (1)

Finally, add the distances and divide the sum by the number of data values. What numbers should be added? (2, 1, 0, 3, 2, 3, 1) **What is the sum of 2, 1, 0, 3, 2, 3, and 1?** (12) **How many data values are there?** (7) **What is 12 divided by 7 rounded to the nearest tenth?** (1.7) **What is the mean absolute deviation?** (1.7)

Problem 5

What is the first step in finding the mean absolute deviation of a data set? (Find the mean of the data set.) **How many data values are there?** (5) **What is the sum of 145, 168, 132, 150, and 165?** (760) **What is 760 divided by 5?** (152) **What is the mean?** (152)

Next, find each data value's distance from the mean. What is the distance between 145 and the mean, 152? (7) **What is the distance between 168 and the mean?** (16) **What is the distance between 132 and the mean?** (20) **What is the distance between 150 and the mean?** (2) **What is the distance between 165 and the mean?** (13)

Finally, add the distances and divide the sum by the number of data values. What numbers should be added? (7, 16, 20, 2, 13) **What is the sum of 7, 16, 20, 2, and 13?** (58) **How many data values are there?** (5) **What is the mean absolute deviation?** (11.6)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 6–14

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 6–9, 12, and 13 **Reminder**

When students are finding the mean absolute deviation, remind them that distance is always positive.

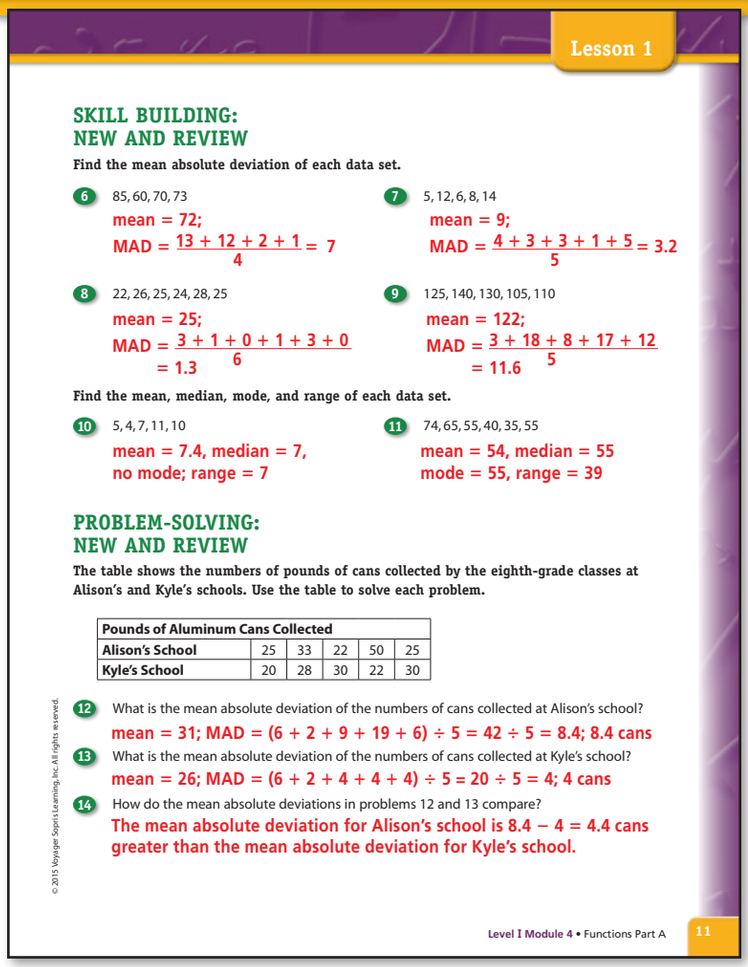
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student chose the correct sum of the distances but did not divide by the number of data values. Use Additional Resources in I.4.1 to review the steps for finding the mean absolute deviation.



Lesson 1

SKILL BUILDING: NEW AND REVIEW

Find the mean absolute deviation of each data set.

6. 85, 60, 70, 73
   mean = 72;
   MAD = $\frac{13 + 12 + 2 + 1}{4} = 7$

7. 5, 12, 6, 8, 14
   mean = 9;
   MAD = $\frac{4 + 3 + 3 + 1 + 5}{5} = 3.2$

8. 22, 26, 25, 24, 28, 25
   mean = 25;
   MAD = $\frac{3 + 1 + 0 + 1 + 3 + 0}{6} = 1.3$

9. 125, 140, 130, 105, 110
   mean = 122;
   MAD = $\frac{3 + 18 + 8 + 17 + 12}{5} = 11.6$

Find the mean, median, mode, and range of each data set.

10. 5, 4, 7, 11, 10
    mean = 7.4, median = 7,
    no mode; range = 7

11. 74, 65, 55, 40, 35, 55
    mean = 54, median = 55
    mode = 55, range = 39

PROBLEM-SOLVING: NEW AND REVIEW

The table shows the numbers of pounds of cans collected by the eighth-grade classes at Alison's and Kyle's schools. Use the table to solve each problem.

Pounds of Aluminum Cans Collected					
Alison's School	25	33	22	50	25
Kyle's School	20	28	30	22	30

12. What is the mean absolute deviation of the numbers of cans collected at Alison's school?
    mean = 31; MAD = (6 + 2 + 9 + 19 + 6) ÷ 5 = 42 ÷ 5 = 8.4; 8.4 cans

13. What is the mean absolute deviation of the numbers of cans collected at Kyle's school?
    mean = 26; MAD = (6 + 2 + 4 + 4 + 4) ÷ 5 = 20 ÷ 5 = 4; 4 cans

14. How do the mean absolute deviations in problems 12 and 13 compare?
    The mean absolute deviation for Alison's school is 8.4 − 4 = 4.4 cans greater than the mean absolute deviation for Kyle's school.

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If student answered 1b or 2d: The student chose the correct mean, but did not continue through the rest of the process for finding the mean absolute deviation. Use Additional Resources in I.4.1 to review the steps for finding the mean absolute deviation.

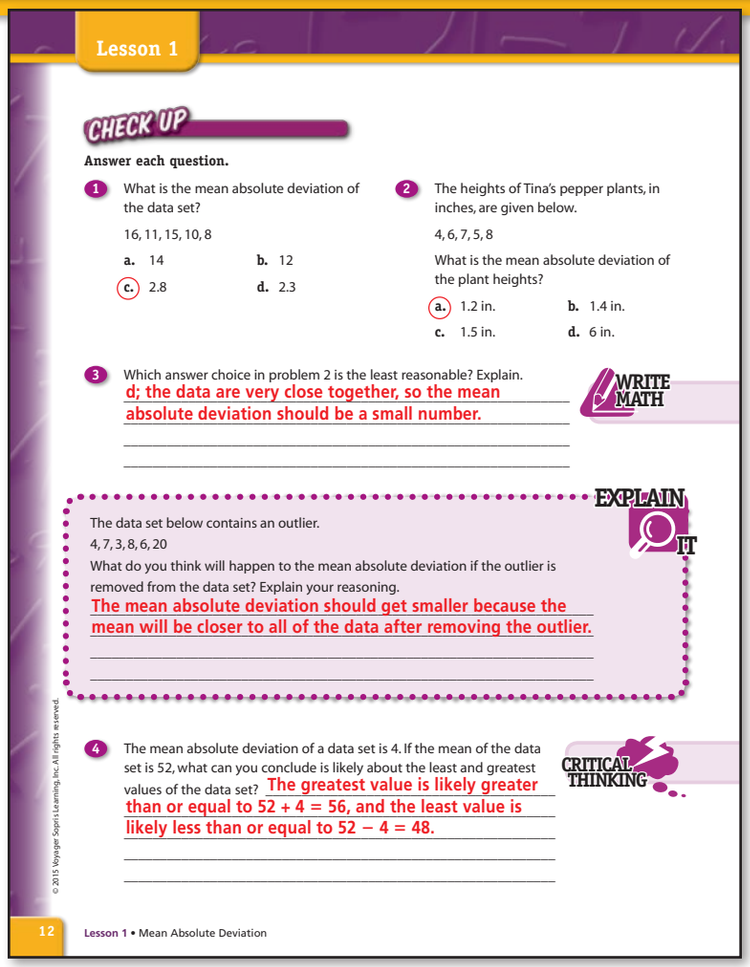
If student answered 1d or 2c: The student divided by an incorrect number of data values. Use Additional Resources in I.4.1 to review the steps for finding the mean absolute deviation.

If student answered 2b: The student calculated a distance incorrectly. Review with the student how to use absolute value to find a distance between two values.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review the meaning of the mean absolute deviation. Use Additional Resources in I.4.1 to review how the mean absolute deviation of a data set relates the mean and the data values.



Lesson 1

CHECK UP

Answer each question.

1. What is the mean absolute deviation of the data set?

   16, 11, 15, 10, 8

   a. 14 **b.** 12

   c. 2.8 **d.** 2.3

2. The heights of Tina's pepper plants, in inches, are given below.

   4, 6, 7, 5, 8

   What is the mean absolute deviation of the plant heights?

   a. 1.2 in. **b.** 1.4 in.

   c. 1.5 in. **d.** 6 in.

3. Which answer choice in problem 2 is the least reasonable? Explain.

   WRITE MATH

   d; the data are very close together, so the mean absolute deviation should be a small number.

EXPLAIN IT

The data set below contains an outlier.

4, 7, 3, 8, 6, 20

What do you think will happen to the mean absolute deviation if the outlier is removed from the data set? Explain your reasoning.

The mean absolute deviation should get smaller because the mean will be closer to all of the data after removing the outlier.

4. The mean absolute deviation of a data set is 4. If the mean of the data set is 52, what can you conclude is likely about the least and greatest values of the data set?

   CRITICAL THINKING

   The greatest value is likely greater than or equal to 52 + 4 = 56, and the least value is likely less than or equal to 52 − 4 = 48.

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EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 4 Lesson 1

Reteach Teacher Edition Module 4 Lesson 1

Extra Practice

Student Book page 58

English Language Learners

Review the process for finding the mean of a data set with students. Then have students look at the words *mean absolute deviation* and discuss how each word helps them remember the process for the calculation. Discuss different synonyms for *deviate* with students.

Students with Special Needs

If students are having difficulty grasping the concept of mean absolute deviation, demonstrate the relationship between this number and the values in the data set using a number line, as was done on the first page of the lesson. Demonstrate an example with a small number of data values so that students can easily find the distance between each data value and the mean and compare this distance with the mean absolute deviation. Point out that this number should be between the greatest and least distances.

Lesson 2

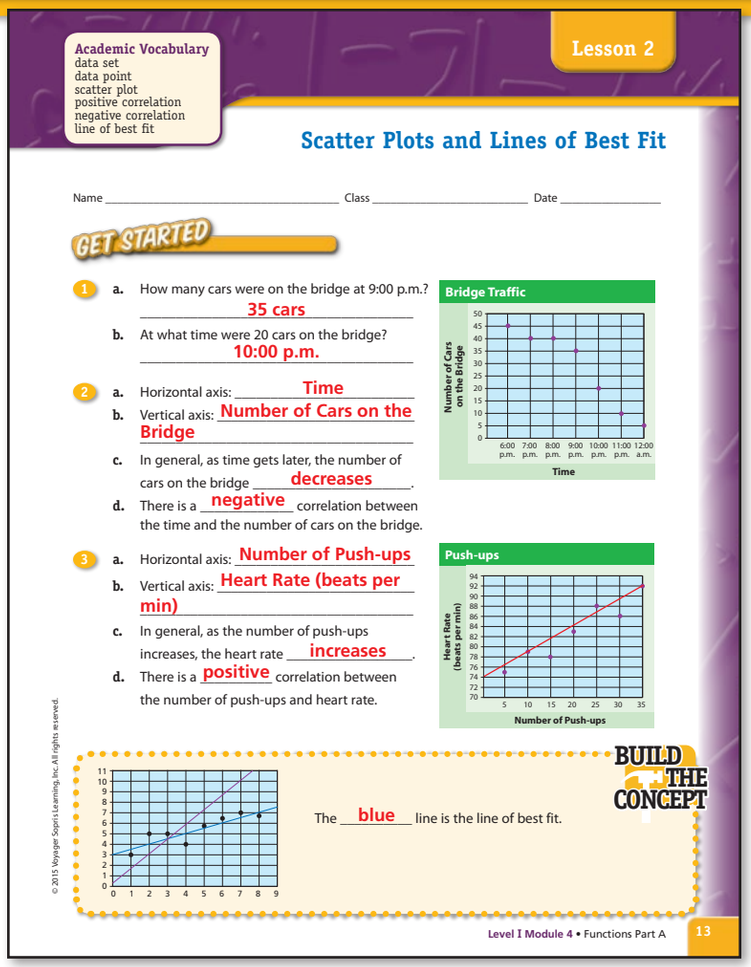
Objective

To interpret a scatter plot

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **data set**
  a collection of related values
- **data point**
  the representation of related values on a graph
- **scatter plot**
  a graph made by plotting points on a coordinate plane to show the relationship between two variables in a data set
- **positive correlation**
  the trend of values increasing as related values increase
- **negative correlation**
  the trend of values decreasing as related values increase
- **line of best fit**
  the line that comes closest to all points on a scatter plot



Academic Vocabulary
data set
data point
scatter plot
positive correlation
negative correlation
line of best fit

Lesson 2

Scatter Plots and Lines of Best Fit

Name _____ Class _____ Date _____

GET STARTED

1 a. How many cars were on the bridge at 9:00 p.m.? 35 cars

b. At what time were 20 cars on the bridge? 10:00 p.m.

2 a. Horizontal axis: Time

b. Vertical axis: Number of Cars on the Bridge

c. In general, as time gets later, the number of cars on the bridge decreases.

d. There is a negative correlation between the time and the number of cars on the bridge.

Bridge Traffic

Number of Cars on the Bridge

6:00 p.m. 7:00 p.m. 8:00 p.m. 9:00 p.m. 10:00 p.m. 11:00 p.m. 12:00 a.m.

Time

3 a. Horizontal axis: Number of Push-ups

b. Vertical axis: Heart Rate (beats per min)

c. In general, as the number of push-ups increases, the heart rate increases.

d. There is a positive correlation between the number of push-ups and heart rate.

Push-ups

Heart Rate (beats per min)

Number of Push-ups

BUILD THE CONCEPT

The blue line is the line of best fit.

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Level I Module 4 • Functions Part A 13

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What is the title of the graph in problem 1? (Bridge Traffic) **This graph shows numbers of cars on a bridge at various times of the evening. To determine the number of cars on the bridge at 9:00 p.m., locate the point for 9:00 p.m. and look across to the vertical axis. The point for 9:00 p.m. lines up with 35 on the vertical axis. How many cars were on the bridge at 9:00 p.m.?** (35 cars)

To find the time when 20 cars were on the bridge, locate 20 on the vertical axis. Follow the 20 grid line to the right to find a point that is on the line. Follow that point down to the horizontal axis. Which time is associated with the point? (10:00 p.m.) **At what time were 20 cars on the bridge?** (10:00 p.m.)

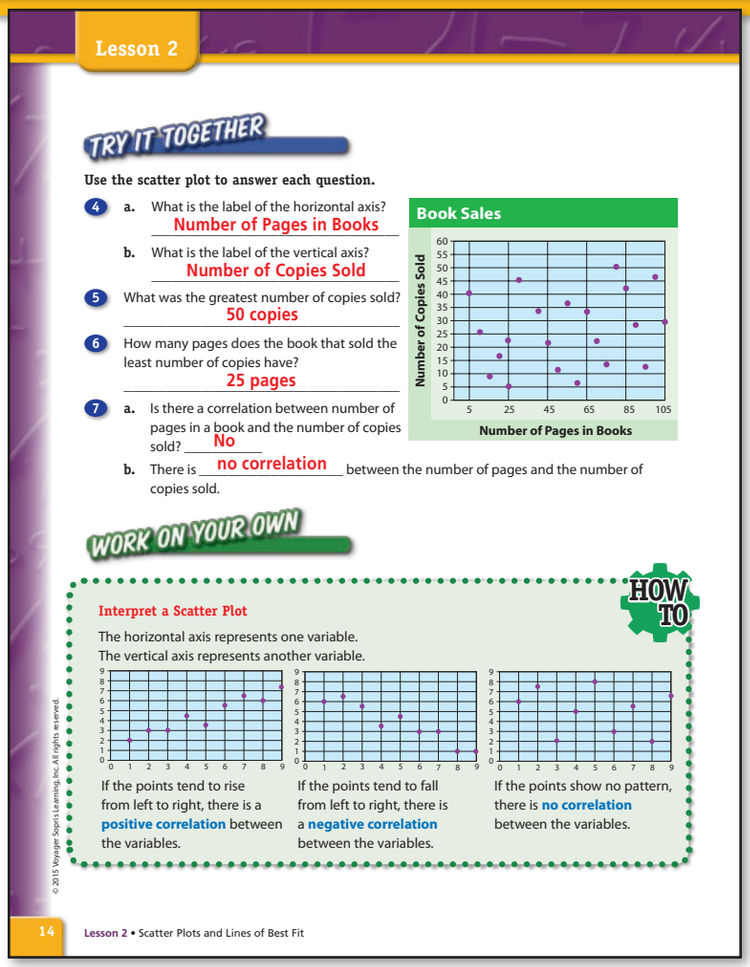
MODEL NEW SKILLS

Problem 2

A scatter plot is a graph made by plotting points on a coordinate plane to show the relationship between two variables in a data set. All the data that is represented on the graph is called the *data set*. Each point on the scatter plot is called a *data point*.

Look at the graph titled Bridge Traffic. What is the label of the horizontal axis? (Time) **What is the label of the vertical axis?** (Number of Cars on the Bridge) **These labels tell the variables, or the quantities that change. This graph is a scatter plot that shows the relationship between two variables: time in the evening and number of cars on the bridge.**

Look at the data points on the scatter plot. There is a pattern to the data points. The points tend to fall from left to right. In general, as the time gets later, what happens to the number of cars on the bridge? (The number of cars decreases.) **This is called a *negative correlation* between the time in the evening and the traffic on the bridge. As the hour of the evening increases, the number of cars on the bridge decreases.**


Lesson 2

TRY IT TOGETHER

Use the scatter plot to answer each question.

4 a. What is the label of the horizontal axis?
Number of Pages in Books

b. What is the label of the vertical axis?
Number of Copies Sold

5 What was the greatest number of copies sold?
50 copies

6 How many pages does the book that sold the least number of copies have?
25 pages

7 a. Is there a correlation between number of pages in a book and the number of copies sold? No

b. There is no correlation between the number of pages and the number of copies sold.

Book Sales

Number of Copies Sold

Number of Pages in Books

WORK ON YOUR OWN

HOW TO

Interpret a Scatter Plot

The horizontal axis represents one variable.
The vertical axis represents another variable.

If the points tend to rise from left to right, there is a **positive correlation** between the variables.

If the points tend to fall from left to right, there is a **negative correlation** between the variables.

If the points show no pattern, there is **no correlation** between the variables.

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14 Lesson 2 • Scatter Plots and Lines of Best Fit

Problem 3

Look at the scatter plot titled Push-ups. What is the label of the horizontal axis? (Number of Push-ups) **What is the label of the vertical axis?** (Heart Rate (beats per min)) **What variables are related in the data set represented on this graph?** (Number of Push-ups and Heart Rate) **This graph is a scatter plot that shows the relationship between the number of push-ups a person does and the person's heart rate.**

Look at the data points on the scatter plot. In general, do the data points tend to rise or fall from left to right? (rise) **What happens to the person's heart rate as the number of push-ups increases?** (The heart rate increases.) **When the data points tend to rise from left to right, there is a positive correlation. What type of correlation is there between the number of push-ups a person does and the person's heart rate?** (positive correlation)

When there is a positive or negative correlation of a data set, the relationship between the two variables can be estimated by using a straight line. The line that comes closest to all points on the scatter plot is called the *line of best fit*. One way to approximate the line of best fit is to lay a ruler or straightedge over the points and move the ruler until it looks closest to all the points.

Use a ruler or straightedge to draw the line of best fit on the scatter plot in problem 3.

BUILD THE CONCEPT

Model how to determine the line of best fit.

There are two lines shown on the scatter plot. The line of best fit is the line that comes closest to all the points on the scatter plot. Look at the point (1, 3). Is the blue line or the purple line closer to the point? (blue line) **Look at the point (2, 5). Which line is closer to the point?** (blue line) **Look at the point (3, 5). Which line is closer to the point?** (neither) **Look at the point (4, 4). Which line is closer to the point?** (blue line) **Look at the point (5, 5.8). Which line is closer to the point?** (blue line) **Look at the point (6, 6.5). Which line is closer to the point?** (blue line) **Look at the point (7, 7). Which line is closer to the point?** (blue line) **Look at the point (8, 6.8). Which line is closer to the point?** (blue line) **Overall, which line is closer to all the points?** (blue line) **So, the blue line is the line of best fit.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

This scatter plot is a graph that shows the relationship between two variables. How is this relationship shown? (by graphing the data points) **What is the title of the scatter plot?** (Book Sales) **What is the label of the horizontal axis of the scatter plot?** (Number of Pages in Books) **What is the label of the vertical axis?** (Number of Copies Sold)

Problem 5

In this scatter plot, the horizontal axis represents the variable, the numbers of pages in books. What variable does the vertical axis represent? (the number of copies sold) **Which data point on the scatter plot indicates the greatest number of copies sold?** (the highest point on the scatter plot)

What number on the vertical axis does the highest point line up with? (50) **What is the greatest number of copies sold?** (50 copies)

Problem 6

Which point on the scatter plot indicates the book that sold the least number of copies? (the lowest point) **How many pages did the book that sold the least number of copies have?** (25 pages)

Problem 7

Look at the data points. Is there any pattern in the data points? (no) **What correlation exists between the number of copies of a book sold and the number of pages the book has?** (no correlation) **Why is there no correlation?** (There is no pattern in the data points.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

PROBLEM-SOLVING This problem illustrates the **Finding a Pattern** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a pattern. The instruction is immediately followed by application of the strategy in problem 13.

Have students read the problem.

Students are to find a pattern from a scatter plot.

Have students describe the trend shown in the scatter plot.

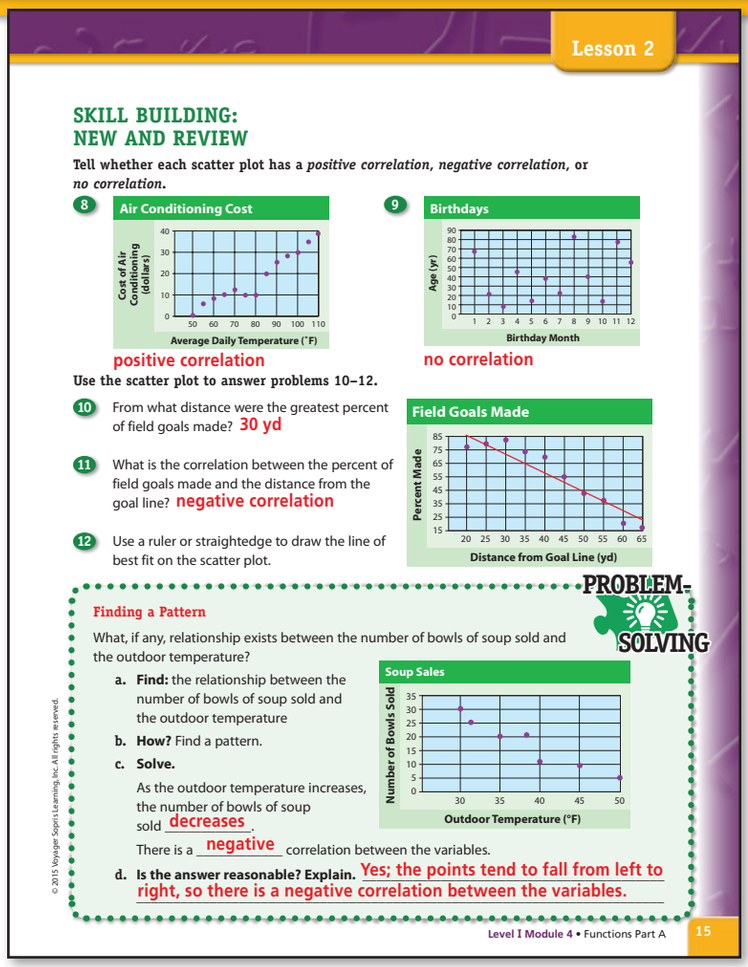
> The number of bowls of soup sold decreases as the temperature increases.

There is a negative correlation between the number of bowls of soup sold and the outdoor temperature.

The answer is reasonable because the points in the scatter plot fall from left to right, indicating a negative correlation.

Problems 8–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Problem 15 Reminder

This problem reviews finding the range. Students must determine the difference between the greatest number of cars sold and the least number of cars sold.

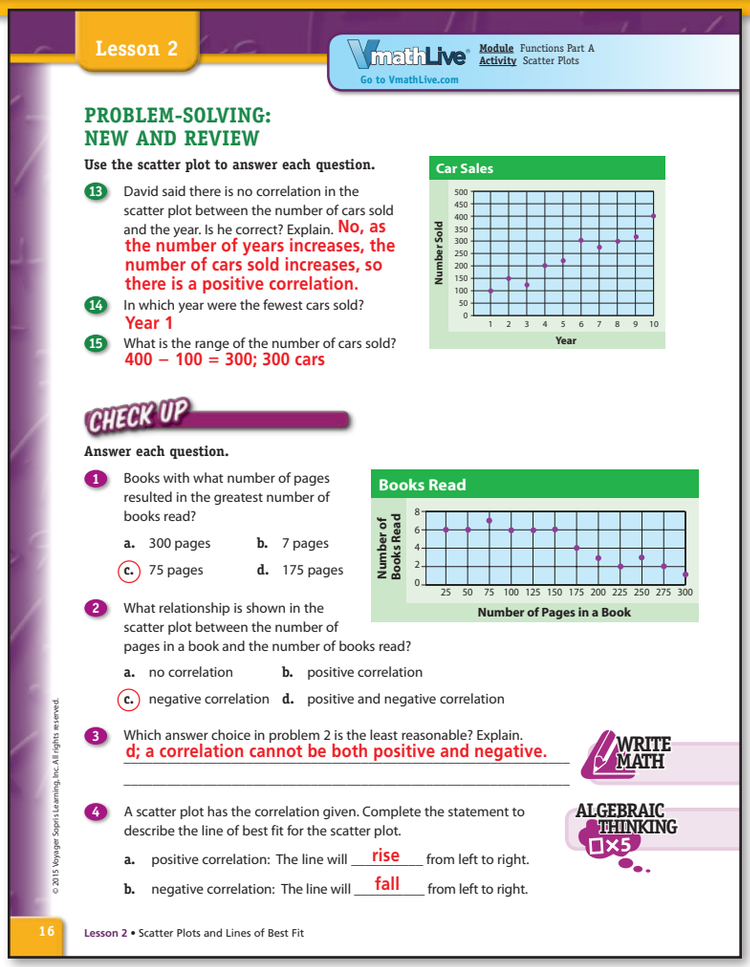
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student chose the number of pages that resulted in the least number of books read. Remind the student that the greatest number of books read will be represented by the highest data point.



Lesson 2

VmathLive Module Functions Part A Activity Scatter Plots Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Use the scatter plot to answer each question.

Car Sales

13 David said there is no correlation in the scatter plot between the number of cars sold and the year. Is he correct? Explain. No, as the number of years increases, the number of cars sold increases, so there is a positive correlation.

14 In which year were the fewest cars sold? Year 1

15 What is the range of the number of cars sold? 400 − 100 = 300; 300 cars

CHECK UP

Answer each question.

Books Read

1 Books with what number of pages resulted in the greatest number of books read?

a. 300 pages b. 7 pages
c. 75 pages d. 175 pages

2 What relationship is shown in the scatter plot between the number of pages in a book and the number of books read?

a. no correlation b. positive correlation
c. negative correlation d. positive and negative correlation

3 Which answer choice in problem 2 is the least reasonable? Explain. d; a correlation cannot be both positive and negative.

WRITE MATH

4 A scatter plot has the correlation given. Complete the statement to describe the line of best fit for the scatter plot.

a. positive correlation: The line will rise from left to right.

b. negative correlation: The line will fall from left to right.

ALGEBRAIC THINKING

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If student answered 1b: The student chose the greatest number of books rather than the number of pages associated with that number of books. Remind the student that in this scatter plot, the number of pages is on the horizontal axis.

If student answered 2a or 2b: The student did not understand that when the points tend to fall from left to right, a negative correlation is indicated. Use Additional Resources in I.4.2 to reteach how to interpret scatter plots.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, explain that a correlation cannot be both positive and negative. If the data points do not show a pattern, there is no correlation.

Technology



Have students review the Animated Glossary on VmathLive for reinforcement of math vocabulary. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Functions Part A
Activity: Scatter Plots

Vmath Reteach
Reteach Student Book Module 4 Lesson 2
Reteach Teacher Edition Module 4 Lesson 2

Extra Practice
Student Book page 58

English Language Learners

Be sure that students are reading graphs from left to right. Tell students that graphs are read like sentences are read. When interpreting a relationship displayed in a scatter plot, encourage students to ask themselves "as the horizontal values increase, what happens to the vertical values?"

Students with Special Needs

Play a matching game. Create several scatter plots, for example, two with a positive correlation, two with a negative correlation, and two with no correlation. Place each graph on an index card. Then write the corresponding titles separately on other index cards. Have students use the axes, data points, and correlations to help them determine which title corresponds with each graph.

Possible titles include:
Outside Temperature vs. Fuel Needed for Heat (negative correlation)
Length of Hair vs. Length of Fingernails (no correlation)
Lemonade Sales vs. Outside Temperature (positive correlation)

Lesson 3

Objective

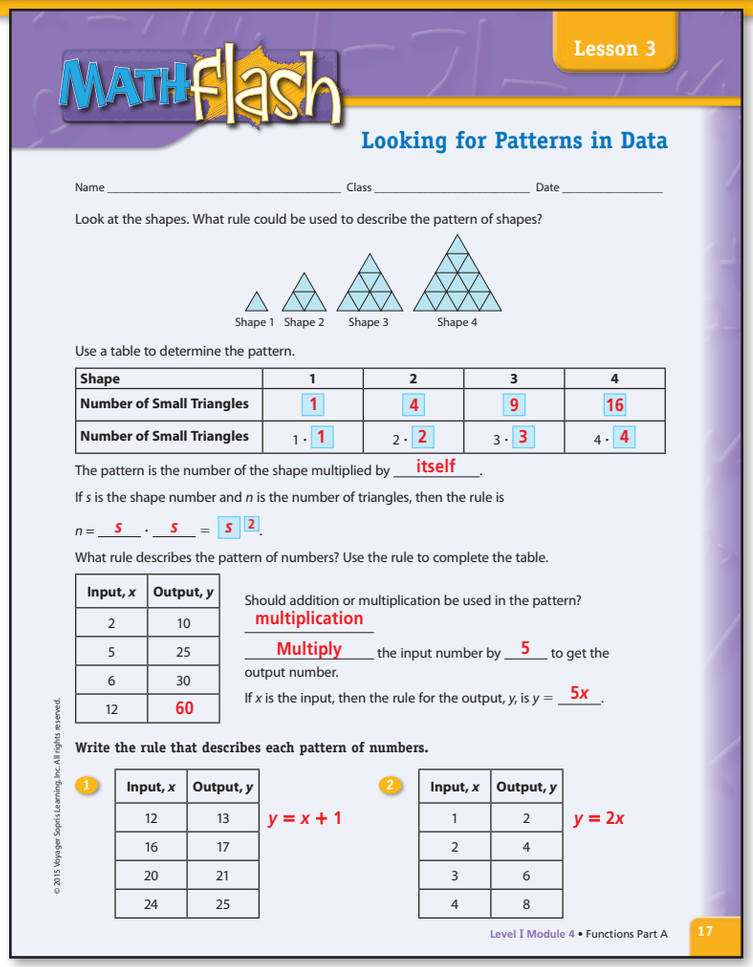
To write an algebraic rule for a pattern

Model the following skills for students.

The shapes shown form a pattern. How many small triangles make Shape 1? (1) **How many small triangles make Shape 2?** (4) **How many small triangles make Shape 3?** (9) **How many small triangles make Shape 4?** (16) **To write a rule for the pattern of shapes, write the relationship between the shape number and the number of small triangles. What number is 1 multiplied by to get 1?** (1) **What number is 2 multiplied by to get 4?** (2) **What number is 3 multiplied by to get 9?** (3) **What number is 4 multiplied by to get 16?** (4) **So, the number of small triangles is equal to the shape number multiplied by itself. If *s* is the shape number and *n* is the number of small triangles, what is the rule for the number of small triangles, *n*, in Shape *s*?** ($n = s \cdot s$, or s^2)

Work with students to complete this skill.

To write a rule for the numbers in the table, find the relationship between the input number and the output number. Because the output number is greater than the input number, use either addition or multiplication to write the rule. What number is added to 2 to get 10? (8) **If 8 is added to 5, is the result 25?** (no) **So, the rule does not use addition. What number is 2 multiplied by to get 10?** (5) **Is this true for the other values of *x*?** (yes) **So, multiply the input by 5 to get the output. If *x* is the input, what is the rule for the output, *y*?** ($y = 5x$) **What is the missing value in the table?** (60)



MATH Flash

Lesson 3

Looking for Patterns in Data

Name _____ Class _____ Date _____

Look at the shapes. What rule could be used to describe the pattern of shapes?

Shape 1 Shape 2 Shape 3 Shape 4

Use a table to determine the pattern.

Shape	1	2	3	4
Number of Small Triangles	1	4	9	16
Number of Small Triangles	1 · 1	2 · 2	3 · 3	4 · 4

The pattern is the number of the shape multiplied by itself.

If *s* is the shape number and *n* is the number of triangles, then the rule is

$n = s \cdot s = s^2$.

What rule describes the pattern of numbers? Use the rule to complete the table.

Input, *x*	Output, *y*
2	10
5	25
6	30
12	60

Should addition or multiplication be used in the pattern?

multiplication

Multiply the input number by 5 to get the output number.

If *x* is the input, then the rule for the output, *y*, is $y = 5x$.

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Write the rule that describes each pattern of numbers.

1

Input, *x*	Output, *y*
12	13
16	17
20	21
24	25

$y = x + 1$

2

Input, *x*	Output, *y*
1	2
2	4
3	6
4	8

$y = 2x$

Math Flash

Lesson 4

Objective

To identify the inverse of a relation in a table and as ordered pairs

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **relation**
  a pairing of numbers in one set, the domain, with numbers in another set, the range
- **domain**
  the set of all the first elements of the ordered pairs of a relation
- **range**
  the set of all the second elements of the ordered pairs of a relation

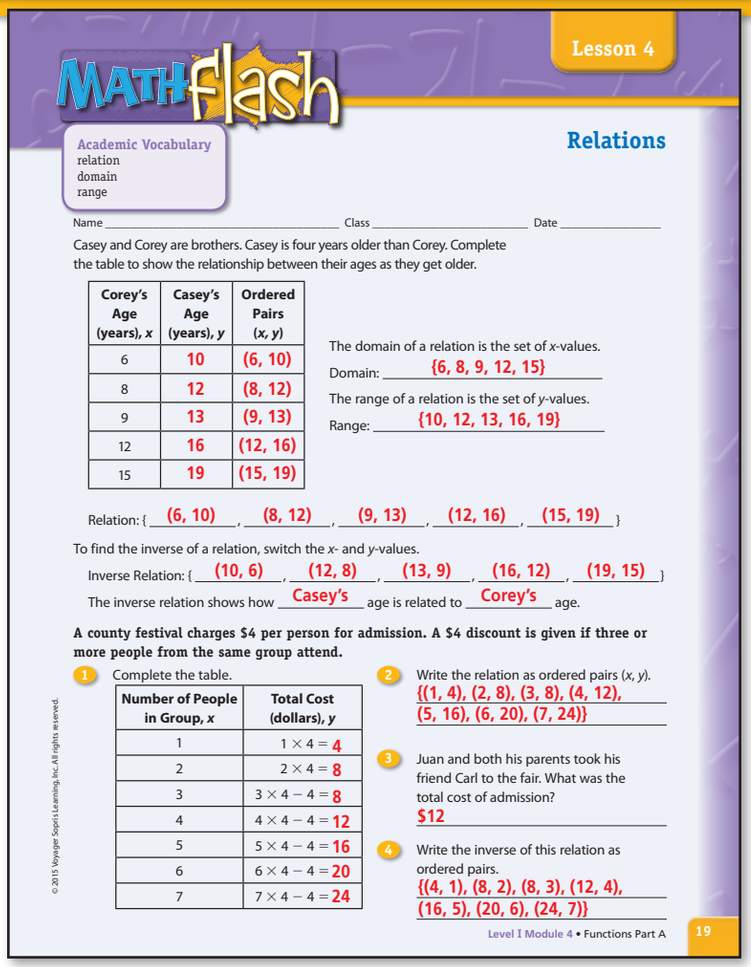
Model the following skills for students.

Casey is four years older than Corey. Complete the table that shows the relationship between their ages. When Corey is 6 years old, how old is Casey? (10) **When Corey is 8?** (12) **When Corey is 9?** (13) **When Corey is 12?** (16) **When Corey is 15?** (19)

A relation is a pairing of numbers in one set, the domain, with numbers in another set, the range. In this relation, the set of Corey's ages is the domain, and the set of Casey's ages is the range. What is the domain of this relation? ({6, 8, 9, 12, 15}) **What is the range of this relation?** ({10, 12, 13, 16, 19}) **Complete the table to write each pair of ages as an ordered pair. A relation can be written as a set of ordered pairs. What is the relation?** ({(6, 10), (8, 12), (9, 13), (12, 16), (15, 19)})

Work with students to complete this skill.

The inverse of a relation is found by switching the *x*- and *y*-values in the ordered pairs. What is the inverse of this relation? ({(10, 6), (12, 8), (13, 9), (16, 12), (19, 15)}) **The inverse relation shows how Casey's age is related to Corey's age.**



Math Flash — Lesson 4

Relations

Academic Vocabulary
relation
domain
range

Name _____ Class _____ Date _____

Casey and Corey are brothers. Casey is four years older than Corey. Complete the table to show the relationship between their ages as they get older.

Corey's Age (years), *x*	Casey's Age (years), *y*	Ordered Pairs (*x*, *y*)
6	10	(6, 10)
8	12	(8, 12)
9	13	(9, 13)
12	16	(12, 16)
15	19	(15, 19)

The domain of a relation is the set of *x*-values.
Domain: {6, 8, 9, 12, 15}

The range of a relation is the set of *y*-values.
Range: {10, 12, 13, 16, 19}

Relation: { (6, 10) , (8, 12) , (9, 13) , (12, 16) , (15, 19) }

To find the inverse of a relation, switch the *x*- and *y*-values.

Inverse Relation: { (10, 6) , (12, 8) , (13, 9) , (16, 12) , (19, 15) }

The inverse relation shows how Casey's age is related to Corey's age.

A county festival charges $4 per person for admission. A $4 discount is given if three or more people from the same group attend.

1. Complete the table.

Number of People in Group, *x*	Total Cost (dollars), *y*
1	1 × 4 = 4
2	2 × 4 = 8
3	3 × 4 − 4 = 8
4	4 × 4 − 4 = 12
5	5 × 4 − 4 = 16
6	6 × 4 − 4 = 20
7	7 × 4 − 4 = 24

2. Write the relation as ordered pairs (*x*, *y*).
{(1, 4), (2, 8), (3, 8), (4, 12), (5, 16), (6, 20), (7, 24)}

3. Juan and both his parents took his friend Carl to the fair. What was the total cost of admission?
$12

4. Write the inverse of this relation as ordered pairs.
{(4, 1), (8, 2), (8, 3), (12, 4), (16, 5), (20, 6), (24, 7)}

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Level I Module 4 • Functions Part A 19

Lesson 5

Objective

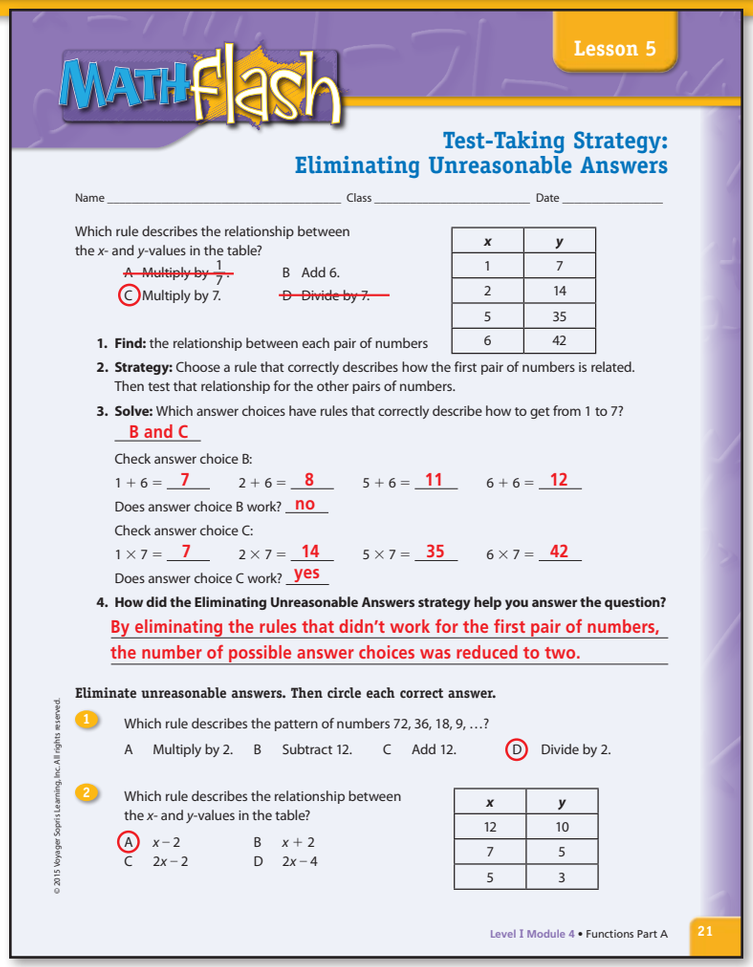
To recognize and eliminate unreasonable answers in a multiple-choice problem

Model the following skills for students.

Read the problem. The correct answer is a rule that works for each pair of *x*- and *y*-values in the table. Look at the table. Which rules work for the first pair of values for *x* and *y*, 1 and 7? (Add 6 or multiply by 7.) **What are the answer choices for these rules?** (answer choices B and C) **Because the rules for answer choices A and D won't work for the first pair, cross them out.**

Check the rule for answer choice B with the other pairs of values. What is 2 plus 6? (8) **What is 5 plus 6?** (11) **What is 6 plus 6?** (12) **Does this rule work for all pairs of values?** (no) **Check the rule for answer choice C next. What is 2 times 7?** (14) **What is 5 times 7?** (35) **What is 6 times 7?** (42) **Does this rule work for all pairs of values?** (yes) **Circle the correct answer choice.**

How did using the Eliminating Unreasonable Answers strategy help you answer the question? (By eliminating the rules that didn't work for the first pair of numbers, the number of possible answer choices was reduced to two.)



MATH Flash — Lesson 5

Test-Taking Strategy: Eliminating Unreasonable Answers

Name _____ Class _____ Date _____

Which rule describes the relationship between the *x*- and *y*-values in the table?

~~A Multiply by $\frac{1}{7}$.~~ B Add 6.

(C) Multiply by 7. ~~D Divide by 7.~~

x	*y*
1	7
2	14
5	35
6	42

1. **Find:** the relationship between each pair of numbers
2. **Strategy:** Choose a rule that correctly describes how the first pair of numbers is related. Then test that relationship for the other pairs of numbers.
3. **Solve:** Which answer choices have rules that correctly describe how to get from 1 to 7?
   B and C

   Check answer choice B:

   1 + 6 = 7 2 + 6 = 8 5 + 6 = 11 6 + 6 = 12

   Does answer choice B work? no

   Check answer choice C:

   1 × 7 = 7 2 × 7 = 14 5 × 7 = 35 6 × 7 = 42

   Does answer choice C work? yes
4. **How did the Eliminating Unreasonable Answers strategy help you answer the question?**
   By eliminating the rules that didn't work for the first pair of numbers, the number of possible answer choices was reduced to two.

Eliminate unreasonable answers. Then circle each correct answer.

1. Which rule describes the pattern of numbers 72, 36, 18, 9, …?

   A Multiply by 2. B Subtract 12. C Add 12. (D) Divide by 2.

2. Which rule describes the relationship between the *x*- and *y*-values in the table?

   (A) $x - 2$ B $x + 2$

   C $2x - 2$ D $2x - 4$

x	*y*
12	10
7	5
5	3

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Technology

Using an online search engine, have students or partners research careers that would need to find the relationship between *x*- and *y*-values. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

Lesson 6

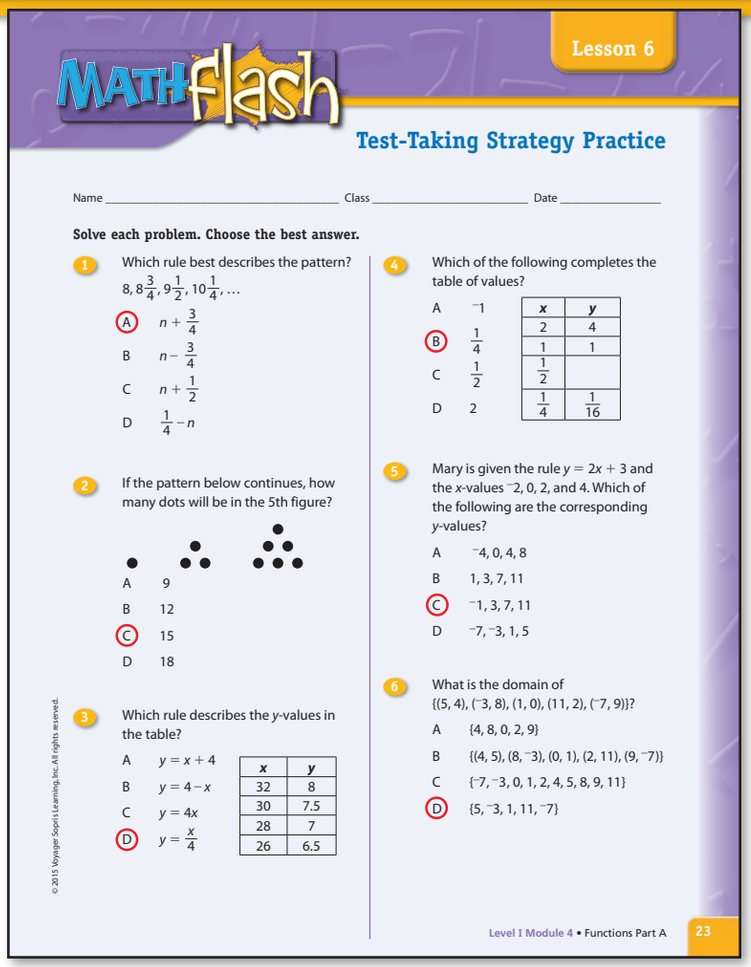
Objective

To use a test-taking strategy to solve a multiple-choice problem

Model the following skills for students.

The problems on pages 23 and 24 give you a chance to practice what you've learned about eliminating unreasonable answers to solve multiple-choice problems. At the same time, you will review the math vocabulary and skills related to linear equations, relations, and patterns.

Make sure you read each problem carefully so that you understand what is being asked. Then analyze the number pairs to determine how they might be related. Next, test your assumptions. After you determine an answer, check to see if there is a multiple-choice answer that expresses your answer.



MATH Flash

Lesson 6

Test-Taking Strategy Practice

Name _____ Class _____ Date _____

Solve each problem. Choose the best answer.

1. Which rule best describes the pattern?
$8, 8\frac{3}{4}, 9\frac{1}{2}, 10\frac{1}{4}, \ldots$

(A) $n + \frac{3}{4}$
B $n - \frac{3}{4}$
C $n + \frac{1}{2}$
D $\frac{1}{4} - n$

2. If the pattern below continues, how many dots will be in the 5th figure?

A 9
B 12
(C) 15
D 18

3. Which rule describes the *y*-values in the table?

A $y = x + 4$
B $y = 4 - x$
C $y = 4x$
(D) $y = \frac{x}{4}$

x	*y*
32	8
30	7.5
28	7
26	6.5

4. Which of the following completes the table of values?

A $^{-}1$
(B) $\frac{1}{4}$
C $\frac{1}{2}$
D 2

x	*y*
2	4
1	1
$\frac{1}{2}$	
$\frac{1}{4}$	$\frac{1}{16}$

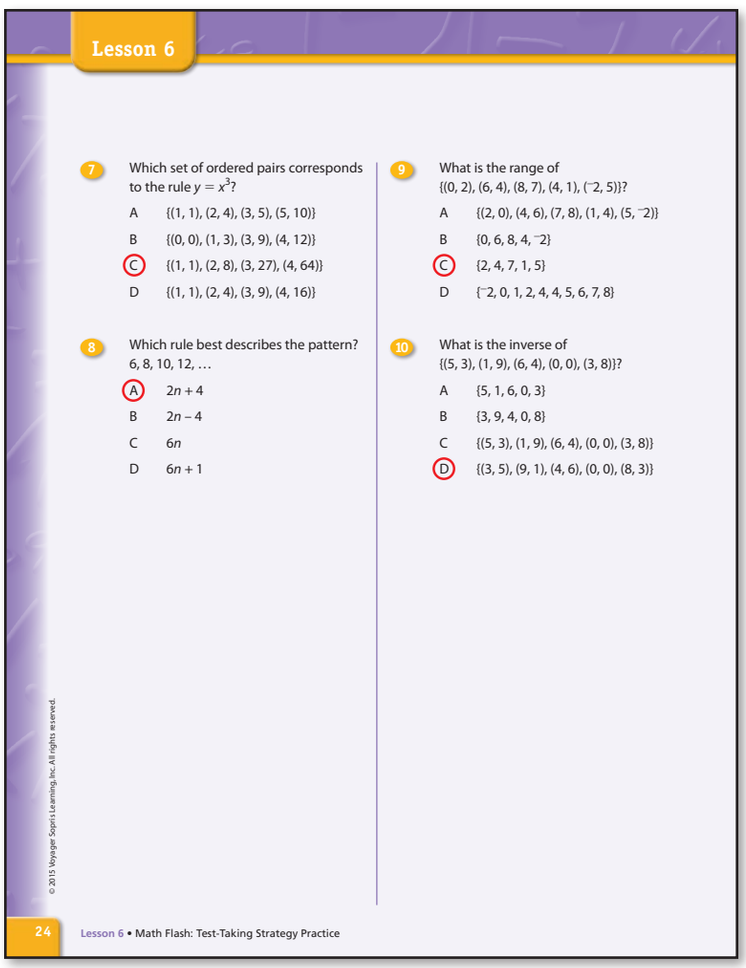
5. Mary is given the rule $y = 2x + 3$ and the *x*-values $^{-}2$, 0, 2, and 4. Which of the following are the corresponding *y*-values?

A $^{-}4, 0, 4, 8$
B $1, 3, 7, 11$
(C) $^{-}1, 3, 7, 11$
D $^{-}7, ^{-}3, 1, 5$

6. What is the domain of $\{(5, 4), (^{-}3, 8), (1, 0), (11, 2), (^{-}7, 9)\}$?

A $\{4, 8, 0, 2, 9\}$
B $\{(4, 5), (8, ^{-}3), (0, 1), (2, 11), (9, ^{-}7)\}$
C $\{^{-}7, ^{-}3, 0, 1, 2, 4, 5, 8, 9, 11\}$
(D) $\{5, ^{-}3, 1, 11, ^{-}7\}$

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Lesson 6

7. Which set of ordered pairs corresponds to the rule $y = x^3$?

A $\{(1, 1), (2, 4), (3, 5), (5, 10)\}$
B $\{(0, 0), (1, 3), (3, 9), (4, 12)\}$
(C) $\{(1, 1), (2, 8), (3, 27), (4, 64)\}$
D $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$

8. Which rule best describes the pattern?
6, 8, 10, 12, …

(A) $2n + 4$
B $2n - 4$
C $6n$
D $6n + 1$

9. What is the range of $\{(0, 2), (6, 4), (8, 7), (4, 1), (^{-}2, 5)\}$?

A $\{(2, 0), (4, 6), (7, 8), (1, 4), (5, ^{-}2)\}$
B $\{0, 6, 8, 4, ^{-}2\}$
(C) $\{2, 4, 7, 1, 5\}$
D $\{^{-}2, 0, 1, 2, 4, 4, 5, 6, 7, 8\}$

10. What is the inverse of $\{(5, 3), (1, 9), (6, 4), (0, 0), (3, 8)\}$?

A $\{5, 1, 6, 0, 3\}$
B $\{3, 9, 4, 0, 8\}$
C $\{(5, 3), (1, 9), (6, 4), (0, 0), (3, 8)\}$
(D) $\{(3, 5), (9, 1), (4, 6), (0, 0), (8, 3)\}$

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Lesson 7

Objective

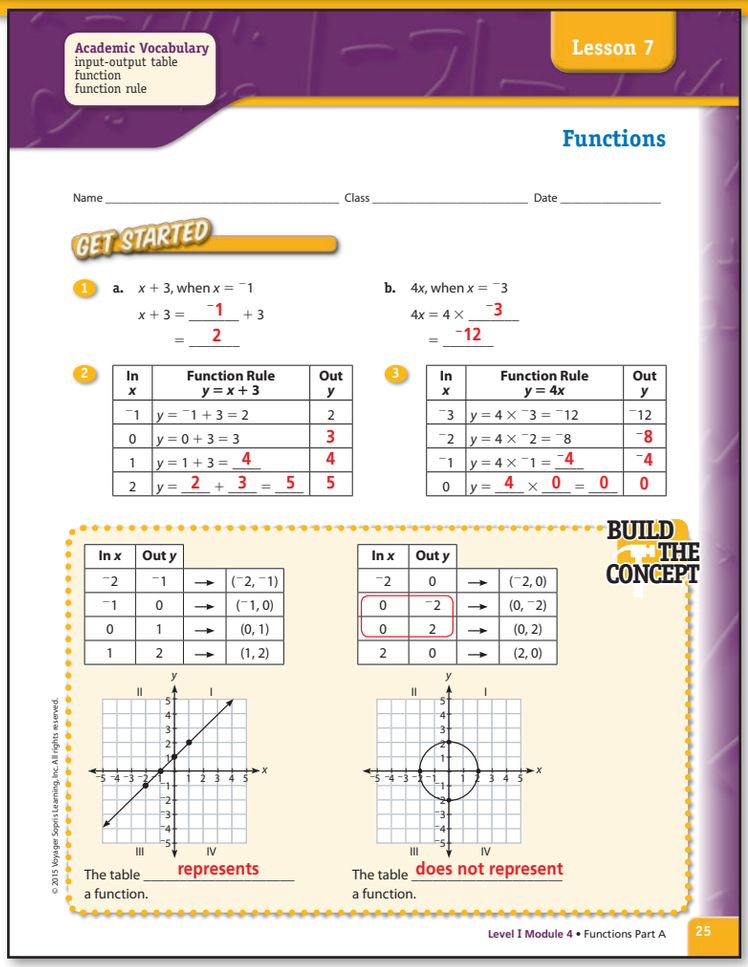
To evaluate a function rule

Preskills	Lesson
Evaluating Algebraic Expressions	I. 3.PL1

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **input-output table**
  a table that shows how an input value, *x*, is paired with an output value, *y*
- **function**
  a pairing of input and output values in which each input value is paired with exactly one output value
- **function rule**
  an equation that shows how an input value, *x*, and an output value, *y*, are related



Academic Vocabulary
input-output table
function
function rule

Lesson 7

Functions

Name _____ Class _____ Date _____

GET STARTED

1 a. $x + 3$, when $x = -1$
$x + 3 = \underline{-1} + 3$
$= \underline{2}$

b. $4x$, when $x = -3$
$4x = 4 \times \underline{-3}$
$= \underline{-12}$

2

In x	Function Rule $y = x + 3$	Out y
-1	$y = -1 + 3 = 2$	2
0	$y = 0 + 3 = 3$	3
1	$y = 1 + 3 = \underline{4}$	4
2	$y = \underline{2} + \underline{3} = \underline{5}$	5

3

In x	Function Rule $y = 4x$	Out y
-3	$y = 4 \times -3 = -12$	-12
-2	$y = 4 \times -2 = -8$	-8
-1	$y = 4 \times -1 = \underline{-4}$	-4
0	$y = \underline{4} \times \underline{0} = \underline{0}$	0

BUILD THE CONCEPT

In x	Out y		
-2	-1	→	$(-2, -1)$
-1	0	→	$(-1, 0)$
0	1	→	$(0, 1)$
1	2	→	$(1, 2)$

In x	Out y		
-2	0	→	$(-2, 0)$
0	-2	→	$(0, -2)$
0	2	→	$(0, 2)$
2	0	→	$(2, 0)$

The table represents a function.

The table does not represent a function.

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Level I Module 4 • Functions Part A 25

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

In problem 1a, substitute $^-1$ for the variable, *x*, and evaluate the expression. What is $^-1$ plus 3? (2) **What is the value of the expression *x* plus 3 when *x* equals $^-1$?** (2) **In problem 1b, substitute $^-3$ for the variable and evaluate the expression. What is the value of the expression 4*x* when *x* equals $^-3$?** ($^-12$)

MODEL NEW SKILLS

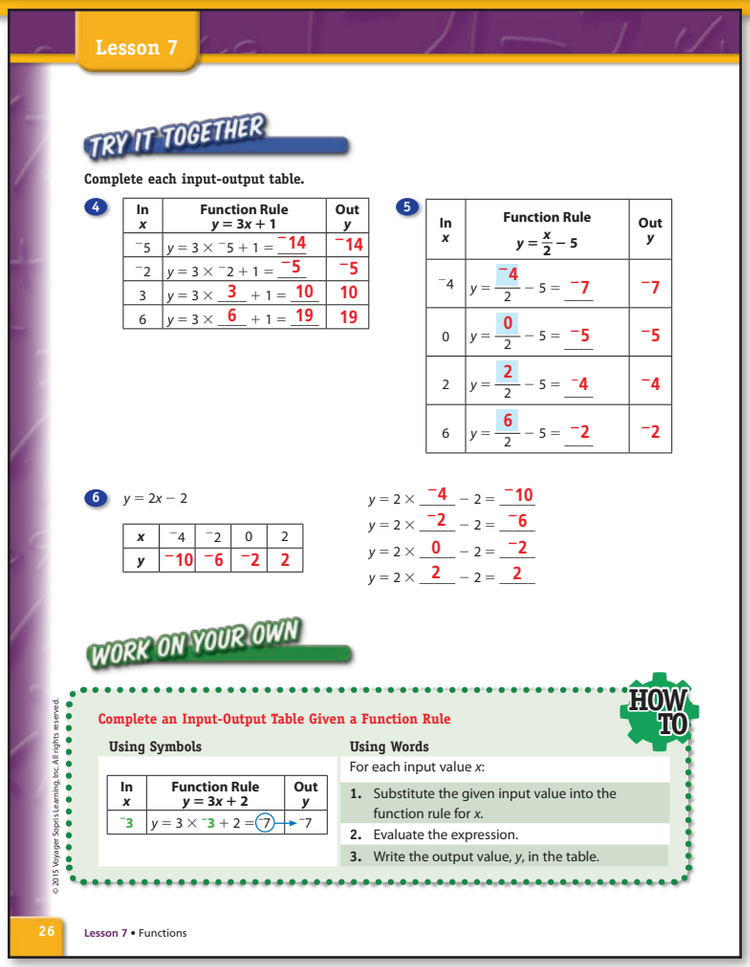
Problem 2

The table in this problem is called an input-output table. An input-output table can be used to show how each input variable, *x*, is paired with an output value, *y*. A function pairs an input value, *x*, with exactly one output value, *y*. A function rule is an equation that shows how to compute the value of *y* when the value of *x* is substituted into the equation. The table has a column for the *x*-values, the input values; a column for computing the output values; and a column to write the output values, the *y*-values. The function rule for this input-output table is *y* equals *x* plus 3. The rule is given in the top row of the second column of the table.

To find the output value *y* when the input value *x* equals $^-1$, substitute $^-1$ for *x*. The sum of $^-1$ and 3 is 2, so the value of *y* when *x* equals $^-1$ is 2. To find the output value *y* when the input value *x* equals zero, substitute zero for *x* in the function rule. The sum of zero and 3 is 3, so the value of *y* when *x* equals zero is 3. Now substitute 1 for *x*. What is 1 plus 3? (4) **What is the output value for this rule when the input value is 1?** (4) **Substitute 2 for *x* in the function rule. What is the output value when *x* equals 2?** (5)

Problem 3

The rule for this input-output table is *y* equals 4*x*. Look at the row of the table for *x* equals $^-3$. When the input value *x* equals $^-3$, the value of *y* is 4 times $^-3$. What is 4 times $^-3$? ($^-12$) **The output value is $^-12$ when the input value is $^-3$. To find the output value *y* when the input value *x* equals $^-2$, substitute $^-2$ for *x* in the function rule. What is 4 times $^-2$?** ($^-8$) **What is the output value when the input value is $^-2$?** ($^-8$) **To find the output value *y* when *x* equals $^-1$, substitute $^-1$ for *x*. What is 4 times $^-1$?** ($^-4$) **What is the output value**



Lesson 7

TRY IT TOGETHER

Complete each input-output table.

4

In x	Function Rule $y = 3x + 1$	Out y
$^-5$	$y = 3 \times {}^-5 + 1 =$ $^-14$	$^-14$
$^-2$	$y = 3 \times {}^-2 + 1 =$ $^-5$	$^-5$
3	$y = 3 \times$ 3 $+ 1 =$ 10	10
6	$y = 3 \times$ 6 $+ 1 =$ 19	19

5

In x	Function Rule $y = \frac{x}{2} - 5$	Out y
$^-4$	$y = \frac{^-4}{2} - 5 =$ $^-7$	$^-7$
0	$y = \frac{0}{2} - 5 =$ $^-5$	$^-5$
2	$y = \frac{2}{2} - 5 =$ $^-4$	$^-4$
6	$y = \frac{6}{2} - 5 =$ $^-2$	$^-2$

6 $y = 2x - 2$

x	$^-4$	$^-2$	0	2
y	$^-10$	$^-6$	$^-2$	2

$y = 2 \times {}^-4 - 2 = {}^-10$
$y = 2 \times {}^-2 - 2 = {}^-6$
$y = 2 \times 0 - 2 = {}^-2$
$y = 2 \times 2 - 2 = 2$

WORK ON YOUR OWN

HOW TO

Complete an Input-Output Table Given a Function Rule

Using Symbols			Using Words
In x	Function Rule $y = 3x + 2$	Out y	For each input value x:
$^-3$	$y = 3 \times {}^-3 + 2 = 7$ →	$^-7$	1. Substitute the given input value into the function rule for x.
			2. Evaluate the expression.
			3. Write the output value, y, in the table.

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when the input value is $^-1$? ($^-4$) Substitute zero for x. What is the output value when the input value x equals zero? (0)

BUILD THE CONCEPT

Model how to evaluate a function rule using a graph.

Look at the input-output table on the left. Is each input value paired with only one output value? (yes) **So, the paired values in the input-output table on the left represent a function. The paired values can be formed into ordered pairs. The graph of the function given by the input-output table on the left is shown below the table. What is the graph of the function?** (a straight line) **The graph shows that the table represents a function because for each value on the x-axis, there is at most one ordered pair on the graph.**

Now look at the input-output table on the right. Is each different input value paired with only one output value? (no) **Which input value is paired with more than one output value?** (0) **Circle these pairs in the table. Because one input value is paired with two output values, the paired values in the table do not represent a function. A graph that represents this input-output table is shown below the table. What is this graph?** (a circle) **Notice that a value can be chosen on the x-axis for which there are two ordered pairs on the graph. This indicates that the table does not represent a function.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What is the function rule for this table? ($y = 3x + 1$) **How will each output value be found?** (by substituting each input value in the table for x into the function rule) **What is the first input value for x in the table?** ($^-5$) **Substitute $^-5$ for x. What is 3 times $^-5$?** ($^-15$) **What is $^-15$ plus 1?** ($^-14$) **What is the output value when x equals $^-5$?** ($^-14$) **The next input value for x is $^-2$. What is 3 times $^-2$?** ($^-6$) **What is $^-6$ plus 1?** ($^-5$) **What is the output value when x equals $^-2$?** ($^-5$) **The next input value for x is 3. What expression needs to be evaluated?** ($3 \times 3 + 1$) **What is 3 times 3 plus 1?** (10) **What is the output value when x equals 3?** (10) **The last input value for x is 6. What is 3 times 6 plus 1?** (19) **What is the output value when x equals 6?** (19)

Problem 5

What is the function rule? ($y = \frac{x}{2} - 5$) **What is the first input value for x in the table?** ($^-4$) **What is $^-4$ divided by 2 minus 5?** ($^-7$) **What is the value of y when x equals $^-4$?** ($^-7$) **Complete the table for x equals zero. What is zero divided by 2 minus 5?** ($^-5$) **What is the output value when x equals zero?** ($^-5$) **What is the output value when x equals 2?** ($^-4$) **What is the output value when x equals 6?** ($^-2$)

Problem 6

Input-output tables can be horizontal or vertical. The input-output table in this problem is horizontal. The x-values, or input values, are in the top row of the table. The y-values, or output values, are in the bottom row. Notice that input-output tables do not always have three columns. The function rule is not always listed inside the table. Sometimes computing the y-value is done outside the table separately. What is the function rule? ($y = 2x - 2$) **What is the first x-value in the table?** ($^-4$) **What is 2 times $^-4$ minus 2?** ($^-10$) **What is the value of y when x equals $^-4$?** ($^-10$) **What is the output value when x equals $^-2$?** ($^-6$) **What is the output value when x equals zero?** ($^-2$) **What is the output value when x equals 2?** (2)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 7–16

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 11–13 Reminder

These problems review evaluating expressions. Students should evaluate each expression for the given values of a and b.

Problem 14 Reminder

Have students complete the input-output table for the function rule $y = 12x$ first, then use the results to decide the number of bags of balloons needed.

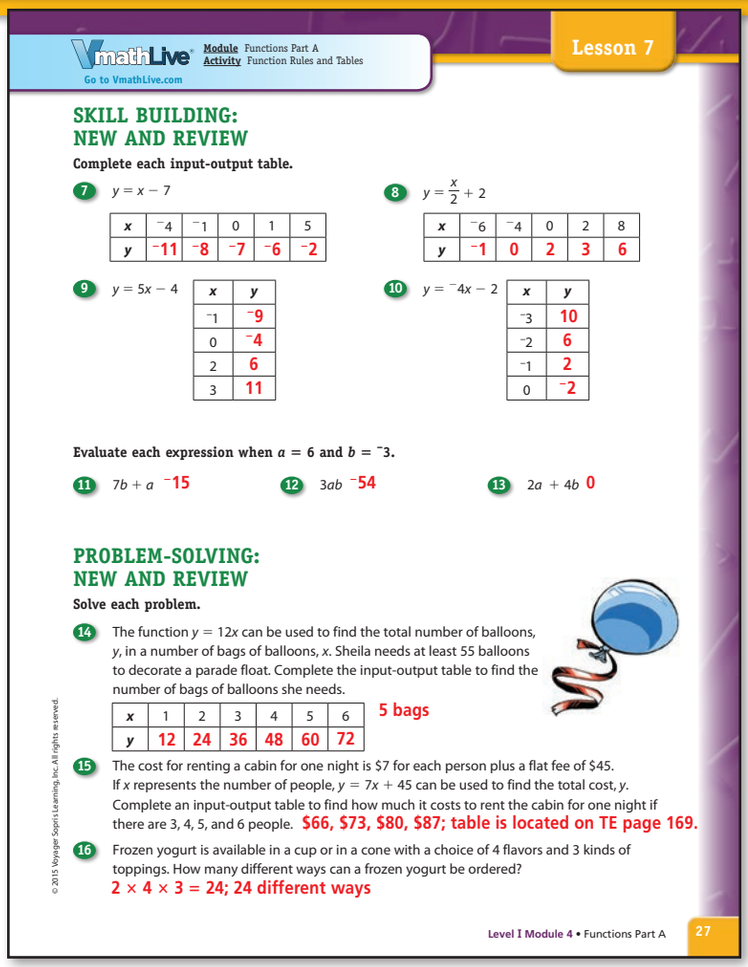
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b or 2c: The student added values instead of subtracting when evaluating the expression. Reteach how to add or subtract integers with the student.



VmathLive Module Functions Part A Activity Function Rules and Tables
Go to VmathLive.com

Lesson 7

SKILL BUILDING: NEW AND REVIEW

Complete each input-output table.

7 $y = x - 7$

x	$^-4$	$^-1$	0	1	5
y	$^-11$	$^-8$	$^-7$	$^-6$	$^-2$

8 $y = \frac{x}{2} + 2$

x	$^-6$	$^-4$	0	2	8
y	$^-1$	0	2	3	6

9 $y = 5x - 4$

x	y
$^-1$	$^-9$
0	$^-4$
2	6
3	11

10 $y = ^-4x - 2$

x	y
$^-3$	10
$^-2$	6
$^-1$	2
0	$^-2$

Evaluate each expression when $a = 6$ and $b = ^-3$.

11 $7b + a$ $^-15$

12 $3ab$ $^-54$

13 $2a + 4b$ 0

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

14 The function $y = 12x$ can be used to find the total number of balloons, y, in a number of bags of balloons, x. Sheila needs at least 55 balloons to decorate a parade float. Complete the input-output table to find the number of bags of balloons she needs.

x	1	2	3	4	5	6
y	12	24	36	48	60	72

5 bags

15 The cost for renting a cabin for one night is $7 for each person plus a flat fee of $45. If x represents the number of people, $y = 7x + 45$ can be used to find the total cost, y. Complete an input-output table to find how much it costs to rent the cabin for one night if there are 3, 4, 5, and 6 people. $66, $73, $80, $87; table is located on TE page 169.

16 Frozen yogurt is available in a cup or in a cone with a choice of 4 flavors and 3 kinds of toppings. How many different ways can a frozen yogurt be ordered?
2 × 4 × 3 = 24; 24 different ways

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Level I Module 4 • Functions Part A 27

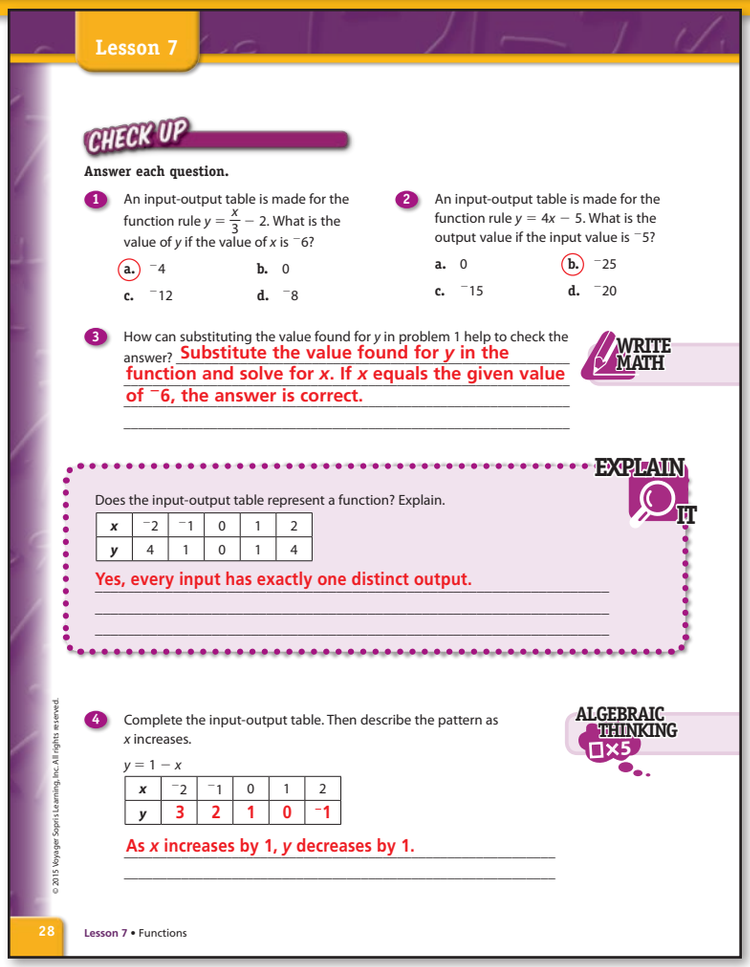
If student answered 1c or 2a: The student substituted the x-value for y instead of x. Use Additional Resources in I.4.7 to reteach input-output tables.

If student answered 1d or 2d: The student did not completely or correctly evaluate the expression. Use Additional Resources in I.3.PL1 to reteach how to evaluate algebraic expressions.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that by substituting the value found for y and solving for x, x should equal the given value of $^-6$.



Lesson 7

CHECK UP

Answer each question.

1. An input-output table is made for the function rule $y = \frac{x}{3} - 2$. What is the value of y if the value of x is $^-6$?

   (a.) $^-4$ b. 0 c. $^-12$ d. $^-8$

2. An input-output table is made for the function rule $y = 4x - 5$. What is the output value if the input value is $^-5$?

   a. 0 (b.) $^-25$ c. $^-15$ d. $^-20$

3. How can substituting the value found for y in problem 1 help to check the answer? Substitute the value found for y in the function and solve for x. If x equals the given value of $^-6$, the answer is correct.

WRITE MATH

EXPLAIN IT

Does the input-output table represent a function? Explain.

x	$^-2$	$^-1$	0	1	2
y	4	1	0	1	4

Yes, every input has exactly one distinct output.

4. Complete the input-output table. Then describe the pattern as x increases.

ALGEBRAIC THINKING

$y = 1 - x$

x	$^-2$	$^-1$	0	1	2
y	3	2	1	0	$^-1$

As x increases by 1, y decreases by 1.

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EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive

Module: Functions Part A

Activity: Function Rules and Tables

Vmath Reteach

Reteach Student Book Module 4 Lesson 7

Reteach Teacher Edition Module 4 Lesson 7

Extra Practice

Student Book page 59

English Language Learners

Write the function $y = x + 6$ on the board or sheet of chart paper. Use a red marker to write the y and a blue marker to write the x. Ask students to tell what x and y equal. Students should see that x and y can have different values. As one variable changes, so does the other. Organize the possibilities in a two-column table, with the first column labeled *If* $x =$ _____ and the second column labeled *Then* $y =$ _____. Ask students to describe any patterns they see in the chart. Ask students if the x- and y-values can be switched. Show students that inputs are very different from outputs, even though the words look very similar.

Students with Special Needs

If students are unable to find correct output values, allow them to practice simplifying expressions using the order of operations. Show students that any time an expression needs to be simplified, the order of operations should be followed. The use of negative numbers can make matters more difficult; it may be helpful to have students work with only positive numbers at first, then move on to include negative numbers.

Problem-Solving: New and Review Answer, Student Book page 27

15. $y = 7x + 45$

x	3	4	5	6
y	$66	$73	$80	$87

Lesson 8

Objective

To find the function rule given an input-output table

Preskills	Lesson
Functions	I.4.7

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The function rule for this input-output table is *y* equals 2*x* plus 3. What is the first input value in the table? (⁻5) **What is 2 times ⁻5 plus 3?** (⁻7) **The output value when *x* equals ⁻5 is ⁻7. What is the next input value in the table?** (⁻3) **The output value when *x* equals ⁻3 is ⁻3. The output value when *x* equals 2 is 7. The output value when *x* equals 8 is 19.**

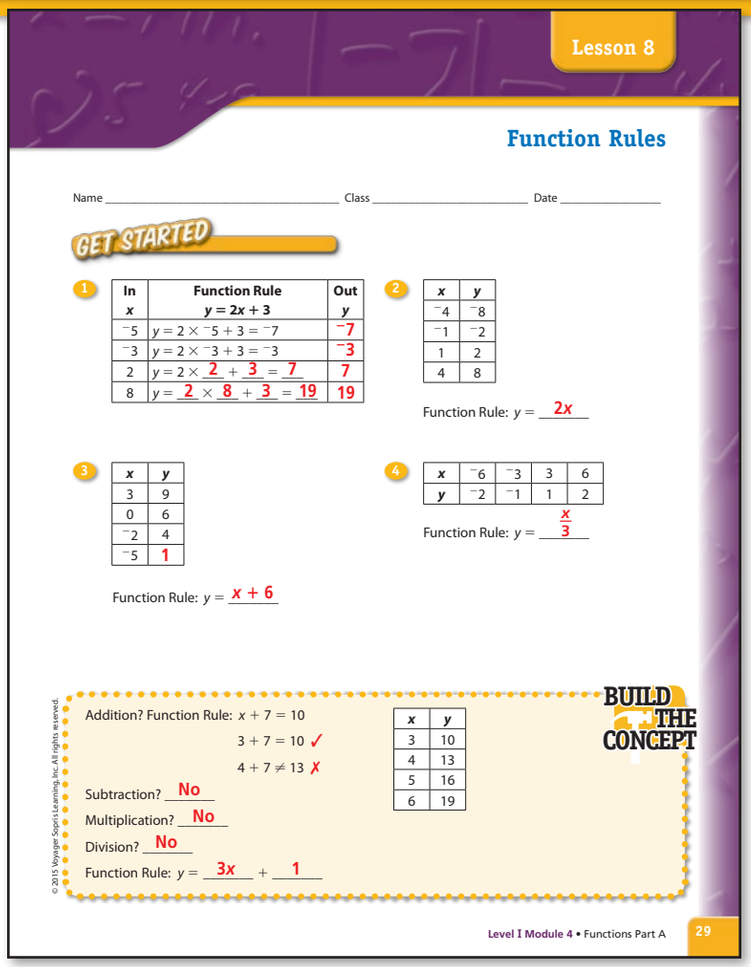
MODEL NEW SKILLS

Problem 2

The table in problem 2 is an input-output table. The table is complete, but the function rule is not given. It is possible to determine the function rule by comparing the input values with the output values.

Look at the first pair of numbers in the table, *x* equals ⁻4 and *y* equals ⁻8. What could be done to ⁻4 to get ⁻8? (⁻4 − 4 = ⁻8 or ⁻4 × 2 = ⁻8) **The function rule cannot be determined without looking at another pair of numbers in the table. What is the next pair of numbers in the table?** (*x* = ⁻1 and *y* = ⁻2) **What could be done to ⁻1 to get ⁻2?** (⁻1 − 1 = ⁻2 or ⁻1 × 2 = ⁻2) **The function rule *multiply the input value by 2* works for the first two sets of numbers in the table. The other rules suggested do not work for both pairs of numbers.**

Check to see if the function rule *multiply the input value by 2* works for the rest of the table. The next input value is 1. What is 1 times 2? (2) **The rule works for the input value *x* equals 1. The last input value is 4. What is 4 times 2?** (8) **The rule works for the input value *x* equals 4. The function rule is *multiply the input value by 2*. The function rule is written as *y* = 2*x*.**



Lesson 8

Function Rules

Name _____ Class _____ Date _____

GET STARTED

1

In *x*	Function Rule *y* = 2*x* + 3	Out *y*
⁻5	*y* = 2 × ⁻5 + 3 = ⁻7	⁻7
⁻3	*y* = 2 × ⁻3 + 3 = ⁻3	⁻3
2	*y* = 2 × 2 + 3 = 7	7
8	*y* = 2 × 8 + 3 = 19	19

2

x	*y*
⁻4	⁻8
⁻1	⁻2
1	2
4	8

Function Rule: *y* = 2*x*

3

x	*y*
3	9
0	6
⁻2	4
⁻5	1

Function Rule: *y* = *x* + 6

4

x	⁻6	⁻3	3	6
y	⁻2	⁻1	1	2

Function Rule: *y* = $\frac{x}{3}$

BUILD THE CONCEPT

Addition? Function Rule: *x* + 7 = 10

3 + 7 = 10 ✓

4 + 7 ≠ 13 ✗

x	*y*
3	10
4	13
5	16
6	19

Subtraction? No

Multiplication? No

Division? No

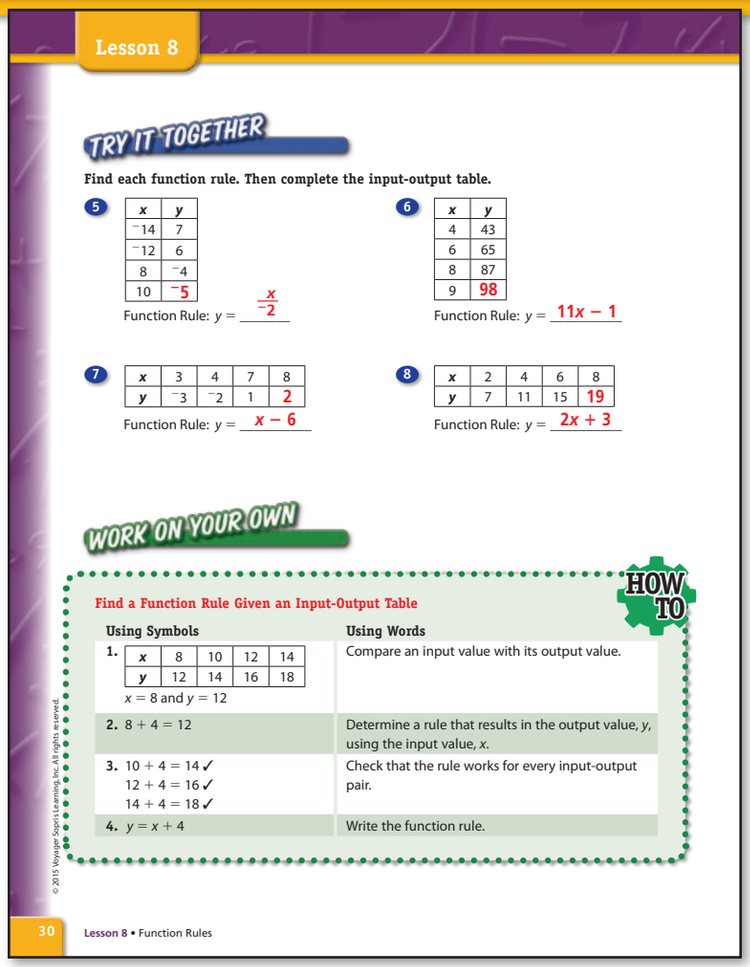
Function Rule: *y* = 3*x* + 1

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Problem 3

What is the first number pair in the table? (*x* = 3 and *y* = 9) **What could be done to 3 to get 9?** (3 + 6 = 9 or 3 × 3 = 9) **See if one of these rules works with the next pair of numbers in the table. What is the next pair of numbers?** (*x* = 0 and *y* = 6) **What could be done to zero to get 6?** (0 + 6 = 6) **The function rule *add 6 to the input value* works with this pair of numbers because zero plus 6 equals 6. The function rule *multiply the input value by 3* does not work because zero times 3 does not equal the output value, 6, when *x* equals zero.**

Check to see if the rule *add 6 to the input value* works with the next number pair given in the table. The input value is ⁻2. What is ⁻2 plus 6? (4) **Does the function rule *y* equals *x* plus 6 work?** (yes) **Use the function rule to find the output value for the last input value in the table, *x* equals ⁻5. What is ⁻5 plus 6?** (1) **The output value when *x* equals ⁻5 is 1.**



Lesson 8

TRY IT TOGETHER

Find each function rule. Then complete the input-output table.

5

x	y
−14	7
−12	6
8	−4
10	−5

Function Rule: $y = \frac{x}{-2}$

6

x	y
4	43
6	65
8	87
9	98

Function Rule: $y = 11x - 1$

7

x	3	4	7	8
y	−3	−2	1	2

Function Rule: $y = x - 6$

8

x	2	4	6	8
y	7	11	15	19

Function Rule: $y = 2x + 3$

WORK ON YOUR OWN

HOW TO

Find a Function Rule Given an Input-Output Table

Using Symbols	Using Words
1. x: 8, 10, 12, 14; y: 12, 14, 16, 18 x = 8 and y = 12	Compare an input value with its output value.
2. 8 + 4 = 12	Determine a rule that results in the output value, y, using the input value, x.
3. 10 + 4 = 14 ✓ 12 + 4 = 16 ✓ 14 + 4 = 18 ✓	Check that the rule works for every input-output pair.
4. y = x + 4	Write the function rule.

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Problem 4

Look at the first pair of numbers in the table to find the function rule. What could be done to ⁻6 to get ⁻2? (⁻6 + 4 = ⁻2 or $\frac{-6}{3} = -2$) **See if one of these rules works with the next pair of numbers in the table.**

The function rule *divide the input value by 3* works with *x* equals ⁻3 and *y* equals ⁻1 because ⁻3 divided by 3 equals ⁻1. The function rule *add 4 to the input value* does not work because ⁻3 plus 4 equals 1, which is not the output value for *x* equals ⁻3.

Check to see if the rule *divide the input value by 3* works with the next number pair given in the table. The input value is 3. What is 3 divided by 3? (1) **Does the function rule *y* equals $\frac{x}{3}$ work?** (yes) **The last input value is 6. What is 6 divided by 3?** (2) **What is the function rule for this input-output table?** ($y = \frac{x}{3}$)

BUILD THE CONCEPT

Model how to find the function rule given an input-output table.

Look at the table. To determine the function rule, first try addition. Is there any number that can be added to 3 to get 10? (yes) **What number added to 3 equals 10?** (7) **Look at the second set of numbers in the table. Does the rule *add 7* work for the *x*-value of 4?** (no) **Does the rule *add 7* work for this function table?** (no)

Is there any number that can be subtracted from 3 to get 10 and can also be subtracted from 4 to get 13? (no) **Is there any number that can be multiplied by 3 to get 10 and also can be multiplied by 4 to get 13?** (no) **Last, try division. Is there any number that 3 can be divided by to get 10 and that 4 can be divided by to get 13?** (no)

When one operation will not work for the function table, a two-step function rule will have to be found. Is there anything that 3 can be multiplied by to get close to 10? (yes) **What?** (3) **What is 3 times 3?** (9) **What has to be done to 9 to get 10?** (Add 1.) **So, the function rule *multiply by 3 plus 1* works for the first set. This is written as *y* = 3*x* + 1.**

Check to see if this rule works for the rest of the table. What is 3 times 4 plus 1? (13) **What is 3 times 5 plus 1?** (16) **What is 3 times 6 plus 1?** (19) **Does this rule work for the rest of the table?** (yes)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What must be done to find the function rule for the table? (Compare input values with output values and find a rule that works for all pairs of values.) **What is the first pair of values in the table?** (*x* = ⁻14 and *y* = 7) **What could be done to ⁻14 to get 7?** (⁻14 + 21 = 7 or $\frac{-14}{-2} = 7$) **What is the second pair of values in the table?** (*x* = ⁻12 and *y* = 6) **Does one of the function rules work for *x* equals ⁻12 and *y* equals 6?** (yes) **Which one?** (Divide the input value by ⁻2.) **Why?** (⁻12 ÷ ⁻2 = 6)

Does the function rule *divide the input value by* ⁻2 work for the next pair of values in the table? (yes) **Why?** (8 ÷ ⁻2 = ⁻4) **How is the function rule written?** ($y = \frac{x}{-2}$) **Use the function rule to find the output value for *x* equals 10. What is 10 divided by ⁻2?** (⁻5) **What is the output value when *x* equals 10?** (⁻5)

Problem 6

What could be done to 4 to get 43? (4 + 39 = 43 or 4 × 11 − 1 = 43) **Which function rule works for this table?** (*y* = 11*x* − 1) **Why?** (Every output value is 1 less than 11 times the corresponding input value.) **What is the output value when the input value is 9?** (98)

Problem 7

What could be done to 3 to get $^-3$? ($3 - 6 = {}^-3$ or $3 \times {}^-1 = {}^-3$) **Which function rule works for the table?** ($y = x - 6$) **What is the output value when the input value is 8?** (2)

Problem 8

How is the function rule written for this table? ($y = 2x + 3$) **What is the output value when the input value is 8?** (19)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using an Equation** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving an equation. The instruction is immediately followed by application of the strategy in problem 16.

Have students read the problem.

Have students read the data in the table.

Students should try to find a rule that works for all values in the table.

Rule 1: "add 80 to each input value"

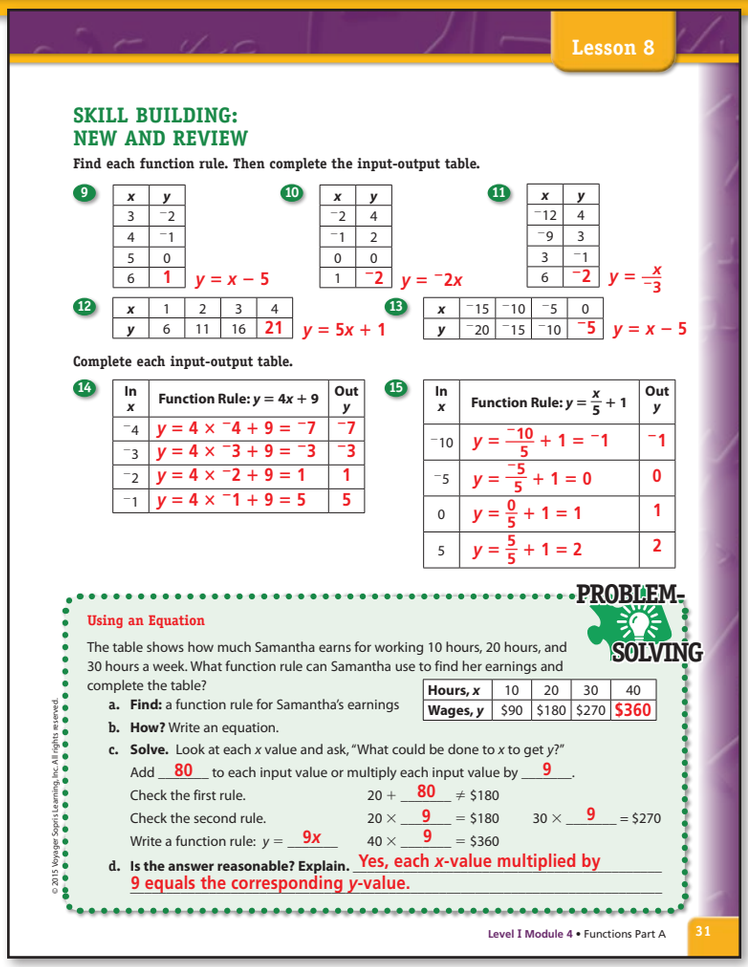
Rule 1 works only when $x = 10$

Rule 2: "multiply each input value by 9"

Rule 2 works for all input values

The function rule is $y = 9x$.

The answer is reasonable because when each input value, x, is multiplied by 9, the product is the same as the corresponding y-value.



Lesson 8

SKILL BUILDING: NEW AND REVIEW

Find each function rule. Then complete the input-output table.

9

x	y
3	$^-2$
4	$^-1$
5	0
6	1

$y = x - 5$

10

x	y
$^-2$	4
$^-1$	2
0	0
1	$^-2$

$y = {}^-2x$

11

x	y
$^-12$	4
$^-9$	3
3	$^-1$
6	$^-2$

$y = \frac{x}{^-3}$

12

x	1	2	3	4
y	6	11	16	21

$y = 5x + 1$

13

x	$^-15$	$^-10$	$^-5$	0
y	$^-20$	$^-15$	$^-10$	$^-5$

$y = x - 5$

Complete each input-output table.

14

In x	Function Rule: $y = 4x + 9$	Out y
$^-4$	$y = 4 \times {}^-4 + 9 = {}^-7$	$^-7$
$^-3$	$y = 4 \times {}^-3 + 9 = {}^-3$	$^-3$
$^-2$	$y = 4 \times {}^-2 + 9 = 1$	1
$^-1$	$y = 4 \times {}^-1 + 9 = 5$	5

15

In x	Function Rule: $y = \frac{x}{5} + 1$	Out y
$^-10$	$y = \frac{^-10}{5} + 1 = {}^-1$	$^-1$
$^-5$	$y = \frac{^-5}{5} + 1 = 0$	0
0	$y = \frac{0}{5} + 1 = 1$	1
5	$y = \frac{5}{5} + 1 = 2$	2

PROBLEM-SOLVING

Using an Equation

The table shows how much Samantha earns for working 10 hours, 20 hours, and 30 hours a week. What function rule can Samantha use to find her earnings and complete the table?

Hours, x	10	20	30	40
Wages, y	$90	$180	$270	$360

a. **Find:** a function rule for Samantha's earnings

b. **How?** Write an equation.

c. **Solve.** Look at each x value and ask, "What could be done to x to get y?"

Add 80 to each input value or multiply each input value by 9.

Check the first rule. $20 + 80 \neq \$180$

Check the second rule. $20 \times 9 = \$180$ $30 \times 9 = \$270$

Write a function rule: $y = 9x$ $40 \times 9 = \$360$

d. **Is the answer reasonable? Explain.** Yes, each x-value multiplied by 9 equals the corresponding y-value.

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Problems 9–19

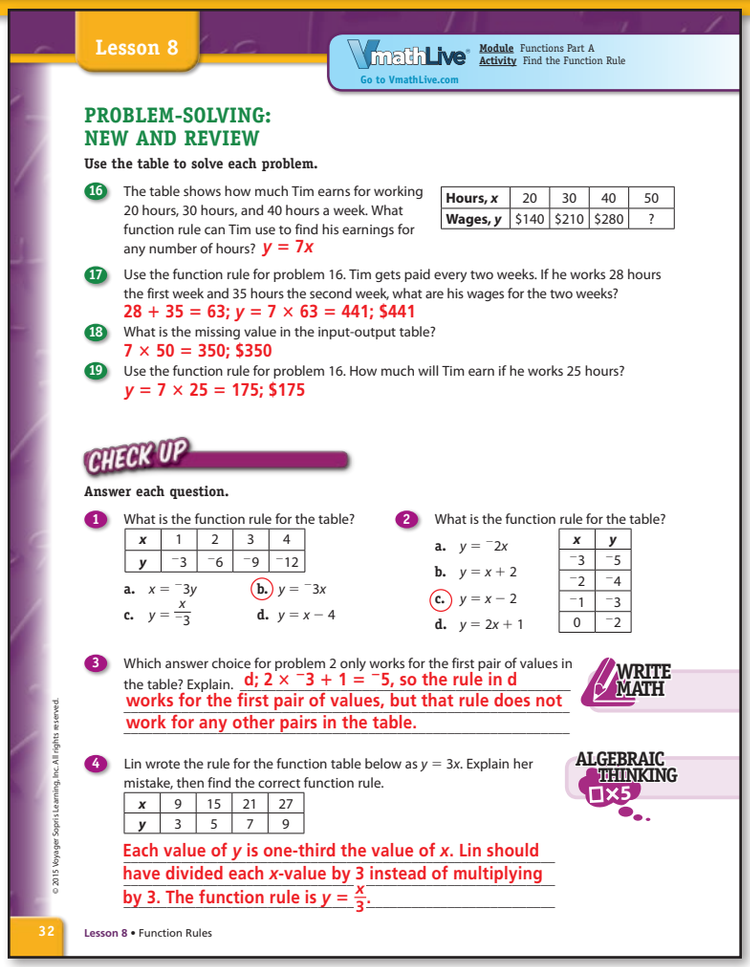
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 14 and 15 Reminder

These problems review using a function rule to complete an input-output table.

Problem 17 Reminder

This is a two-step problem. Be sure students first add to find the total number of hours, then multiply by 7.



Lesson 8

VmathLive Module Functions Part A Activity Find the Function Rule
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Use the table to solve each problem.

16 The table shows how much Tim earns for working 20 hours, 30 hours, and 40 hours a week. What function rule can Tim use to find his earnings for any number of hours? $y = 7x$

Hours, x	20	30	40	50
Wages, y	$140	$210	$280	?

17 Use the function rule for problem 16. Tim gets paid every two weeks. If he works 28 hours the first week and 35 hours the second week, what are his wages for the two weeks?
$28 + 35 = 63$; $y = 7 \times 63 = 441$; $441

18 What is the missing value in the input-output table?
$7 \times 50 = 350$; $350

19 Use the function rule for problem 16. How much will Tim earn if he works 25 hours?
$y = 7 \times 25 = 175$; $175

CHECK UP

Answer each question.

1 What is the function rule for the table?

x	1	2	3	4
y	-3	-6	-9	-12

a. $x = -3y$ (b.) $y = -3x$
c. $y = \frac{x}{-3}$ d. $y = x - 4$

2 What is the function rule for the table?

a. $y = -2x$
b. $y = x + 2$
(c.) $y = x - 2$
d. $y = 2x + 1$

x	y
-3	-5
-2	-4
-1	-3
0	-2

3 Which answer choice for problem 2 only works for the first pair of values in the table? Explain. d; $2 \times -3 + 1 = -5$, so the rule in d works for the first pair of values, but that rule does not work for any other pairs in the table.

WRITE MATH

4 Lin wrote the rule for the function table below as $y = 3x$. Explain her mistake, then find the correct function rule.

ALGEBRAIC THINKING

x	9	15	21	27
y	3	5	7	9

Each value of y is one-third the value of x. Lin should have divided each x-value by 3 instead of multiplying by 3. The function rule is $y = \frac{x}{3}$.

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CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1c: The student divided the input variable by -3 rather than multiplying the input variable by -3. Use the Additional Resources in I.4.8 to reteach how to find a function rule given an input-output table.

If student answered 1d or 2d: The student wrote a rule that only works for the first input-output pair. Remind the student to make sure the rule works for every pair in the table.

If student answered 2b: The student added 2 to the input value rather than subtracting 2. Use Additional Resources in I.4.8 to reteach how to find a function rule given an input-output table.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to check all the function rules correctly to answer the question. Remind the student that some rules will work for some input-output pairs, but for a rule to be the function rule, it must work for all input-output pairs in the table.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Functions Part A
Activity: Find the Function Rule

Vmath Reteach
Reteach Student Book Module 4 Lesson 8
Reteach Teacher Edition Module 4 Lesson 8

Extra Practice
Student Book page 59

English Language Learners

Have students write the terms *input*, *output*, and *function rule* on index cards. Write a table of values and the corresponding function rule on the board. Point to different numbers in the table and have students hold up the appropriate index card to identify the number. Provide ongoing practice identifying input, output, and function rules so students become familiar with the terms associated with input-output tables.

Students with Special Needs

Write on the board: *Kevin spends $2 each day for lunch. How much will he have spent in 5 days?* Guide students to use counters to represent how much he spends each day. Help students write the equation that represents spending $2 each day, $y = 2x$. Some students will write $y = x + 2$. Explain why $y = x + 2$ is incorrect. Review the use and meaning of each operation.

Lesson 9

Objective

To solve problems by finding a pattern

Preskills	Lesson
Functions	I.4.7
Function Rules	I.4.8

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

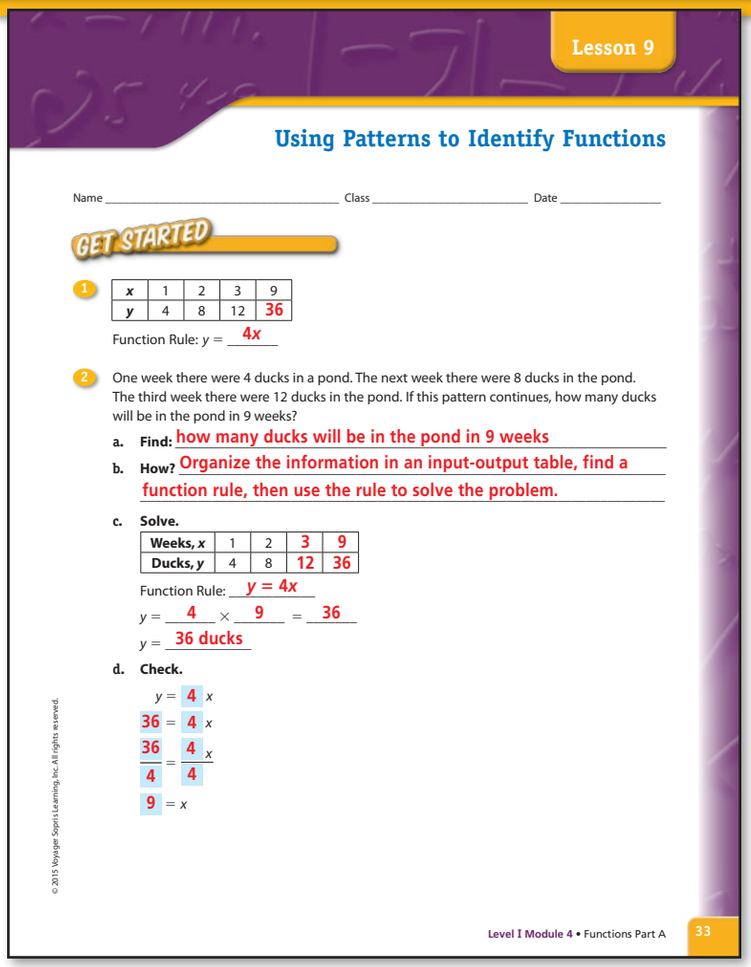
Problem 1

The table in this problem is a horizontal input-output table. The input values are in the top row. The output values are in the bottom row. In a horizontal table like this one, each output value is directly below its corresponding input value. To write a function rule for the table, compare an input value with its output value. What is the first input-output pair? ($x = 1$ and $y = 4$)

What could be done to 1 to get 4? ($1 + 3 = 4$ or $1 \times 4 = 4$) **Check to see if one of these rules works for the other input-output pairs in the table.**

The function rule *add 3 to the input value* does not work for the next input value, x equals 2, because 2 plus 3 equals 5, not 8. The function rule *multiply the input value by 4* does work for x equals 2 because 4 times 2 equals 8.

Check the function rule *multiply the input value by 4* with the next pair, x equals 3 and y equals 12. Does the function rule work? (yes) **Why?** ($4 \times 3 = 12$) **The function rule is y equals $4x$. Use the function rule to find the output value when the input value is 9. What is 4 times 9?** (36) **What is the output value when x equals 9?** (36)



Lesson 9

Using Patterns to Identify Functions

Name _____ Class _____ Date _____

GET STARTED

1

x	1	2	3	9
y	4	8	12	36

Function Rule: $y =$ 4x

2 One week there were 4 ducks in a pond. The next week there were 8 ducks in the pond. The third week there were 12 ducks in the pond. If this pattern continues, how many ducks will be in the pond in 9 weeks?

a. **Find:** how many ducks will be in the pond in 9 weeks

b. **How?** Organize the information in an input-output table, find a function rule, then use the rule to solve the problem.

c. **Solve.**

Weeks, x	1	2	3	9
Ducks, y	4	8	12	36

Function Rule: $y = 4x$

$y = 4 \times 9 = 36$

$y =$ 36 ducks

d. **Check.**

$y = 4x$

$36 = 4x$

$\frac{36}{4} = \frac{4x}{4}$

$9 = x$

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Level I Module 4 • Functions Part A 33

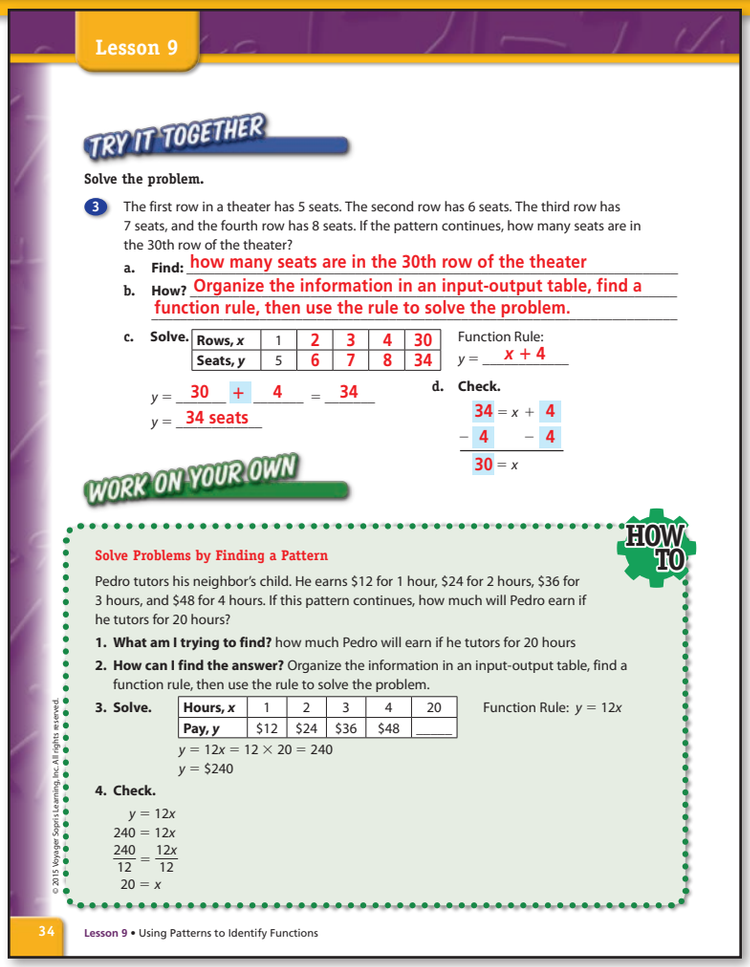
MODEL NEW SKILLS

Problem 2

To solve this problem, first ask, "What am I trying to find?" What needs to be found? (how many ducks will be in the pond in 9 weeks) **Now ask, "How can I find the answer?" Organize the information in an input-output table, find a function rule for the input-output pairs, and use the function rule to find the number of ducks in the pond in 9 weeks. Use the number of weeks as the input, or x-values, in the table.**

What will be the output, or y-values, in the table? (the number of ducks) **The first two pairs of values are entered in the table: first, x equals 1 and y equals 4, and second, x equals 2 and y equals 8. What is the next pair of input-output values?** ($x = 3$ and $y = 12$)

Look at problem 1 to find the function rule for this input-output table. The function rule is *multiply the input value by 4*. How is the function rule written as an equation? ($y = 4x$) **Solve the problem. Substitute 9 for x in the rule and solve for y. What is the equation with 9 substituted for x?** ($y = 4 \times 9$) **What is the solution?** ($y = 36$) **When x is 9, y is 36. How many ducks will be in the pond in 9 weeks?** (36 ducks)



Lesson 9

TRY IT TOGETHER

Solve the problem.

3 The first row in a theater has 5 seats. The second row has 6 seats. The third row has 7 seats, and the fourth row has 8 seats. If the pattern continues, how many seats are in the 30th row of the theater?

a. Find: how many seats are in the 30th row of the theater

b. How? Organize the information in an input-output table, find a function rule, then use the rule to solve the problem.

c. Solve.

Rows, x	1	2	3	4	30
Seats, y	5	6	7	8	34

Function Rule: $y =$ $x + 4$

$y =$ 30 + 4 = 34

$y =$ 34 seats

d. Check.

34 = x + 4

− 4 − 4

30 = x

WORK ON YOUR OWN

HOW TO

Solve Problems by Finding a Pattern

Pedro tutors his neighbor's child. He earns \$12 for 1 hour, \$24 for 2 hours, \$36 for 3 hours, and \$48 for 4 hours. If this pattern continues, how much will Pedro earn if he tutors for 20 hours?

1. **What am I trying to find?** how much Pedro will earn if he tutors for 20 hours
2. **How can I find the answer?** Organize the information in an input-output table, find a function rule, then use the rule to solve the problem.
3. **Solve.**

Hours, x	1	2	3	4	20
Pay, y	\$12	\$24	\$36	\$48	____

Function Rule: $y = 12x$

$y = 12x = 12 \times 20 = 240$

$y =$ \$240

4. **Check.**

$y = 12x$

$240 = 12x$

$\frac{240}{12} = \frac{12x}{12}$

$20 = x$

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Now ask, "Is my answer correct?" To check the question, substitute 36 for y in the function rule and solve for x. What is the function rule with 36 substituted for y? ($36 = 4x$) **Divide both sides by 4. What is 36 divided by 4?** (9) **This is the corresponding x-value given in the problem, so the answer is correct.**

TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 3

First ask, "What am I trying to find?" What needs to be found? (the number of seats in the 30th row of the theater) **Now ask, "How can I find the answer?" How can the answer be found?** (Organize the information in an input-output table, find a function rule for the input-output pairs, then use the function rule to find the number of seats in the 30th row of the theater.) **Use rows as the input, or x-values, in the table. What will be the output, or y-values, in the table?** (seats)

The first pair of values is entered in the table as x equals 1 and y equals 5. What is the next pair of input-output values? ($x = 2$ and $y = 6$) **What is the third pair of input-output values?** ($x = 3$ and $y = 7$) **What is the fourth pair of input-output values?** ($x = 4$ and $y = 8$) **Now find the function rule. What can be done to each input value to get the corresponding output value?** (Add 4.)

How is the function rule written as an equation? ($y = x + 4$) **Now solve the problem. Substitute 30 for x in the function rule and solve for y. What is the equation with 30 substituted for x?** ($y = 30 + 4$) **What is the value of y?** ($y = 34$) **How many seats are in the 30th row?** (34 seats)

To check the answer, substitute 34 for y in the function rule and solve for x. What is the function rule when 34 is substituted for y? ($34 = x + 4$) **Subtract 4 from both sides. What does x equal?** (30) **Is the solution correct?** (yes) **Why?** (This x-value matches the x-value that corresponds to $y = 34$ in the problem.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words for each process step.

Problems 4–9

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 4, 5, and 7–9 **Reminder**

Encourage students to make input-output tables to solve these problems.

CHECK UP

ASSESS INFORMALLY

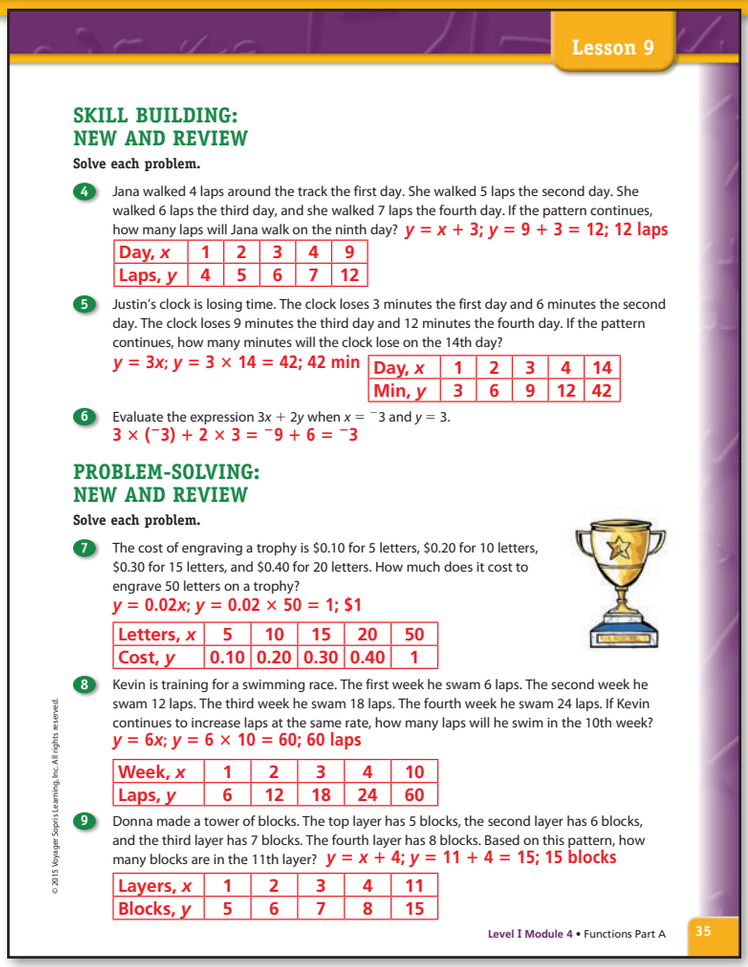
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1d: The student chose a rule that only works for one pair of values. Remind the student to find a function rule that works for every pair of values in the table.

If student answered 1b: The student multiplied by 2 instead of 3, which is the rule. Use Additional Resources in I.4.8 to reteach how to find a function rule given an input-output table.

If student answered 2b: The student continued the pattern of the numbers in the second row of the table to find the missing value instead of using the function rule. Use Additional Resources in I.4.9 to reteach how to use a table to solve problems involving functions.



Lesson 9

SKILL BUILDING: NEW AND REVIEW

Solve each problem.

4. Jana walked 4 laps around the track the first day. She walked 5 laps the second day. She walked 6 laps the third day, and she walked 7 laps the fourth day. If the pattern continues, how many laps will Jana walk on the ninth day? $y = x + 3$; $y = 9 + 3 = 12$; 12 laps

Day, x	1	2	3	4	9
Laps, y	4	5	6	7	12

5. Justin's clock is losing time. The clock loses 3 minutes the first day and 6 minutes the second day. The clock loses 9 minutes the third day and 12 minutes the fourth day. If the pattern continues, how many minutes will the clock lose on the 14th day?
$y = 3x$; $y = 3 \times 14 = 42$; 42 min

Day, x	1	2	3	4	14
Min, y	3	6	9	12	42

6. Evaluate the expression $3x + 2y$ when $x = {}^{-}3$ and $y = 3$.
$3 \times ({}^{-}3) + 2 \times 3 = {}^{-}9 + 6 = {}^{-}3$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

7. The cost of engraving a trophy is \$0.10 for 5 letters, \$0.20 for 10 letters, \$0.30 for 15 letters, and \$0.40 for 20 letters. How much does it cost to engrave 50 letters on a trophy?
$y = 0.02x$; $y = 0.02 \times 50 = 1$; \$1

Letters, x	5	10	15	20	50
Cost, y	0.10	0.20	0.30	0.40	1

8. Kevin is training for a swimming race. The first week he swam 6 laps. The second week he swam 12 laps. The third week he swam 18 laps. The fourth week he swam 24 laps. If Kevin continues to increase laps at the same rate, how many laps will he swim in the 10th week?
$y = 6x$; $y = 6 \times 10 = 60$; 60 laps

Week, x	1	2	3	4	10
Laps, y	6	12	18	24	60

9. Donna made a tower of blocks. The top layer has 5 blocks, the second layer has 6 blocks, and the third layer has 7 blocks. The fourth layer has 8 blocks. Based on this pattern, how many blocks are in the 11th layer? $y = x + 4$; $y = 11 + 4 = 15$; 15 blocks

Layers, x	1	2	3	4	11
Blocks, y	5	6	7	8	15

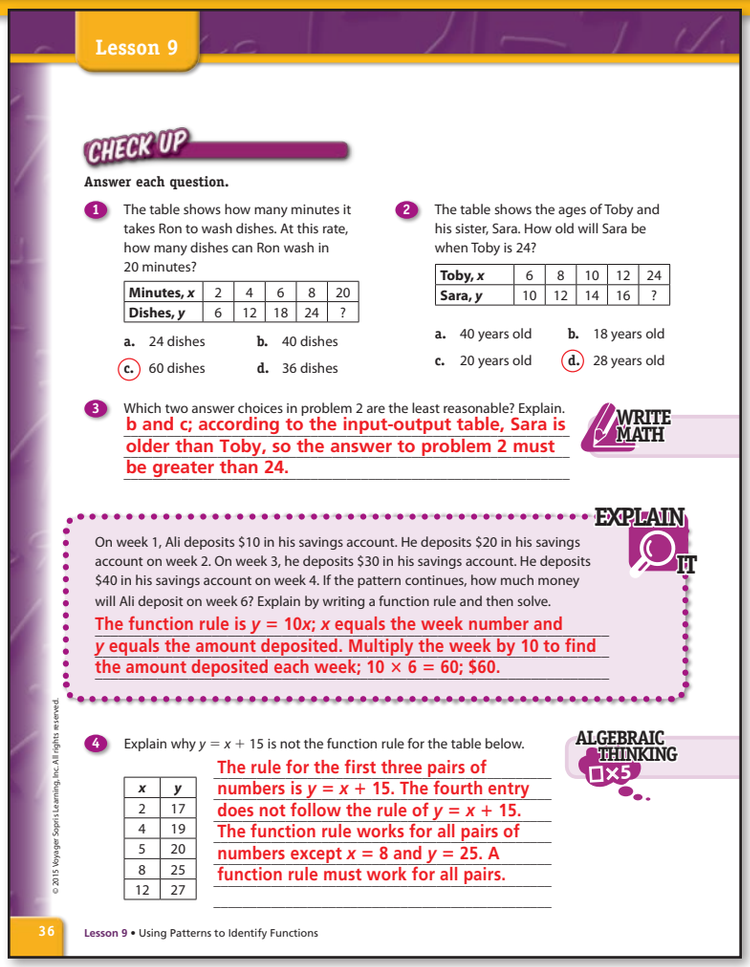
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WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the correct answer must be a number greater than 24. Review with the student how to find the function rule for the table and how to use the rule to solve the problem.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 9

CHECK UP

Answer each question.

1. The table shows how many minutes it takes Ron to wash dishes. At this rate, how many dishes can Ron wash in 20 minutes?

Minutes, x	2	4	6	8	20
Dishes, y	6	12	18	24	?

a. 24 dishes b. 40 dishes
c. 60 dishes d. 36 dishes

2. The table shows the ages of Toby and his sister, Sara. How old will Sara be when Toby is 24?

Toby, x	6	8	10	12	24
Sara, y	10	12	14	16	?

a. 40 years old b. 18 years old
c. 20 years old d. 28 years old

3. Which two answer choices in problem 2 are the least reasonable? Explain.
b and c; according to the input-output table, Sara is older than Toby, so the answer to problem 2 must be greater than 24.

WRITE MATH

EXPLAIN IT

On week 1, Ali deposits $10 in his savings account. He deposits $20 in his savings account on week 2. On week 3, he deposits $30 in his savings account. He deposits $40 in his savings account on week 4. If the pattern continues, how much money will Ali deposit on week 6? Explain by writing a function rule and then solve.
The function rule is $y = 10x$; x equals the week number and y equals the amount deposited. Multiply the week by 10 to find the amount deposited each week; $10 \times 6 = 60$; $60.

4. Explain why $y = x + 15$ is not the function rule for the table below.

x	y
2	17
4	19
5	20
8	25
12	27

The rule for the first three pairs of numbers is $y = x + 15$. The fourth entry does not follow the rule of $y = x + 15$. The function rule works for all pairs of numbers except $x = 8$ and $y = 25$. A function rule must work for all pairs.

ALGEBRAIC THINKING

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DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 4 Lesson 9
Reteach Teacher Edition Module 4 Lesson 9

Extra Practice
Student Book page 60

English Language Learners

Review the names and functions of the four operation symbols. Then write the four operation symbols on the board: $\times$, $\div$, $+$, $-$. Tell students that in a function table, when the value of y is greater than x, the function rule will typically include multiplication ($\times$) or addition ($+$). When y is less than x, the function rule will typically include division ($\div$) or subtraction ($-$).

Students with Special Needs

Create a function table with a middle column for students to record possible rules for each pair of values. Model writing possible rules in the middle column. For example, when $x = 1$ and $y = 3$, write $+ 2$ or $\times 3$ in the middle column. Explain that students can write the rules for getting from x to y in each row, circle the common rule, and write the circled rule as the function rule for the table.

HANDS-ON GUIDED DISCOVERY

Lesson 10

Objective

To use grid paper to discover the relationship between the side length and perimeter of a regular polygon

CONCEPTUAL DEVELOPMENT

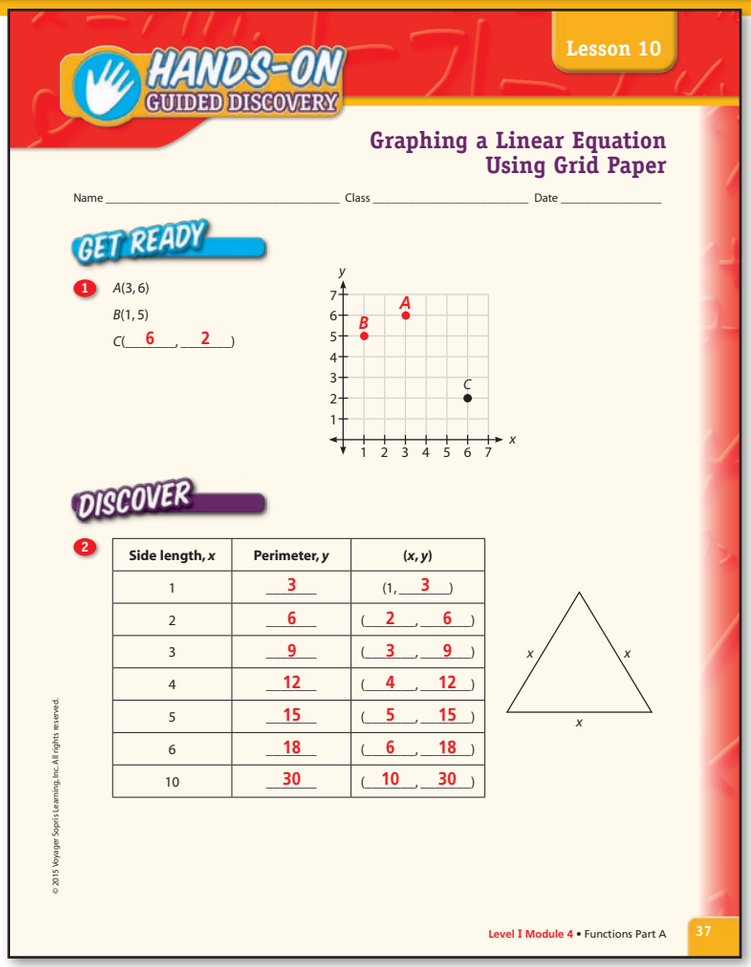
Materials

- Copy Master: Grid Paper
- straightedge

Lesson Notes

Before beginning the lesson, be sure each student has a Student Book, three copies of Copy Master: Grid Paper, and a straightedge.

In problems 2–4, make sure students draw their coordinate planes large enough to accommodate the points in the tables. Also make sure they use one grid square as one square unit.



HANDS-ON GUIDED DISCOVERY

Lesson 10

Graphing a Linear Equation Using Grid Paper

Name _____ Class _____ Date _____

GET READY

1. $A(3, 6)$
   $B(1, 5)$
   $C(\underline{6}, \underline{2})$

DISCOVER

2.

Side length, x	Perimeter, y	(x, y)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 12)
5	15	(5, 15)
6	18	(6, 18)
10	30	(10, 30)

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GET READY

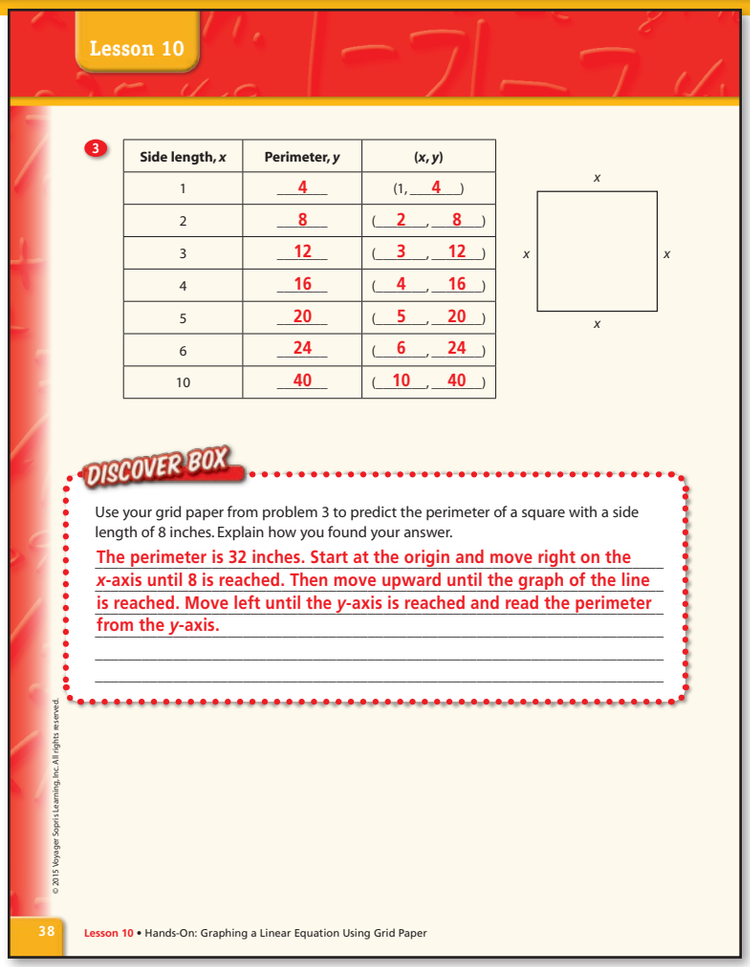
Problem 1

On the coordinate plane shown in problem 1, the point at which the *x*-axis and the *y*-axis intersect is the origin. To graph the ordered pair (3, 6), start at the origin. Move 3 units to the right. From there, move 6 units up. Graph the point and label it *A*. To graph the ordered pair (1, 5), start at the origin. How many units to the right of the origin is point *B*? (1) **How many units up from the origin is point *B*?** (5) **Graph the point and label it *B*. To write an ordered pair that locates point *C* on the grid, follow the point down to the *x*-axis. What point on the *x*-axis lines up with point *C*?** (6) **The *x*-coordinate of point *C* is 6. Locate point *C* again. Follow the point across to the *y*-axis. What point on the *y*-axis lines up with point *C*?** (2) **The *y*-coordinate of point *C* is 2. The ordered pair is (6, 2).**

DISCOVER

Problem 2

In problem 2, the relationship between the side length of equilateral triangles and perimeter will be described graphically. How is the perimeter of an equilateral triangle found? (by adding the side lengths or multiplying a side length by 3 because the sides are equal) **If the side length of the equilateral triangle is 1 unit, what is the perimeter?** (3 units) **Record the perimeter in the second column of the table. If the side length of the equilateral triangle is 2 units, what is the perimeter?** (6 units) **Record the perimeter in the table. If the side length of the equilateral triangle is 3 units, what is the perimeter?** (9 units) **Record the perimeter in the table. If the side length of the equilateral triangle is 4 units, what is the perimeter?** (12 units) **Record the perimeter in the table. If the side length of the equilateral triangle is 5 units, what is the perimeter?** (15 units) **Record the perimeter in the table. If the side length of the equilateral triangle is 6 units, what is the perimeter?** (18 units) **Record the perimeter in the table. If the side length of the equilateral triangle is 10 units, what is the perimeter?** (30 units) **Record the perimeter in the table.**



Lesson 10

3

Side length, x	Perimeter, y	(x, y)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)
5	20	(5, 20)
6	24	(6, 24)
10	40	(10, 40)

DISCOVER BOX

Use your grid paper from problem 3 to predict the perimeter of a square with a side length of 8 inches. Explain how you found your answer.

The perimeter is 32 inches. Start at the origin and move right on the x-axis until 8 is reached. Then move upward until the graph of the line is reached. Move left until the y-axis is reached and read the perimeter from the y-axis.

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To describe the relationship between side length and perimeter graphically, consider the side length and the corresponding perimeter as an ordered pair. Suppose the *x*-coordinate of the ordered pair represents side length and the *y*-coordinate represents perimeter. What ordered pair can be written for an equilateral triangle with a side length of 1 and perimeter of 3? ((1, 3)) **Complete the third column of the table for the remaining side lengths and perimeters. On grid paper, draw a coordinate plane like the one in problem 1. Place the origin at the bottom left corner. The *x*-coordinates will go to 10, and the *y*-coordinates will go to 30. Graph the points from the table. Is there any pattern to the points?** (yes) **What is the pattern?** (The points lie in a straight line.) **Use a straightedge to draw a line that passes through all the points. Because all the points lie on a straight line, the relationship between side length and perimeter is linear.**

Problem 3

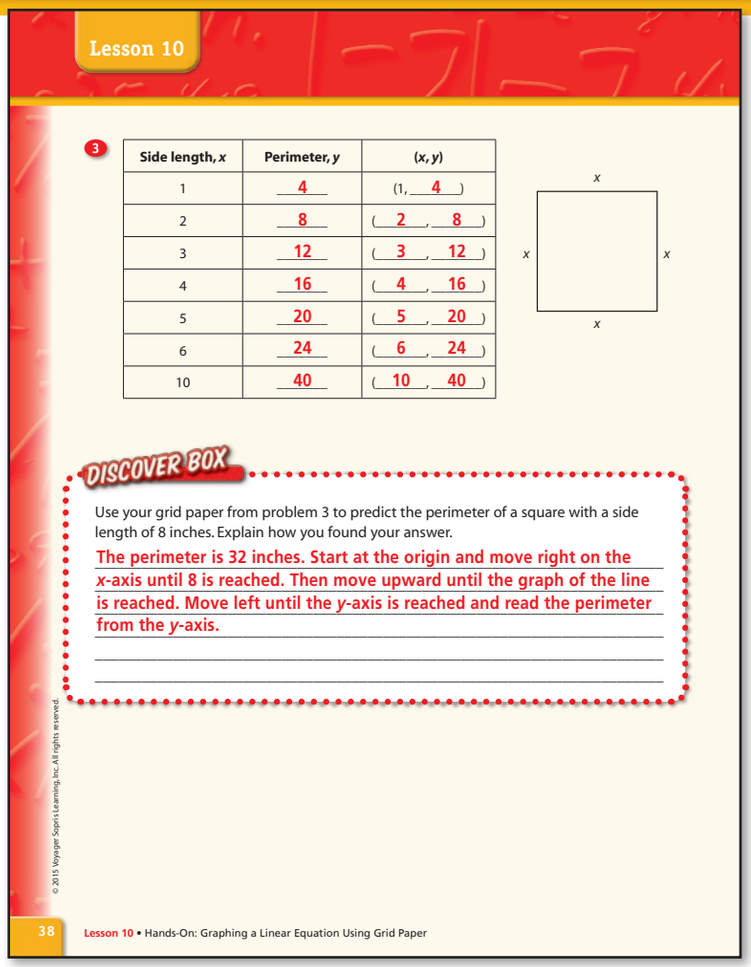
In problem 3, the relationship between the side length of squares and perimeter will be described graphically. How is the perimeter of a square found? (by adding the side lengths or multiplying a side length by 4) **If the side length of the square is 1 unit, what is the perimeter?** (4 units) **Record the perimeter in the second column of the table. If the side length of the square is 2 units, what is the perimeter?** (8 units) **Record the perimeter in the table. If the side length of the square is 3 units, what is the perimeter?** (12 units) **Record the perimeter in the table. If the side length of the square is 4 units, what is the perimeter?** (16 units) **Record the perimeter in the table. If the side length of the square is 5 units, what is the perimeter?** (20 units) **Record the perimeter in the table. If the side length of the square is 6 units, what is the perimeter?** (24 units) **Record the perimeter in the table. If the side length of the square is 10 units, what is the perimeter?** (40 units) **Record the perimeter in the table.**

To describe the relationship between side length and perimeter graphically, consider the side length and the corresponding perimeter as an ordered pair. Suppose the *x*-coordinate of the ordered pair represents side length and the *y*-coordinate represents perimeter. What ordered pair can be written for a square with a side length of 1 and perimeter of 4? ((1, 4)) **Complete the third column of the table for the remaining side lengths and perimeters.**

On the second sheet of grid paper, draw a coordinate plane like the one in problem 1. Place the origin at the bottom left corner. The *x*-coordinates will go to 10, and the *y*-coordinates will go to 40. Plot the points from the table. Is there any pattern to the points? (yes) **What is the pattern?** (The points lie in a straight line.) **Use a straightedge to draw a line that passes through all the points. Because all the points lie on a straight line, the relationship between side length and perimeter is linear.**

DISCOVER BOX

How do you find the perimeter of a square? (add the side lengths or multiply a side length by 4) **What is the perimeter of a square with a side length of 8 units?** (32 units) **Look at your graph from problem 3. Which axis represents the side length of a square?** (*x*-axis) **Which axis represents the perimeter of a square?** (*y*-axis) **How can you use your line to find the perimeter of a square with a side length of 8 units rather than calculating the perimeter?** (Start at the origin and move right on the *x*-axis until 8 is reached. Then move upward until the graph of the line is reached. Then move left until the *y*-axis is reached and read the perimeter from the *y*-axis.)



Lesson 10

3

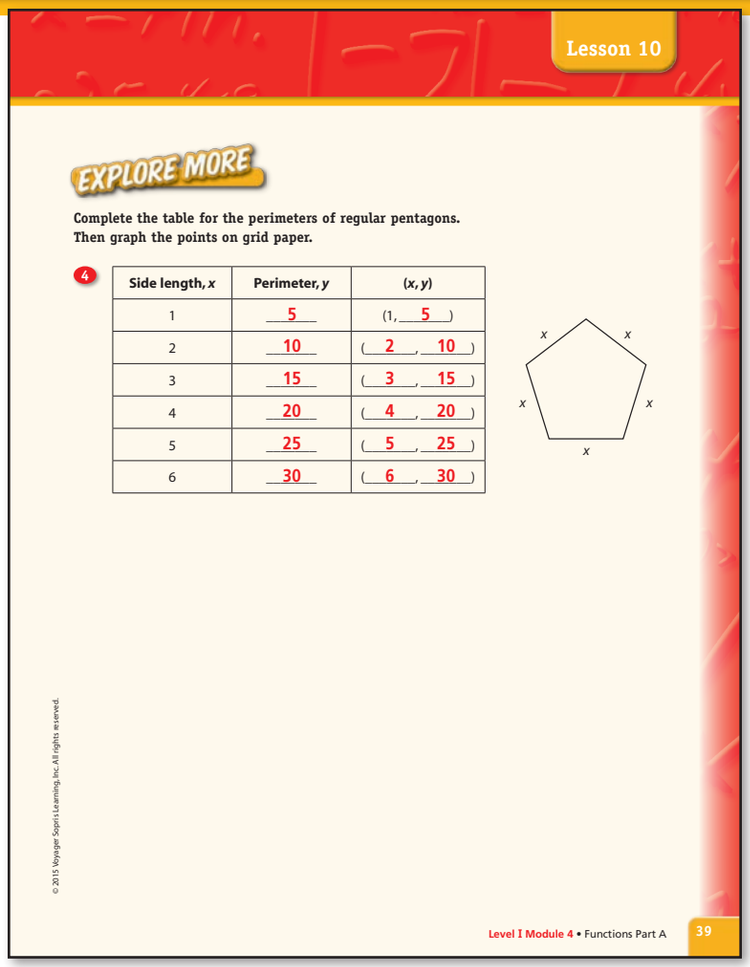
Side length, *x*	Perimeter, *y*	(*x*, *y*)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)
5	20	(5, 20)
6	24	(6, 24)
10	40	(10, 40)

DISCOVER BOX

Use your grid paper from problem 3 to predict the perimeter of a square with a side length of 8 inches. Explain how you found your answer.

The perimeter is 32 inches. Start at the origin and move right on the *x*-axis until 8 is reached. Then move upward until the graph of the line is reached. Move left until the *y*-axis is reached and read the perimeter from the *y*-axis.

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Lesson 10

EXPLORE MORE

Complete the table for the perimeters of regular pentagons. Then graph the points on grid paper.

4

Side length, x	Perimeter, y	(x, y)
1	5	(1, 5)
2	10	(2, 10)
3	15	(3, 15)
4	20	(4, 20)
5	25	(5, 25)
6	30	(6, 30)

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EXPLORE MORE

Problem 4

Have students work independently. Students should repeat the process used in problems 2 and 3 for regular pentagons.

Extension: Have students or partners explore more graphs of linear equations by graphing the function rules in Lesson 4.8 on grid paper.

Lesson Adaptations for Module 4

Lesson 4.11

This lesson focuses on graphing linear equations. It may help students to graph the equations on grid paper.

Lesson 11

Objective

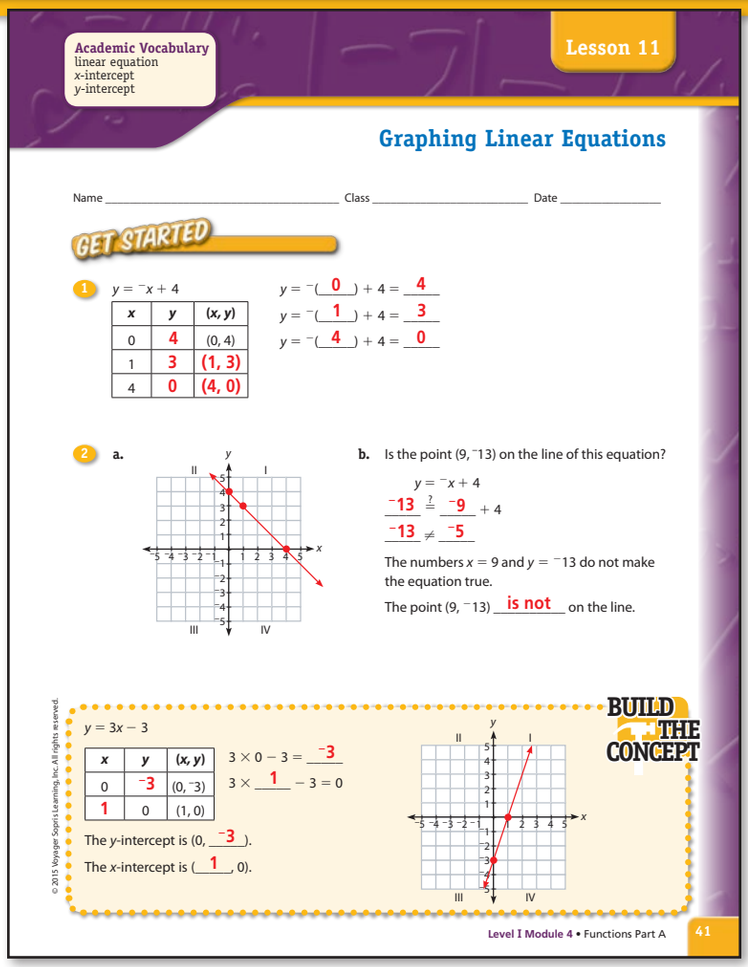
To graph a linear equation

Preskills	Lesson
Evaluating Algebraic Expressions	I.3.PL1
Coordinate Planes in Four Quadrants	I.4.PL1
Functions	I.4.7

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **linear equation**
  an equation whose graph is a straight line
- ***x*-intercept**
  the point where a graph crosses the *x*-axis
- ***y*-intercept**
  the point where a graph crosses the *y*-axis



Academic Vocabulary
linear equation
x-intercept
y-intercept

Lesson 11

Graphing Linear Equations

Name _____ Class _____ Date _____

GET STARTED

1 $y = -x + 4$

x	y	(x, y)
0	4	(0, 4)
1	3	(1, 3)
4	0	(4, 0)

$y = -(0) + 4 = 4$
$y = -(1) + 4 = 3$
$y = -(4) + 4 = 0$

2 a.

b. Is the point (9, −13) on the line of this equation?

$y = -x + 4$
$-13 \stackrel{?}{=} -9 + 4$
$-13 \neq -5$

The numbers $x = 9$ and $y = -13$ do not make the equation true.

The point (9, −13) is not on the line.

BUILD THE CONCEPT

$y = 3x - 3$

x	y	(x, y)
0	−3	(0, −3)
1	0	(1, 0)

$3 \times 0 - 3 = -3$
$3 \times 1 - 3 = 0$

The *y*-intercept is (0, −3).
The *x*-intercept is (1, 0).

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The function rule for this input-output table is *y* equals −*x* plus 4. The output value *y* when *x* equals zero is found by substituting zero for *x* in the function rule. What is zero plus 4? (4) **The output value is 4. The *x*-value in the first column of the table is the *x*-coordinate in an ordered pair. The *y*-value in the second column of the table is the *y*-coordinate in an ordered pair. When *x* equals zero, *y* equals 4. The ordered pair is written (0, 4).**

What is the next input value for *x*? (1) **Substitute 1 for *x* in the function rule. What is the value of *y* when *x* equals 1?** (3) **The *x*-value in the first column of the table is the *x*-coordinate in an ordered pair. The *y*-value in the second column of the table is the *y*-coordinate in an ordered pair. What is the ordered pair?** ((1, 3))

Now, substitute 4 for *x* in the function rule. What is the value of *y* when *x* equals 4? (0) **What is the ordered pair?** ((4, 0))

MODEL NEW SKILLS

Problem 2

In problem 2a, graph points for the three ordered pairs from problem 1 on the coordinate plane. Draw a line through the three points on the graph. Do the three points lie in a straight line? (yes) **Then the equation for the function rule *y* equals −*x* plus 4 is a linear equation.**

Look at problem 2b. The question asks if the point (9, −13) is on the line graphed in problem 2a. The grid is not big enough to see if the line goes through this point. So, to answer the question, check to see if the values *x* equals 9 and *y* equals −13 make the equation of the line true. The equation is true if both sides have the same value.

What is the equation of the line? ($y = -x + 4$) **What value should be substituted for *y*?** (−13) **What value should be substituted for *x*?** (9) **What is −9 plus 4?** (−5) **Negative 13 does not equal −5; this is shown by an equal sign with a slash through it. Both sides of the equation are not equal to the same value. The values substituted for *x* and *y* do not make the equation true. So, the point (9, −13) is not on the line.**

TRY IT TOGETHER

Complete the input-output table, then graph the equation.

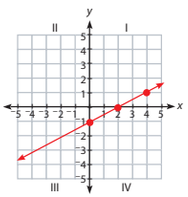
3 a. $y = \frac{x}{2} - 1$

x	y	(x, y)
0	−1	(0, −1)
2	0	(2, 0)
4	1	(4, 1)

$y = \frac{0}{2} - 1 = \underline{-1}$

$\frac{2}{2} - 1 = 0 \rightarrow x = \underline{2}$

$y = \frac{4}{2} - 1 = \underline{1}$



b. Is the point (8, 4) on the line of this equation?

$y = \frac{x}{2} - 1$

$4 \stackrel{?}{=} \frac{8}{2} - 1$

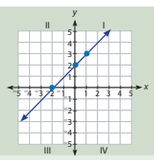
$4 \neq 3$

No, point (8, 4) is not on this line.

WORK ON YOUR OWN

HOW TO

Graph a Linear Equation

Using Symbols	Using Words
1. $y = x + 2$	Make an input-output table consisting of at least three ordered pairs. Include the *y*-intercept and the *x*-intercept. ***y*-intercept:** Find *y* when *x* is 0. ***x*-intercept:** Find *x* when *y* is 0. **Note:** When the *x*- and *y*-intercepts both occur at (0, 0), only one intercept will be used in the table.
2. 	Graph the points for the ordered pairs. Draw a line through the points.

x	y	(x, y)	
0	2	(0, 2)	← *y*-intercept
−2	0	(−2, 0)	← *x*-intercept
1	3	(1, 3)	

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BUILD THE CONCEPT

Model how to graph a linear equation with an *x*- and *y*-intercept.

Look at the line graphed in problem 2a. At what point does the line cross the *y*-axis? ((0, 4)) **The point at which a graph crosses the *y*-axis is called the *y*-intercept. So the point (0, 4) is the *y*-intercept of the graph of *y* equals −*x* plus 4. What is the *x*-coordinate of this point?** (0) **Any *y*-intercept will always have an *x*-coordinate of zero. Why?** (It is on the *y*-axis.)

Look at the line in problem 2a again. At what point does the graph cross the *x*-axis? ((4, 0)) **The point at which a graph crosses the *x*-axis is called the *x*-intercept. What is the *y*-coordinate of this point?** (0) **Any *x*-intercept will always have a *y*-coordinate of zero. Why?** (It is on the *x*-axis.)

Now, consider the equation *y* equals 3*x* minus 3. The *y*-intercept of the graph of this equation will have an *x*-coordinate of zero. When *x* is zero, what is *y*? (−3) **So, the *y*-intercept of the graph of *y* equals 3*x* minus 3 is (0, −3). Graph this point.**

The *x*-intercept of the graph of *y* equals 3*x* minus 3 will have a *y*-coordinate of zero. This means that a number must be found so that 3 times the number minus 3 is zero. The result is zero when 3 is subtracted from what number? (3) **So, 3 times what number is 3?** (1) **When *x* is 1, what is the value of *y*?** (0)

The *x*-intercept of the graph of *y* equals 3*x* minus 3 is (1, 0). Graph this point. Now, draw a straight line through the points to graph the equation.

TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 3

To complete the input-output table, first find the *y*-intercept. When *x* is zero, what is *y*? (−1) **What is the ordered pair?** ((0, −1)) **Now find the *x*-intercept. What value of *x* makes $\frac{x}{2}$ minus 1 equal to zero?** (2) **What is the ordered pair?** ((2, 0))

Any number can be chosen for the *x*-values in an input-output table. It is easier to graph integers rather than fractions or decimals, so choose *x*-values that will ensure the *y*-values are integers. Because a number is being divided by 2 in this equation, choose the last *x*-value so that it is a multiple of 2. When the *x*-value is divided by 2, the quotient will be an integer. For example, choose 4 as the other *x*-value. If 4 is used as the *x*-value, what is the ordered pair? ((4, 1))

What should be done next? (Graph the points for the ordered pairs.) **Graph each point and draw a line through the three points on the graph. Why is the equation for the function rule *y* equals $\frac{x}{2}$ minus 1 a linear equation?** (The points lie in a straight line.)

In problem 3b, to decide if (8, 4) is a point on the line, substitute *x* equals 8 and *y* equals 4 into the equation for the line, *y* equals $\frac{x}{2}$ minus 1, then simplify. If both sides of the equation equal the same value, then *x* equals 8 and *y* equals 4 is a solution of the equation, and (8, 4) is a point on the line. If both sides are not equal to the same value, then *x* equals 8 and *y* equals 4 is not a solution of the equation, and (8, 4) is not on the line.

How is the equation written with the values *x* equals 8 and *y* equals 4 substituted? ($4 = \frac{8}{2} - 1$) **What is 8 divided by 2 minus 1?** (3) **Are both sides of the equation equal to the same value?** (no) **The values *x* equals 8 and *y* equals 4 do not make the equation true, so (8, 4) is not on the line.**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Graph** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a graph. The instruction is immediately followed by application of the strategy in problem 7.

Have students read the problem.

Have students complete the graph by multiplying each *x*-value by 12 and finding the corresponding ordered pair.

Then have students graph the values on the coordinate graph.

To find the cost of 5 pizzas, find 5 on the horizontal axis and follow the line up to the graph.

The cost of 5 pizzas is $60.

The answer is reasonable because using the function rule, $y = 12x$, $y = 12 \times 5 = 60$.

Problems 4–9

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 9 Reminder

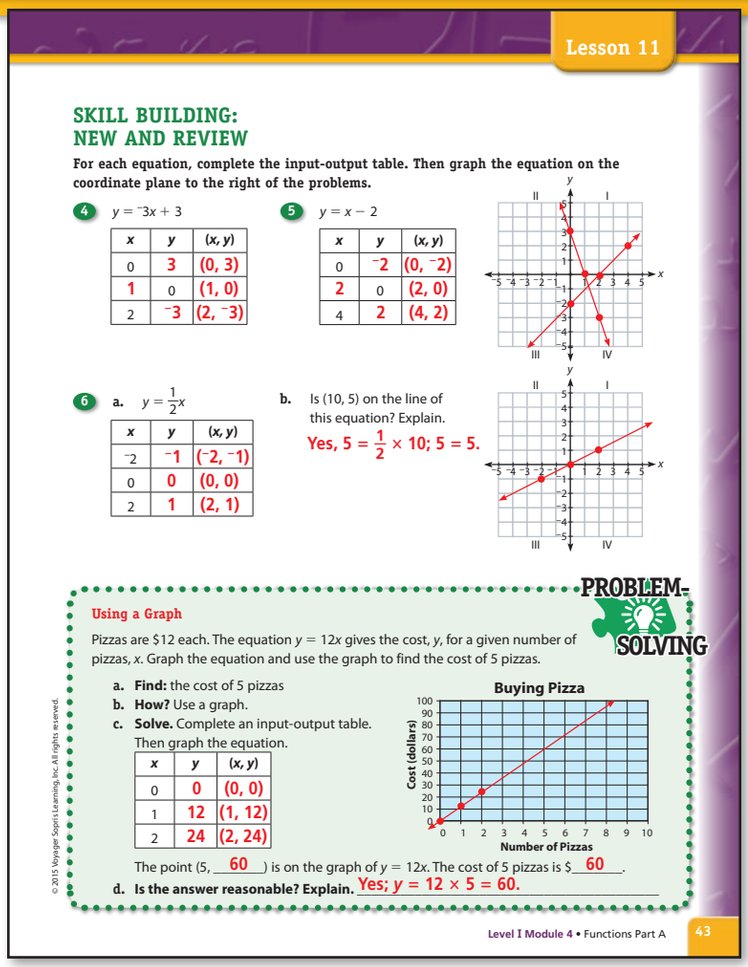
This problem reviews how to write equations.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.



Lesson 11

SKILL BUILDING: NEW AND REVIEW

For each equation, complete the input-output table. Then graph the equation on the coordinate plane to the right of the problems.

4 $y = -3x + 3$

x	y	(x, y)
0	3	(0, 3)
1	0	(1, 0)
2	−3	(2, −3)

5 $y = x - 2$

x	y	(x, y)
0	−2	(0, −2)
2	0	(2, 0)
4	2	(4, 2)

6 a. $y = \frac{1}{2}x$

x	y	(x, y)
−2	−1	(−2, −1)
0	0	(0, 0)
2	1	(2, 1)

b. Is (10, 5) on the line of this equation? Explain.

Yes, $5 = \frac{1}{2} \times 10$; $5 = 5$.

PROBLEM-SOLVING

Using a Graph

Pizzas are $12 each. The equation $y = 12x$ gives the cost, *y*, for a given number of pizzas, *x*. Graph the equation and use the graph to find the cost of 5 pizzas.

a. **Find:** the cost of 5 pizzas
b. **How?** Use a graph.
c. **Solve.** Complete an input-output table. Then graph the equation.

x	y	(x, y)
0	0	(0, 0)
1	12	(1, 12)
2	24	(2, 24)

Buying Pizza

Cost (dollars)

Number of Pizzas

The point (5, 60) is on the graph of $y = 12x$. The cost of 5 pizzas is $60.

d. **Is the answer reasonable? Explain.** Yes; $y = 12 \times 5 = 60$.

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If student answered 1b or 2a: The student reversed *x* and *y* when evaluating whether the ordered pair lies on the line. Review with the student how to use the equation to find the coordinates of points on a line. Use Additional Resources in I.4.11 to reteach how to graph linear equations.

If student answered problem 1c, 1d, 2b, or 2c: The student made a sign error when finding the value of *y* for a given value of *x*. Use Additional Resources in I.3.PL1 to reteach how to evaluate algebraic expressions.



WRITE MATH **If student answered problem 3 incorrectly:** While students work together, remind the student that the values of *x* and *y* that are solutions of the equation of the line must both be positive or both be negative. Use Additional Resources in I.3.PL1 to reteach how to evaluate algebraic expressions.

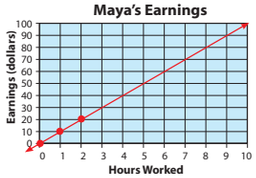
VmathLive Module Functions Part A Activity Graphing from a Table of Values 2 Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

7. Maya earns $10 an hour at her part-time job. The equation $y = 10x$ gives the earnings, y, in dollars for a given number of hours worked, x. Complete the input-output table for the equation. Then graph the equation and use the graph to find how much Maya earns after working 9 hours. $90

x	y	(x, y)
0	0	(0, 0)
1	10	(1, 10)
2	20	(2, 20)



8. Is the equation in problem 7 a linear equation? Explain. Yes; the graph is a straight line, so the equation is a linear equation.

9. Write the statement *26 is 2 more than 3 times x* as an equation. $26 = 3x + 2$

CHECK UP

Answer each question.

1. Which ordered pair lies on the graph of the equation $y = -2x + 4$?
   (a.) (1, 2) b. (2, 1) c. (1, 6) d. (−1, 2)

2. Which ordered pair lies on the graph of the equation $y = 5x$?
   a. (30, 6) b. (−6, 30) c. (6, −30) (d.) (6, 30)

3. Which two answer choices in problem 2 can be eliminated based on signs? Explain. b and c; x and y must both be positive or both be negative for the equation $y = 5x$ to be true.

WRITE MATH

4. Find the y-intercept and x-intercept of the graph of $y = \frac{x}{3} + 1$. Explain how the intercepts are found. To find the y-intercept, find the value of y when x is equal to 0. $y = \frac{0}{3} + 1$, $y = 1$, (0, 1). To find the x-intercept, find the value of x that makes y equal to 0. $0 = \frac{x}{3} + 1$, $x = -3$, (−3, 0).

CRITICAL THINKING

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DIFFERENTIATION

Additional Resources

VmathLive
Module: Functions Part A
Activity: Graphing from a Table of Values 2

Vmath Reteach
Reteach Student Book Module 4 Lesson 11
Reteach Teacher Edition Module 4 Lesson 11

Extra Practice
Student Book page 60

English Language Learners

The ordered pair for any intercept always has at least one coordinate that is equal to zero. Some students might have trouble remembering exactly which coordinate of an ordered pair is zero. Write these sentences on the board and have students copy them to use as a reference: *To find the x-intercept, find the x-coordinate. The y-coordinate is 0. To find the y-intercept, find the y-coordinate. The x-coordinate is 0.*

Students with Special Needs

Use the following activity to build the concept that when points form a line, the x- and y-coordinates all have the same relationship.

Show the graph of $y = x + 1$ to students. Make sure the numbers on the axes are clearly labeled. Ask students to name as many points as they can that are on the line. Have students list the values in an input-output table. Ask questions like, *What patterns can be found in the table? Is y related to x always in the same way? How are they related? How does this relationship compare to the equation?*

Lesson 12

Objective

To find the slope of a line

Work with students to complete these skills.

The slope of a line describes the steepness of the line. If a line rises from left to right, the slope is positive. If a line falls from left to right, the slope is negative. If a line is flat, then the slope of the line is 0. To find the slope of a line, first identify any two points on the line. Start with points *A* and *B*. What is the ordered pair for *A*? ((⁻4, 6)) **What is the ordered pair for *B*?** ((2, 3)) **The next step is to find the rise. The rise is the vertical change between the two points. Use point *A* as the starting point. Do you go up or down to get to point *B*?** (down) **How many do you go down?** (3) **So, the rise from point *A* to point *B* is ⁻3. Now find the run. The run is the horizontal change between the two points. Do you go to the left or to the right?** (right) **How many?** (6) **So, the run from point *A* to point *B* is 6. Now write the ratio of the rise to the run. What is the slope of the line?** ($\frac{-3}{6}$) **What is this written in simplest form?** ($\frac{-1}{2}$)

MATH Flash — Lesson 12

Slope

Name _____ Class _____ Date _____

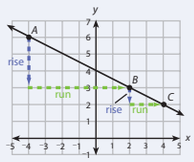
The slope of a line describes the steepness of the line.

A line that rises from left to right has a **positive slope**.

A line that falls from left to right has a **negative slope**.

A line that is horizontal has **zero slope**.

Use the following steps to find the slope of a line.



1. Identify any two points on the line.	Use A(⁻4, 6) and B(2, 3).	Use B(2, 3) and C(4, 2).
2. Find the rise between the two points. Rise is positive when moving up and negative when moving down.	Rise = down 3 Rise = ⁻3	Rise = down 1 Rise = ⁻1
3. Find the run between the two points. Run is positive when moving to the right and negative when moving to the left.	Run = right 6 Run = 6	Run = right 2 Run = 2
4. Write the ratio of the rise to the run. Simplify the ratio if possible.	Slope = $\frac{\text{rise}}{\text{run}} = \frac{-3}{6} = \frac{-1}{2}$	Slope = $\frac{\text{rise}}{\text{run}} = \frac{-1}{2}$

The line has a negative slope.

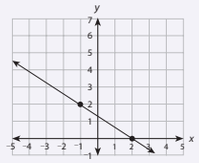
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Lesson 12

Find the slope of each line. Tell whether the slope is *positive*, *negative*, or *zero*.

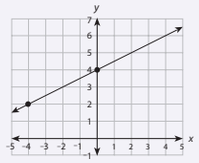
1



Slope = $\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$

negative slope

2



Slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$

positive slope

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Now find the slope using points *B* and *C*. What is the ordered pair for *B*? ((2, 3)) **What is the ordered pair for *C*?** ((4, 2)) **What is the rise from point *B* to point *C*?** ($^{-}1$) **What is the run from point *B* to point *C*?** (2) **What is the slope of the line written in simplest form?** ($\frac{^{-}1}{2}$) **Notice that the slope is the same no matter which two points on the line you use. Is the slope of this line positive, negative, or zero?** (negative) **How does the graph show that the slope is negative?** (The line falls from left to right.)

Lesson 13

Objective

To practice an open-ended response item

Work with students to complete these skills.

Part 1: Organize your solution: What do I know? **The first part of answering an open-ended question is to organize your solution. Read the problem. Find the given information from the problem. What number must the slope of a wheelchair ramp be less than or equal to?** ($\frac{1}{12}$) **What is the height of Leena's ramp?** (4 feet) **What is the length of the ramp?** (20 feet)

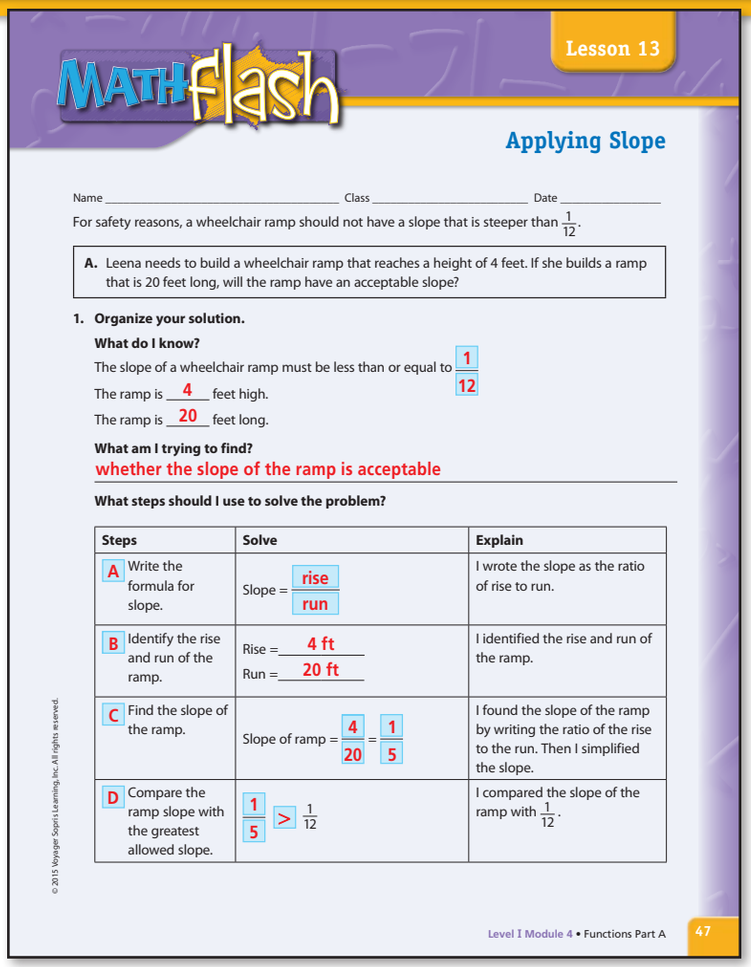
What am I trying to find? Read the problem again. What does the problem ask for? (whether the slope of the ramp will be acceptable)

What steps should I use to solve the problem? Now that you have found all the given information, you can break down the problem into steps to find whether the slope of the ramp will be acceptable.

Direct students' attention to the table. **When solving a problem, you can use a table like this one to organize your work. Each row represents a step you take to solve the problem.** Direct students' attention to the first row. **Write the letter *A* in the first box in the first row. What is Step A?** (Write the formula for slope.) **What is the formula for slope?** (slope = $\frac{\text{rise}}{\text{run}}$) **Does the sentence in the Explain column confirm your work in the Solve column?** (yes)

Direct students' attention to the second row. **Write the letter *B* in the box in the second row. What is Step B?** (Identify the rise and run of the ramp.) **What is the rise?** (4 ft) **What is the run?** (20 ft)

Direct students' attention to the third row. **Write the letter *C* in the first box in the third row. What is Step C?** (Find the slope of the ramp.) **What is the ratio of the rise to the run written as a fraction?** ($\frac{4}{20}$) **What is the slope of the ramp written as a fraction in simplest form?** ($\frac{1}{5}$)



MATHFlash — Lesson 13

Applying Slope

Name _____ Class _____ Date _____

For safety reasons, a wheelchair ramp should not have a slope that is steeper than $\frac{1}{12}$.

A. Leena needs to build a wheelchair ramp that reaches a height of 4 feet. If she builds a ramp that is 20 feet long, will the ramp have an acceptable slope?

1. Organize your solution.

What do I know?

The slope of a wheelchair ramp must be less than or equal to $\frac{1}{12}$

The ramp is 4 feet high.

The ramp is 20 feet long.

What am I trying to find?

whether the slope of the ramp is acceptable

What steps should I use to solve the problem?

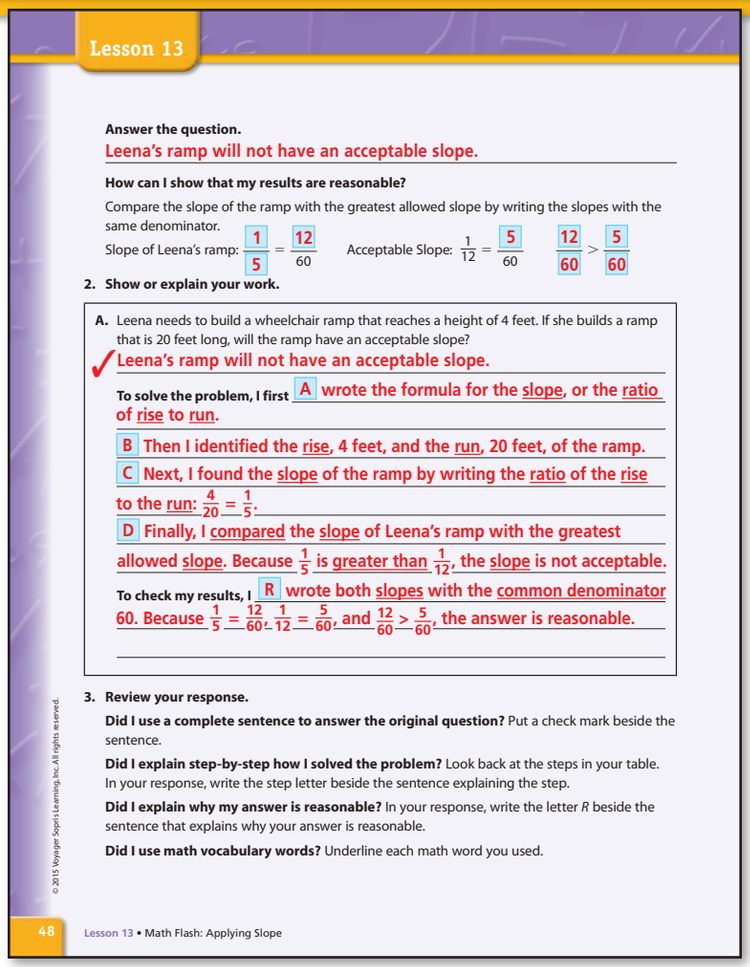
Steps	Solve	Explain
A Write the formula for slope.	Slope = $\frac{\text{rise}}{\text{run}}$	I wrote the slope as the ratio of rise to run.
B Identify the rise and run of the ramp.	Rise = 4 ft Run = 20 ft	I identified the rise and run of the ramp.
C Find the slope of the ramp.	Slope of ramp = $\frac{4}{20} = \frac{1}{5}$	I found the slope of the ramp by writing the ratio of the rise to the run. Then I simplified the slope.
D Compare the ramp slope with the greatest allowed slope.	$\frac{1}{5} > \frac{1}{12}$	I compared the slope of the ramp with $\frac{1}{12}$.

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Direct students' attention to the fourth row. **Write the letter *D* in the first box in the fourth row. What is Step D?** (Compare the slope of the ramp with the greatest allowed slope.) **What is the greatest allowed slope?** ($\frac{1}{12}$) **Is $\frac{1}{5}$ greater than or less than $\frac{1}{12}$?** (greater than)

Answer the question. Answer the original question using a complete sentence. If she builds a ramp that is 20 feet long, will the ramp have an acceptable slope? (Leena's ramp will not have an acceptable slope.)

How can I show that my results are reasonable? To check for reasonableness, write the slope of the ramp and the greatest allowed slope with the same denominator. Then compare the fractions. Write the fractions using the least common denominator, 60. What is $\frac{1}{5}$ written as a fraction with a denominator of 60? ($\frac{12}{60}$) **What is $\frac{1}{12}$ written as a fraction with a denominator of 60?** ($\frac{5}{60}$) **How does $\frac{12}{60}$ compare with $\frac{5}{60}$?** ($\frac{12}{60}$ is greater than $\frac{5}{60}$.) **Is the slope of Leena's ramp greater than the greatest allowed slope?** (yes)



Lesson 13

Answer the question.

Leena's ramp will not have an acceptable slope.

How can I show that my results are reasonable?

Compare the slope of the ramp with the greatest allowed slope by writing the slopes with the same denominator.

Slope of Leena's ramp: $\frac{1}{5} = \frac{12}{60}$ Acceptable Slope: $\frac{1}{12} = \frac{5}{60}$ $\frac{12}{60} > \frac{5}{60}$

2. Show or explain your work.

A. Leena needs to build a wheelchair ramp that reaches a height of 4 feet. If she builds a ramp that is 20 feet long, will the ramp have an acceptable slope?

✓ Leena's ramp will not have an acceptable slope.

To solve the problem, I first A **wrote the formula for the slope, or the ratio of rise to run.**

B Then I identified the rise, 4 feet, and the run, 20 feet, of the ramp.

C Next, I found the slope of the ramp by writing the ratio of the rise to the run: $\frac{4}{20} = \frac{1}{5}$.

D Finally, I compared the slope of Leena's ramp with the greatest allowed slope. Because $\frac{1}{5}$ is greater than $\frac{1}{12}$, the slope is not acceptable.

To check my results, I R **wrote both slopes with the common denominator 60. Because** $\frac{1}{5} = \frac{12}{60}$, $\frac{1}{12} = \frac{5}{60}$, and $\frac{12}{60} > \frac{5}{60}$, **the answer is reasonable.**

3. Review your response.

Did I use a complete sentence to answer the original question? Put a check mark beside the sentence.

Did I explain step-by-step how I solved the problem? Look back at the steps in your table. In your response, write the step letter beside the sentence explaining the step.

Did I explain why my answer is reasonable? In your response, write the letter *R* beside the sentence that explains why your answer is reasonable.

Did I use math vocabulary words? Underline each math word you used.

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Part 2: Show or explain your work. Now that you are organized, the next part is to show or explain your work. Write your response to the open-ended question using math vocabulary words that describe how you got your answer.

First, answer the original question using a complete sentence. If she builds a ramp that is 20 feet long, will the ramp have an acceptable slope? (Leena's ramp will not have an acceptable slope.) Next, describe in words how you solved the problem. To write these steps in words, use your answers from the table and finish the statement, "To solve the problem, I first …." Be sure to use complete sentences. Next, use complete sentences to describe how you checked the results for reasonableness. To do this, complete the sentence, "To check my results, I …."

Part 3: Review your response. Finally, review the written response by reading through the questions in Part 3. After you have read the questions and followed the instructions, you should have a check mark; the letters *A*, *B*, *C*, *D*, and *R*; and underlined vocabulary words in your response.

Lesson 14

Objective

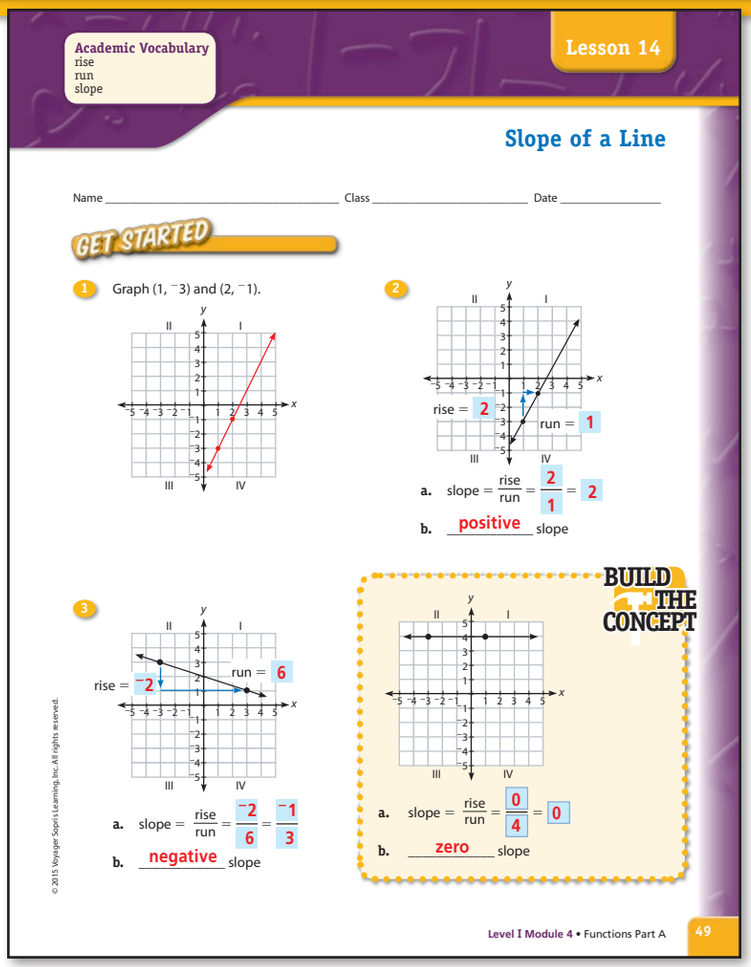
To find the slope of a line

Preskills	Lesson
Coordinate Planes in Four Quadrants	I.4.PL1

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **rise**
  the vertical change between any two points on a line
- **run**
  the horizontal change between any two points on a line
- **slope**
  the ratio of the vertical change, rise, to the horizontal change, run, between any two points on a line


Academic Vocabulary
rise
run
slope

Lesson 14

Slope of a Line

Name _____ Class _____ Date _____

GET STARTED

1 Graph (1, −3) and (2, −1).

2

rise = 2 run = 1

a. slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

b. positive slope

3

rise = −2 run = 6

a. slope = $\frac{\text{rise}}{\text{run}} = \frac{-2}{6} = \frac{-1}{3}$

b. negative slope

BUILD THE CONCEPT

a. slope = $\frac{\text{rise}}{\text{run}} = \frac{0}{4} = 0$

b. zero slope

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Graph the point for the ordered pair (1, −3) on the coordinate plane. Now, graph the point for the ordered pair (2, −1) on the same coordinate plane. Last, draw a line through the points.

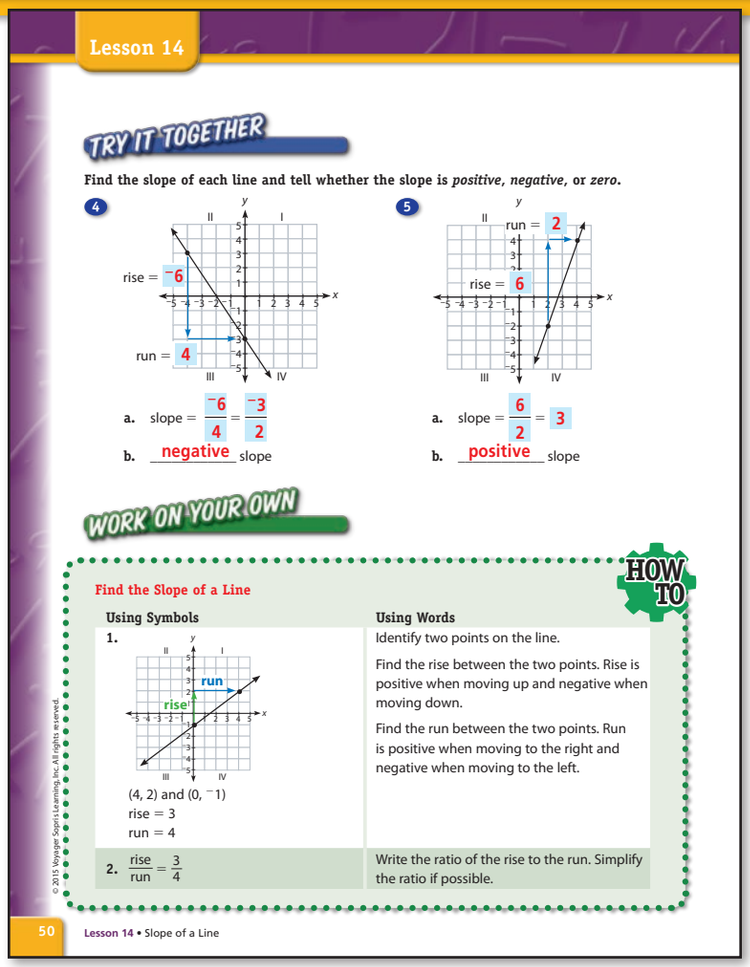
MODEL NEW SKILLS

Problem 2

The slope of a line is the ratio of the vertical change to the horizontal change between any two points on the line. Every nonvertical line has a slope. To find the slope of a line, first identify two points on the line. Any two points on the line will work, but it is most convenient to select two points at intersecting grid lines on the coordinate plane. What two points are marked on the line graphed in problem 2? ((1, −3) and (2, −1))

Now, find the rise. The rise is the vertical change between the two points. To find the rise, start at the point on the left. For this problem, start at (1, −3). Count the number of units that the second point is above or below the first point. If the second point is above the first point, then the rise is positive. If the second point is below the first point, then the rise is negative. The rise is indicated on the graph with a vertical blue arrow. Is the second point above or below the first point? (above) **Is the rise positive or negative?** (positive) **How many units above (1, −3) is (2, −1)?** (2) **The rise is 2.**

Next, find the run. The run is the horizontal change between two points. To find the run, start at the end of the arrow indicating the rise. For this problem, the rise arrow ends at (1, −1). Count from this point the number of units that the second point is to the left or to the right of the arrow's endpoint. If the second point is to the left of the first point, then the run is negative. If the second point is to the right of the first point, then the run is positive. The run is indicated on the graph with a horizontal blue arrow. Is the second point to the left or right of the first point? (to the right) **Is the run positive or negative?** (positive) **How many units to the right of (1, −3) is (2, −1)?** (1) **The run is 1.**


Lesson 14

TRY IT TOGETHER

Find the slope of each line and tell whether the slope is *positive*, *negative*, or *zero*.

4

rise = −6
run = 4

a. slope = $\frac{-6}{4} = \frac{-3}{2}$

b. negative slope

5

run = 2
rise = 6

a. slope = $\frac{6}{2} = 3$

b. positive slope

WORK ON YOUR OWN

HOW TO

Find the Slope of a Line

Using Symbols	Using Words
1. (4, 2) and (0, −1) rise = 3 run = 4	Identify two points on the line. Find the rise between the two points. Rise is positive when moving up and negative when moving down. Find the run between the two points. Run is positive when moving to the right and negative when moving to the left.
2. $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$	Write the ratio of the rise to the run. Simplify the ratio if possible.

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Last, compute the ratio of the rise to the run by making a fraction with the rise as the numerator and the run as the denominator. This is the slope of the line. What is the ratio of the rise to the run? ($\frac{2}{1}$) **Always simplify the ratio if possible. The ratio $\frac{2}{1}$ written in simplest form is 2. The slope of the line is 2. A line such as this one that rises from the left of the graph to the right of the graph has a positive slope.**

Problem 3

To find the slope of the line in problem 3, first select two points on the line. Use the points (−3, 3) and (3, 1) that are marked on the graph. To find the rise, start at (−3, 3). Count the number of units that (3, 1) is above or below (−3, 3). Is (3, 1) above or below (−3, 3)? (below) **Is the rise positive or negative?** (negative) **Why?** (The second point is below the first point.) **What is the rise?** (−2)

To find the run, start at the end of the rise arrow at (−3, 1), and follow the arrow to the second point indicating the run. Is the run positive or negative? (positive) **Why?** (The second point is to the right of the first point.) **What is the run?** (6)

The slope is the ratio of the rise to the run. What is the slope of the line? ($\frac{-2}{6}$) **What is the slope of the line written in simplest form?** ($\frac{-1}{3}$) **A line such as this one that falls from the left of the graph to the right of the graph has negative slope.**

BUILD THE CONCEPT

Model how to find the ratio of rise to run of a horizontal line.

What is noticeable about the line graphed on the coordinate plane? (It is a horizontal line.) **What is the rise between these two points?** (0) **What is the run between these two points?** (4) **What is the ratio of the rise to the run?** ($\frac{0}{4}$) **A horizontal line has a slope of zero. The slope of a vertical line, such as the *y*-axis, is not defined and therefore does not have a slope.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What is the first thing that must be done to find the slope of this line? (Find two points on the line.) **Use (−4, 3) and (0, −3). What is the next step?** (Find the rise.) **What is the rise?** (the vertical change between the two points) **Start with the point (−4, 3). What is the rise?** (−6) **Why is the rise negative?** ((0, −3) is below (−4, 3).) **Last, what is the run?** (4) **Why is the run positive?** ((0, −3) is to the right of (−4, 3).) **The line falls from the left of the graph to the right of the graph. Is the slope of this line positive, negative, or zero?** (negative) **What is the slope of the line?** ($\frac{-6}{4}$) **Is the ratio written in simplest form?** (no) **Simplify $\frac{-6}{4}$ but leave the ratio as an improper fraction. Do not write slope ratios as mixed numbers. What is the slope of this line written in simplest form?** ($\frac{-3}{2}$)

Problem 5

Use (2, −2) and (4, 4) to find the slope of the line in this problem. What is the rise? (6) **What is the run?** (2) **Is the slope of this line positive, negative, or zero?** (positive) **Why?** (The line rises from the left of the graph to the right of the graph.) **What is the slope of the line?** ($\frac{6}{2}$) **Is the ratio written in simplest form?** (no) **What is the slope of this line written in simplest form?** (3)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 6–13

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 9 Reminder

Have students complete the input-output table to generate coordinates to graph the line.

Problem 11 Reminder

Students should find the slope, then see how the error could have been made.

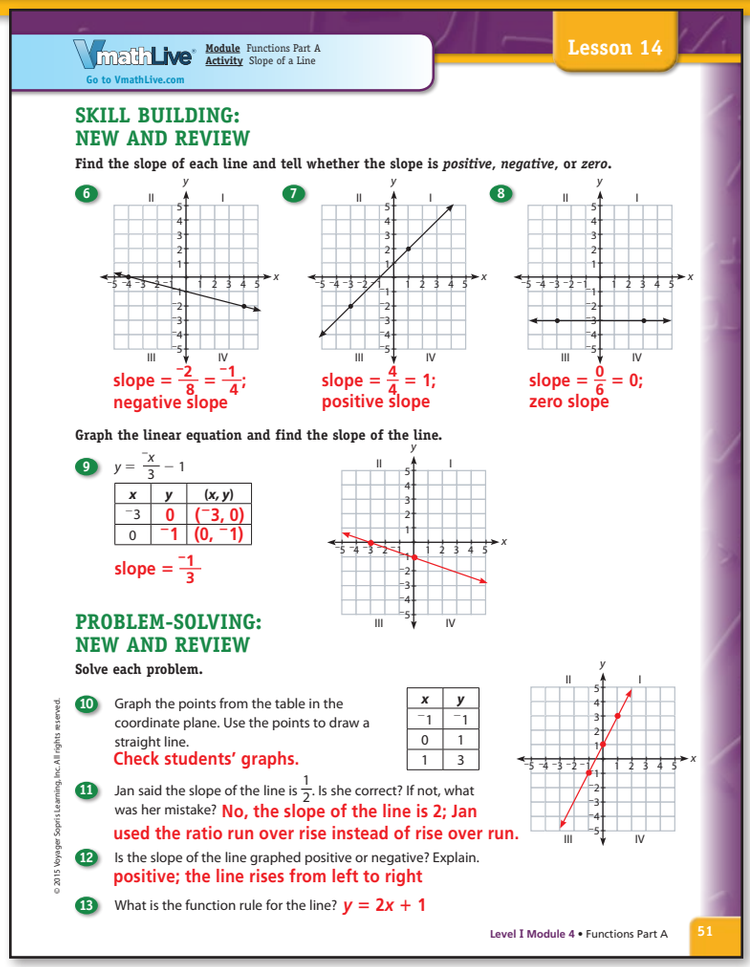
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2a: The student found the opposite value of the ratio of the run to the rise instead of finding the ratio of the rise to the run. Use Additional Resources in I.4.14 to reteach how to find the slope of a line.



VmathLive Module Activity Functions Part A Slope of a Line
Go to VmathLive.com

Lesson 14

SKILL BUILDING: NEW AND REVIEW

Find the slope of each line and tell whether the slope is *positive*, *negative*, or *zero*.

6 slope = $\frac{-2}{8} = \frac{-1}{4}$; negative slope

7 slope = $\frac{4}{4} = 1$; positive slope

8 slope = $\frac{0}{6} = 0$; zero slope

Graph the linear equation and find the slope of the line.

9 $y = \frac{-x}{3} - 1$

x	y	(x, y)
−3	0	(−3, 0)
0	−1	(0, −1)

slope = $\frac{-1}{3}$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

10 Graph the points from the table in the coordinate plane. Use the points to draw a straight line.
Check students' graphs.

x	y
−1	−1
0	1
1	3

11 Jan said the slope of the line is $\frac{1}{2}$. Is she correct? If not, what was her mistake? No, the slope of the line is 2; Jan used the ratio run over rise instead of rise over run.

12 Is the slope of the line graphed positive or negative? Explain.
positive; the line rises from left to right

13 What is the function rule for the line? $y = 2x + 1$

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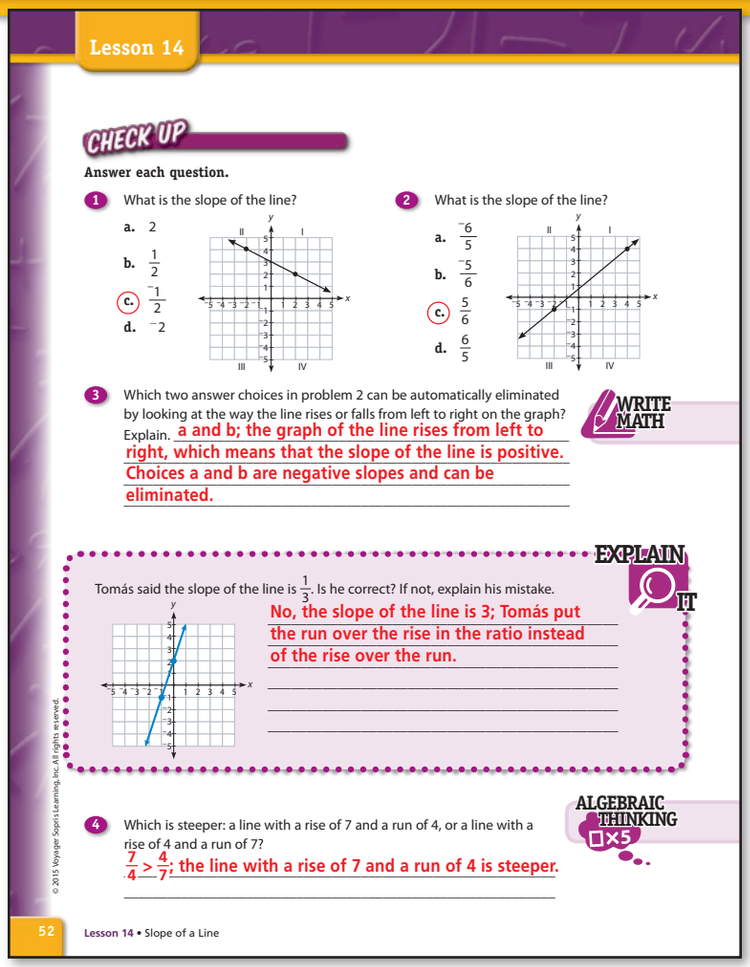
If student answered 1b or 2b: The student found the opposite value of the slope. Remind the student that the slope is positive if the line rises from left to right and negative if the line falls.

If student answered 1d or 2d: The student found the ratio of the run to the rise instead of the rise to the run. Use Additional Resources in I.4.14 to reteach how to find the slope of a line.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that a line that rises from left to right has positive slope. Any answer choice that is negative for the slope of the line in problem 2 can be eliminated.



Lesson 14

CHECK UP

Answer each question.

1 What is the slope of the line?

a. 2

b. $\frac{1}{2}$

c. $\frac{-1}{2}$

d. -2

2 What is the slope of the line?

a. $\frac{-6}{5}$

b. $\frac{-5}{6}$

c. $\frac{5}{6}$

d. $\frac{6}{5}$

3 Which two answer choices in problem 2 can be automatically eliminated by looking at the way the line rises or falls from left to right on the graph? Explain. a and b; the graph of the line rises from left to right, which means that the slope of the line is positive. Choices a and b are negative slopes and can be eliminated.

WRITE MATH

EXPLAIN IT

Tomás said the slope of the line is $\frac{1}{3}$. Is he correct? If not, explain his mistake.

No, the slope of the line is 3; Tomás put the run over the rise in the ratio instead of the rise over the run.

4 Which is steeper: a line with a rise of 7 and a run of 4, or a line with a rise of 4 and a run of 7?

$\frac{7}{4} > \frac{4}{7}$; the line with a rise of 7 and a run of 4 is steeper.

ALGEBRAIC THINKING

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52 Lesson 14 • Slope of a Line

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

Technology

Have students practice math fluency while competing against one another online in VmathLive activities. Online videos in VmathLive reinforce math concepts. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive

Module: Functions Part A

Activity: Slope of a Line

Vmath Reteach

Reteach Student Book Module 4 Lesson 14

Reteach Teacher Edition Module 4 Lesson 14

Extra Practice

Student Book page 61

English Language Learners

Students may not be familiar with the terms *rise, run,* and *slope* in a mathematical context. Review the meaning of each term. Assign each student a partner. Have students draw a picture that represents the rise, run, and slope of a line. For example, students may draw a ski slope and say the slope of the hill represents the slope of a line.

Students with Special Needs

Provide a concrete example of slope. Give each student a sheet of grid paper and a piece of uncooked spaghetti. Students can manipulate the spaghetti to pass through two points on the grid, count the rise and run, and determine the slope of the line.

Lesson 15

Objective

To graph and find the slope of a vertical or horizontal line

Preskills	Lesson
Graphing Linear Equations	I.4.11
Slope of a Line	I.4.14



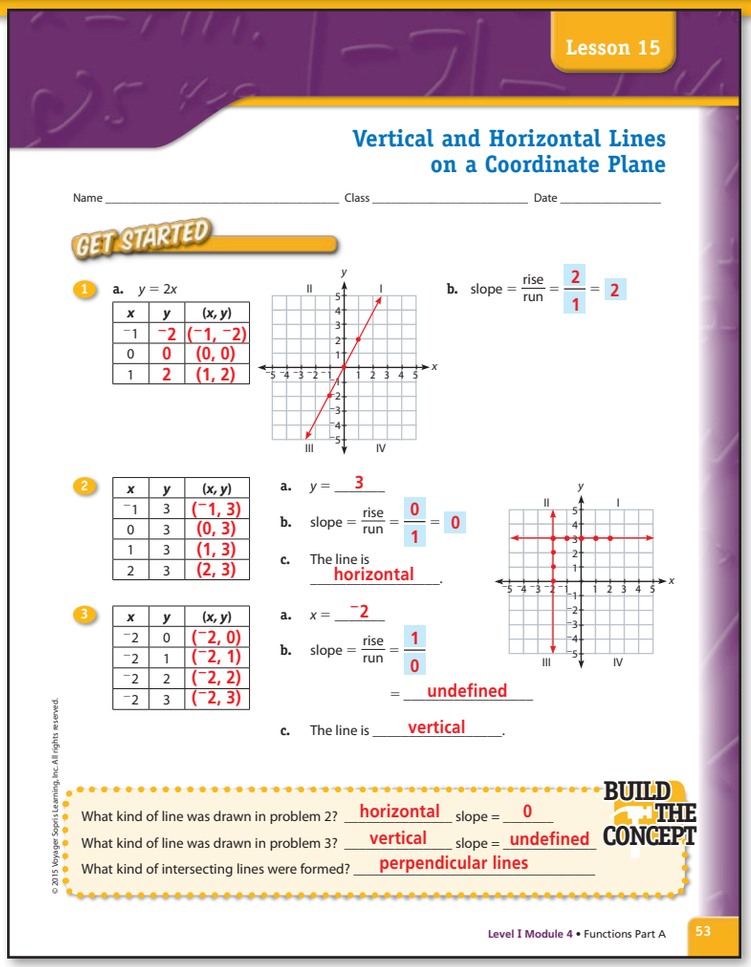
Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Look at problem 1a. What is the function rule for this input-output table? ($y = 2x$) **The output value y when x equals $^-1$ is found by substituting $^-1$ for x in the function rule. What is 2 times $^-1$?** ($^-2$) **The output value is $^-2$. The x-value in the table is the x-coordinate in an ordered pair. The y-value in the table is the y-coordinate in an ordered pair. When x equals $^-1$, y equals $^-2$. The ordered pair is written ($^-1, ^-2$). What is the next input value of x?** (0) **Substitute zero for x in the function rule. What is the value of y when x equals zero?** (0) **What is the ordered pair?** ((0, 0)) **Now, substitute 1 for x in the function rule. What is the value of y when x equals 1?** (2) **What is the ordered pair?** ((1, 2))

Graph the points for the ordered pairs from the table. What is the first ordered pair? (($^-1, ^-2$)) **Start at the origin, move left on the x-axis 1 unit for $^-1$, then move down 2 units for $^-2$. Graph the point. What is the next ordered pair?** ((0, 0)) **The ordered pair (0, 0) is graphed as a point at the origin. Graph the point at the origin. What is the next ordered pair?** ((1, 2)) **Graph the point at (1, 2). The three points on the coordinate plane lie in a straight line. Draw a line through the three points.**



Lesson 15

Vertical and Horizontal Lines on a Coordinate Plane

Name _____ Class _____ Date _____

GET STARTED

1 a. $y = 2x$

x	y	(x, y)
$^-1$	$^-2$	($^-1, ^-2$)
0	0	(0, 0)
1	2	(1, 2)

b. slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

2

x	y	(x, y)
$^-1$	3	($^-1$, 3)
0	3	(0, 3)
1	3	(1, 3)
2	3	(2, 3)

a. $y =$ 3

b. slope = $\frac{\text{rise}}{\text{run}} = \frac{0}{1} = 0$

c. The line is horizontal.

3

x	y	(x, y)
$^-2$	0	($^-2$, 0)
$^-2$	1	($^-2$, 1)
$^-2$	2	($^-2$, 2)
$^-2$	3	($^-2$, 3)

a. $x =$ $^-2$

b. slope = $\frac{\text{rise}}{\text{run}} = \frac{1}{0}$ = undefined

c. The line is vertical.

BUILD THE CONCEPT

What kind of line was drawn in problem 2? horizontal slope = 0

What kind of line was drawn in problem 3? vertical slope = undefined

What kind of intersecting lines were formed? perpendicular lines

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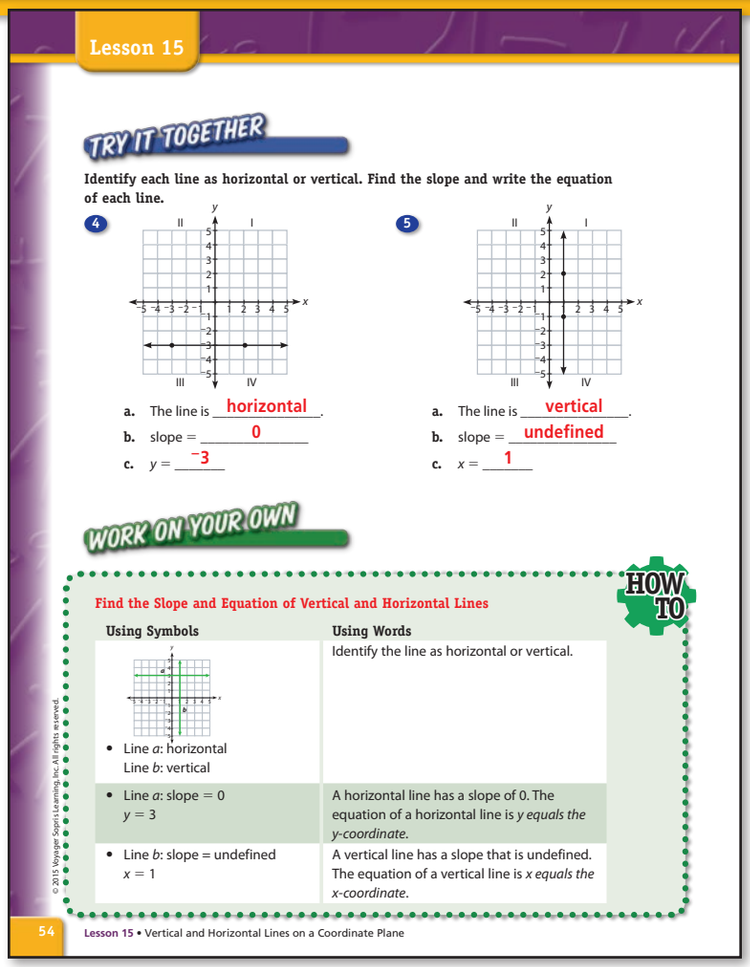
Look at problem 1b. The slope of a line is the ratio of the vertical change to the horizontal change between any two points on the line. Use the ordered pairs (0, 0) and (1, 2) to find the slope of the line. The rise is the vertical change between the two points. Start with the point (0, 0). What is the rise? (2) **The run is the horizontal change between the two points. What is the run?** (1) **What is the slope of the line?** ($\frac{2}{1}$) **What is $\frac{2}{1}$ written in simplest form?** (2) **The slope is 2.**

MODEL NEW SKILLS

Problem 2

Look at the input-output table in problem 2. Notice the value of y is always 3. When the value of x is $^-1$, what is the value of y? (3) **What is the ordered pair?** (($^-1$, 3)) **Graph the point for the ordered pair ($^-1$, 3). When the value of x is zero, what is the value of y?** (3) **What is the ordered pair?** ((0, 3)) **Graph the point for (0, 3).**

When the value of x is 1, what is the value of y? (3) **What is the ordered pair?** ((1, 3)) **Graph the point for (1, 3). If the value of x is 2, what is the value of y?** (3) **What is the ordered pair?** ((2, 3)) **Graph the point for (2, 3).**


Lesson 15

TRY IT TOGETHER

Identify each line as horizontal or vertical. Find the slope and write the equation of each line.

4

a. The line is horizontal.
b. slope = 0
c. $y = -3$

5

a. The line is vertical.
b. slope = undefined
c. $x = 1$

WORK ON YOUR OWN

HOW TO

Find the Slope and Equation of Vertical and Horizontal Lines

Using Symbols	Using Words
• Line *a*: horizontal Line *b*: vertical	Identify the line as horizontal or vertical.
• Line *a*: slope = 0 $y = 3$	A horizontal line has a slope of 0. The equation of a horizontal line is *y equals the y-coordinate.*
• Line *b*: slope = undefined $x = 1$	A vertical line has a slope that is undefined. The equation of a vertical line is *x equals the x-coordinate.*

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Draw a line through the points. What type of line is formed when the points are connected? (horizontal) **The equation of this line is *y* equals 3 because the *y*-coordinate is always 3. When each *y*-coordinate is the same in a set of ordered pairs, the points for these ordered pairs lie on a horizontal line. Use the ordered pairs (0, 3) and (1, 3) to find the slope of the line. What is the rise?** (0) **What is the run?** (1) **What is the slope of the line?** ($\frac{0}{1} = 0$) **The slope of any horizontal line is always zero. The equation for any horizontal line is *y equals the y-coordinate.***

Problem 3

Look at the input-output table in problem 3. Notice the value of *x* is always ⁻2. When the value of *y* is zero, what is the value of *x*? (⁻2) **What is the ordered pair?** ((⁻2, 0)) **Using the same coordinate plane as for problem 2, graph the point for (⁻2, 0). When the value of *y* is 1, what is the value of *x*?** (⁻2) **What is the ordered pair?** ((⁻2, 1)) **Graph the point for (⁻2, 1). When the value of *y* is 2, what is the value of *x*?** (⁻2) **What is the ordered pair?** ((⁻2, 2)) **Graph the point for (⁻2, 2). When the value of *y* is 3, what is the value of *x*?** (⁻2) **What is the ordered pair?** ((⁻2, 3)) **Graph the point for (⁻2, 3).**

Draw a line through the points. What type of line is formed when the points are connected? (vertical) **The equation of this line is *x* equals ⁻2 because the *x*-coordinate is always ⁻2. When each *x*-coordinate is the same in a set of ordered pairs, the points for these ordered pairs lie on a vertical line.**

Use the ordered pairs (⁻2, 0) and (⁻2, 1) to find the slope of the line. What is the rise? (1) **What is the run?** (0) **What is the slope of the line?** ($\frac{1}{0}$) **Any fraction with a denominator of zero is undefined. So the slope of this line is undefined. The slope of any vertical line is always undefined. The equation of any vertical line is *x equals the x-coordinate.***

BUILD THE CONCEPT

Model how to recognize intersecting vertical and horizontal lines as perpendicular.

Look at the graph in problems 2 and 3. What kind of line was drawn for problem 2, horizontal or vertical? (horizontal) **What kind of line was drawn for problem 3?** (vertical)

Do these lines intersect? (yes) **At what angle do they intersect?** (right angle) **So, when a line with a slope of zero and a line with an undefined slope cross, what kind of intersecting lines do they form?** (perpendicular lines)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

Is the line in problem 4 horizontal or vertical? (horizontal) **What is the slope of the line?** (0) **Look at the points to find the equation of the line. What two points are marked on the line?** ((⁻3, ⁻3) and (2, ⁻3))

Does the *x*- or the *y*-coordinate stay the same? (*y*-coordinate) **When the *y*-coordinates are the same value in a set of ordered pairs, the points for these ordered pairs lie on a horizontal line. What is the value of the *y*-coordinate?** (⁻3) **What is the equation of the line?** ($y = -3$)

Problem 5

Is the line in problem 5 horizontal or vertical? (vertical) **What is the slope of the line?** (undefined) **Look at the points to find the equation of the line. What two points are marked on the line?** ((1, 2) and (1, ⁻1))

Does the *x*- or the *y*-coordinate stay the same? (*x*-coordinate) **When the *x*-coordinates are the same value in a set of ordered pairs, the points for these ordered pairs lie on a vertical line. What is the value of the *x*-coordinate?** (1) **What is the equation of the line?** ($x = 1$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

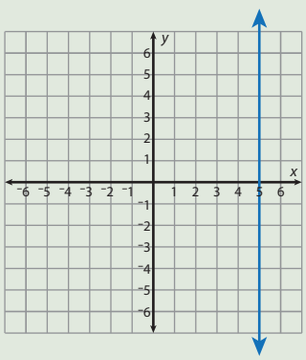
Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using Reasonableness** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 12.

Have students read the problem.

It may be helpful for students to graph the line:



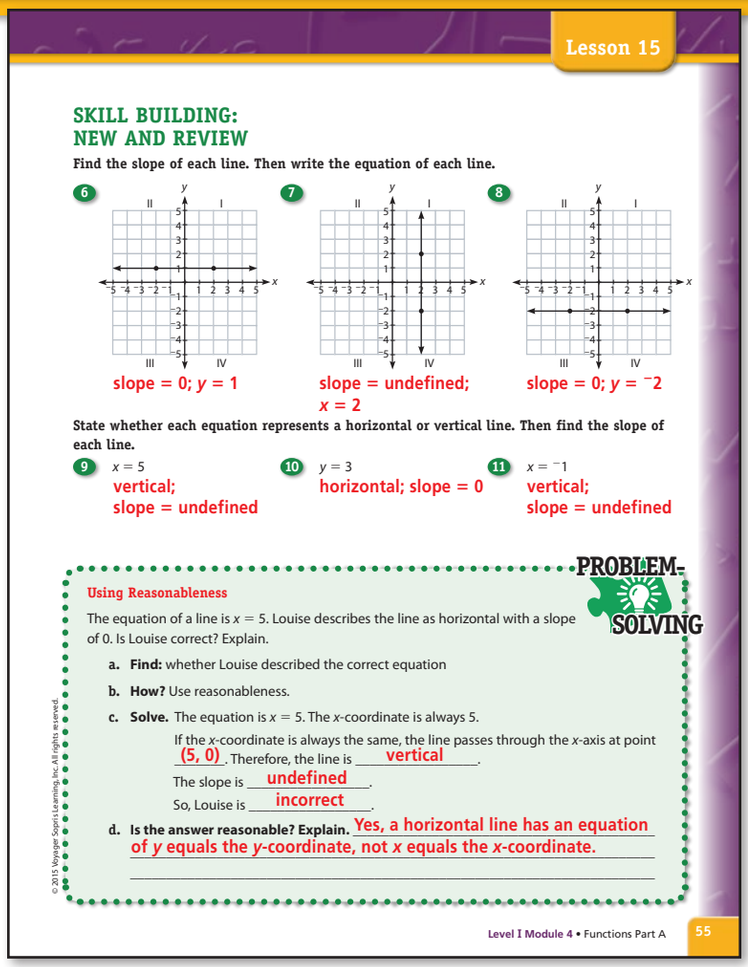
The line is vertical so the slope is undefined.

So, Louise is incorrect.

The answer is reasonable because a vertical line has an undefined slope; a horizontal line has a slope of zero.

Problems 6–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 15

SKILL BUILDING: NEW AND REVIEW

Find the slope of each line. Then write the equation of each line.

6 slope = 0; $y = 1$

7 slope = undefined; $x = 2$

8 slope = 0; $y = -2$

State whether each equation represents a horizontal or vertical line. Then find the slope of each line.

9 $x = 5$ vertical; slope = undefined

10 $y = 3$ horizontal; slope = 0

11 $x = -1$ vertical; slope = undefined

PROBLEM-SOLVING

Using Reasonableness

The equation of a line is $x = 5$. Louise describes the line as horizontal with a slope of 0. Is Louise correct? Explain.

a. **Find:** whether Louise described the correct equation

b. **How?** Use reasonableness.

c. **Solve.** The equation is $x = 5$. The *x*-coordinate is always 5.

If the *x*-coordinate is always the same, the line passes through the *x*-axis at point (5, 0). Therefore, the line is vertical.

The slope is undefined.

So, Louise is incorrect.

d. **Is the answer reasonable? Explain.** Yes, a horizontal line has an equation of *y* equals the *y*-coordinate, not *x* equals the *x*-coordinate.

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Level I Module 4 • Functions Part A 55

Problem 12 Reminder

Make sure students know that a line with an equation represented by $x = a$, where a represents any real number, is a vertical line. Review with students that vertical lines have a slope that is undefined.

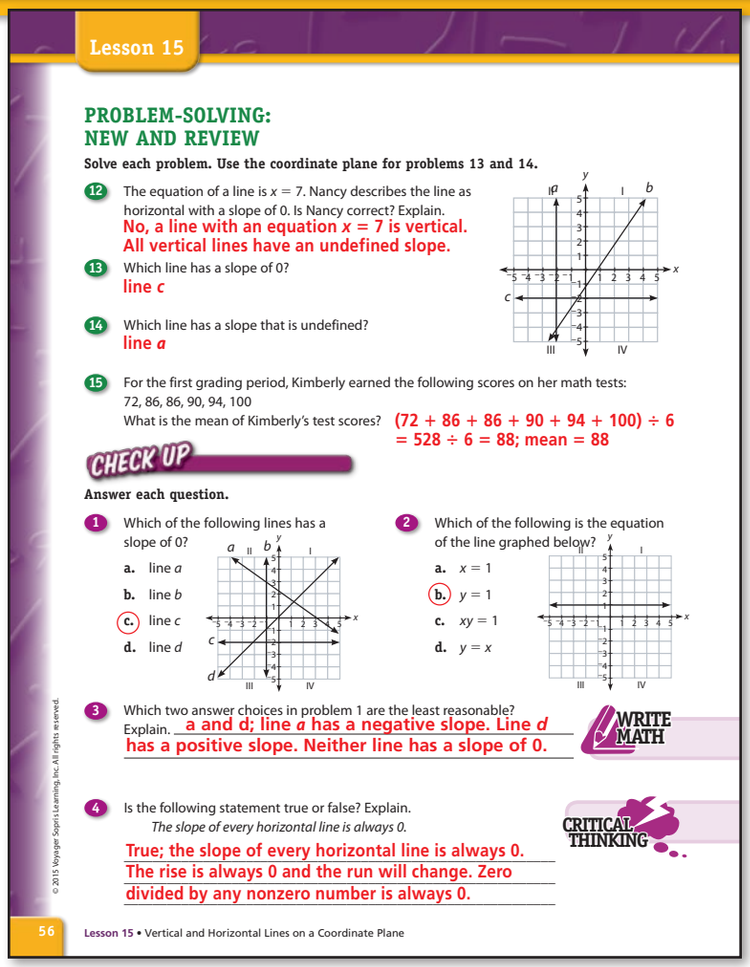
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly: The student did not understand that horizontal lines have a slope of zero. Use Additional Resources in I.4.15 to reteach how to find the slope of horizontal and vertical lines.



Lesson 15

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem. Use the coordinate plane for problems 13 and 14.

12 The equation of a line is $x = 7$. Nancy describes the line as horizontal with a slope of 0. Is Nancy correct? Explain.
No, a line with an equation $x = 7$ is vertical. All vertical lines have an undefined slope.

13 Which line has a slope of 0?
line *c*

14 Which line has a slope that is undefined?
line *a*

15 For the first grading period, Kimberly earned the following scores on her math tests:
72, 86, 86, 90, 94, 100
What is the mean of Kimberly's test scores? (72 + 86 + 86 + 90 + 94 + 100) ÷ 6 = 528 ÷ 6 = 88; mean = 88

CHECK UP

Answer each question.

1 Which of the following lines has a slope of 0?
a. line *a*
b. line *b*
c. line *c*
d. line *d*

2 Which of the following is the equation of the line graphed below?
a. $x = 1$
b. $y = 1$
c. $xy = 1$
d. $y = x$

3 Which two answer choices in problem 1 are the least reasonable? Explain. a and d; line *a* has a negative slope. Line *d* has a positive slope. Neither line has a slope of 0.

WRITE MATH

4 Is the following statement true or false? Explain.
The slope of every horizontal line is always 0.
True; the slope of every horizontal line is always 0. The rise is always 0 and the run will change. Zero divided by any nonzero number is always 0.

CRITICAL THINKING

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If student answered problem 2 incorrectly: The student did not understand how to find the equation of horizontal lines. Use Additional Resources in I.4.15 to reteach how to find the equation of horizontal and vertical lines.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to determine a line with a slope of zero. Use Additional Resources in I.4.15 to reteach how to find the slope of horizontal and vertical lines.

Technology



Using an online search engine, have students or partners research careers that would need to find slope or work with horizontal and vertical lines. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 4 Lesson 15
Reteach Teacher Edition Module 4 Lesson 15

Extra Practice
Student Book page 61

English Language Learners

Review the definitions of *horizontal* and *vertical*. Have students find examples in the classroom of horizontal and vertical lines. For example: The top of the board is a horizontal line. The side of the classroom door is a vertical line.

Students with Special Needs

Write the formula for slope on the board. Remind students that slope is the ratio of the vertical change to the horizontal change between any two points on the line. It can be found by dividing the change in *y* by the change in *x*. Write several input-output tables with *x*- and *y*-values on the board (the tables should plot vertical or horizontal lines). Use the formula for slope to find the slope of each line defined in the tables. Emphasize that 0 divided by any number equals 0, and that any fraction with a denominator of 0 is undefined.

Extra Practice Answer Key

Extra Practice

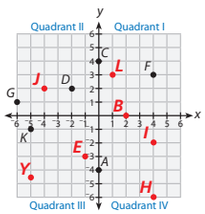
Name _____ Class _____ Date _____

Lesson PL1 Coordinate Planes in Four Quadrants

Use the coordinate plane for problems 1–12.

Write the ordered pair for each point.

1. Point *A* (0, −4)
2. Point *C* (0, 4)
3. Point *D* (−2, 2)
4. Point *F* (4, 3)
5. Point *G* (−6, 1)
6. Point *K* (−5, −1)



Graph each point on the coordinate plane.

7. *B*(2, 0)
8. *E*(−1, −3)
9. *H*(4, −6)
10. *I*(4, −2)
11. *J*(−4, 2)
12. *L*(1, 3)

Lesson PL2 Ratios

Write each ratio two other ways. Then write the ratio as a fraction in simplest form.

1. 5 to 1 5 : 1; $\frac{5}{1}$
2. 5 : 20 5 to 20; $\frac{5}{20} = \frac{1}{4}$
3. $\frac{75}{100}$ 75 : 100; 75 to 100; $\frac{3}{4}$

Solve each problem.

4. Compare the ratios in problems 2 and 3. Which is less? Explain.
   $\frac{5}{20} = \frac{1}{4}$ and $\frac{75}{100} = \frac{3}{4}$; $\frac{1}{4} < \frac{3}{4}$; the ratio 5 : 20 is less than the ratio $\frac{75}{100}$.
5. Compare the ratio 80 : 120 with the ratio in problem 3. Which is greater? Explain.
   $\frac{80}{120} = \frac{2}{3} = \frac{8}{12}$ and $\frac{75}{100} = \frac{3}{4} = \frac{9}{12}$; $\frac{9}{12} > \frac{8}{12}$; the ratio $\frac{75}{100}$ is greater.
6. The ratio of right-handed people to left-handed people in one class is 35 : 5. The ratio of right-handed people to left-handed people in another class is 21 : 3. Compare the two ratios using common denominators. 35 : 5 = $\frac{7}{1}$; 21 : 3 = $\frac{7}{1}$; the two ratios are the same, so the ratios are equivalent.

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Extra Practice

Name _____ Class _____ Date _____

Lesson 1 Mean Absolute Deviation

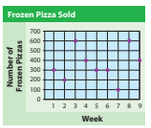
Find the mean absolute deviation of each data set.

1. 40, 55, 48, 61
   mean = 51;
   MAD = $\frac{11 + 4 + 3 + 10}{4}$ = 7
2. 13, 15, 7, 4, 11
   mean = 10;
   MAD = $\frac{3 + 5 + 3 + 6 + 1}{5}$ = 3.6
3. 33, 38, 35, 42, 38, 30
   mean = 36;
   MAD = $\frac{3 + 2 + 1 + 6 + 2 + 6}{6}$ = 3.3
4. 110, 125, 120, 115, 100
   mean = 114;
   MAD = $\frac{4 + 11 + 6 + 1 + 14}{5}$ = 7.2

Lesson 2 Scatter Plots and Lines of Best Fit

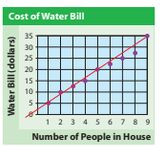
Tell whether each scatter plot has a *positive correlation*, *negative correlation*, or *no correlation*.

1. 

positive correlation
2. 

no correlation

Use the scatter plot to solve problems 3–6.



3. How much is the water bill for a house with five people?
   $20
4. Does a pattern exist among the points on the scatter plot? If so, what is the pattern? Yes; as the number of people in the household increases, the cost of the water bill increases.
5. What is the correlation between the number of people in a household and the cost of the water bill? positive correlation
6. Use a ruler or straightedge to draw the line of best fit on the scatter plot.

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Extra Practice

Name _____ Class _____ Date _____

Lesson 7 Functions

Complete each input-output table.

1. $y = \frac{x}{3} - 6$

x	−12	−6	0	3	9
y	−10	−8	−6	−5	−3

2. $y = 12x - 6$

x	−4	−1	0	2	5
y	−54	−18	−6	18	54

3. $y = -2x + 5$

x	y
−10	25
−5	15
0	5
3	−1
11	−17

4. $y = x + 14$

x	y
−25	−11
−14	0
0	14
7	21
66	80

Lesson 8 Function Rules

Find each function rule. Then complete the input-output table.

1.

x	y
−2	8
−1	4
0	0
1	−4
2	−8

$y = -4x$

2.

x	y
−1	6
0	7
1	8
2	9
3	10

$y = x + 7$

3.

x	y
−18	3
−6	1
6	−1
12	−2
24	−4

$y = \frac{x}{-6}$

4.

x	1	2	3	4	5
y	6	12	18	24	30

$y = 6x$

5.

x	−12	−9	−6	−3	0
y	−15	−12	−9	−6	−3

$y = x - 3$

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Extra Practice

Name _____ Class _____ Date _____

Lesson 9 Using Patterns to Identify Functions

Solve each problem.

1. The local library charges fees for overdue books. If a book is 1 day late, the fee is $0.10. If the book is 2 days late, the fee is $0.20. If the book is 3 days late, the fee is $0.30. If the book is 4 days late, the fee is $0.40. How much will the fee be if a book is 12 days late?
   $y = 0.10x$; $y = 0.10 \times 12 = 1.2$; $1.20

Days, x	1	2	3	4	12
Fee, y	0.10	0.20	0.30	0.40	1.20

2. A car can travel 20 miles with 1 gallon of gas. It can travel 40 miles with 2 gallons of gas. It can travel 60 miles with 3 gallons of gas. It can travel 80 miles with 4 gallons of gas. How far can the car travel with 14 gallons of gas?
   $y = 20x$; $y = 20 \times 14 = 280$; 280 mi

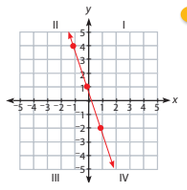
Gal, x	1	2	3	4	14
Mi, y	20	40	60	80	280

Lesson 11 Graphing Linear Equations

For each equation, complete the input-output table. Then graph the equation.

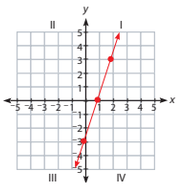
1. $y = -3x + 1$

x	y
−1	4
0	1
1	−2



2. $y = 3x - 3$

x	y
0	−3
1	0
2	3



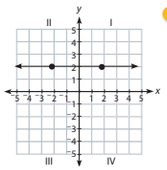
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Extra Practice

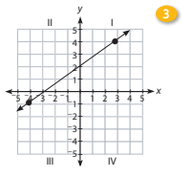
Name _____ Class _____ Date _____

Lesson 14 Slope of a Line

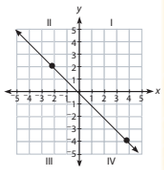
Find the slope of each line and tell whether the slope is *positive*, *negative*, or *zero*.

1. 

slope = $\frac{0}{4} = 0$; zero slope

2. 

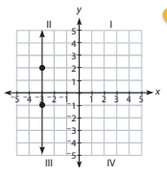
slope = $\frac{5}{7}$; positive slope

3. 

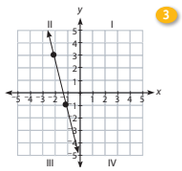
slope = $\frac{-6}{6} = -1$; negative slope

Lesson 15 Vertical and Horizontal Lines on a Coordinate Plane

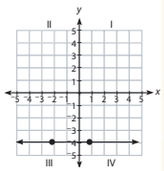
Find the slope of each line.

1. 

slope = undefined

2. 

slope = $\frac{-4}{1} = -4$

3. 

slope = $\frac{0}{3} = 0$

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Glossary

coordinate plane
a graph formed by two number lines intersecting at a right angle

data point
the representation of related values on a graph

data set
a collection of related values

domain
the set of all the first elements of the ordered pairs of a relation

equivalent ratios
ratios that can be written as equivalent fractions

function
a pairing of input and output values in which each input value is paired with exactly one output value

function rule
an equation that shows how an input value, *x*, and an output value, *y*, are related

input-output table
a table that shows how an input value, *x*, is paired with an output value, *y*

line of best fit
the line that comes closest to all points on a scatter plot

linear equation
an equation whose graph is a straight line

mean absolute deviation
the average distance between each data value and the mean

negative correlation
the trend of values decreasing as related values increase

ordered pair
a pair of numbers used to locate a point on a coordinate plane

origin
the point where the *x*-axis and *y*-axis intersect

positive correlation
the trend of values increasing as related values increase

quadrants
the four sections formed by the intersection of the *x*-axis and *y*-axis; labeled with Roman numerals

range
the set of all the second elements of the ordered pairs of a relation

ratio
a comparison of two quantities by division

relation
a pairing of numbers in one set, the domain, with numbers in another set, the range

rise
the vertical change between any two points on a line

run
the horizontal change between any two points on a line

scatter plot
a graph made by plotting points on a coordinate plane to show the relationship between two variables in a data set

slope
the ratio of the vertical change, rise, to the horizontal change, run, between any two points on a line

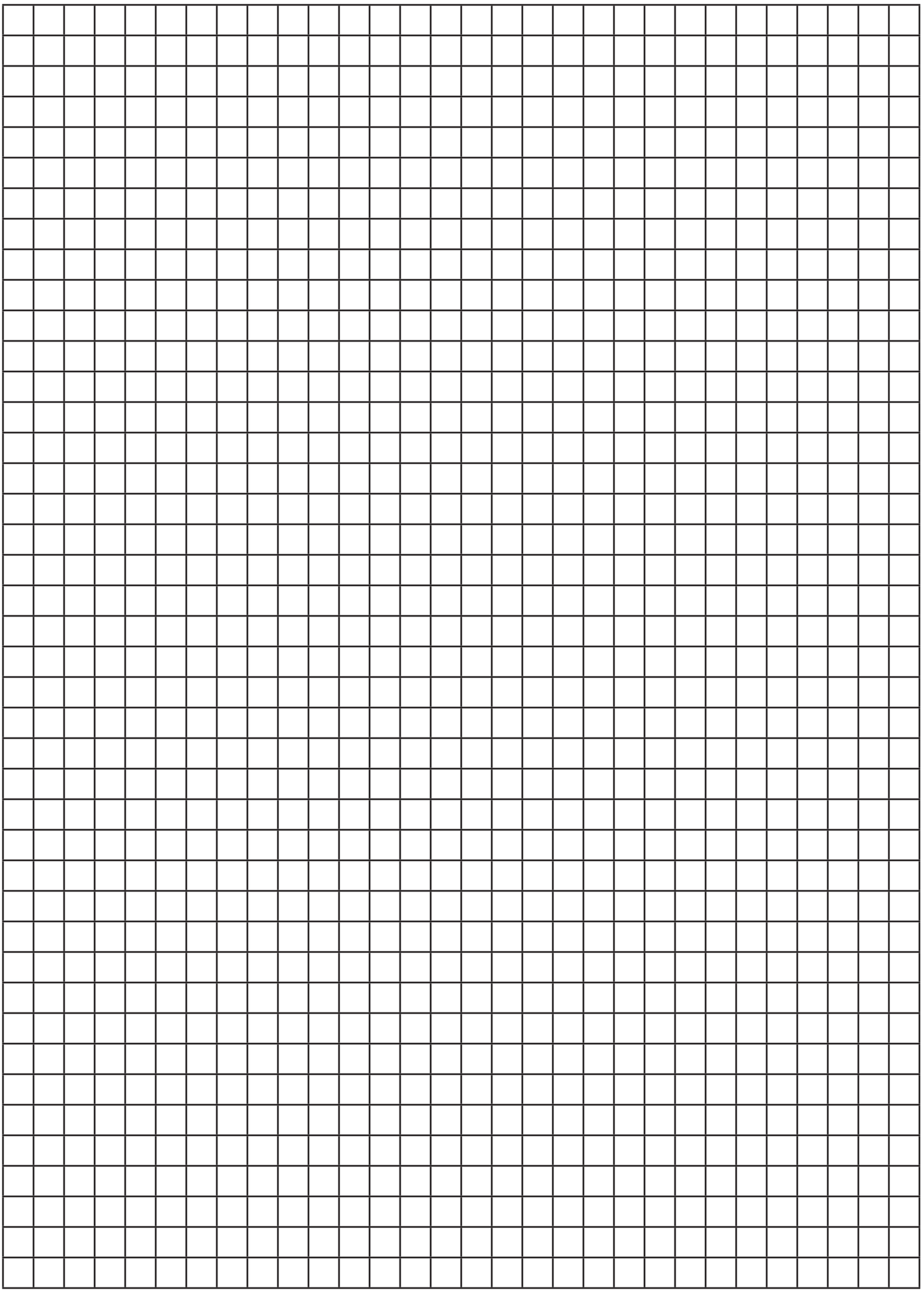
***x*-axis**
the horizontal number line of a coordinate plane

***x*-intercept**
the point where a graph crosses the *x*-axis

***y*-axis**
the vertical number line of a coordinate plane

***y*-intercept**
the point where a graph crosses the *y*-axis

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Functions Part B

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Lesson PL1

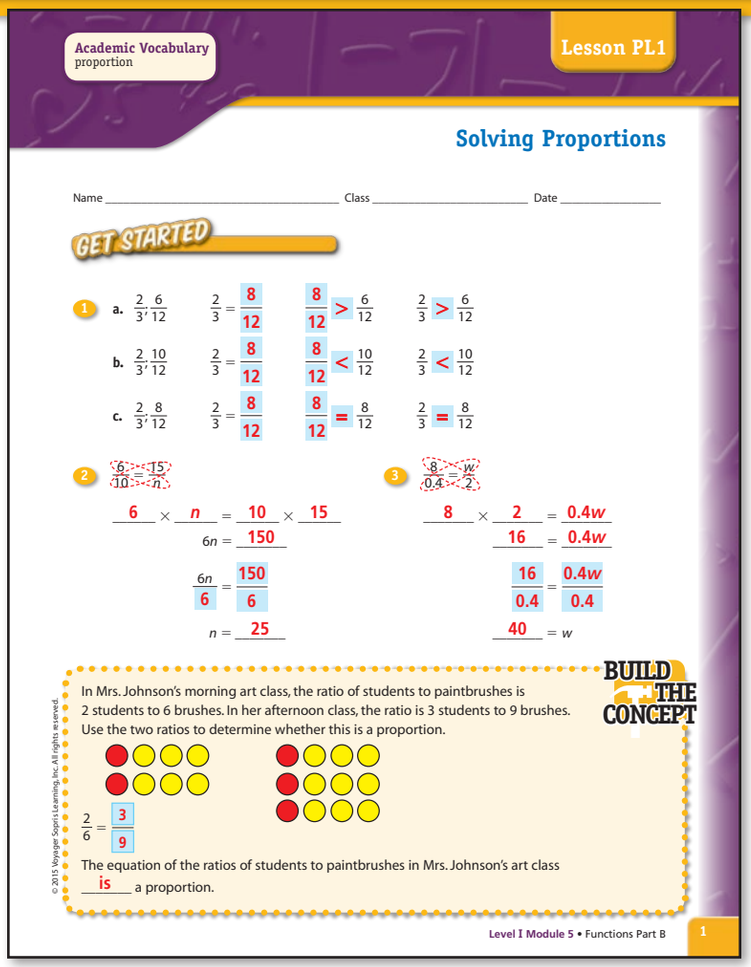
Objective

To solve a proportion using cross multiplication

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **proportion**
  an equation that shows two ratios are equivalent



Academic Vocabulary
proportion

Lesson PL1

Solving Proportions

Name _____ Class _____ Date _____

GET STARTED

1 a. $\frac{2}{3}, \frac{6}{12}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{8}{12} > \frac{6}{12}$ $\frac{2}{3} > \frac{6}{12}$

b. $\frac{2}{3}, \frac{10}{12}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{8}{12} < \frac{10}{12}$ $\frac{2}{3} < \frac{10}{12}$

c. $\frac{2}{3}, \frac{8}{12}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{8}{12} = \frac{8}{12}$ $\frac{2}{3} = \frac{8}{12}$

2 $\frac{6}{10} = \frac{15}{n}$

$6 \times n = 10 \times 15$

$6n = 150$

$\frac{6n}{6} = \frac{150}{6}$

$n = 25$

3 $\frac{8}{0.4} = \frac{w}{2}$

$8 \times 2 = 0.4w$

$16 = 0.4w$

$\frac{16}{0.4} = \frac{0.4w}{0.4}$

$40 = w$

BUILD THE CONCEPT

In Mrs. Johnson's morning art class, the ratio of students to paintbrushes is 2 students to 6 brushes. In her afternoon class, the ratio is 3 students to 9 brushes. Use the two ratios to determine whether this is a proportion.

$\frac{2}{6} = \frac{3}{9}$

The equation of the ratios of students to paintbrushes in Mrs. Johnson's art class is a proportion.

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To compare the ratio $\frac{2}{3}$ with the ratio $\frac{6}{12}$, write equivalent ratios with a common denominator of 12. What ratio is equivalent to $\frac{2}{3}$ with a denominator of 12? ($\frac{8}{12}$) How does $\frac{8}{12}$ compare with $\frac{6}{12}$? (It is greater than $\frac{6}{12}$.) Write the greater than symbol between the two ratios.

To compare the ratio $\frac{2}{3}$ with the ratio $\frac{10}{12}$, write equivalent ratios with a common denominator of 12. How does $\frac{8}{12}$ compare with $\frac{10}{12}$? (It is less than $\frac{10}{12}$.) Write the less than symbol between the two ratios.

To compare the ratio $\frac{2}{3}$ with the ratio $\frac{8}{12}$, write equivalent ratios with a common denominator of 12. How does $\frac{8}{12}$ compare with $\frac{8}{12}$? (They are equal.) Write an equal sign between the two ratios.

MODEL NEW SKILLS

Problem 2

A proportion is an equation that shows two ratios are equivalent. What is the variable in the expression of two equal ratios in problem 2? (n) The equation in problem 2 represents a proportion. It shows that the ratio 6 to 10 is equivalent to the ratio 15 to n, where n is an unknown number. This proportion is read as *6 is to 10 as 15 is to n*.

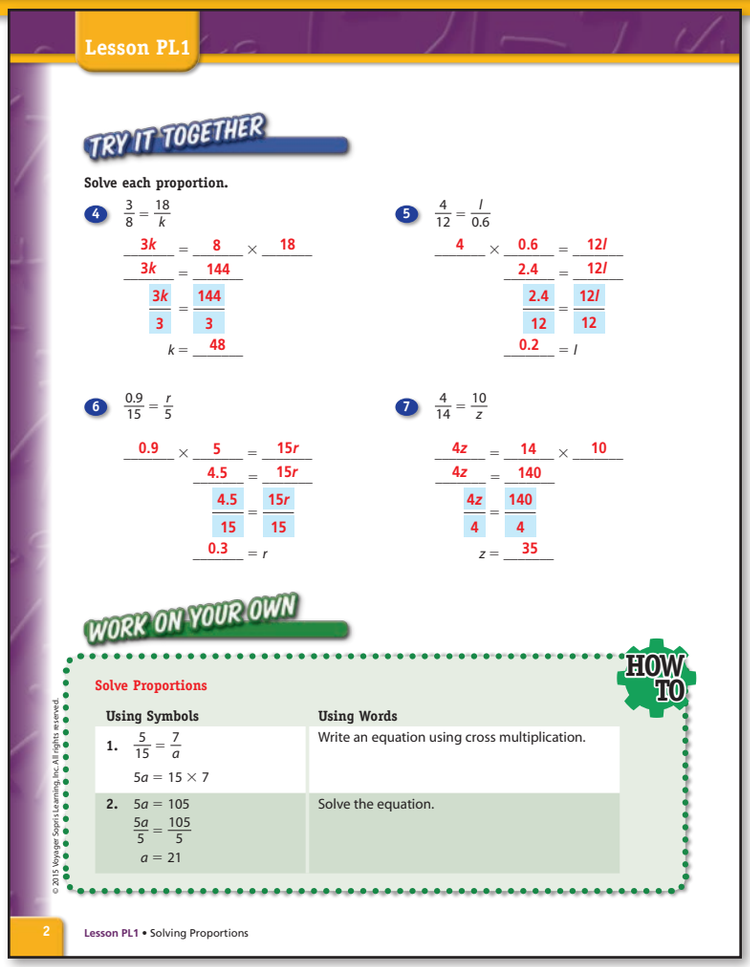
To find the value of n, write an equation using cross multiplication. To cross multiply, multiply the numerator of the ratio on the left by the denominator of the ratio on the right. Draw a ring around the numbers to be multiplied.

Then multiply the denominator of the ratio on the left by the numerator of the ratio on the right. Draw a ring around the numbers to be multiplied. These products are called cross products. Write *6 times n* on the left side of the equation. Write *10 times 15* on the right side of the equation.

Now solve the equation. What is 6 times n? ($6n$) What is 10 times 15? (150) What must be done to both sides of the equation to get the variable, n, by itself? (Divide by 6.) What is $6n$ divided by 6? (n) What is 150 divided by 6? (25) What is the value of n? ($n = 25$) The proportion is read as *six is to 10 as 15 is to 25.*

Problem 3

This proportion is read as *eight is to four tenths as w is to two.* If the numerator of a ratio is greater than the denominator of the ratio, then an equivalent ratio also will have a numerator that is greater than the denominator. In this problem, the numerator 8 of the first ratio is greater


Lesson PL1

TRY IT TOGETHER

Solve each proportion.

4 $\frac{3}{8} = \frac{18}{k}$

$\frac{3k}{3k} = \frac{8}{144} \times \frac{18}{}$

$\frac{3k}{3} = \frac{144}{3}$

$k = 48$

5 $\frac{4}{12} = \frac{l}{0.6}$

$\frac{4}{} \times \frac{0.6}{2.4} = \frac{12l}{12l}$

$\frac{2.4}{12} = \frac{12l}{12}$

$0.2 = l$

6 $\frac{0.9}{15} = \frac{r}{5}$

$\frac{0.9}{} \times \frac{5}{4.5} = \frac{15r}{15r}$

$\frac{4.5}{15} = \frac{15r}{15}$

$0.3 = r$

7 $\frac{4}{14} = \frac{10}{z}$

$\frac{4z}{4z} = \frac{14}{140} \times \frac{10}{}$

$\frac{4z}{4} = \frac{140}{4}$

$z = 35$

WORK ON YOUR OWN

HOW TO

Solve Proportions

Using Symbols	Using Words
1. $\frac{5}{15} = \frac{7}{a}$ $5a = 15 \times 7$	Write an equation using cross multiplication.
2. $5a = 105$ $\frac{5a}{5} = \frac{105}{5}$ $a = 21$	Solve the equation.

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2 Lesson PL1 • Solving Proportions

than the denominator 0.4, so the numerator *w* of the equivalent ratio will be greater than the denominator 2. To find the value of *w*, write an equation using cross multiplication. Draw a ring around the numbers that should be multiplied. Write *8 times 2* on the left side of the equation. Write *0.4w* on the right side of the equation. What is 8 times 2? (16) **Divide both sides by 0.4. What is 16 divided by 0.4?** (40) **What is the value of *w*?** ($w = 40$) **So, the proportion is read as *eight is to four tenths as 40 is to two.***

BUILD THE CONCEPT

Model how to use proportions.

Modeling a proportion is like modeling equivalent fractions. Read the problem about Mrs. Johnson's art class. The situation is modeled here. The first group of counters shows the ratio of 2 students (red counters) to 6 paintbrushes (yellow counters) in the morning class. Look at the second group of counters. How many red counters are there? (3) **How many yellow counters are there?** (9) **How is this written as a ratio of red counters to yellow counters?** (3 : 9) **This is the ratio of students to paintbrushes in Mrs. Johnson's afternoon class. In both groups, there is 1 red counter for every group of 3 yellow counters. So, 2 to 6 and 3 to 9 form a proportion. Cross multiplication can also be used to check whether the ratios form a proportion. What is 2 times 9?** (18) **What is 6 times 3?** (18) **The cross products are equal, so the equation of the ratios is a proportion.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What is the proportion in problem 4? (3 is to 8 as 18 is to *k*) **Will the value of *k* be greater than or less than 18?** (greater than) **Why?** (The denominator of $\frac{3}{8}$ is greater than the numerator; in the equivalent ratio, the denominator also will be greater than the numerator.)

What is the first step in solving the proportion and finding the value of *k*? (Write an equation using cross multiplication.) **What goes on the left side of the equation?** (*3k*) **What goes on the right side of the equation?** (8×18)

What should be done next? (Solve the equation.) **What is 8 times 18?** (144) **What is 144 divided by 3?** (48) **What is the value of *k*?** ($k = 48$) **How is the proportion read?** (3 is to 8 as 18 is to 48)

Problem 5

What is the proportion in problem 5? (4 is to 12 as *l* is to 0.6) **Will the value of *l* be greater than or less than 0.6?** (less than) **Why?** (The numerator of $\frac{4}{12}$ is less than the denominator, so in the equivalent ratio, the numerator also will be less than the denominator.)

What goes on the left side of the equation? (4×0.6) **What goes on the right side of the equation?** ($12l$) **What is 4 times 0.6?** (2.4) **What is 2.4 divided by 12?** (0.2) **What is the value of *l*?** ($l = 0.2$) **How is the proportion read?** (4 is to 12 as 0.2 is to 0.6)

Problem 6

Will the value of *r* be greater than or less than 5? (less than) **What goes on the left side of the equation?** (0.9×5) **What goes on the right side of the equation?** ($15r$) **What is 0.9 times 5?** (4.5) **What is 4.5 divided by 15?** (0.3) **What is the value of *r*?** ($r = 0.3$) **How is the proportion read?** (0.9 is to 15 as 0.3 is to 5)

Problem 7

What goes on the left side of the equation? ($4z$) **What goes on the right side of the equation?** (14×10) **What is 14 times 10?** (140) **What is 140 divided by 4?** (35) **What is the value of *z*?** ($z = 35$) **How is the proportion read?** (4 is to 14 as 10 is to 35)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using Reasonableness** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 17.

In this problem, students are asked to find whether two ratios, $\frac{3}{36}$ and $\frac{7}{84}$, form a proportion. To do that, students will first need to cross multiply. Three times 84 equals 252. Thirty-six times 7 equals 252. Because 252 equals 252, the ratios form a proportion.

Problems 8–20

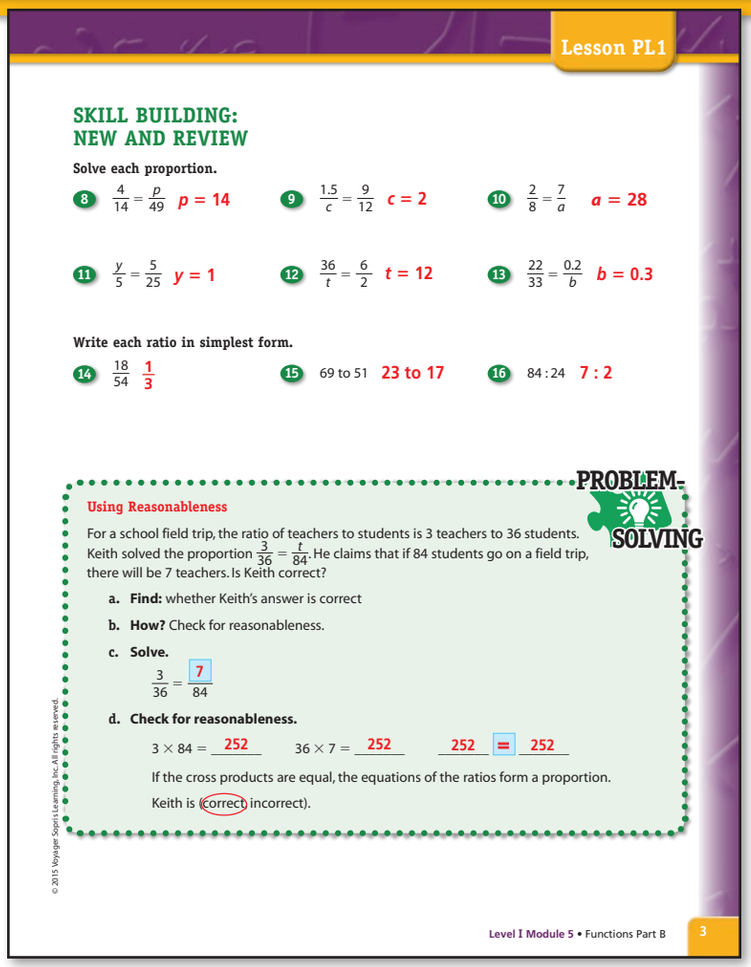
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 17 Reminder

Students need to find the cross products of the proportion and verify they are equal.

Problem 18 Reminder

Students need to compare ratios to solve this problem. The ratio of vacation albums to total albums should be written for each person. Then students can find equivalent ratios with the same denominator before comparing.



Lesson PL1

SKILL BUILDING: NEW AND REVIEW

Solve each proportion.

8. $\frac{4}{14} = \frac{p}{49}$ $p = 14$
9. $\frac{1.5}{c} = \frac{9}{12}$ $c = 2$
10. $\frac{2}{8} = \frac{7}{a}$ $a = 28$
11. $\frac{y}{5} = \frac{5}{25}$ $y = 1$
12. $\frac{36}{t} = \frac{6}{2}$ $t = 12$
13. $\frac{22}{33} = \frac{0.2}{b}$ $b = 0.3$

Write each ratio in simplest form.

14. $\frac{18}{54}$ $\frac{1}{3}$
15. 69 to 51 23 to 17
16. 84 : 24 7 : 2

PROBLEM-SOLVING

Using Reasonableness

For a school field trip, the ratio of teachers to students is 3 teachers to 36 students. Keith solved the proportion $\frac{3}{36} = \frac{t}{84}$. He claims that if 84 students go on a field trip, there will be 7 teachers. Is Keith correct?

a. **Find:** whether Keith's answer is correct

b. **How?** Check for reasonableness.

c. **Solve.**

$\frac{3}{36} = \frac{7}{84}$

d. **Check for reasonableness.**

$3 \times 84 =$ 252 $36 \times 7 =$ 252 252 = 252

If the cross products are equal, the equations of the ratios form a proportion.

Keith is (correct) incorrect).

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Level I Module 5 • Functions Part B 3

CHECK UP

ASSESS INFORMALLY

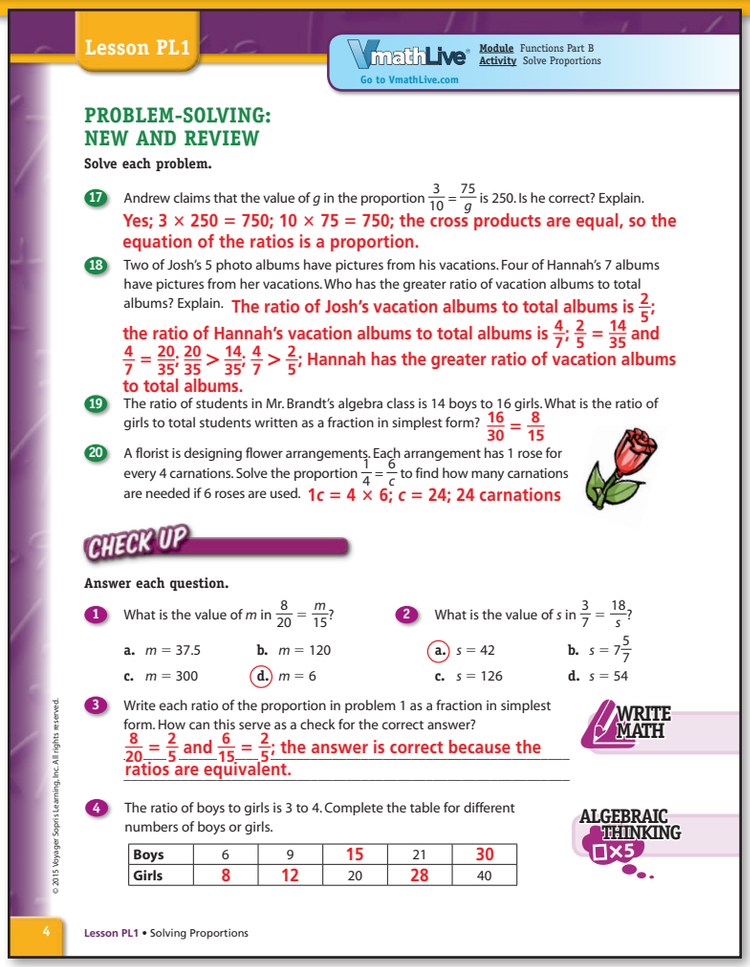
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2b: The student multiplied numerators and denominators rather than cross multiplying. Use Additional Resources in I.5.PL1 to reteach how to cross multiply to solve a proportion.

If student answered 1b or 2c: The student did not cross multiply correctly. Use Additional Resources in I.5.PL1 to reteach how to cross multiply to solve a proportion.

If student answered 1c: The student multiplied the denominators only. Use Additional Resources in I.5.PL1 to reteach how to cross multiply to solve a proportion.



Lesson PL1

VmathLive Module Functions Part B Activity Solve Proportions Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

17 Andrew claims that the value of g in the proportion $\frac{3}{10} = \frac{75}{g}$ is 250. Is he correct? Explain.
Yes; $3 \times 250 = 750$; $10 \times 75 = 750$; the cross products are equal, so the equation of the ratios is a proportion.

18 Two of Josh's 5 photo albums have pictures from his vacations. Four of Hannah's 7 albums have pictures from her vacations. Who has the greater ratio of vacation albums to total albums? Explain. The ratio of Josh's vacation albums to total albums is $\frac{2}{5}$; the ratio of Hannah's vacation albums to total albums is $\frac{4}{7}$; $\frac{2}{5} = \frac{14}{35}$ and $\frac{4}{7} = \frac{20}{35}$; $\frac{20}{35} > \frac{14}{35}$; $\frac{4}{7} > \frac{2}{5}$; Hannah has the greater ratio of vacation albums to total albums.

19 The ratio of students in Mr. Brandt's algebra class is 14 boys to 16 girls. What is the ratio of girls to total students written as a fraction in simplest form? $\frac{16}{30} = \frac{8}{15}$

20 A florist is designing flower arrangements. Each arrangement has 1 rose for every 4 carnations. Solve the proportion $\frac{1}{4} = \frac{6}{c}$ to find how many carnations are needed if 6 roses are used. $1c = 4 \times 6$; $c = 24$; 24 carnations

CHECK UP

Answer each question.

1 What is the value of m in $\frac{8}{20} = \frac{m}{15}$?
a. $m = 37.5$ b. $m = 120$ c. $m = 300$ (d.) $m = 6$

2 What is the value of s in $\frac{3}{7} = \frac{18}{s}$?
(a.) $s = 42$ b. $s = 7\frac{5}{7}$ c. $s = 126$ d. $s = 54$

3 Write each ratio of the proportion in problem 1 as a fraction in simplest form. How can this serve as a check for the correct answer?
$\frac{8}{20} = \frac{2}{5}$ and $\frac{6}{15} = \frac{2}{5}$; the answer is correct because the ratios are equivalent.

WRITE MATH

4 The ratio of boys to girls is 3 to 4. Complete the table for different numbers of boys or girls.

ALGEBRAIC THINKING

Boys	6	9	15	21	30
Girls	8	12	20	28	40

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4 Lesson PL1 • Solving Proportions

If student answered 2d: The student multiplied the numerators only. Use Additional Resources in I.5.PL1 to reteach how to cross multiply to solve a proportion.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the two ratios that form a proportion are equivalent ratios. In simplest form, the two ratios will be the same.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Functions Part B
Activity: Solve Proportions

Vmath Reteach
Reteach Student Book Module 5 Lesson PL1
Reteach Teacher Edition Module 5 Lesson PL1

Extra Practice
Student Book page 39

English Language Learners

Relate the term *proportion* to the term *equivalent ratios.* Show that a proportion is a statement that shows that one ratio is equal to another. Count the number of boys and girls in the class and write the results as a ratio. Guide students to find equivalent ratios. Set two of the ratios equal to each other. Point out the equal sign and write *equivalent* below the ratios. Identify the two ratios as forming a proportion. Write other pairs of ratios on the board and have students tell whether they are equivalent ratios and whether they form a proportion.

Students with Special Needs

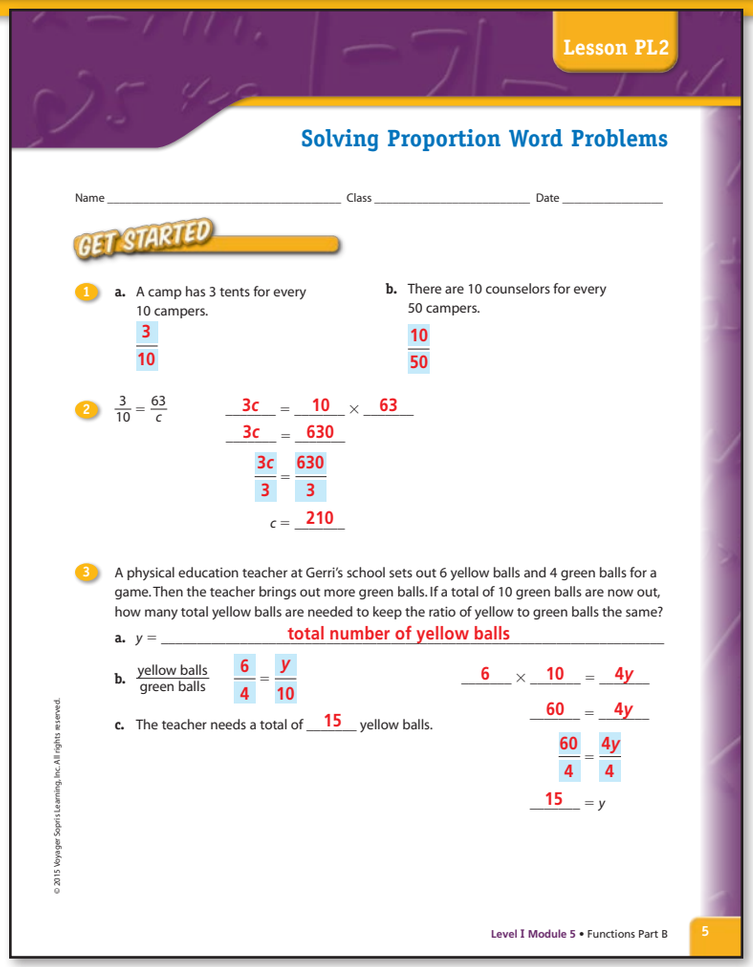
Provide students with two-color counters. Have pairs of students place 3 red counters and 4 yellow counters on a sheet of paper. Ask them to write the ratio of red to yellow counters. (3 : 4) On another sheet of paper have students place 6 red counters. Have them determine how many yellow counters are needed to show an equivalent ratio and write the equivalent ratio. Ask each pair to explain how the answer was found.

Lesson PL2

Objective

To use a proportion to solve a word problem

Preskills	Lesson
Solving Proportions	I.5.PL1





Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What ratio compares tents with campers? ($\frac{3}{10}$) **Look at problem 1b. What ratio compares counselors with campers?** ($\frac{10}{50}$)

Problem 2

The proportion in problem 2 is read as *three is to 10 as 63 is to c*. To find the value of *c* in the proportion, first cross multiply to write an equation. What is the equation? ($3c = 10 \times 63$)

Now solve the equation for *c*. What is 10 times 63? (630) **What is 630 divided by 3?** (210) **What is the value of *c*?** ($c = 210$)

MODEL NEW SKILLS

Problem 3

This word problem can be solved by using a proportion. The initial ratio of yellow balls to green balls needs to be equivalent to the ratio of yellow balls to green balls after more balls are brought into the game. To solve the proportion word problem, use a variable to represent the number in the proportion that is not known. What part of the proportion is not known? (the total number of yellow balls) **Let *y* represent the total number of yellow balls in the game. What is the ratio of yellow balls to green balls initially?** (6 to 4) **What is the ratio of yellow balls to green balls after more balls are brought into the game?** (*y* to 10) **Set the ratios equal to each other and solve the equation. Cross multiply to write an equation. What is the equation?** ($6 \times 10 = 4y$) **What is 6 times 10?** (60) **What is 60 divided by 4?** (15) **What is the value of *y*?** ($y = 15$) **How many total yellow balls are needed for the game?** (15)



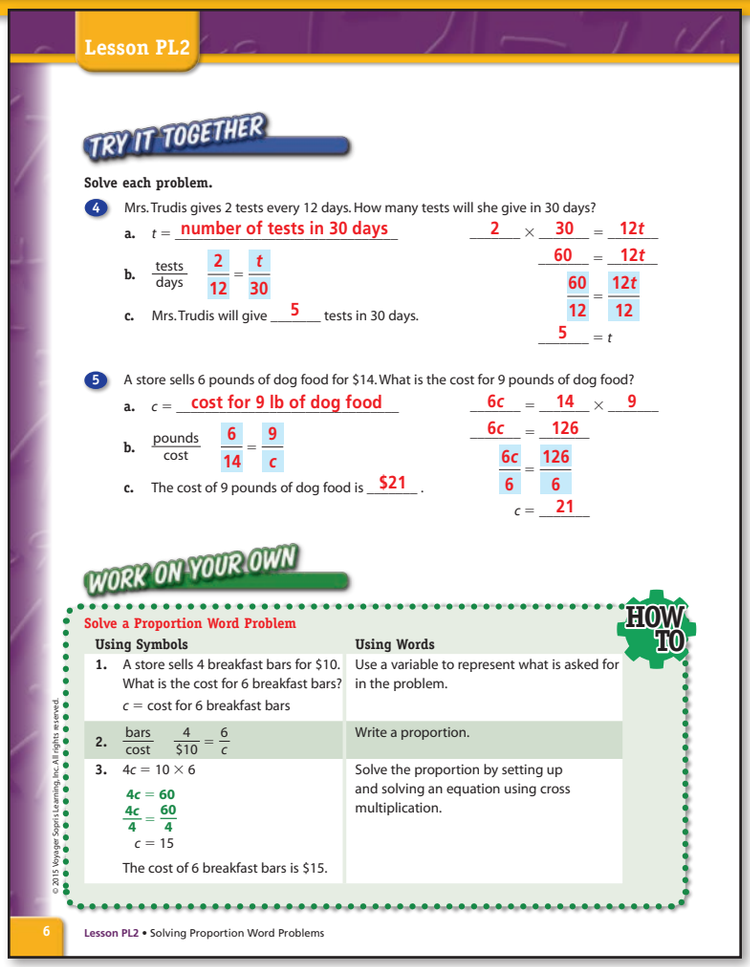
Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What ratios are equivalent in this word problem? (the number of tests in 12 days and the number of tests in 30 days) **How many tests are given in 12 days?** (2) **What part of the proportion is not known?** (the number of tests in 30 days) **Let *t* represent the number of tests in 30 days. What is the proportion?** (2 is to 12 as *t* is to 30)

What should be done next? (Use cross multiplication to write an equation.) **What is the equation?** ($2 \times 30 = 12t$) **What is 2 times 30?** (60) **What is 60 divided by 12?** (5) **What is the value of *t*?** ($t = 5$) **How many tests will Mrs. Trudis give in 30 days?** (5)



Lesson PL2

TRY IT TOGETHER

Solve each problem.

4 Mrs. Trudis gives 2 tests every 12 days. How many tests will she give in 30 days?

a. $t =$ number of tests in 30 days

b. $\frac{\text{tests}}{\text{days}}$ $\frac{2}{12} = \frac{t}{30}$

c. Mrs. Trudis will give 5 tests in 30 days.

$2 \times 30 = 12t$
$60 = 12t$
$\frac{60}{12} = \frac{12t}{12}$
$5 = t$

5 A store sells 6 pounds of dog food for $14. What is the cost for 9 pounds of dog food?

a. $c =$ cost for 9 lb of dog food

b. $\frac{\text{pounds}}{\text{cost}}$ $\frac{6}{14} = \frac{9}{c}$

c. The cost of 9 pounds of dog food is $21.

$6c = 14 \times 9$
$6c = 126$
$\frac{6c}{6} = \frac{126}{6}$
$c = 21$

WORK ON YOUR OWN

Solve a Proportion Word Problem

Using Symbols	Using Words
1. A store sells 4 breakfast bars for $10. What is the cost for 6 breakfast bars? c = cost for 6 breakfast bars	Use a variable to represent what is asked for in the problem.
2. $\frac{\text{bars}}{\text{cost}}$ $\frac{4}{\$10} = \frac{6}{c}$	Write a proportion.
3. $4c = 10 \times 6$ $\frac{4c}{4} = \frac{60}{4}$ $c = 15$ The cost of 6 breakfast bars is $15.	Solve the proportion by setting up and solving an equation using cross multiplication.

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Problem 5

What ratios are equivalent in this word problem? (the cost of 6 lb of dog food and the cost of 9 lb of dog food) **How much do 6 pounds of dog food cost?** ($14) **What will the variable represent in this problem?** (the cost for 9 lb of dog food) **Let *c* represent this cost. What is the proportion?** (6 is to 14 as 9 is to *c*)

What is the equation after cross multiplying? ($6c = 14 \times 9$) **What is 14 times 9?** (126) **What is 126 divided by 6?** (21) **What is the value of *c*?** ($c = 21$) **What is the cost for 9 pounds of dog food?** ($21)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 6–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 7 Reminder

Students should realize that the first ratio is 610 miles per 10 gallons. The second ratio is some number of miles per 1 gallon.

Problem 14 Reminder

Students must first find how many acres Stephen can mow in 10 hours. Then they need to compare the number of acres mowed in 10 hours in problem 13.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly: The student incorrectly set up one or both of the ratios in the proportion. Use Additional Resources in I.5.PL2 to reteach how to solve word problems using proportions.

If student answered 2a: The student wrote one ratio incorrectly. Remind the student that both ratios in the proportion must compare amounts in the same way. Use Additional Resources in I.5.PL1 to reteach how to solve proportions.

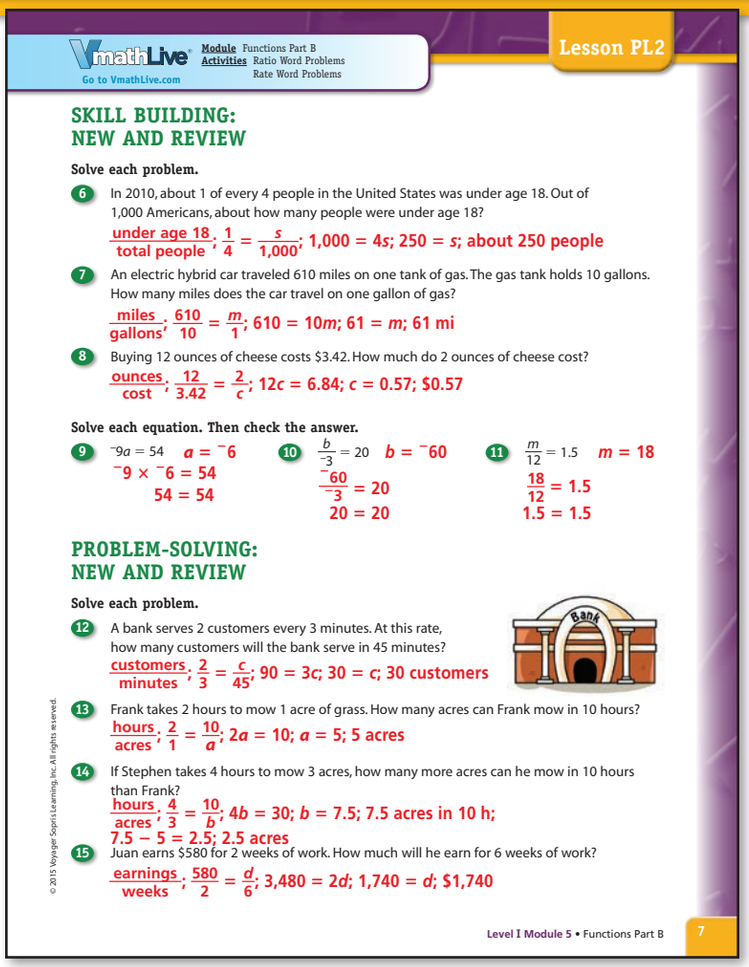
If student answered 2b: The student divided 9 by 3. Use Additional Resources in I.5.PL1 to reteach how to solve proportions.

If student answered 2c: The student multiplied the numbers of lilies only. Use Additional Resources in I.5.PL1 to reteach how to solve proportions.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review how to write ratios and remind students that if the first number in a ratio is less than the second number, this relationship will be the same with an equivalent ratio. In this problem, there are more carnations than lilies. So, if the arrangement has 9 lilies, the number of carnations must be greater than 9.



VmathLive Go to VmathLive.com — Module Activities: Functions Part B; Ratio Word Problems; Rate Word Problems

Lesson PL2

SKILL BUILDING: NEW AND REVIEW

Solve each problem.

6. In 2010, about 1 of every 4 people in the United States was under age 18. Out of 1,000 Americans, about how many people were under age 18?
$\frac{\text{under age 18}}{\text{total people}}$; $\frac{1}{4} = \frac{s}{1{,}000}$; $1{,}000 = 4s$; $250 = s$; about 250 people

7. An electric hybrid car traveled 610 miles on one tank of gas. The gas tank holds 10 gallons. How many miles does the car travel on one gallon of gas?
$\frac{\text{miles}}{\text{gallons}}$; $\frac{610}{10} = \frac{m}{1}$; $610 = 10m$; $61 = m$; 61 mi

8. Buying 12 ounces of cheese costs $3.42. How much do 2 ounces of cheese cost?
$\frac{\text{ounces}}{\text{cost}}$; $\frac{12}{3.42} = \frac{2}{c}$; $12c = 6.84$; $c = 0.57$; $0.57

Solve each equation. Then check the answer.

9. $-9a = 54$ $a = -6$
$-9 \times -6 = 54$
$54 = 54$

10. $\frac{b}{-3} = 20$ $b = -60$
$\frac{-60}{-3} = 20$
$20 = 20$

11. $\frac{m}{12} = 1.5$ $m = 18$
$\frac{18}{12} = 1.5$
$1.5 = 1.5$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

12. A bank serves 2 customers every 3 minutes. At this rate, how many customers will the bank serve in 45 minutes?
$\frac{\text{customers}}{\text{minutes}}$; $\frac{2}{3} = \frac{c}{45}$; $90 = 3c$; $30 = c$; 30 customers

13. Frank takes 2 hours to mow 1 acre of grass. How many acres can Frank mow in 10 hours?
$\frac{\text{hours}}{\text{acres}}$; $\frac{2}{1} = \frac{10}{a}$; $2a = 10$; $a = 5$; 5 acres

14. If Stephen takes 4 hours to mow 3 acres, how many more acres can he mow in 10 hours than Frank?
$\frac{\text{hours}}{\text{acres}}$; $\frac{4}{3} = \frac{10}{b}$; $4b = 30$; $b = 7.5$; 7.5 acres in 10 h;
$7.5 - 5 = 2.5$; 2.5 acres

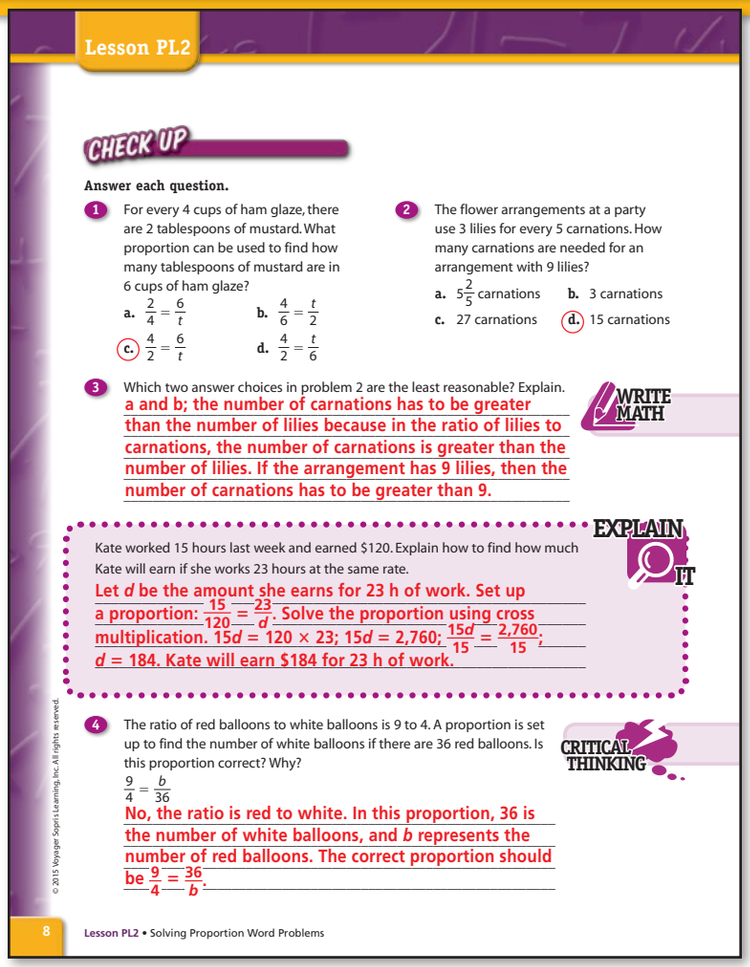
15. Juan earns $580 for 2 weeks of work. How much will he earn for 6 weeks of work?
$\frac{\text{earnings}}{\text{weeks}}$; $\frac{580}{2} = \frac{d}{6}$; $3{,}480 = 2d$; $1{,}740 = d$; $1,740

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Level I Module 5 • Functions Part B 7

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson PL2

CHECK UP

Answer each question.

1. For every 4 cups of ham glaze, there are 2 tablespoons of mustard. What proportion can be used to find how many tablespoons of mustard are in 6 cups of ham glaze?
   a. $\frac{2}{4} = \frac{6}{t}$ b. $\frac{4}{6} = \frac{t}{2}$
   c. $\frac{4}{2} = \frac{6}{t}$ d. $\frac{4}{2} = \frac{t}{6}$

2. The flower arrangements at a party use 3 lilies for every 5 carnations. How many carnations are needed for an arrangement with 9 lilies?
   a. $5\frac{2}{5}$ carnations b. 3 carnations
   c. 27 carnations d. 15 carnations

3. Which two answer choices in problem 2 are the least reasonable? Explain.
   a and b; the number of carnations has to be greater than the number of lilies because in the ratio of lilies to carnations, the number of carnations is greater than the number of lilies. If the arrangement has 9 lilies, then the number of carnations has to be greater than 9.

WRITE MATH

EXPLAIN IT

Kate worked 15 hours last week and earned $120. Explain how to find how much Kate will earn if she works 23 hours at the same rate.
Let d be the amount she earns for 23 h of work. Set up a proportion: $\frac{15}{120} = \frac{23}{d}$. Solve the proportion using cross multiplication. $15d = 120 \times 23$; $15d = 2{,}760$; $\frac{15d}{15} = \frac{2{,}760}{15}$; $d = 184$. Kate will earn $184 for 23 h of work.

4. The ratio of red balloons to white balloons is 9 to 4. A proportion is set up to find the number of white balloons if there are 36 red balloons. Is this proportion correct? Why?
   $\frac{9}{4} = \frac{b}{36}$
   No, the ratio is red to white. In this proportion, 36 is the number of white balloons, and b represents the number of red balloons. The correct proportion should be $\frac{9}{4} = \frac{36}{b}$.

CRITICAL THINKING

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Technology

Find and preview short online videos that show proportions being used in a career. Show the videos to students, then discuss other related careers that could use the same skill. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Functions Part B
Activities: Ratio Word Problems
Rate Word Problems

Vmath Reteach
Reteach Student Book Module 5 Lesson PL2
Reteach Teacher Edition Module 5 Lesson PL2

Extra Practice
Student Book page 39

English Language Learners

Talk about the meaning of *proportion*. Remind students that a proportion is an equation that shows two ratios are equivalent. Using the students in the classroom, write several ratios (boys to girls, girls to boys, boys to total, girls to total) on the board. Ask students questions such as, *To maintain the ratio of boys to girls, how many boys would be in a class with three times as many girls?*

Students with Special Needs

Encourage students to include labels in their ratios to help them write correct proportions. For example, instead of writing $\frac{4}{5} = \frac{x}{20}$, students can write $\frac{4 \text{ boys}}{5 \text{ girls}} = \frac{x \text{ boys}}{20 \text{ girls}}$.

It may also be helpful to ask students to write a "let statement" for each variable in a word problem. This means that whenever students use a variable to represent a missing value in a word problem, they should write a note that tells what the variable represents. For the example, the note might be, *Let x be the number of boys.*

MATH Flash

Lesson 1

Objective

To compare different representations of linear functions

Academic Vocabulary

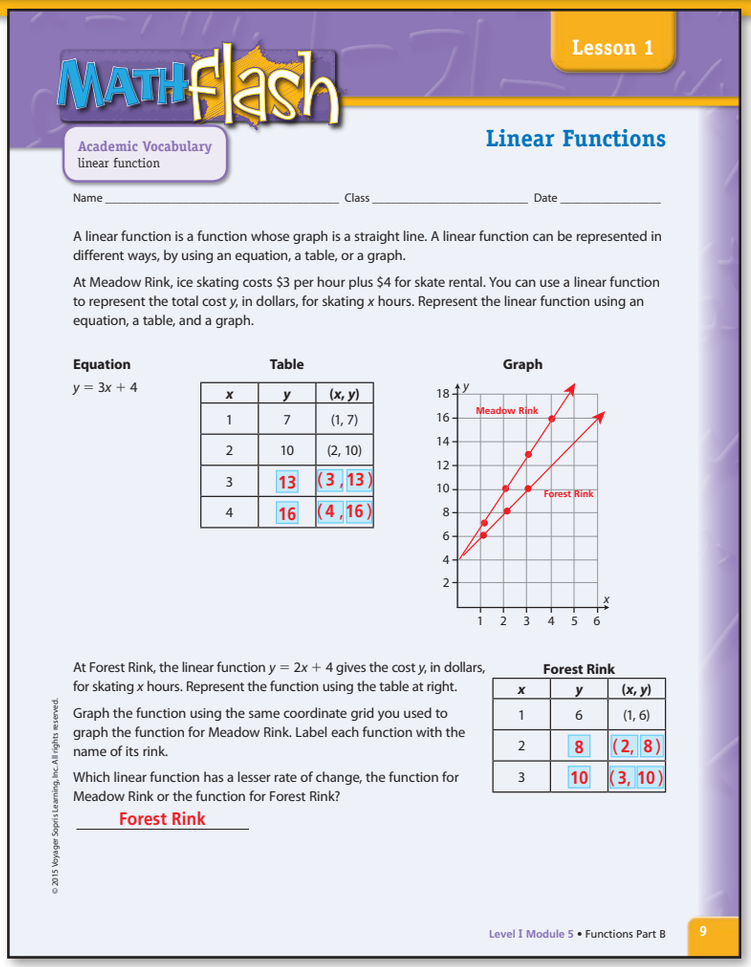
Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **linear function**
  a function whose graph is a line

Model the following skills for students.

A linear function is a function whose graph is a straight line. You can use an equation, a table, or a graph to represent a linear function. The relationship between the cost of skating and the number of hours skated is a linear function. What is the given equation of the linear function? ($y = 3x + 4$) **The 3 is the slope because it is the rate of change per hour and the 4 is the *y*-intercept of the graph because it is the initial starting cost of the skate rental.**

The first two rows of the table are already completed. When *x* is 3, what is *y*? (13) **What is the ordered pair?** ((3, 13)) **When *x* is 4, what is *y*?** (16) **What is the ordered pair?** ((4, 16)) **Graph the points for the ordered pairs on the coordinate grid. Draw a line through the points to show the linear function.**



MATH Flash

Lesson 1

Academic Vocabulary
linear function

Linear Functions

Name _____ Class _____ Date _____

A linear function is a function whose graph is a straight line. A linear function can be represented in different ways, by using an equation, a table, or a graph.

At Meadow Rink, ice skating costs $3 per hour plus $4 for skate rental. You can use a linear function to represent the total cost *y*, in dollars, for skating *x* hours. Represent the linear function using an equation, a table, and a graph.

Equation

$y = 3x + 4$

Table

x	*y*	(*x*, *y*)
1	7	(1, 7)
2	10	(2, 10)
3	13	(3, 13)
4	16	(4, 16)

Graph

At Forest Rink, the linear function $y = 2x + 4$ gives the cost *y*, in dollars, for skating *x* hours. Represent the function using the table at right.

Graph the function using the same coordinate grid you used to graph the function for Meadow Rink. Label each function with the name of its rink.

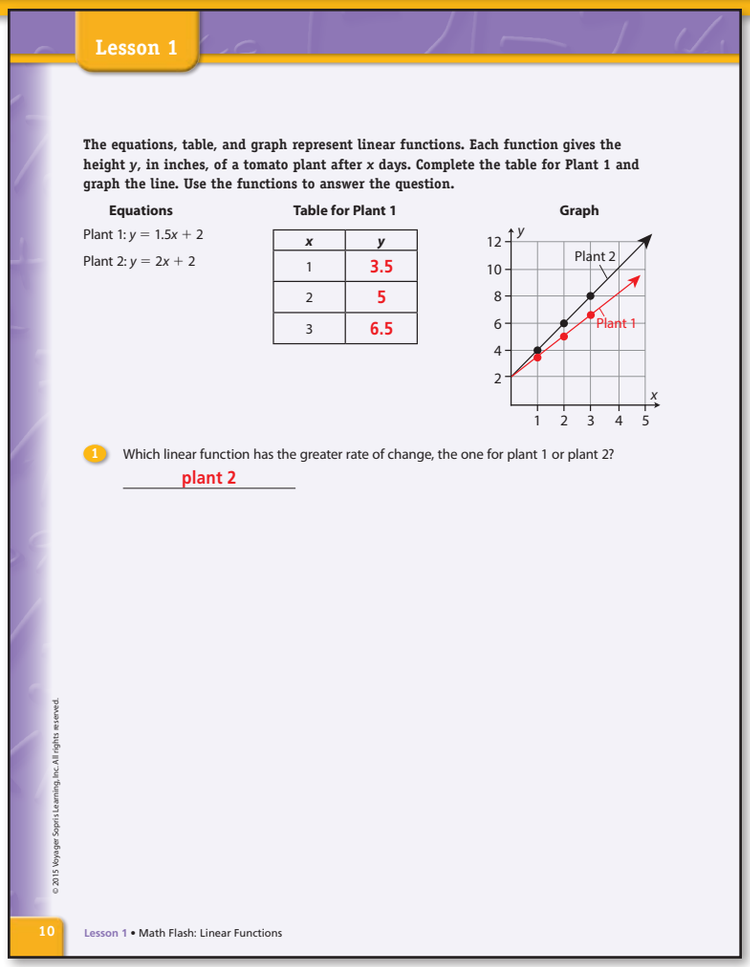
Which linear function has a lesser rate of change, the function for Meadow Rink or the function for Forest Rink?

Forest Rink

Forest Rink

x	*y*	(*x*, *y*)
1	6	(1, 6)
2	8	(2, 8)
3	10	(3, 10)

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Lesson 1

The equations, table, and graph represent linear functions. Each function gives the height *y*, in inches, of a tomato plant after *x* days. Complete the table for Plant 1 and graph the line. Use the functions to answer the question.

Equations

Plant 1: $y = 1.5x + 2$

Plant 2: $y = 2x + 2$

Table for Plant 1

x	*y*
1	3.5
2	5
3	6.5

Graph

1 Which linear function has the greater rate of change, the one for plant 1 or plant 2?

plant 2

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Work with students to complete the following skills.

Complete the table for the linear function for Forest Rink, *y* equals 2*x* plus 4. Then graph the function on the same coordinate grid above. Label the first line you drew *Meadow Rink* and the second line *Forest Rink*. The question asks which linear function has the lesser rate of change. In other words, which line rises less steeply, the line for Meadow Rink or the line for Forest Rink? (Forest Rink) **The steepness of a line is its slope, or rate of change. Which linear function has a lesser rate of change?**

(Forest Rink)

Technology

Have students use an online search engine to discover other experiments they could do that require creating a function table. Additional digital content is available through this feature in the eBook.

Lesson 2

Objective

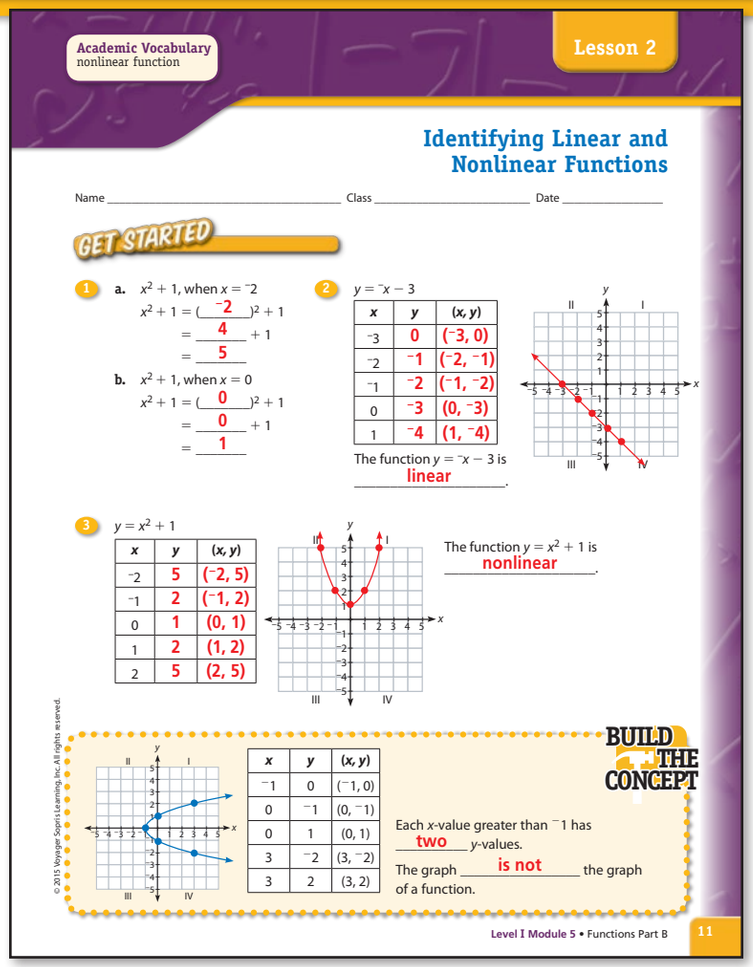
To identify a linear and a nonlinear function

Preskills	Lesson
Evaluating Algebraic Expressions	I.3.PL1

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **nonlinear function**
  a function whose graph is not a line



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

In problem 1a, substitute $^-2$ for the variable, x, and evaluate the expression. How is the square of $^-2$ found? (Multiply $^-2$ times $^-2$.) **What is the square of $^-2$?** (4) **What is 4 plus 1?** (5) **What is the value of the expression $x^2 + 1$ when x equals $^-2$?** (5)

In problem 1b, substitute 0 for the variable and evaluate the expression. What is the value of the expression $x^2 + 1$ when x equals 0? (1)

MODEL NEW SKILLS

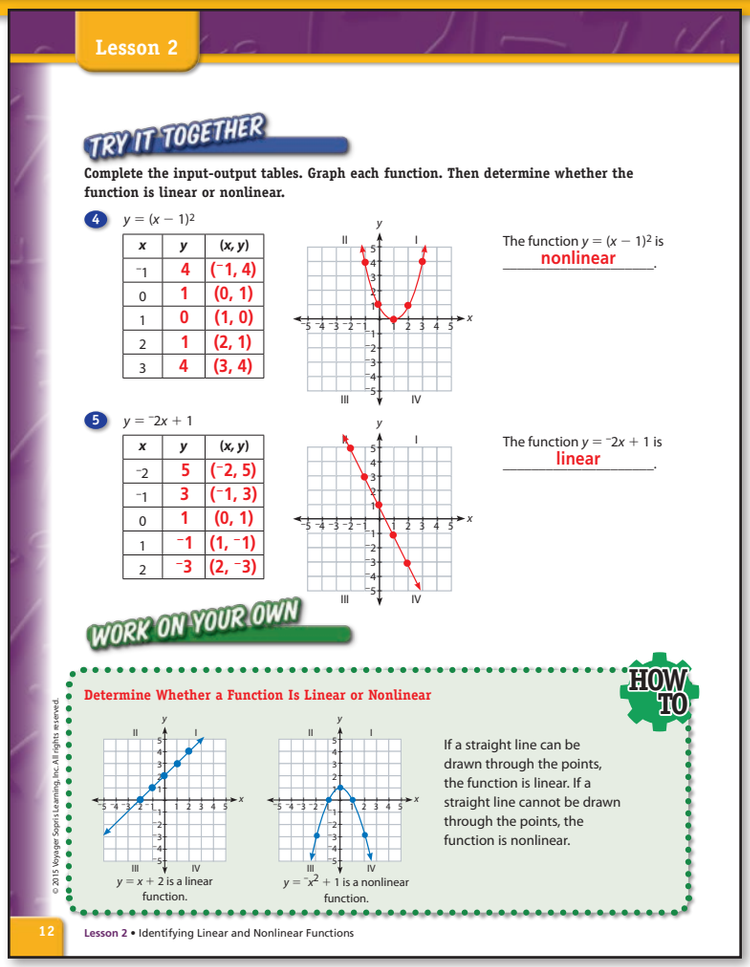
Problem 2

In problem 2, consider the function $y = {}^-x - 3$. Graphing a function is exactly the same as graphing an equation. Start by completing an input-output table. In problem 2, ^-x means $^-1$ times x. To solve for y, first multiply the x-value by $^-1$, then subtract 3. What is the function rule with $^-3$ substituted for x? ($^-1 \times {}^-3 - 3$) **When x is $^-3$, what is y?** (0) **What is the ordered pair?** ($(^-3, 0)$) **What is the next input value for x?** ($^-2$) **What is the function rule with $^-2$ substituted for x?** ($^-1 \times {}^-2 - 3$) **When x is $^-2$, what is y?** ($^-1$) **What is the ordered pair?** ($(^-2, ^-1)$) **When x is $^-1$, what is y?** ($^-2$) **What is the ordered pair?** ($(^-1, ^-2)$) **When x is 0, what is y?** ($^-3$) **What is the ordered pair?** ($(0, ^-3)$) **When x is 1, what is y?** ($^-4$) **What is the ordered pair?** ($(1, ^-4)$)

Graph the points for the ordered pairs on the coordinate plane. Use a ruler or straightedge to determine whether the points fall in a straight line. Do the points fall in a straight line? (yes) **Draw the line on the graph. Because the graph of the function $y = {}^-x - 3$ is a straight line, the function $y = {}^-x - 3$ is called a linear function.**

Problem 3

In problem 3, graph the function $y = x^2 + 1$. Start by completing an input-output table. In problem 1a, the value of y was found for this function when x is $^-2$. When x is $^-2$, what is y? (5) **What is the ordered pair?** ($(^-2, 5)$) **What is the next input value for x?** ($^-1$) **What is the function rule with $^-1$ substituted for x?** ($(^-1)^2 + 1$) **When x is $^-1$, what is y?** (2) **What is the ordered pair?** ($(^-1, 2)$) **When x is 0, what is y?** (1) **What is the ordered pair?** ($(0, 1)$) **When x is 1, what is y?** (2) **What is the ordered pair?** ($(1, 2)$) **When x is 2, what is y?** (5) **What is the ordered pair?** ($(2, 5)$)



Lesson 2

TRY IT TOGETHER

Complete the input-output tables. Graph each function. Then determine whether the function is linear or nonlinear.

4 $y = (x - 1)^2$

x	y	(x, y)
$^-1$	4	$(^-1, 4)$
0	1	(0, 1)
1	0	(1, 0)
2	1	(2, 1)
3	4	(3, 4)

The function $y = (x - 1)^2$ is nonlinear.

5 $y = ^-2x + 1$

x	y	(x, y)
$^-2$	5	$(^-2, 5)$
$^-1$	3	$(^-1, 3)$
0	1	(0, 1)
1	$^-1$	$(1, ^-1)$
2	$^-3$	$(2, ^-3)$

The function $y = ^-2x + 1$ is linear.

WORK ON YOUR OWN

HOW TO

Determine Whether a Function Is Linear or Nonlinear

If a straight line can be drawn through the points, the function is linear. If a straight line cannot be drawn through the points, the function is nonlinear.

$y = x + 2$ is a linear function.

$y = ^-x^2 + 1$ is a nonlinear function.

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Graph the points for the ordered pairs on the coordinate plane. Use a ruler or straightedge to determine whether the points fall in a straight line. Do the points fall in a straight line? (no)

These points form a graph that is in the shape of a U. Graph the function by connecting the points to form a U. Because the graph of the function $y = x^2 + 1$ is not a straight line, the function $y = x^2 + 1$ is called a nonlinear function.

BUILD THE CONCEPT

Model how to identify whether a graph is a function or not.

Look at the graph. Is this graph a straight line? (no) **Just because the graph is not a straight line does not mean that it automatically can be classified as a nonlinear function. Remember that a function is a pairing of input and output values in which each input value can be paired with only one output value. Some of the points on the graph are given as ordered pairs in the input-output table. Does any x-value correspond to more than one y-value?** (yes) **Which ones?** (0 and 3) **This means that the graph is not the graph of a function.**

The graph in problem 3 is also U-shaped, but it does represent the graph of a function. Notice that in a function, two different x-values can have the same y-value as in problem 3, but one x-value cannot have two different y-values.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

To determine whether the function $y = (x - 1)^2$ is a linear function or a nonlinear function, find the ordered pairs, then graph the function. First, complete an input-output table by substituting the x-values to find the y-values. When x is $^-1$, what is y? (4) **Why?** ($(^-1 - 1)^2 = (^-2)^2 = 4$) **What is the ordered pair?** ($(^-1, 4)$)

When x is 0, what is y? (1) **Why?** ($(0 - 1)^2 = (^-1)^2 = 1$) **What is the ordered pair?** ((0, 1)) **When x is 1, what is y?** (0) **Why?** ($(1 - 1)^2 = 0^2 = 0$) **What is the ordered pair?** ((1, 0)) **When x is 2, what is y?** (1) **What is the ordered pair?** ((2, 1)) **When x is 3, what is y?** (4) **What is the ordered pair?** ((3, 4))

Graph the points for the ordered pairs on the coordinate plane. Use a straightedge or ruler to try to draw a straight line through all the points. Can a straight line be drawn through all the points? (no) **Is the function $y = (x - 1)^2$ a linear function or a nonlinear function?** (nonlinear) **Why?** (The points do not make a straight line.)

Problem 5

What is the first step in determining whether $y = ^-2x + 1$ is a linear function or a nonlinear function? (Complete an input-output table.) **When x is $^-2$, what is y?** (5) **What is the ordered pair?** ($(^-2, 5)$) **When x is $^-1$, what is y?** (3) **What is the ordered pair?** ($(^-1, 3)$) **When x is 0, what is y?** (1) **What is the ordered pair?** ((0, 1)) **When x is 1, what is y?** ($^-1$) **What is the ordered pair?** ($(1, ^-1)$) **When x is 2, what is y?** ($^-3$) **What is the ordered pair?** ($(2, ^-3)$)

Graph the points for the ordered pairs on the coordinate plane. Use a straightedge or ruler to try to draw a straight line through all the points. Can a straight line be drawn through all the points? (yes) **Is the function $y = ^-2x + 1$ a linear function or a nonlinear function?** (linear) **Why?** (The points make a straight line.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Graph** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a graph. The instruction is immediately followed by application of the strategy in problem 11.

Have students read the problem.

Have students recall the definition of a linear function:

The points of a linear function lie in a straight line.

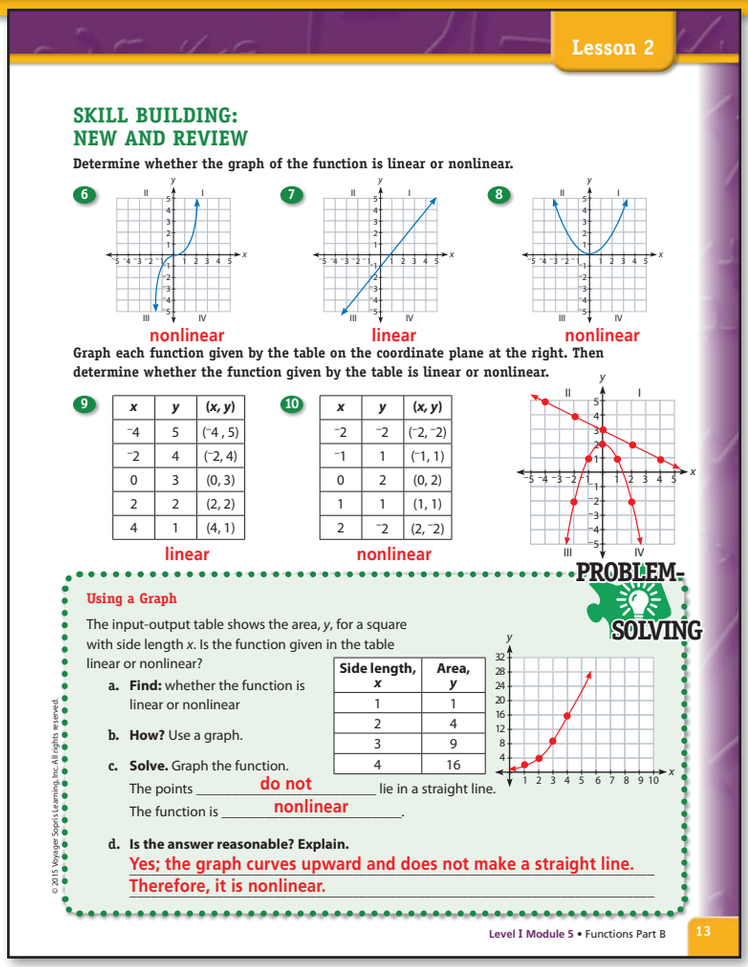
Have students graph the function. The function is parabolic.

The function is not linear because the points do not lie in a straight line.

The answer is reasonable because the graph curves upward and does not make a straight line. So, the function is not linear.

Problems 6–13

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 2

SKILL BUILDING: NEW AND REVIEW

Determine whether the graph of the function is linear or nonlinear.

6 nonlinear

7 linear

8 nonlinear

Graph each function given by the table on the coordinate plane at the right. Then determine whether the function given by the table is linear or nonlinear.

9

x	y	(x, y)
−4	5	(−4, 5)
−2	4	(−2, 4)
0	3	(0, 3)
2	2	(2, 2)
4	1	(4, 1)

linear

10

x	y	(x, y)
−2	−2	(−2, −2)
−1	1	(−1, 1)
0	2	(0, 2)
1	1	(1, 1)
2	−2	(2, −2)

nonlinear

PROBLEM-SOLVING

Using a Graph

The input-output table shows the area, *y*, for a square with side length *x*. Is the function given in the table linear or nonlinear?

Side length, x	Area, y
1	1
2	4
3	9
4	16

a. **Find:** whether the function is linear or nonlinear

b. **How?** Use a graph.

c. **Solve.** Graph the function.

The points do not lie in a straight line.

The function is nonlinear.

d. **Is the answer reasonable? Explain.**

Yes; the graph curves upward and does not make a straight line. Therefore, it is nonlinear.

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CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 or problem 2 incorrectly:
The student did not understand the meaning of a linear function. Use Additional Resources in I.5.2 to reteach how to determine if a function is linear or nonlinear.



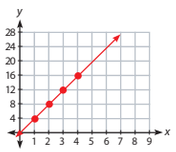
WRITE MATH **If student answered problem 3 incorrectly:** While students work together, remind the student that in order for the graph to be a function, each *x*-value must only have one *y*-value.

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

11. The input-output table shows the perimeter, *y*, for a square with side length *x*. Is the function given in the table linear or nonlinear?

Side length, *x*	1	2	3	4
Perimeter, *y*	4	8	12	16

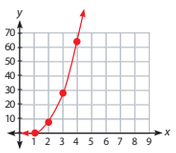


linear

12. Write a function rule for the input-output table in problem 11. $y = 4x$

13. The input-output table shows the volume, *y*, of cube-shaped packing boxes with side length *x*. Is the function given in the table linear or nonlinear?

Side length, *x*	1	2	3	4
Volume, *y*	1	8	27	64



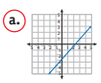
nonlinear

CHECK UP

Answer each question.

1. Which statement is true of all linear functions?
   - a. Both the *x*-values and the *y*-values of the function increase.
   - (b.) The points for its ordered pairs all fall in a straight line.
   - c. Its graph is in the shape of a U.
   - d. The intercepts of its graph are both (0, 0).

2. Which graph below is the graph of a linear function?

(a.)  b. 

c.  d. 

3. Which graph in problem 2 is not the graph of a function? Explain.
   d; to be a function, each *x*-value must have exactly one *y*-value.

WRITE MATH

4. Complete the statement using *always*, *sometimes*, or *never*.
   The graph of a nonvertical straight line is always the graph of a function.

CRITICAL THINKING

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DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 5 Lesson 2

Reteach Teacher Edition Module 5 Lesson 2

Extra Practice

Student Book page 40

English Language Learners

Write the words *linear* and *nonlinear* on the board. Write the word *linear* in a straight line, but write the word *nonlinear* in a parabolic shape. Discuss the meaning of the prefix *non-*. Have students think of other words that contain this prefix.

Students with Special Needs

If students have trouble graphing points, they will not be able to successfully determine whether a function is linear. Be sure that students can plot ordered pairs correctly. If students plot many points in a straight line, but one or two of the points don't seem to fit, encourage them to go back and check each point before deciding that the function is nonlinear.

Math Flash

Lesson 3

Objective

To write a linear equation in slope-intercept form

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

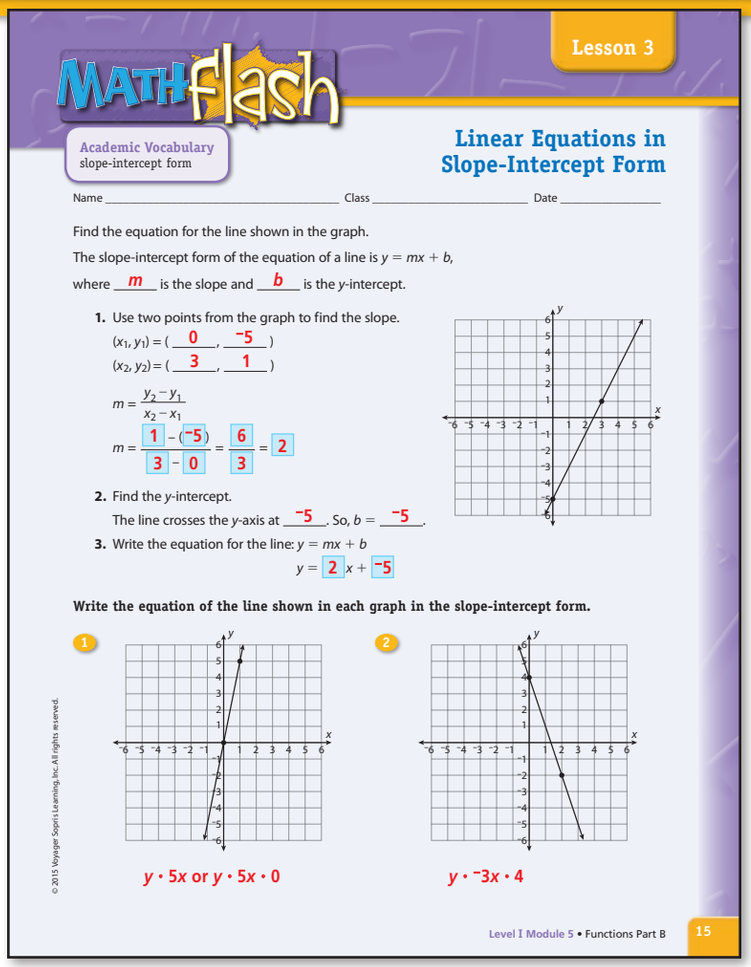
- **slope-intercept form**
  the equation of a straight line in the form $y = mx + b$, where m is the slope of the line and b is its y-intercept

Model the following skills for students.

What is a linear equation? (an algebraic equation which when graphed forms a straight line) **What is slope?** (the steepness of a line) **What is the *y*-intercept of a graph?** (the point at which the graph crosses the *y*-axis) **The slope-intercept form of the equation of a line is an equation of the form *y* · *mx* · *b*, where *m* is the slope of the line and *b* is the *y*-intercept.**

To write the equation of a line in slope-intercept form, first find the slope of the line. Look at the graph of the line. What two points are graphed? ((0, ⁻5) and (3, 1)) **Let (x_1, y_1) represent (0, ⁻5) and let (x_2, y_2) represent (3, 1). Remember that the slope is the ratio of the rise to the run. Find the rise, or change in *y*, first. What is 1 minus ⁻5?** (6) **Now find the run, or change in *x*. What is 3 minus 0?** (3) **What is the slope, *m*?** ($\frac{6}{3}$) **What is the slope written in simplest form?** (2)

At what *y*-value does the line cross the *y*-axis? (⁻5) **What is the *y*-intercept, *b*?** (⁻5) **Finally, substitute the values for *m* and *b* in the formula *y* · *mx* · *b*. What is the equation of the line?** ($y = 2x + ^{-}5$)



MATH Flash — Lesson 3

Academic Vocabulary
slope-intercept form

Linear Equations in Slope-Intercept Form

Name _____ Class _____ Date _____

Find the equation for the line shown in the graph.

The slope-intercept form of the equation of a line is $y = mx + b$,

where ___m___ is the slope and ___b___ is the *y*-intercept.

1. Use two points from the graph to find the slope.
   $(x_1, y_1) = (_0_, _{-5}_)$
   $(x_2, y_2) = (_3_, _1_)$

   $m = \frac{y_2 - y_1}{x_2 - x_1}$

   $m = \frac{1 - (^{-}5)}{3 - 0} = \frac{6}{3} = 2$

2. Find the *y*-intercept.
   The line crosses the *y*-axis at ___⁻5___. So, $b =$ ___⁻5___.
3. Write the equation for the line: $y = mx + b$
   $y = 2x + ^{-}5$

Write the equation of the line shown in each graph in the slope-intercept form.

1

$y = 5x$ or $y = 5x + 0$

2

$y = ^{-}3x + 4$

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Level I Module 5 • Functions Part B 15

Lesson 4

Objective

To determine whether a linear relationship, nonlinear relationship, or no relationship exists between bivariate sets of data

Preskills	Lesson
Scatter Plots and Lines of Best Fit	I.4.2
Identifying Linear and Nonlinear Functions	I.5.2

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

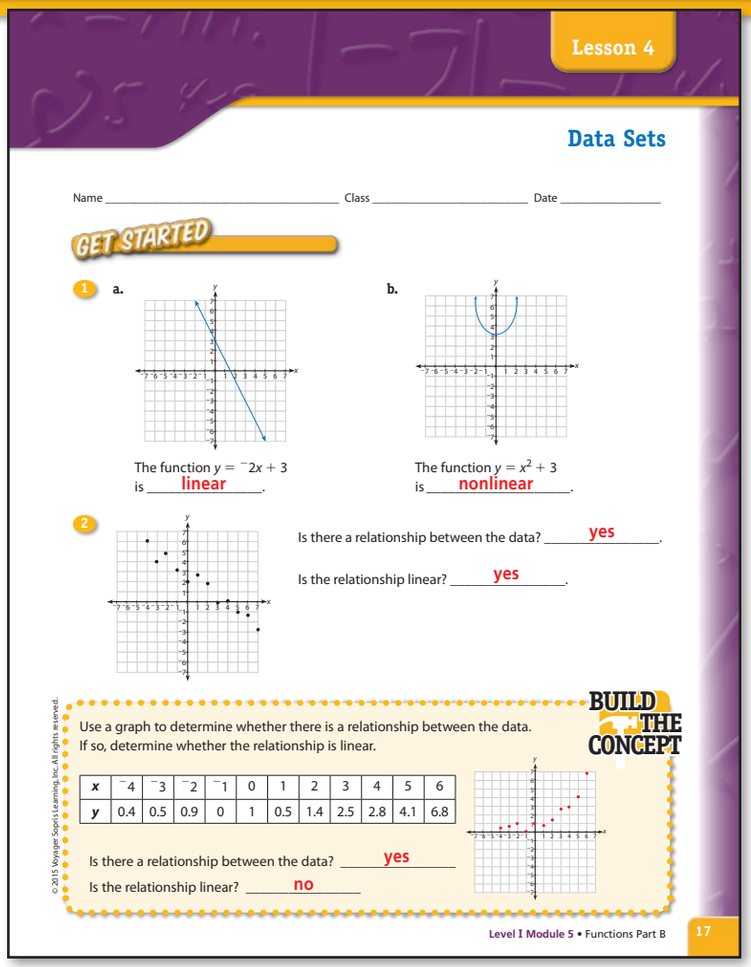
Look at the graph of $y = {}^-2x + 3$ in part a. Is the function a linear function? (yes) **Why?** (The graph is a straight line.)

Look at the graph of $y = x^2 + 3$ in part b. Is the function a linear function? (no) **Why not?** (The graph is not a straight line.) **What kind of function is $y = x^2 + 3$?** (a nonlinear function)

MODEL NEW SKILLS

Problem 2

Look at the data points on the scatter plot. Is there a pattern to the data points? (yes) **So, there is a relationship between the data. Look at the graph from left to right. In general, what happens to the *y*-values as the *x*-values get greater, or increase?** (The *y*-values decrease.) **Could a straight line be drawn that is very close to most of the points?** (yes) **So, the relationship is linear.**



Lesson 4

Data Sets

Name _____ Class _____ Date _____

GET STARTED

1 a.

The function $y = {}^-2x + 3$ is linear.

b.

The function $y = x^2 + 3$ is nonlinear.

2

Is there a relationship between the data? yes.

Is the relationship linear? yes.

BUILD THE CONCEPT

Use a graph to determine whether there is a relationship between the data. If so, determine whether the relationship is linear.

x	−4	−3	−2	−1	0	1	2	3	4	5	6
y	0.4	0.5	0.9	0	1	0.5	1.4	2.5	2.8	4.1	6.8

Is there a relationship between the data? yes

Is the relationship linear? no

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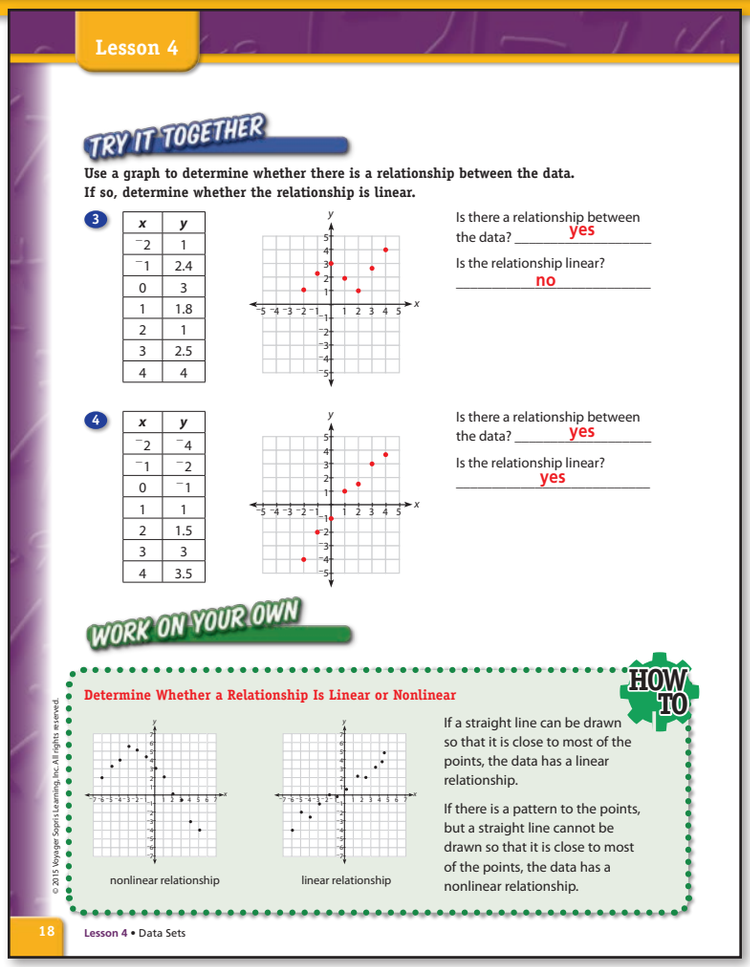
Level I Module 5 • Functions Part B 17

BUILD THE CONCEPT

Model how to use a scatter plot to determine the relationship between bivariate data.

Look at the data in the table. A good way to determine whether there is a relationship between the data is to draw a scatter plot. Graph the data given by the table on the coordinate plane.

Look at the scatter plot. Is there a pattern to the data points? (yes) **So, there is a relationship between the data. Look at the graph from left to right. In general, what happens to the *y*-values as the *x*-values get greater, or increase?** (The *y*-values increase.) Encourage students to use a straightedge to determine the answer to the next question. **Could a straight line be drawn that is very close to most of the points?** (no) **So, the relationship between the data in the table is nonlinear.**



Lesson 4

TRY IT TOGETHER

Use a graph to determine whether there is a relationship between the data. If so, determine whether the relationship is linear.

3

x	y
−2	1
−1	2.4
0	3
1	1.8
2	1
3	2.5
4	4

Is there a relationship between the data? yes

Is the relationship linear? no

4

x	y
−2	−4
−1	−2
0	−1
1	1
2	1.5
3	3
4	3.5

Is there a relationship between the data? yes

Is the relationship linear? yes

WORK ON YOUR OWN

HOW TO

Determine Whether a Relationship Is Linear or Nonlinear

If a straight line can be drawn so that it is close to most of the points, the data has a linear relationship.

If there is a pattern to the points, but a straight line cannot be drawn so that it is close to most of the points, the data has a nonlinear relationship.

nonlinear relationship

linear relationship

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18 Lesson 4 • Data Sets

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 3

Look at the table. To determine whether there is a relationship between these data, graph the data on the coordinate plane.

Look at the scatter plot. Is there a relationship between the data? (yes) **How do you know?** (There is a pattern to the points.) **Look at the graph from left to right. In general, what happens to the *y*-values as the *x*-values get greater, or increase?** (The *y*-values increase, then decrease, then increase again.) **Could a straight line be drawn that is very close to most of the points?** (no) **So, the relationship is nonlinear.**

Problem 4

Look at the table. How can a relationship between the data be determined? (Graph the data.) **Graph the data on the coordinate plane.**

Look at the scatter plot. Is there a relationship between the data? (yes) **What is the relationship?** (In general, the *y*-values increase as the *x*-values increase.) **Is the relationship linear?** (yes) **Why?** (A straight line could be drawn that is very close to most of the points.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 5–9

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 5 and 6 Reminder

Encourage students to use a straightedge to determine whether the relationship is linear or nonlinear.

CHECK UP

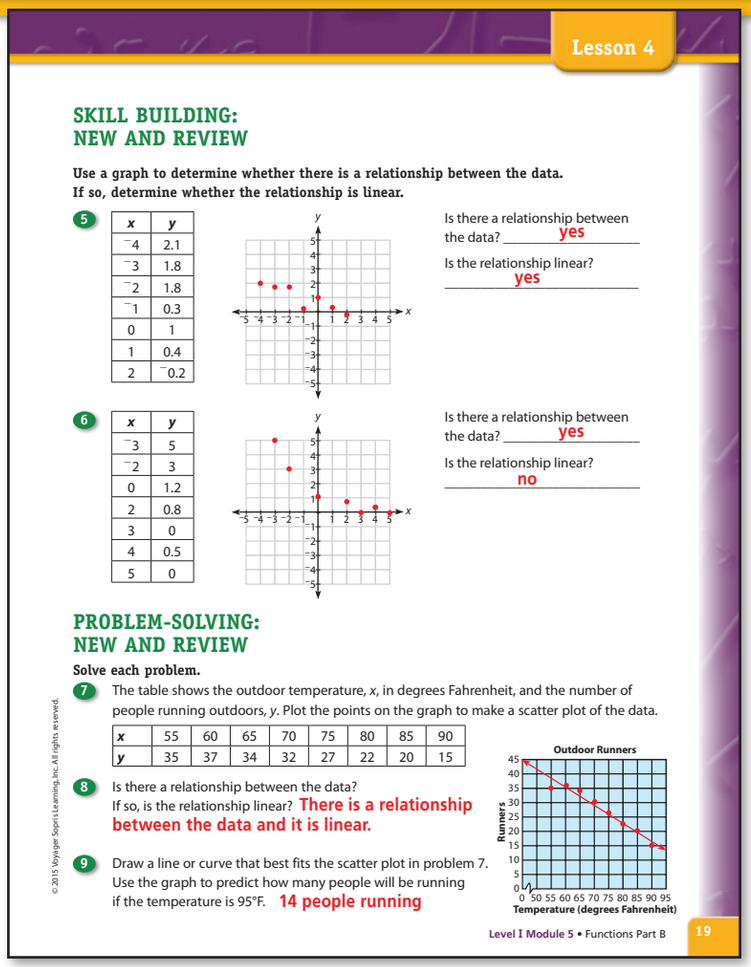
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b, 1c, 2a, or 2c: The student did not understand the meaning of a linear relationship. Use Additional Resources in I.5.4 to review the difference between a linear relationship and a nonlinear relationship.

If student answered 1d or 2b: The student was not able to distinguish between a linear and nonlinear relationship in which *y*-values decrease or increase as the *x*-value increase. Use Additional Resources in I.5.4 to review the difference between a linear relationship and a nonlinear relationship.



Lesson 4

SKILL BUILDING: NEW AND REVIEW

Use a graph to determine whether there is a relationship between the data. If so, determine whether the relationship is linear.

5

x	y
−4	2.1
−3	1.8
−2	1.8
−1	0.3
0	1
1	0.4
2	−0.2

Is there a relationship between the data? yes

Is the relationship linear? yes

6

x	y
−3	5
−2	3
0	1.2
2	0.8
3	0
4	0.5
5	0

Is there a relationship between the data? yes

Is the relationship linear? no

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

7 The table shows the outdoor temperature, *x*, in degrees Fahrenheit, and the number of people running outdoors, *y*. Plot the points on the graph to make a scatter plot of the data.

x	55	60	65	70	75	80	85	90
y	35	37	34	32	27	22	20	15

Outdoor Runners

Runners

Temperature (degrees Fahrenheit)

8 Is there a relationship between the data? If so, is the relationship linear? There is a relationship between the data and it is linear.

9 Draw a line or curve that best fits the scatter plot in problem 7. Use the graph to predict how many people will be running if the temperature is 95°F. 14 people running

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Level I Module 5 • Functions Part B 19

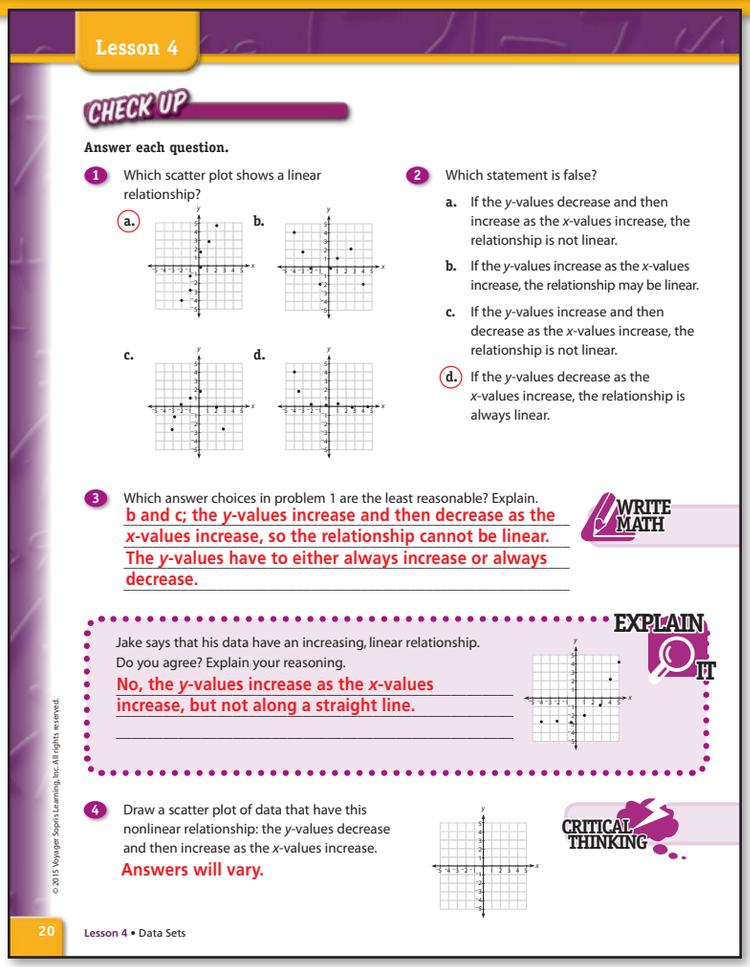


WRITE MATH

If student answered problem 3 incorrectly: While students work together, review linear relationships. Use Additional Resources in I.5.4 to review the difference between a linear relationship and a nonlinear relationship.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 4

CHECK UP

Answer each question.

1. Which scatter plot shows a linear relationship?

(a.) b. c. d.

2. Which statement is false?

a. If the *y*-values decrease and then increase as the *x*-values increase, the relationship is not linear.

b. If the *y*-values increase as the *x*-values increase, the relationship may be linear.

c. If the *y*-values increase and then decrease as the *x*-values increase, the relationship is not linear.

(d.) If the *y*-values decrease as the *x*-values increase, the relationship is always linear.

3. Which answer choices in problem 1 are the least reasonable? Explain.

WRITE MATH

b and c; the *y*-values increase and then decrease as the *x*-values increase, so the relationship cannot be linear. The *y*-values have to either always increase or always decrease.

EXPLAIN IT

Jake says that his data have an increasing, linear relationship. Do you agree? Explain your reasoning.

No, the *y*-values increase as the *x*-values increase, but not along a straight line.

4. Draw a scatter plot of data that have this nonlinear relationship: the *y*-values decrease and then increase as the *x*-values increase.

CRITICAL THINKING

Answers will vary.

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DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 5 Lesson 4

Reteach Teacher Edition Module 5 Lesson 4

Extra Practice

Student Book page 40

English Language Learners

Review what it means for a graph to be increasing or decreasing. Remind students to read graphs from left to right. Draw arrows on simple scatter plots to demonstrate different increasing relationships and different decreasing relationships.

Students with Special Needs

Some students may have difficulty differentiating a linear relationship with a negative slope from a decreasing exponential relationship. Show an example of a scatter plot with data that have a relationship than can be modeled by a linear function with a negative slope. Then show an example of a scatter plot with data that is best modeled by a decreasing exponential function. Demonstrate how a line is not appropriate to use as a model in the latter example. Repeat this process for linear relationships with increasing data.

Lesson 5

Objective

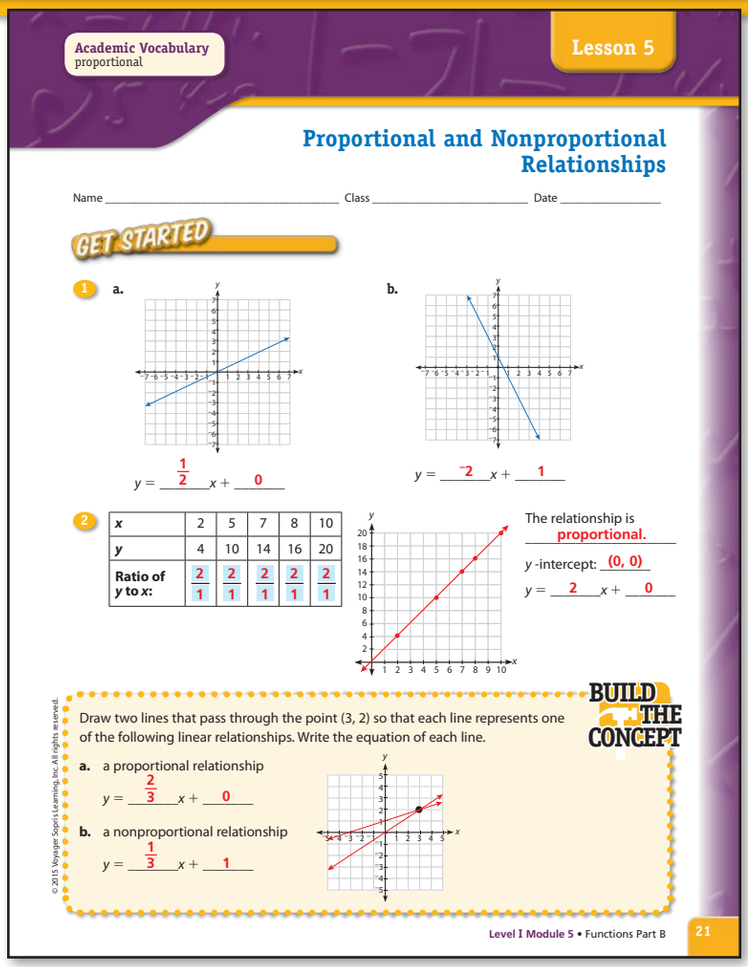
To distinguish between proportional and nonproportional relationships

Preskills	Lesson
Function Rules	I.4.8
Graphing Linear Equations	I.4.11
Math Flash: Linear Equations in Slope-Intercept Form	I.5.3

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **proportional**
  a linear relationship that can be represented in the form $y = mx$


Academic Vocabulary
proportional

Lesson 5

Proportional and Nonproportional Relationships

Name _____ Class _____ Date _____

GET STARTED

1 a.

$y = \frac{1}{2}x + 0$

b.

$y = -2x + 1$

2

x	2	5	7	8	10
y	4	10	14	16	20
Ratio of y to x:	2/1	2/1	2/1	2/1	2/1

The relationship is proportional.

y-intercept: (0, 0)

$y = 2x + 0$

BUILD THE CONCEPT

Draw two lines that pass through the point (3, 2) so that each line represents one of the following linear relationships. Write the equation of each line.

a. a proportional relationship

$y = \frac{2}{3}x + 0$

b. a nonproportional relationship

$y = \frac{1}{3}x + 1$

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Look at the graph in part a. The graph of what kind of function is shown? (linear function) **The slope-intercept form of a linear equation, $y = mx + b$, can be used to write the function for this graph. What are m and b?** (the slope and y-intercept) **Look at the graph of the line. What is the slope?** ($\frac{1}{2}$) **What is the y-coordinate of the y-intercept?** (0) **What is the equation for the line?** ($y = \frac{1}{2}x + 0$ or $y = \frac{1}{2}x$)

Look at the graph in part b. The graph of what kind of function is shown? (linear function) **What is the slope of the line?** ($^-2$) **What is the y-coordinate of the y-intercept?** (1) **What is the equation for the line written in slope-intercept form?** ($y = {}^-2x + 1$)

MODEL NEW SKILLS

Problem 2

Look at the table. One way to determine whether there is a relationship between the variables x and y is to write each pair of values as a ratio of y to x. What is the first pair of values? ($x = 2$ and $y = 4$)

What is the ratio of 4 to 2 written as a fraction in simplest form? ($\frac{2}{1}$) **What is the ratio of 10 to 5 written as a fraction in simplest form?** ($\frac{2}{1}$) **What is the ratio of 14 to 7 written as a fraction in simplest form?** ($\frac{2}{1}$) **What is the ratio of 16 to 8 written as a fraction in simplest form?** ($\frac{2}{1}$) **What is the ratio of 20 to 10 written as a fraction in simplest form?** ($\frac{2}{1}$) **Because the ratios are equal, this table of values represents a proportional relationship.**

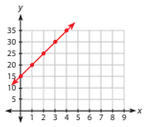
Graph the pairs of values. Do the points form a straight line? (yes) **Proportional relationships are linear. Draw a line through the points. What is the y-intercept of the graph?** ((0, 0)) **A linear relationship is proportional when the y-intercept is the origin. What is the slope of the line?** (2) **Use the slope-intercept form to write an equation for this proportional relationship. What is the equation?** ($y = 2x + 0$ or $y = 2x$)

TRY IT TOGETHER

Create a graph and write an equation for each data set. Then determine whether the relationship is proportional or nonproportional.

3

Number of days, x	0	1	2	3	4
Rental cost (dollars), y	15	20	25	30	35

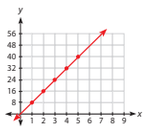


$y =$ 5 $x +$ 15

The relationship is nonproportional.

4

Hours worked, x	1	2	3	4	5
Earnings (dollars), y	8	16	24	32	40



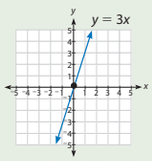
$y =$ 8 $x +$ 0

The relationship is proportional.

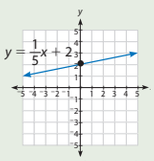
WORK ON YOUR OWN

HOW TO

Determine Whether a Relationship Is Proportional or Nonproportional



proportional relationship



nonproportional relationship

A linear relationship is proportional if the y-intercept of its graph is 0. The equation has the form $y = mx$.

A linear relationship is nonproportional if the y-intercept of its graph is not 0. The equation has the form $y = mx + b$, where b is not 0.

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BUILD THE CONCEPT

Model how to graph and write an equation of a proportional relationship and a nonproportional relationship.

What does the graph of a proportional relationship look like? (It is a line with a y-intercept that is the origin.) **Use a straightedge to draw a line that passes through the origin and the point (3, 2). What is the slope of this line?** ($\frac{2}{3}$) **What is the equation of the line?** ($y = \frac{2}{3}x$)

When a relationship is linear but is not proportional, the graph is a line with a y-intercept that is not the origin. To draw a line with a nonproportional relationship that passes through the point (3, 2), choose a point for the y-intercept that is not the origin. Choose the point (0, 1). Use a straightedge to draw a line that passes through (0, 1) and (3, 2). What is the slope of this line? ($\frac{1}{3}$) **What is the y-coordinate of the y-intercept?** (1) **What is the equation of the line?** ($y = \frac{1}{3}x + 1$)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 3

Look at the table. These data describe the total cost, y, to rent a piece of equipment for x days. To determine whether the relationship between the number of days and total cost is proportional or nonproportional, graph the data on the coordinate plane. What points should be graphed? ((0, 15), (1, 20), (2, 25), (3, 30), (4, 35)) **Graph the points. Then draw the line that passes through the points.**

Now write an equation for the line. What is the slope? (5) **What is the y-coordinate of the y-intercept?** (15) **What is the equation that relates the number of days, x, and the rental cost, y?** ($y = 5x + 15$)

Is the relationship proportional or nonproportional? (nonproportional) **Why is the relationship nonproportional?** (The graph is a line that does not pass through the origin.)

Problem 4

Look at the table. These data describe the earnings, y, for working x hours. To determine whether the relationship between the number of hours and the earnings is proportional or nonproportional, graph the data on the coordinate plane. Then draw the line that passes through the points.

Now write an equation for the line. What is the slope? (8) **What is the y-coordinate of the y-intercept?** (0) **What is the equation that relates the number of hours, x, and the earnings, y?** ($y = 8x$)

Is the relationship proportional or nonproportional? (proportional) **Why is the relationship proportional?** (The graph is a line that passes through the origin.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using a Graph** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a linear relationship. The instruction is immediately followed by application of the strategy in problem 7.

Have students read the problem. Suggest that creating a graph can help solve the problem.

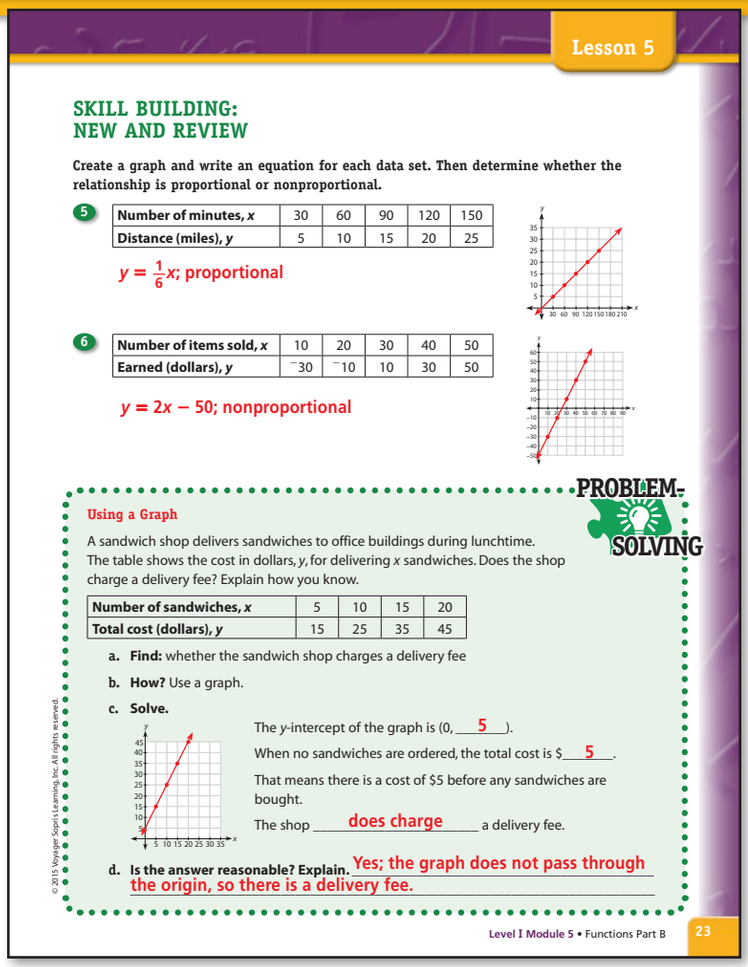
After students plot the points and draw the line, have them identify what each variable represents. Then have them identify and explain the meaning of the *y*-intercept in the context of the problem. Ask students what it means when there is a cost of $5 before any sandwiches are bought. Students should interpret this cost as a delivery fee.

Problems 5–9

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 9 Reminder

Students should use their equation from problem 7 to find how much money is in the account after 36 weeks.



Lesson 5

SKILL BUILDING: NEW AND REVIEW

Create a graph and write an equation for each data set. Then determine whether the relationship is proportional or nonproportional.

5

Number of minutes, x	30	60	90	120	150
Distance (miles), y	5	10	15	20	25

$y = \frac{1}{6}x$; proportional

6

Number of items sold, x	10	20	30	40	50
Earned (dollars), y	−30	−10	10	30	50

$y = 2x - 50$; nonproportional

PROBLEM-SOLVING

Using a Graph

A sandwich shop delivers sandwiches to office buildings during lunchtime. The table shows the cost in dollars, y, for delivering x sandwiches. Does the shop charge a delivery fee? Explain how you know.

Number of sandwiches, x	5	10	15	20
Total cost (dollars), y	15	25	35	45

a. **Find:** whether the sandwich shop charges a delivery fee

b. **How?** Use a graph.

c. **Solve.**

The y-intercept of the graph is (0, 5).

When no sandwiches are ordered, the total cost is $5.

That means there is a cost of $5 before any sandwiches are bought.

The shop does charge a delivery fee.

d. **Is the answer reasonable? Explain.** Yes; the graph does not pass through the origin, so there is a delivery fee.

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CHECK UP

ASSESS INFORMALLY

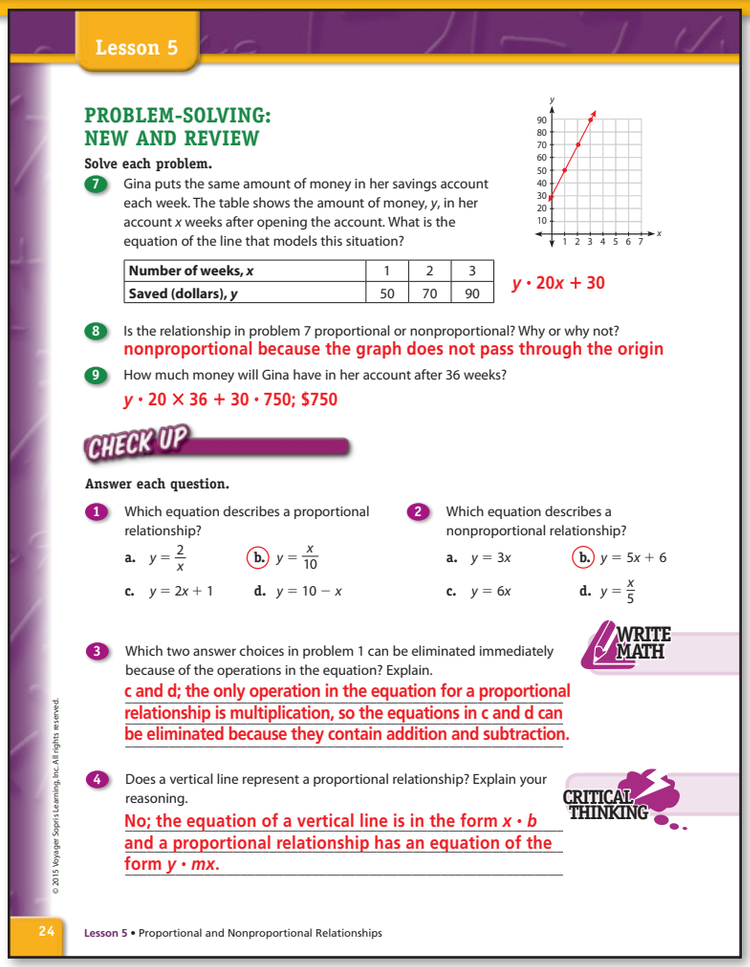
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student chose a nonlinear relationship. Use Additional Resources in I.5.5 to reinforce that a proportional relationship must be linear.

If student answered 1c or 1d: The student chose a linear, but nonproportional relationship. Use Additional Resources in I.5.5 to review the difference between a proportional relationship and a nonproportional relationship.

If student answered problem 2 incorrectly: The student was not able to distinguish between a proportional and a nonproportional relationship. Use Additional Resources in I.5.5 to review the difference between a proportional relationship and a nonproportional relationship.



Lesson 5

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

7. Gina puts the same amount of money in her savings account each week. The table shows the amount of money, *y*, in her account *x* weeks after opening the account. What is the equation of the line that models this situation?

Number of weeks, *x*	1	2	3
Saved (dollars), *y*	50	70	90

$y = 20x + 30$

8. Is the relationship in problem 7 proportional or nonproportional? Why or why not?
nonproportional because the graph does not pass through the origin

9. How much money will Gina have in her account after 36 weeks?
$y = 20 \times 36 + 30 = 750$; \$750

CHECK UP

Answer each question.

1. Which equation describes a proportional relationship?
   a. $y = \frac{2}{x}$ (b.) $y = \frac{x}{10}$
   c. $y = 2x + 1$ d. $y = 10 - x$

2. Which equation describes a nonproportional relationship?
   a. $y = 3x$ (b.) $y = 5x + 6$
   c. $y = 6x$ d. $y = \frac{x}{5}$

WRITE MATH

3. Which two answer choices in problem 1 can be eliminated immediately because of the operations in the equation? Explain.
c and d; the only operation in the equation for a proportional relationship is multiplication, so the equations in c and d can be eliminated because they contain addition and subtraction.

CRITICAL THINKING

4. Does a vertical line represent a proportional relationship? Explain your reasoning.
No; the equation of a vertical line is in the form $x = b$ and a proportional relationship has an equation of the form $y = mx$.

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24 Lesson 5 • Proportional and Nonproportional Relationships



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review proportional relationships. Use Additional Resources in I.5.5 to compare equations of proportional relationships with equations of linear nonproportional relationships.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 5 Lesson 5
Reteach Teacher Edition Module 5 Lesson 5

Extra Practice
Student Book page 41

English Language Learners

Use the VmathLive Animated Glossary to review the terms *linear equation*, *slope*, and *y-intercept*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Make sure that students don't confuse comparing linear and nonlinear relationships with proportional and nonproportional relationships. In this lesson, students are focusing on linear relationships and categorizing these relationships as proportional or nonproportional.

Students with Special Needs

Some students may assume that all proportional relationships are linear functions with positive slope. While this is true for many real-life situations, students should recognize that the distinguishing characteristic is the *y*-intercept. Have students use a coordinate plane to draw several examples of proportional and nonproportional linear relationships, as is done in Build the Concept for this lesson.

Lesson 6

Objective

To model proportional and nonproportional relationships

Preskills	Lesson
Math Flash: Linear Equations in Slope-Intercept Form	I.5.3
Proportional and Nonproportional Relationships	I.5.5

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

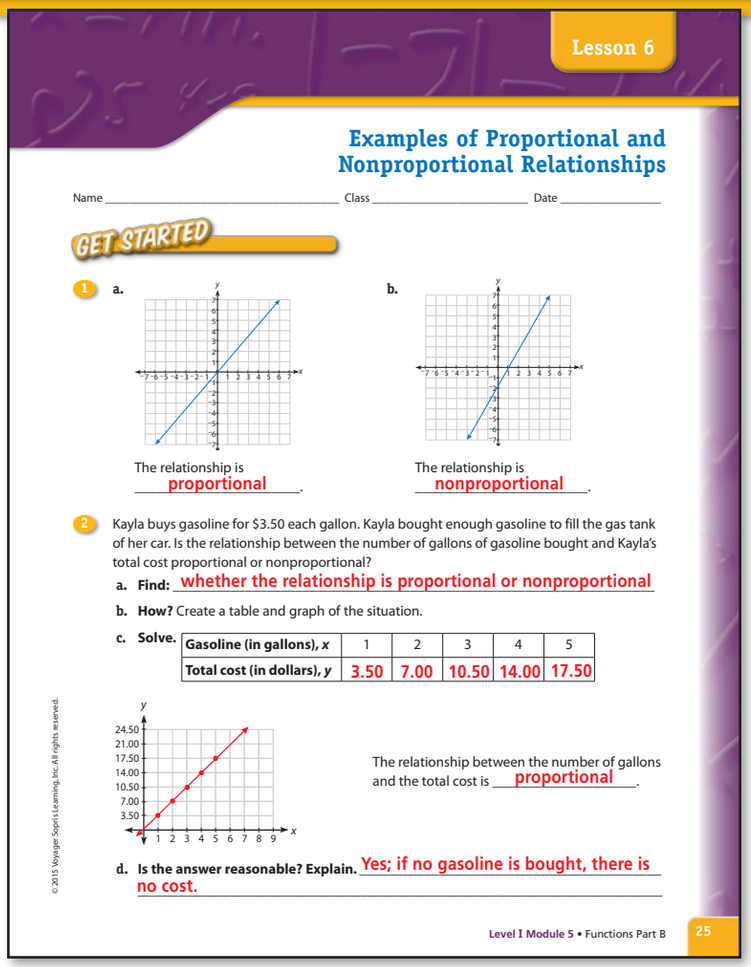
Look at the graph in part a. The graph of what kind of function is shown? (linear function) **Does this graph represent a proportional or nonproportional relationship?** (proportional) **Why does the graph represent a proportional relationship?** (The graph is a line that passes through the origin.)

Look at the graph in part b. The graph of what kind of function is shown? (linear function) **Does this graph represent a proportional or nonproportional relationship?** (nonproportional) **Why does the graph represent a nonproportional relationship?** (The graph does not pass through the origin.)

MODEL NEW SKILLS

Problem 2

Read the problem. The first step to solving this problem is to identify what is to be found. What is to be found? (whether the relationship is proportional or nonproportional) **What is true about the graphs of proportional relationships?** (The graph is a line that passes through the origin.) **To solve the problem, first create a table of values from the information in the problem. Then use the table to create a graph.**



Lesson 6

Examples of Proportional and Nonproportional Relationships

Name _____ Class _____ Date _____

GET STARTED

1 a. b.

The relationship is proportional. The relationship is nonproportional.

2 Kayla buys gasoline for $3.50 each gallon. Kayla bought enough gasoline to fill the gas tank of her car. Is the relationship between the number of gallons of gasoline bought and Kayla's total cost proportional or nonproportional?

a. **Find:** whether the relationship is proportional or nonproportional

b. **How?** Create a table and graph of the situation.

c. **Solve.**

Gasoline (in gallons), x	1	2	3	4	5
Total cost (in dollars), y	3.50	7.00	10.50	14.00	17.50

The relationship between the number of gallons and the total cost is proportional.

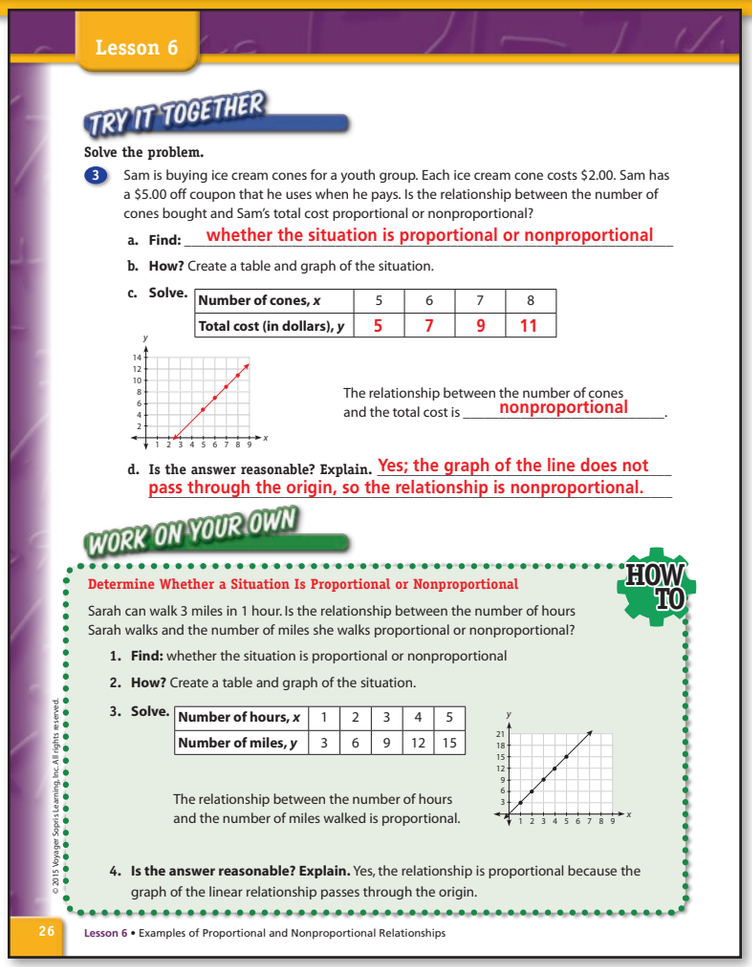
d. **Is the answer reasonable? Explain.** Yes; if no gasoline is bought, there is no cost.

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What is the problem about? (number of gallons of gasoline bought and the total cost of the gasoline) **Do you know how many gallons Kayla bought?** (no) **Look at the table. Which variable represents the amount of gasoline?** (x) **Which variable represents the cost?** (y) **How much does one gallon of gasoline cost?** ($3.50) **Record the cost in the table. If Kayla bought one more gallon of gasoline for a total of 2 gallons, how can the cost be calculated?** (by adding $3.50 to $3.50) **What is 3.50 plus 3.50?** (7) **What is the cost of 2 gallons of gasoline?** ($7) **Record this cost in the table. If Kayla bought one more gallon of gasoline for a total of 3 gallons, how can the cost be calculated?** (by adding 3.50 to 7) **What is 3.50 plus 7?** (10.50) **What is the cost of 3 gallons of gasoline?** ($10.50) **What is the cost of 4 gallons of gasoline?** ($14.00) **What is the cost of 5 gallons of gasoline?** ($17.50)

Next, graph the values in the table. Do the points form a straight line? (yes) **Draw the line. Is the relationship between the number of gallons and total cost proportional or nonproportional?** (proportional) **Why is the relationship proportional?** (The graph is a straight line that passes through the origin.)



Lesson 6

TRY IT TOGETHER

Solve the problem.

3 Sam is buying ice cream cones for a youth group. Each ice cream cone costs $2.00. Sam has a $5.00 off coupon that he uses when he pays. Is the relationship between the number of cones bought and Sam's total cost proportional or nonproportional?

a. **Find:** whether the situation is proportional or nonproportional

b. **How?** Create a table and graph of the situation.

c. **Solve.**

Number of cones, x	5	6	7	8
Total cost (in dollars), y	5	7	9	11

The relationship between the number of cones and the total cost is nonproportional.

d. **Is the answer reasonable? Explain.** Yes; the graph of the line does not pass through the origin, so the relationship is nonproportional.

WORK ON YOUR OWN

HOW TO

Determine Whether a Situation Is Proportional or Nonproportional

Sarah can walk 3 miles in 1 hour. Is the relationship between the number of hours Sarah walks and the number of miles she walks proportional or nonproportional?

1. **Find:** whether the situation is proportional or nonproportional
2. **How?** Create a table and graph of the situation.
3. **Solve.**

Number of hours, x	1	2	3	4	5
Number of miles, y	3	6	9	12	15

The relationship between the number of hours and the number of miles walked is proportional.

4. **Is the answer reasonable? Explain.** Yes, the relationship is proportional because the graph of the linear relationship passes through the origin.

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TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 3

Read the problem. What is to be found? (whether the relationship is proportional or nonproportional) **How can this problem be solved?** (Make a graph from the problem information and use the graph to determine whether the relationship is proportional or nonproportional.)

What is the problem about? (number of ice cream cones bought and the total cost) **Do you know how many ice cream cones Sam bought?** (no) **Look at the table. What does *x* represent?** (the number of ice cream cones) **What does *y* represent?** (the total cost) **How much does one ice cream cone cost?** ($2) **If one ice cream cone costs $2, how much do 5 ice cream cones cost?** ($10) **If Sam uses his $5 off coupon, how much will he pay for the 5 ice cream cones?** ($5) **Why?** ($10 – $5 = $5) **Record this cost in the table. If one ice cream cone costs $2, how much do 6 ice cream cones cost?** ($12) **If Sam uses his $5 off coupon, how much will he pay for the 6 ice cream cones?** ($7) **Why?** ($12 – $5 = $7) **Record this cost in the table.**

If one ice cream cone costs $2, how much do 7 ice cream cones cost? ($14) **If Sam uses his $5 off coupon, how much will he pay for the 7 ice cream cones?** ($9) **Why?** ($14 – $5 = $9) **Record this cost in the table. If one ice cream cone costs $2, how much do 8 ice cream cones cost?** ($16) **If Sam uses his $5 off coupon, how much will he pay for the 8 ice cream cones?** ($11) **Why?** ($16 – $5 = $11) **Record this cost in the table.**

Next, graph the values in the table. Do the points form a straight line? (yes) **Draw the line. Is the relationship between the number of ice cream cones and total cost proportional or nonproportional?** (nonproportional) **Why is the relationship nonproportional?** (The line does not pass through the origin.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 4–7

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 7 Reminder

Students should use their graph in problem 6 to find the time, *x*, when the distance, *y*, is 0.

CHECK UP

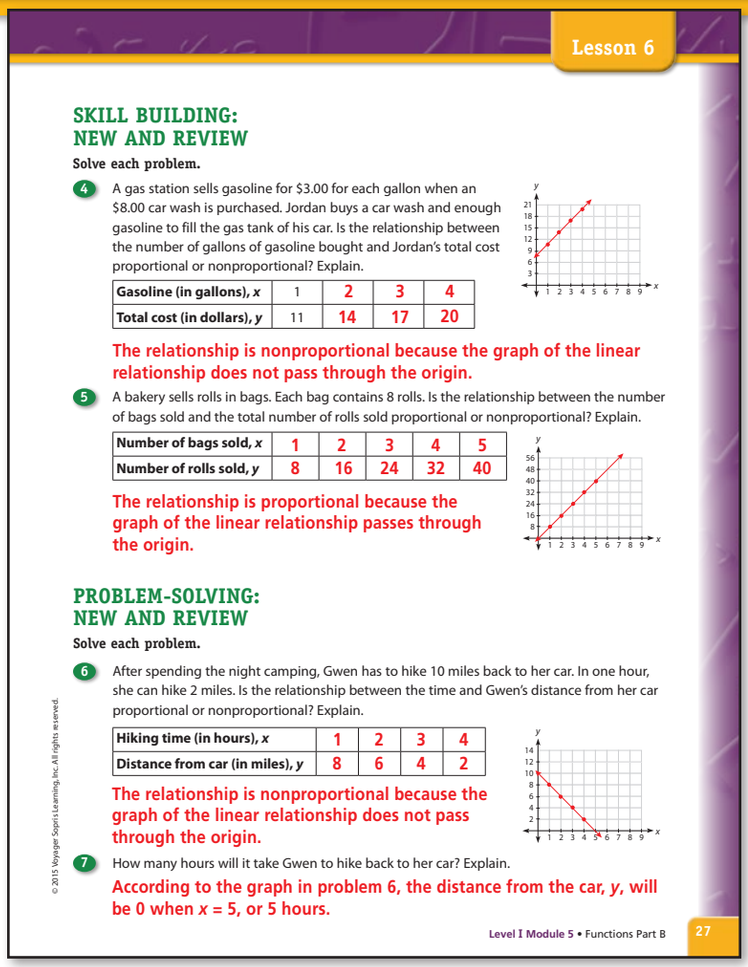
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly: The student chose an equation that could not model the problem situation. Encourage the student to think about how distance is related to time. Ask the student to think about how the distance can be obtained if the time is known.

If student answered problem 2 incorrectly: The student chose an equation that could not model the problem situation. Ask the student to consider what the number of months is when Emily opens her account and the amount in the account when she opens it.



Lesson 6

SKILL BUILDING: NEW AND REVIEW

Solve each problem.

4 A gas station sells gasoline for $3.00 for each gallon when an $8.00 car wash is purchased. Jordan buys a car wash and enough gasoline to fill the gas tank of his car. Is the relationship between the number of gallons of gasoline bought and Jordan's total cost proportional or nonproportional? Explain.

Gasoline (in gallons), *x*	1	2	3	4
Total cost (in dollars), *y*	11	14	17	20

The relationship is nonproportional because the graph of the linear relationship does not pass through the origin.

5 A bakery sells rolls in bags. Each bag contains 8 rolls. Is the relationship between the number of bags sold and the total number of rolls sold proportional or nonproportional? Explain.

Number of bags sold, *x*	1	2	3	4	5
Number of rolls sold, *y*	8	16	24	32	40

The relationship is proportional because the graph of the linear relationship passes through the origin.

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

6 After spending the night camping, Gwen has to hike 10 miles back to her car. In one hour, she can hike 2 miles. Is the relationship between the time and Gwen's distance from her car proportional or nonproportional? Explain.

Hiking time (in hours), *x*	1	2	3	4
Distance from car (in miles), *y*	8	6	4	2

The relationship is nonproportional because the graph of the linear relationship does not pass through the origin.

7 How many hours will it take Gwen to hike back to her car? Explain.

According to the graph in problem 6, the distance from the car, *y*, will be 0 when *x* = 5, or 5 hours.

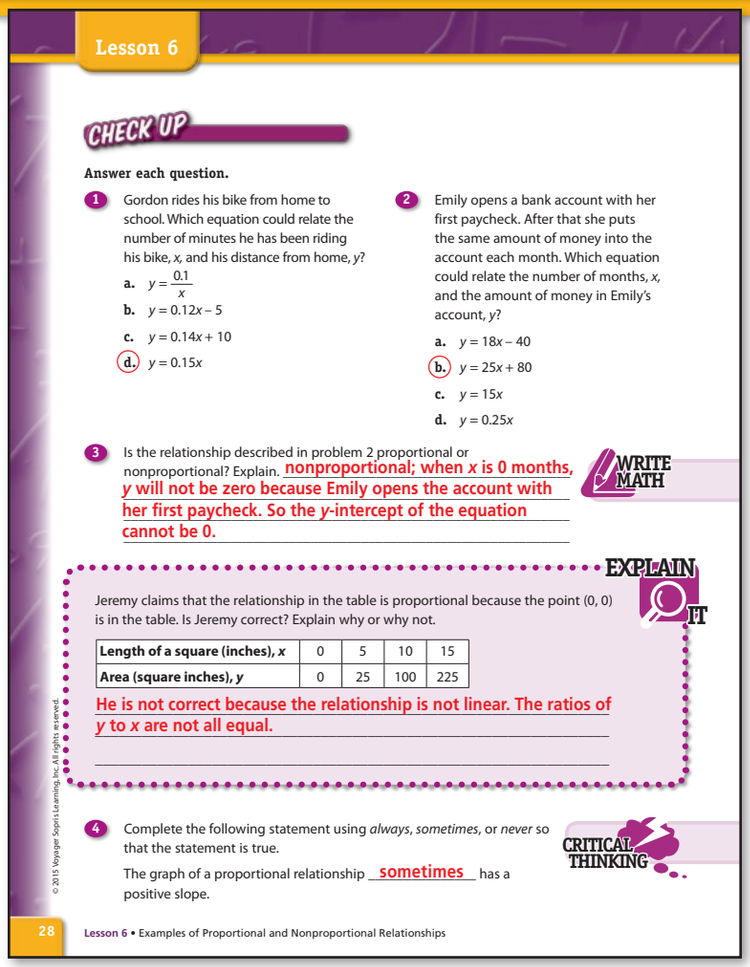
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WRITE MATH

If student answered problem 3 incorrectly: While students work together, review relationships that are proportional and nonproportional. Use Additional Resources in I.5.6 to review how to model proportional relationships and nonproportional relationships with equations.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 6

CHECK UP

Answer each question.

1. Gordon rides his bike from home to school. Which equation could relate the number of minutes he has been riding his bike, *x*, and his distance from home, *y*?
   - a. $y = \frac{0.1}{x}$
   - b. $y = 0.12x - 5$
   - c. $y = 0.14x + 10$
   - (d.) $y = 0.15x$

2. Emily opens a bank account with her first paycheck. After that she puts the same amount of money into the account each month. Which equation could relate the number of months, *x*, and the amount of money in Emily's account, *y*?
   - a. $y = 18x - 40$
   - (b.) $y = 25x + 80$
   - c. $y = 15x$
   - d. $y = 0.25x$

3. Is the relationship described in problem 2 proportional or nonproportional? Explain. nonproportional; when *x* is 0 months, *y* will not be zero because Emily opens the account with her first paycheck. So the *y*-intercept of the equation cannot be 0.

WRITE MATH

EXPLAIN IT

Jeremy claims that the relationship in the table is proportional because the point (0, 0) is in the table. Is Jeremy correct? Explain why or why not.

Length of a square (inches), *x*	0	5	10	15
Area (square inches), *y*	0	25	100	225

He is not correct because the relationship is not linear. The ratios of *y* to *x* are not all equal.

4. Complete the following statement using *always*, *sometimes*, or *never* so that the statement is true.

   The graph of a proportional relationship sometimes has a positive slope.

CRITICAL THINKING

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Technology

Using an online search engine, have students or partners research careers that would require them to describe and categorize situations as different kinds of relationships. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 5 Lesson 6

Reteach Teacher Edition Module 5 Lesson 6

Extra Practice

Student Book page 41

English Language Learners

When students are reading through the problems, remind them that the word *each* means *for every one*. For example, $3.50 for each gallon means $3.50 for every one gallon. So, if there are 2 gallons, $3.50 is added twice or multiplied by 2. If necessary, draw a diagram to help students understand this concept.

Students with Special Needs

If students are having difficulty with completing the tables, suggest that they draw a picture of the problem situation. For example, in problem 3, have them draw 5 ice cream cones and a coupon. Under each ice cream cone, they can draw a price tag with $2 and for the coupon they can write *$5 off*. From the picture, have students count the total cost by adding the costs of cones, then subtracting the value of the coupon.

MATH Flash

Lesson 7

Objective

To write a linear equation in point-slope form

Academic Vocabulary

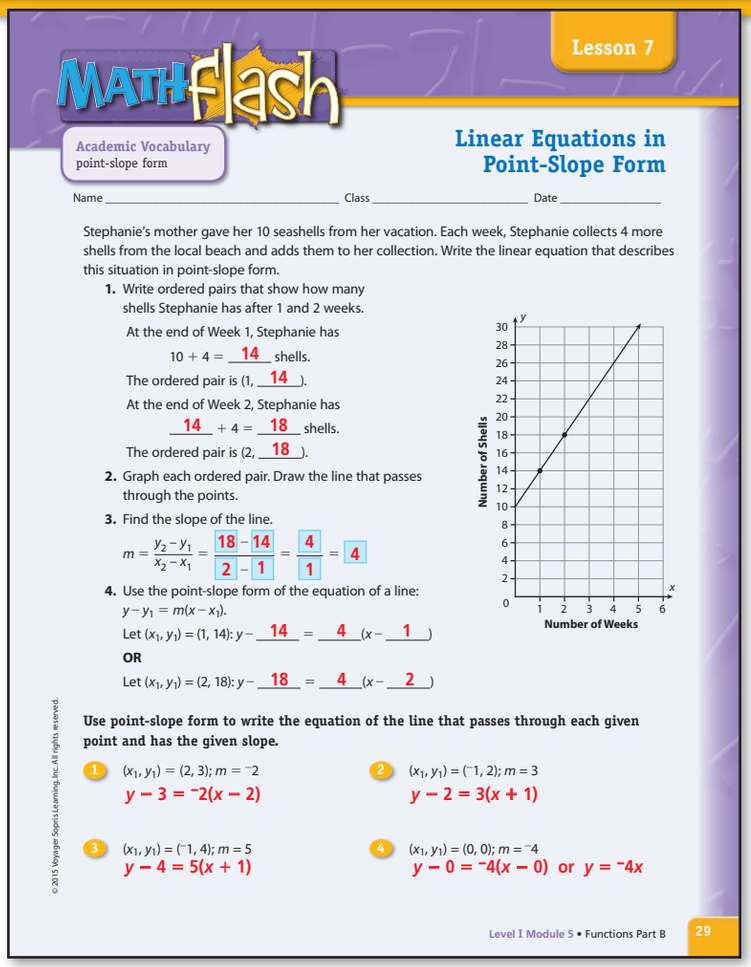
Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **point-slope form**
  the equation of a straight line in the form $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) are the coordinates of a given point on the line

Model the following skills for students.

The situation of the problem can be described by a linear equation. To write the linear equation, first find two points on the line. How many shells does Stephanie start with? (10) **How many shells does she add each week?** (4) **What numbers should be added to get the number of shells she has at the end of Week 1?** (10 and 4) **What is 10 plus 4?** (14) **So, the ordered pair (1, 14) represents this situation. How many shells will Stephanie add in Week 2?** (4) **What is 14 plus 4?** (18) **What ordered pair represents the number of shells she has in Week 2?** ((2, 18))

Next, graph the points and draw a straight line through them. This graph represents the situation. Find the slope of this line using the ordered pairs. What is (x_1, y_1)? ((1, 14)) **What is (x_2, y_2)?** ((2, 18)) **Subtract the *y*-values. What is 18 minus 14?** (4) **What is 2 minus 1?** (1) **What is the slope of the line?** (4)



MATH Flash — Lesson 7

Academic Vocabulary
point-slope form

Linear Equations in Point-Slope Form

Name _____ Class _____ Date _____

Stephanie's mother gave her 10 seashells from her vacation. Each week, Stephanie collects 4 more shells from the local beach and adds them to her collection. Write the linear equation that describes this situation in point-slope form.

1. Write ordered pairs that show how many shells Stephanie has after 1 and 2 weeks.
   At the end of Week 1, Stephanie has
   $10 + 4 =$ 14 shells.
   The ordered pair is (1, 14).
   At the end of Week 2, Stephanie has
   14 + 4 = 18 shells.
   The ordered pair is (2, 18).
2. Graph each ordered pair. Draw the line that passes through the points.
3. Find the slope of the line.
   $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18 - 14}{2 - 1} = \frac{4}{1} = 4$
4. Use the point-slope form of the equation of a line: $y - y_1 = m(x - x_1)$.
   Let $(x_1, y_1) = (1, 14)$: $y -$ 14 $=$ 4 $(x -$ 1 $)$
   OR
   Let $(x_1, y_1) = (2, 18)$: $y -$ 18 $=$ 4 $(x -$ 2 $)$

Use point-slope form to write the equation of the line that passes through each given point and has the given slope.

1. $(x_1, y_1) = (2, 3); m = -2$
   $y - 3 = -2(x - 2)$
2. $(x_1, y_1) = (-1, 2); m = 3$
   $y - 2 = 3(x + 1)$
3. $(x_1, y_1) = (-1, 4); m = 5$
   $y - 4 = 5(x + 1)$
4. $(x_1, y_1) = (0, 0); m = -4$
   $y - 0 = -4(x - 0)$ or $y = -4x$

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Level I Module 5 • Functions Part B 29

Now, write the equation of the line. The point-slope form of a line is $y - y_1 = m(x - x_1)$. Let (x_1, y_1) represent the point (1, 14). What is *m*? (4) **What is the equation in point-slope form?** ($y - 14 = 4(x - 1)$) **The other point, (2, 18), could also be used to write the equation. What is the equation in point-slope form using (2, 18)?** ($y - 18 = 4(x - 2)$) **Both of these equations represent the same line, even though they look different.**

Lesson 8

Objective

To write a linear equation in standard form

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **standard form**
  the equation of a straight line in the form $ax + by = c$, where $a \neq 0$ and $b \neq 0$

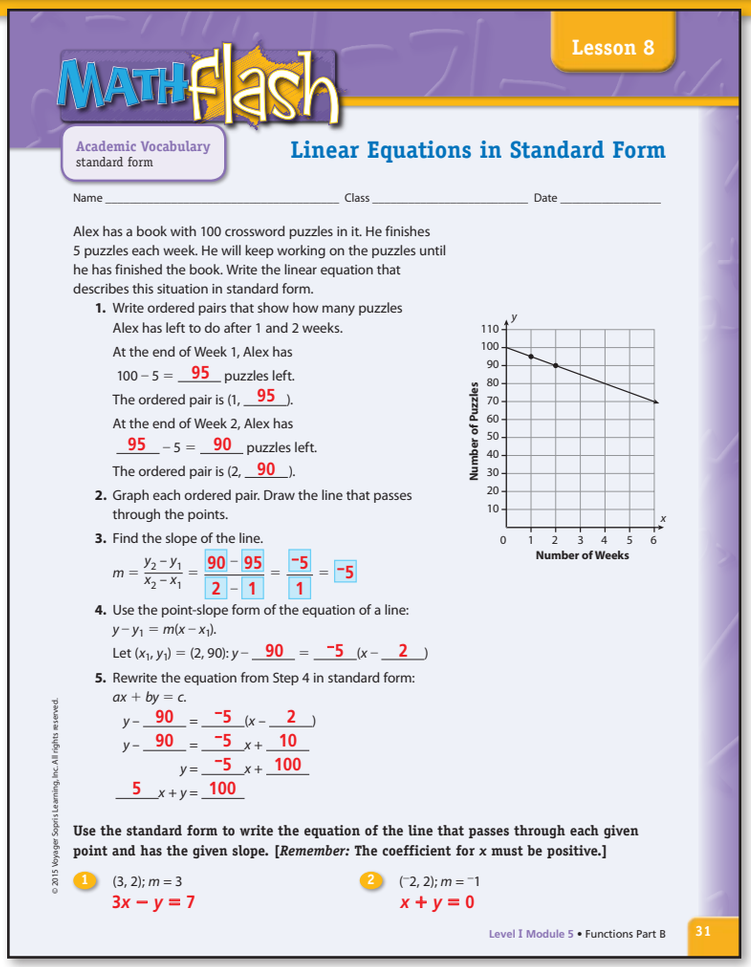
Model the following skills for students.

Read the problem. Because Alex completes the same number of puzzles each week, this situation can be described by a linear equation. To write the linear equation, first find two points on the line from the problem statement.

At the end of Week 1, how many puzzles does Alex have left to do? (95) **Why?** (He starts with 100 puzzles and finishes 5 puzzles during the week, so 95 puzzles are left at the end of Week 1.) **What ordered pair represents this?** ((1, 95)) **Find the number of puzzles left at the end of Week 2. What is 95 minus 5?** (90) **What ordered pair can be formed?** ((2, 90))

Now graph the line. Should the slope of this line be positive or negative? (negative) **Why?** (The graph falls from left to right.) **What is the slope of this line?** ($^-5$) **What is the equation in point-slope form if the point (2, 90) is used?** ($y - 90 = {}^-5(x - 2)$)

Now, write the equation from Step 4 in standard form, $ax \cdot by \cdot c$. First, multiply $^-5$ and $(x - 2)$. What is $^-5$ times x? (^-5x) **What is $^-5$ times $^-2$?** (10) **Now add 90 to each side. What is the equation?** ($y = {}^-5x + 100$) **Finally, add $5x$ to each side. Look at the coefficient for the term with x. If it is positive, it is in standard form. If it is negative, change it to positive, then change each terms sign to its opposite. Is the coefficient for $5x$ a positive or negative?** (positive) **So, no sign changing needs to be done for the standard form. What is the equation in standard form?** ($5x + y = 100$)



MATH Flash

Lesson 8

Academic Vocabulary
standard form

Linear Equations in Standard Form

Name _____ Class _____ Date _____

Alex has a book with 100 crossword puzzles in it. He finishes 5 puzzles each week. He will keep working on the puzzles until he has finished the book. Write the linear equation that describes this situation in standard form.

1. Write ordered pairs that show how many puzzles Alex has left to do after 1 and 2 weeks.
   At the end of Week 1, Alex has
   100 − 5 = 95 puzzles left.
   The ordered pair is (1, 95).
   At the end of Week 2, Alex has
   95 − 5 = 90 puzzles left.
   The ordered pair is (2, 90).

2. Graph each ordered pair. Draw the line that passes through the points.
3. Find the slope of the line.
   $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 95}{2 - 1} = \frac{-5}{1} = -5$
4. Use the point-slope form of the equation of a line:
   $y - y_1 = m(x - x_1)$.
   Let $(x_1, y_1) = (2, 90)$: $y - 90 = -5(x - 2)$
5. Rewrite the equation from Step 4 in standard form:
   $ax + by = c$.
   $y - 90 = -5(x - 2)$
   $y - 90 = -5x + 10$
   $y = -5x + 100$
   $5x + y = 100$

Use the standard form to write the equation of the line that passes through each given point and has the given slope. [*Remember:* The coefficient for x must be positive.]

1. (3, 2); $m = 3$
   $3x - y = 7$
2. ($^-2$, 2); $m = {}^-1$
   $x + y = 0$

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Technology

Have students think about math in their world, using an online search engine to find references describing situations that could be described by a linear equation. Additional digital content is available through this feature in the eBook.

Lesson 9

Objective

To find the solution of a system of linear equations by graphing

Preskills	Lesson
Graphing Linear Equations	I.4.11

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **system of linear equations**
  two or more linear equations considered together
- **solution of a system of equations**
  an ordered pair that is a solution of every equation in the system

GET STARTED

Model the following skills for students.

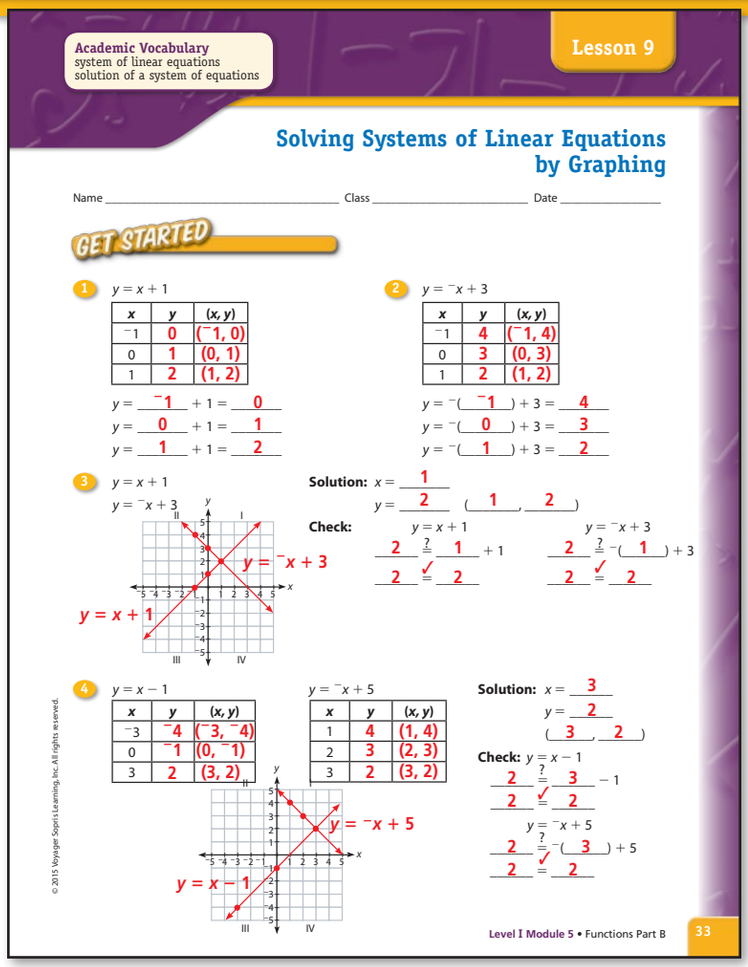
REVIEW PRESKILLS

Problem 1

To graph the equation $y = x + 1$, use an input-output table. The input-output table generates ordered pairs that can be used to graph the equation. What is the first value in the table? ($^-1$) What is the value of y when x equals $^-1$? (0) What is the corresponding ordered pair? (($^-1, 0$)) What is the value of y when x equals 0? (1) What is the corresponding ordered pair? ((0, 1)) What is the value of y when x equals 1? (2) What is the corresponding ordered pair? ((1, 2))

Problem 2

To graph the equation $y = {}^-x + 3$, use an input-output table. A negative sign in front of the variable is the same as multiplying the variable by $^-1$. What is the value of y when x equals $^-1$? (4) What is the corresponding ordered pair? (($^-1, 4$)) What is the value of y when x equals 0? (3) What is the corresponding ordered pair? ((0, 3)) What is the value of y when x equals 1? (2) What is the corresponding ordered pair? ((1, 2))



Academic Vocabulary
system of linear equations
solution of a system of equations

Lesson 9

Solving Systems of Linear Equations by Graphing

Name _____ Class _____ Date _____

GET STARTED

1 $y = x + 1$

x	y	(x, y)
$^-1$	0	($^-1$, 0)
0	1	(0, 1)
1	2	(1, 2)

$y = {}^-1 + 1 = 0$
$y = 0 + 1 = 1$
$y = 1 + 1 = 2$

2 $y = {}^-x + 3$

x	y	(x, y)
$^-1$	4	($^-1$, 4)
0	3	(0, 3)
1	2	(1, 2)

$y = {}^-(^-1) + 3 = 4$
$y = {}^-(0) + 3 = 3$
$y = {}^-(1) + 3 = 2$

3 $y = x + 1$
$y = {}^-x + 3$

Solution: $x = 1$
$y = 2$ (1, 2)

Check: $y = x + 1$: $2 \stackrel{?}{=} 1 + 1$, $2 \checkmark 2$
$y = {}^-x + 3$: $2 \stackrel{?}{=} {}^-(1) + 3$, $2 \checkmark 2$

4 $y = x - 1$

x	y	(x, y)
$^-3$	$^-4$	($^-3$, $^-4$)
0	$^-1$	(0, $^-1$)
3	2	(3, 2)

$y = {}^-x + 5$

x	y	(x, y)
1	4	(1, 4)
2	3	(2, 3)
3	2	(3, 2)

Solution: $x = 3$
$y = 2$
(3, 2)

Check: $y = x - 1$: $2 \stackrel{?}{=} 3 - 1$, $2 \checkmark 2$
$y = {}^-x + 5$: $2 \stackrel{?}{=} {}^-(3) + 5$, $2 \checkmark 2$

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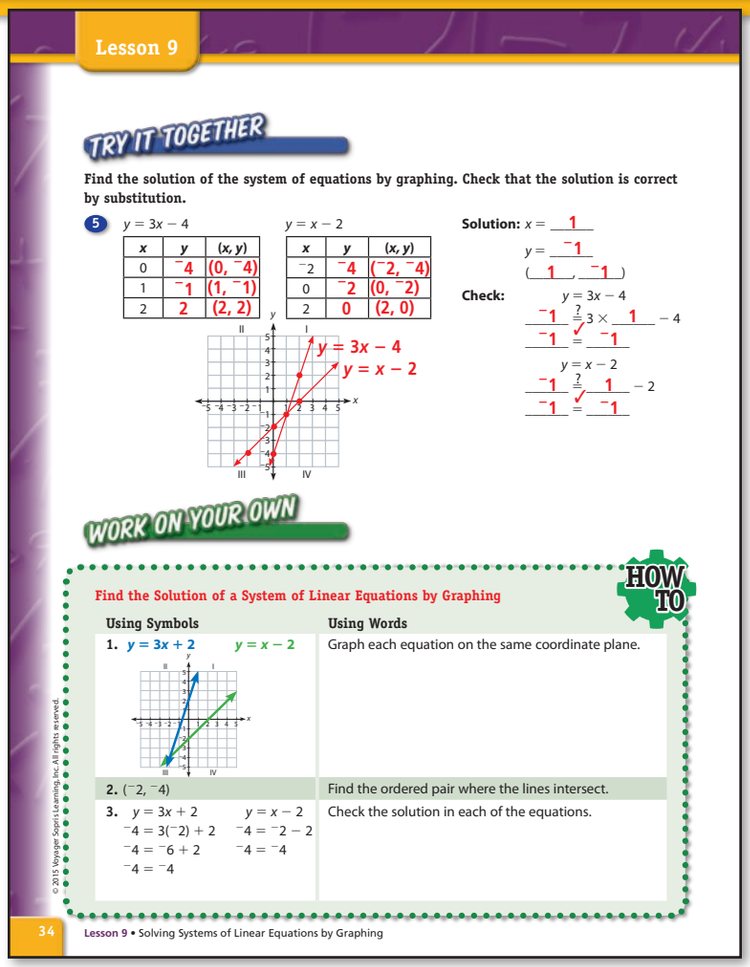
MODEL NEW SKILLS

Problem 3

A system of equations is a set of two or more equations considered together. The solution of a system of equations is an ordered pair that is a solution to each equation in the system. Systems of linear equations can have no solution, one solution, or infinite solutions. The system has infinite solutions only if the equations each describe the same line. One way to find the solution of a system of equations is to graph the equations on the same coordinate plane. The point at which the lines intersect is the solution of the system of equations.

Look at problem 3. The equations $y = x + 1$ and $y = {}^-x + 3$ make up a system of equations. To find the solution of $y = x + 1$ and $y = {}^-x + 3$, graph the equations on the same coordinate plane. Use the ordered pairs from problems 1 and 2. Label each line with its equation. At what point do the two lines intersect? ((1, 2)) The solution of the system of equations is (1, 2). That means that for both equations, when x equals 1, y equals 2.

To check that an ordered pair is the solution of a system of equations, substitute the values for x and y in each equation in the system. If both equations are true with


Lesson 9

TRY IT TOGETHER

Find the solution of the system of equations by graphing. Check that the solution is correct by substitution.

5 $y = 3x - 4$

x	y	(x, y)
0	$^-4$	(0, $^-4$)
1	$^-1$	(1, $^-1$)
2	2	(2, 2)

$y = x - 2$

x	y	(x, y)
$^-2$	$^-4$	($^-2$, $^-4$)
0	$^-2$	(0, $^-2$)
2	0	(2, 0)

Solution: $x =$ 1
$y =$ $^-1$
(1, $^-1$)

Check:
$y = 3x - 4$
$^-1 \stackrel{?}{=} 3 \times 1 - 4$
$^-1 \checkmark ^-1$

$y = x - 2$
$^-1 \stackrel{?}{=} 1 - 2$
$^-1 = ^-1$

WORK ON YOUR OWN

HOW TO

Find the Solution of a System of Linear Equations by Graphing

Using Symbols	Using Words
1. $y = 3x + 2$ $y = x - 2$	Graph each equation on the same coordinate plane.
2. ($^-2$, $^-4$)	Find the ordered pair where the lines intersect.
3. $y = 3x + 2$ $^-4 = 3(^-2) + 2$ $^-4 = ^-6 + 2$ $^-4 = ^-4$ $y = x - 2$ $^-4 = ^-2 - 2$ $^-4 = ^-4$	Check the solution in each of the equations.

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these values, the solution is correct. Check that (1, 2) is a solution of $y = x + 1$ and $y = ^-x + 3$. What is $y = x + 1$ with the values x equals 1 and y equals 2 substituted? ($2 = 1 + 1$) Does 2 equal 1 plus 1? (yes) What is $y = ^-x + 3$ with the values x equals 1 and y equals 2 substituted? ($2 = ^-1 + 3$) Does 2 equal $^-1$ plus 3? (yes) Because both equations are true with these values, the solution (1, 2) is correct.

Problem 4

The equations $y = x - 1$ and $y = ^-x + 5$ form a system of linear equations. To find the solution of the system, graph the equations on the coordinate plane. Complete the input-output tables for each equation to generate ordered pairs. Remember, a negative sign in front of the variable is the same as multiplying the variable by $^-1$. What are the ordered pairs for $y = x - 1$? (($^-3$, $^-4$), (0, $^-1$), (3, 2)) What are the ordered pairs for $y = ^-x + 5$? ((1, 4), (2, 3), (3, 2)) Graph the equations on the same coordinate plane. At which point do the lines intersect? ((3, 2))

To check that this solution is correct, substitute the values x equals 3 and y equals 2 into the equations. Does 2 equal 3 minus 1? (yes) Does 2 equal $^-3$ plus 5? (yes) Because both equations are true, the solution (3, 2) is correct.

TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 5

What equations are included in this system of equations? ($y = 3x - 4$ and $y = x - 2$) What is the first step in finding the solution of a system of equations? (Graph both equations on the same coordinate plane.) Use the input-output tables to generate ordered pairs. What are the ordered pairs for $y = 3x - 4$? ((0, $^-4$), (1, $^-1$), (2, 2)) What are the ordered pairs for $y = x - 2$? (($^-2$, $^-4$), (0, $^-2$), (2, 0)) What is the next step? (Graph the lines on the same coordinate plane.) Graph the lines. At what point do the lines intersect? ((1, $^-1$)) What is the next step? (Check the solution.)

Substitute the values for x and y into the equations and check that the equations are true. What is the first equation with the values for x and y substituted? ($^-1 = 3 \times 1 - 4$) Does $^-1$ equal 3 times 1 minus 4? (yes) What is the second equation with the values for x and y substituted? ($^-1 = 1 - 2$) Does $^-1$ equal 1 minus 2? (yes) Is the solution correct? (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 6–13

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 6–10 Reminder

Tell students to graph the lines by making input-output tables to generate ordered pairs.

CHECK UP

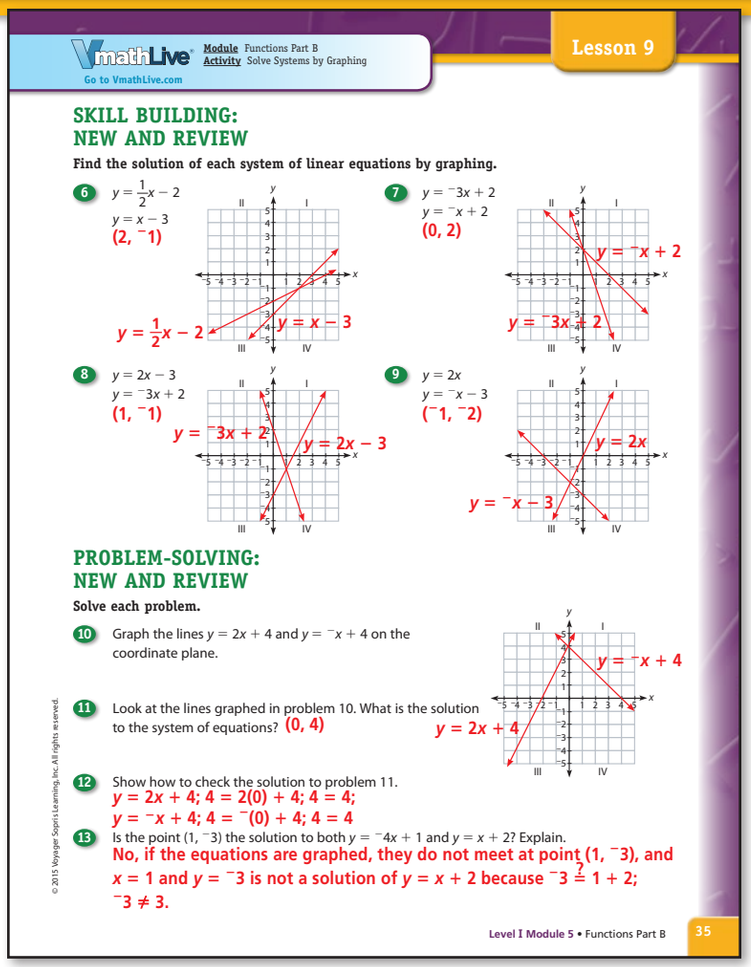
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2a: The student reversed the order of the *x*- and *y*-coordinates of the point of intersection of the lines. Remind the student to name the *x*-coordinate first and the *y*-coordinate second in an ordered pair.

If student answered 1b or 1d: The student chose the coordinates of a point where one line crosses an axis. Use Additional Resources in I.5.9 to reteach how to solve a system of equations by graphing.



VmathLive Module Functions Part B Activity Solve Systems by Graphing
Go to VmathLive.com

Lesson 9

SKILL BUILDING: NEW AND REVIEW

Find the solution of each system of linear equations by graphing.

6. $y = \frac{1}{2}x - 2$, $y = x - 3$; (2, ⁻1); $y = \frac{1}{2}x - 2$; $y = x - 3$
7. $y = ⁻3x + 2$, $y = ⁻x + 2$; (0, 2); $y = ⁻x + 2$; $y = ⁻3x + 2$
8. $y = 2x - 3$, $y = ⁻3x + 2$; (1, ⁻1); $y = ⁻3x + 2$; $y = 2x - 3$
9. $y = 2x$, $y = ⁻x - 3$; (⁻1, ⁻2); $y = 2x$; $y = ⁻x - 3$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

10. Graph the lines $y = 2x + 4$ and $y = ⁻x + 4$ on the coordinate plane. $y = ⁻x + 4$; $y = 2x + 4$
11. Look at the lines graphed in problem 10. What is the solution to the system of equations? (0, 4)
12. Show how to check the solution to problem 11. $y = 2x + 4$; $4 = 2(0) + 4$; $4 = 4$; $y = ⁻x + 4$; $4 = ⁻(0) + 4$; $4 = 4$
13. Is the point (1, ⁻3) the solution to both $y = ⁻4x + 1$ and $y = x + 2$? Explain. No, if the equations are graphed, they do not meet at point (1, ⁻3), and $x = 1$ and $y = ⁻3$ is not a solution of $y = x + 2$ because $⁻3 \stackrel{?}{=} 1 + 2$; $⁻3 \neq 3$.

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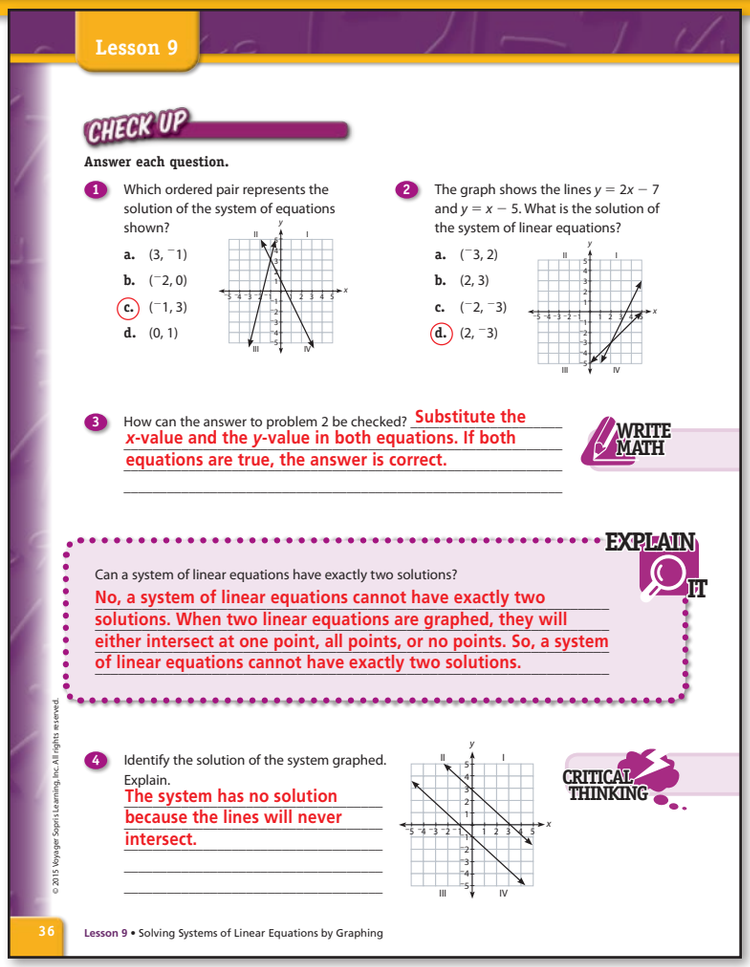
If student answered 2b: The student chose a positive value for *y* instead of a negative value. Remind the student that values of *y* below the *x*-axis are negative.

If student answered 2c: The student chose a negative value for *x* instead of a positive value. Remind the student that values of *x* to the right of the *y*-axis are positive.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review how to check the solution of a system of equations by substituting the *x*-value and *y*-value of the ordered pair in both of the original equations.



Lesson 9

CHECK UP

Answer each question.

1. Which ordered pair represents the solution of the system of equations shown?

a. $(3, -1)$
b. $(-2, 0)$
c. $(-1, 3)$
d. $(0, 1)$

2. The graph shows the lines $y = 2x - 7$ and $y = x - 5$. What is the solution of the system of linear equations?

a. $(-3, 2)$
b. $(2, 3)$
c. $(-2, -3)$
d. $(2, -3)$

3. How can the answer to problem 2 be checked? Substitute the *x*-value and the *y*-value in both equations. If both equations are true, the answer is correct.

WRITE MATH

EXPLAIN IT

Can a system of linear equations have exactly two solutions?

No, a system of linear equations cannot have exactly two solutions. When two linear equations are graphed, they will either intersect at one point, all points, or no points. So, a system of linear equations cannot have exactly two solutions.

4. Identify the solution of the system graphed. Explain.

The system has no solution because the lines will never intersect.

CRITICAL THINKING

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EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Functions Part B
Activity: Solve Systems by Graphing

Vmath Reteach
Reteach Student Book Module 5 Lesson 9
Reteach Teacher Edition Module 5 Lesson 9

Extra Practice
Student Book page 42

English Language Learners

Write the term *system of linear equations* on the board. Mention to students that this term simply means two or more linear equations in the same problem, generally with both being graphed to find the point of intersection. Mention that when lines intersect, they do so at a point. This point is the solution to the system.

Students with Special Needs

Write a system of equations on the board. Have students write each of the equations with a different colored pencil. Have them solve each linear equation by creating a table of values for each equation, completing each table with the same color pencil as the corresponding equation. Have them graph each equation using the same color pencil they used for the equation. Using a third color, have the students draw the point of intersection of the two equations. Have them substitute the solution into the original equations. Have the students use this as a model in solving other problems until they become more comfortable with the concept.

Lesson 10

Objective

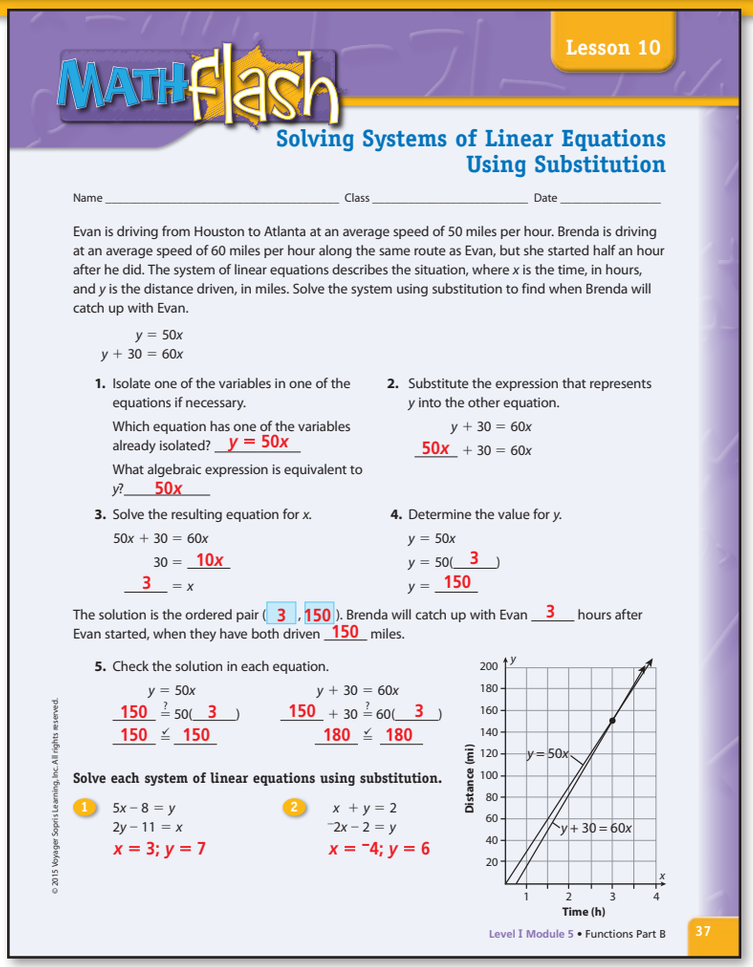
To solve a system of linear equations by using substitution

Model the following skills for students.

The solution of a system of equations is an ordered pair that is the solution of each equation in the system. One way to find the solution of a system of equations is to use substitution.

Read the problem about Evan and Brenda. Which equation has one of the variables already isolated, or by itself, on one side of the equation? ($y = 50x$) **What is *y* equal to?** ($50x$) **Substitute this expression, $50x$, for *y* in the other equation, $y + 30 = 60x$. What is the new equation?** ($50x + 30 = 60x$) **Now solve this equation. First, get the *x*'s on one side of the equation. Subtract $50x$ from each side. What is $60x$ minus $50x$?** ($10x$) **Next, divide each side by 10. What is 30 divided by 10?** (3) **What is the value of *x*?** (3) **Find the value of *y* by substituting 3 for *x* in the first equation, $y = 50x$. What is 50 times 3?** (150) **What is the value of *y*?** (150) **What ordered pair is the solution of the system?** ((3, 150))

Because *x* represents time and *y* represents the distance, Brenda will catch up with Evan 3 hours after Evan started, when they have each driven 150 miles. Check your answer by substituting the values for *x* and *y* into each equation. Are both equations true? (yes) **Look at the graph of the system of equations. What is the point of intersection?** (3, 150)



MATH Flash

Lesson 10

Solving Systems of Linear Equations Using Substitution

Name _____ Class _____ Date _____

Evan is driving from Houston to Atlanta at an average speed of 50 miles per hour. Brenda is driving at an average speed of 60 miles per hour along the same route as Evan, but she started half an hour after he did. The system of linear equations describes the situation, where *x* is the time, in hours, and *y* is the distance driven, in miles. Solve the system using substitution to find when Brenda will catch up with Evan.

$y = 50x$
$y + 30 = 60x$

1. Isolate one of the variables in one of the equations if necessary.

Which equation has one of the variables already isolated? $y = 50x$

What algebraic expression is equivalent to *y*? $50x$

2. Substitute the expression that represents *y* into the other equation.

$y + 30 = 60x$
$50x + 30 = 60x$

3. Solve the resulting equation for *x*.

$50x + 30 = 60x$
$30 = 10x$
$3 = x$

4. Determine the value for *y*.

$y = 50x$
$y = 50(3)$
$y = 150$

The solution is the ordered pair (3 , 150). Brenda will catch up with Evan 3 hours after Evan started, when they have both driven 150 miles.

5. Check the solution in each equation.

$y = 50x$
$150 \stackrel{?}{=} 50(3)$
$150 \checkmark 150$

$y + 30 = 60x$
$150 + 30 \stackrel{?}{=} 60(3)$
$180 \checkmark 180$

Solve each system of linear equations using substitution.

1. $5x - 8 = y$
   $2y - 11 = x$
   $x = 3; y = 7$

2. $x + y = 2$
   $-2x - 2 = y$
   $x = -4; y = 6$

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Extra Practice Answer Key

Extra Practice

Name _____ Class _____ Date _____

Lesson PL1 Solving Proportions

Solve each proportion.

1. $\frac{5}{6} = \frac{a}{36}$ $a = 30$
2. $\frac{1}{3} = \frac{11}{b}$ $b = 33$
3. $\frac{c}{8} = \frac{64}{16}$ $c = 32$
4. $\frac{7}{d} = \frac{1}{5}$ $d = 35$
5. $\frac{25}{40} = \frac{j}{16}$ $j = 10$
6. $\frac{f}{4} = \frac{21}{84}$ $f = 1$
7. $\frac{3}{4} = \frac{12}{x}$ $x = 16$
8. $\frac{2}{3} = \frac{k}{18}$ $k = 12$
9. $\frac{x}{12} = \frac{7}{4}$ $x = 21$
10. $\frac{8}{20} = \frac{y}{5}$ $y = 2$
11. $\frac{10}{6} = \frac{60}{p}$ $p = 36$
12. $\frac{1}{w} = \frac{8}{24}$ $w = 3$
13. $\frac{a}{50} = \frac{3}{2}$ $a = 75$
14. $\frac{20}{b} = \frac{15}{30}$ $b = 40$
15. $\frac{3}{400} = \frac{9}{c}$ $c = 1{,}200$
16. $\frac{3}{5} = \frac{x}{20}$ $x = 12$
17. $\frac{5}{6} = \frac{15}{y}$ $y = 18$
18. $\frac{z}{8} = \frac{9}{24}$ $z = 3$

Lesson PL2 Solving Proportion Word Problems

Solve each problem.

1. Sam can answer 42 questions in 14 minutes. At this rate, how long will it take Sam to answer 15 questions? $\frac{\text{questions}}{\text{minutes}}$; $\frac{42}{14} = \frac{15}{x}$; $42x = 210$; $x = 5$; 5 min
2. Sam is allowed 25 minutes to complete a 66-question test. If he can answer 42 questions in 14 minutes, will he have enough time to complete the test?
   $\frac{\text{questions}}{\text{minutes}}$; $\frac{42}{14} = \frac{66}{x}$; $42x = 924$; $x = 22$; 22 min to answer 66 questions; he will have enough time to complete the test.
3. Dave can row 2.5 miles in 15 minutes. If he rows for 45 minutes, how far will Dave have traveled? $\frac{\text{miles}}{\text{minutes}}$; $\frac{2.5}{15} = \frac{x}{45}$; $112.5 = 15x$; $7.5 = x$; 7.5 mi
4. Sally can read 24 pages in 60 minutes. How long will it take her to read a 288-page book?
   $\frac{\text{pages}}{\text{minutes}}$; $\frac{24}{60} = \frac{288}{x}$; $24x = 17{,}280$; $x = 720$; 720 min
5. Six apples cost $3.60. How much does 1 apple cost?
   $\frac{\text{apples}}{\text{cost}}$; $\frac{6}{3.60} = \frac{1}{x}$; $6x = 3.60$; $x = 0.60$; $0.60

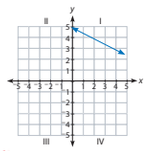
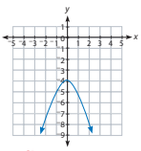
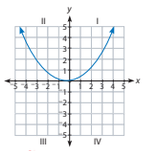
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Extra Practice

Name _____ Class _____ Date _____

Lesson 2 Identifying Linear and Nonlinear Functions

Determine whether the graph of the function is linear or nonlinear.

1. 
   linear
2. 
   nonlinear
3. 
   nonlinear

Graph each function given by the table on the coordinate plane at the right. Then determine whether the function given by the table is linear or nonlinear.

4.

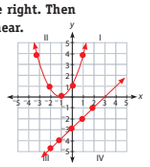
x	y	(x, y)
−2	−5	(−2, −5)
−1	−4	(−1, −4)
0	−3	(0, −3)
1	−2	(1, −2)
2	−1	(2, −1)

linear

5.

x	y	(x, y)
−3	4	(−3, 4)
−2	1	(−2, 1)
−1	0	(−1, 0)
0	1	(0, 1)
1	4	(1, 4)

nonlinear

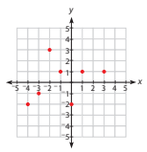


Lesson 4 Data Sets

Use a graph to determine whether there is a relationship between the data. If so, determine whether the relationship is linear.

1.

x	y
−4	−2
−3	−1
−2	3
−1	1
0	−2
1	1
3	1

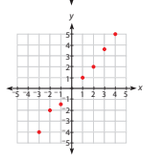


Is there a relationship between the data? no

Is the relationship linear? no

2.

x	y
−3	−4
−2	−2
−1	1.5
1	1
2	2
3	3.5
4	5



Is there a relationship between the data? yes

Is the relationship linear? yes

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Extra Practice

Name _____ Class _____ Date _____

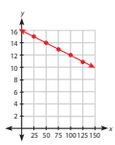
Lesson 5 Proportional and Nonproportional Relationships

Create a graph and write an equation for each data set. Then determine whether the relationship is proportional or nonproportional.

1.

Miles driven, x	25	50	75	100	125
Gasoline left (gallons), y	15	14	13	12	11

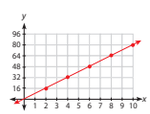
$y = \frac{-1}{25}x + 16$; nonproportional



2.

Number of cars washed, x	2	4	6	8	10
Earned (dollars), y	16	32	48	64	80

$y = 8x$; proportional



Lesson 6 Examples of Proportional and Nonproportional Relationships

Solve each problem.

1. One mile is the same as about 1.6 kilometers. Is the relationship between the number of miles and the number of kilometers proportional or nonproportional? Explain.

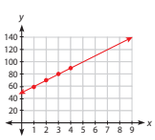
Number of miles, x	1	2	3	4	5
Number of kilometers, y	1.6	3.2	4.8	6.4	8



The relationship is proportional because the graph of the linear relationship passes through the origin.

2. Maya bought a cell phone for $50. She pays $10 per month to make calls. Is the relationship between the number of months Maya uses the phone and the total cost of the cell phone proportional or nonproportional? Explain.

Number of months, x	1	2	3	4
Total cost (in dollars), y	60	70	80	90



The relationship is nonproportional because the graph of the linear relationship does not pass through the origin.

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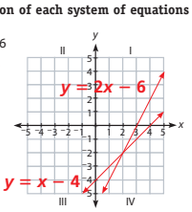
Extra Practice

Name _____ Class _____ Date _____

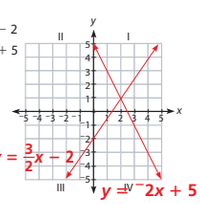
Lesson 9 Solving Systems of Linear Equations by Graphing

Find the solution of each system of equations by graphing.

1. $y = 2x - 6$
   $y = x - 4$
   (2, −2)



2. $y = \frac{3}{2}x - 2$
   $y = -2x + 5$
   (2, 1)



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Glossary

linear function
a function whose graph is a line

nonlinear function
a function whose graph is not a line

point-slope form
the equation of a straight line in the form $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) are the coordinates of a given point on the line

proportion
an equation that shows two ratios are equivalent

proportional
a linear relationship that can be represented in the form $y = mx$

slope-intercept form
the equation of a straight line in the form $y = mx + b$, where m is the slope of the line and b is its y-intercept

solution of a system of equations
an ordered pair that is a solution of every equation in the system

standard form
the equation of a straight line in the form $ax + by = c$, where $a \neq 0$ and $b \neq 0$

system of linear equations
two or more linear equations considered together

Transforming Geometry

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Lesson PL1

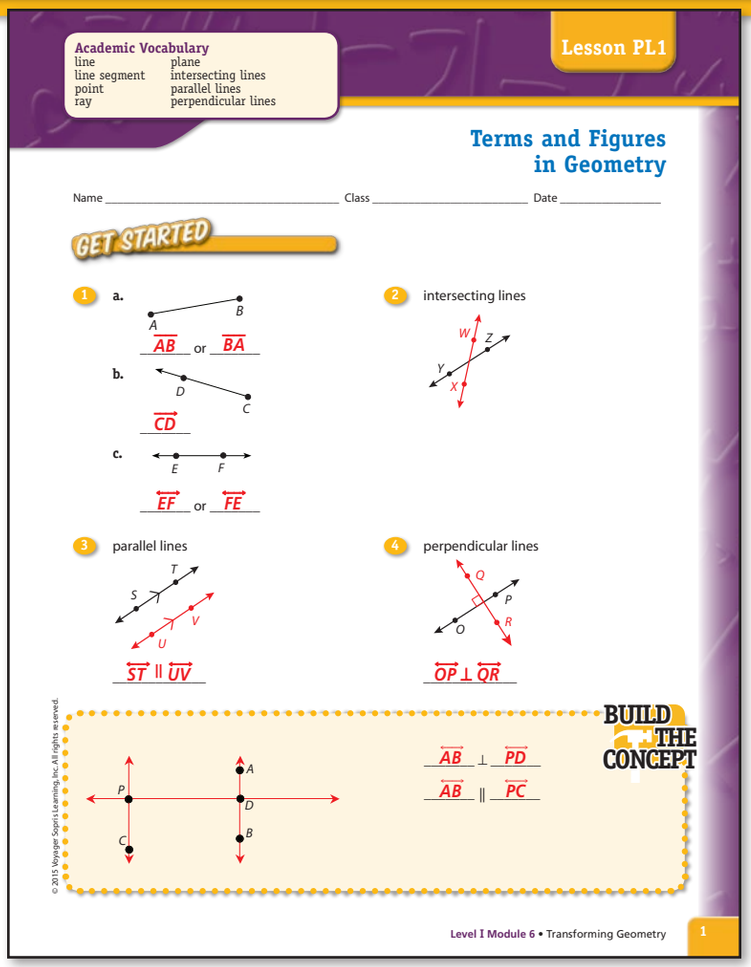
Objective

To use the correct symbols to identify and name points, lines, line segments, rays, planes, and the relationship between intersecting, parallel, and perpendicular lines

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **line**
  a straight path with no endpoints
- **line segment**
  part of a line with two endpoints
- **point**
  a location in space that has no size
- **ray**
  part of a line with one endpoint
- **plane**
  a flat surface with no boundaries
- **intersecting lines**
  lines that cross at one point
- **parallel lines**
  lines that are in the same plane and never intersect; the symbol ∥ means "is parallel to"
- **perpendicular lines**
  lines that intersect to form 90° angles; the symbol ⊥ means "is perpendicular to"



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To name line segments, rays, and lines, use two points and the correct symbol. Points are designated with capital letters. What kind of figure is shown in problem 1a? (line segment) **To name a line segment, use the endpoints of the line segment and the line segment symbol. One endpoint is *A*. The other endpoint is *B*. The endpoints can be listed in either order. The name of the line segment is segment *AB* or segment *BA*. How would the name of the line segment be written using the correct symbol?** ($\overline{AB}$ or $\overline{BA}$)

What kind of figure is shown in problem 1b? (ray) **To name a ray, use the endpoint, any other point on the ray, and the ray symbol. What is the endpoint?** (*C*) **What is another point on the ray?** (*D*) **When naming a ray, always list the endpoint first. The name of the ray is ray *CD*. How would the name of the ray be written using the correct symbol?** ($\overrightarrow{CD}$)

What kind of figure is shown in problem 1c? (line) **To name a line, use any two points on the line and the line symbol. What is one point?** (*E*) **What is another point?** (*F*) **The points can be listed in either order. The name of the line is line *EF* or line *FE*. How would the name of the line be written using the correct symbol?** ($\overleftrightarrow{EF}$ or $\overleftrightarrow{FE}$)

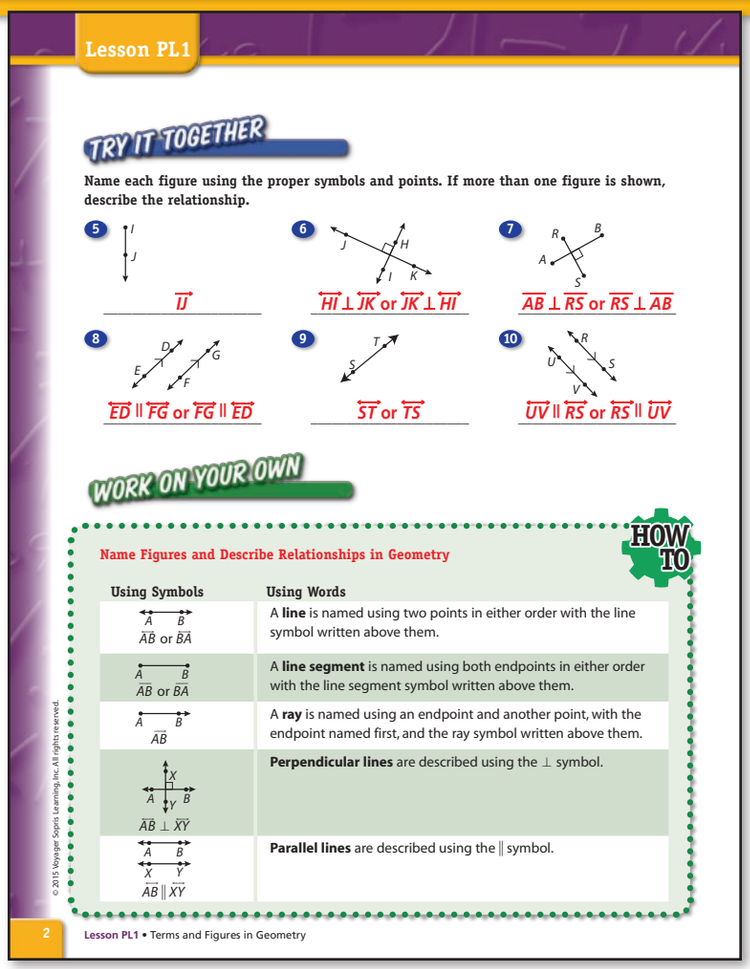
Problem 2

Lines that intersect, or cross, at a point are intersecting lines. What line is shown? ($\overleftrightarrow{YZ}$ or $\overleftrightarrow{ZY}$) **Draw a line *XW* that intersects $\overleftrightarrow{YZ}$. Label points *X* and *W* on the line drawn.**

MODEL NEW SKILLS

Problem 3

A plane is a flat surface without boundaries. Lines in the same plane that never intersect are parallel lines. What line is shown? ($\overleftrightarrow{ST}$ or $\overleftrightarrow{TS}$) **Draw $\overleftrightarrow{UV}$ parallel to $\overleftrightarrow{ST}$. Label points *U* and *V* on the line drawn. Notice the black arrow in the middle of $\overleftrightarrow{ST}$. Put a small arrow on $\overleftrightarrow{UV}$ pointing in the same**


Lesson PL1

TRY IT TOGETHER

Name each figure using the proper symbols and points. If more than one figure is shown, describe the relationship.

5 $\overrightarrow{IJ}$

6 $\overleftrightarrow{HI} \perp \overleftrightarrow{JK}$ or $\overleftrightarrow{JK} \perp \overleftrightarrow{HI}$

7 $\overline{AB} \perp \overline{RS}$ or $\overline{RS} \perp \overline{AB}$

8 $\overline{ED} \parallel \overline{FG}$ or $\overline{FG} \parallel \overline{ED}$

9 $\overline{ST}$ or $\overline{TS}$

10 $\overline{UV} \parallel \overline{RS}$ or $\overline{RS} \parallel \overline{UV}$

WORK ON YOUR OWN

HOW TO

Name Figures and Describe Relationships in Geometry

Using Symbols	Using Words
$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$	A **line** is named using two points in either order with the line symbol written above them.
$\overline{AB}$ or $\overline{BA}$	A **line segment** is named using both endpoints in either order with the line segment symbol written above them.
$\overrightarrow{AB}$	A **ray** is named using an endpoint and another point, with the endpoint named first, and the ray symbol written above them.
$\overleftrightarrow{AB} \perp \overleftrightarrow{XY}$	**Perpendicular lines** are described using the $\perp$ symbol.
$\overleftrightarrow{AB} \parallel \overleftrightarrow{XY}$	**Parallel lines** are described using the $\parallel$ symbol.

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2 Lesson PL1 • Terms and Figures in Geometry

direction as the arrow on $\overleftrightarrow{ST}$. This is how parallel lines are indicated in a figure.

The parallel line symbol is a set of two small vertical lines written close to each other. When this symbol is written between the names of two lines, it means that the lines are parallel to each other. To write $\overleftrightarrow{ST}$ *is parallel to* $\overleftrightarrow{UV}$, name the lines and write the parallel line symbol between them. How would this be written in problem 3? ($\overleftrightarrow{ST} \parallel \overleftrightarrow{UV}$) It does not matter which line comes before the symbol and which comes after the symbol. It also is correct to say $\overleftrightarrow{UV} \parallel \overleftrightarrow{ST}$.

Problem 4

Intersecting lines that intersect to form 90° angles, or right angles, are perpendicular lines. What line is shown? ($\overleftrightarrow{OP}$ or $\overleftrightarrow{PO}$) Draw $\overleftrightarrow{QR}$ perpendicular to $\overleftrightarrow{OP}$. Label points *Q* and *R* on the line drawn. Draw a small square at the point of intersection to indicate that the angle is a 90° angle; this is the right angle symbol.

The perpendicular lines symbol is a small upside down capital *T*. When this symbol is written between the names of two lines, it means that the lines are perpendicular. To write $\overleftrightarrow{OP}$ *is perpendicular to* $\overleftrightarrow{QR}$, name the lines and write the perpendicular lines symbol between them. How would this be written in problem 4? ($\overleftrightarrow{OP} \perp \overleftrightarrow{QR}$) It does not matter which line comes before the symbol and which comes after the symbol. It also is correct to say $\overleftrightarrow{QR} \perp \overleftrightarrow{OP}$.

BUILD THE CONCEPT

Model how to understand parallel and perpendicular lines.

Use a ruler or straightedge to draw a line that passes through points *A*, *D*, and *B*. What is one way to name this line? (Possible answer: $\overleftrightarrow{AB}$) Use a ruler or straightedge to draw a line that passes through points *P* and *D*. What is one way to name this line? (Possible answer: $\overleftrightarrow{PD}$) Do the lines cross? (yes) These lines are intersecting lines. Do the angles formed by the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{PD}$ appear to be right angles? (yes) Then $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{PD}$. Use a ruler or straightedge to draw a line that passes through points *P* and *C*. What is the name of this line? (Possible answer: $\overleftrightarrow{PC}$) What is the relationship between $\overleftrightarrow{AB}$ and $\overleftrightarrow{PC}$? (The lines are parallel.) Why? (The lines are in the same plane and do not intersect.)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What is this figure? (a ray) Why is it a ray? (It is part of a line, it has one endpoint, and it extends without end in one direction.) How is a ray named? (Use the endpoint, any other point on the ray, and the ray symbol.) Which point is listed first? (the endpoint) What is the name of the ray? ($\overrightarrow{IJ}$)

Problem 6

What figures are shown? ($\overleftrightarrow{HI}$ and $\overleftrightarrow{JK}$) What kind of lines are they? (perpendicular lines) What indicates that the lines are perpendicular? (the right angle symbol) Using symbols, how would $\overleftrightarrow{HI}$ *is perpendicular to* $\overleftrightarrow{JK}$ be written? ($\overleftrightarrow{HI} \perp \overleftrightarrow{JK}$ or $\overleftrightarrow{JK} \perp \overleftrightarrow{HI}$)

Problem 7

What figures are shown? ($\overline{AB}$ and $\overline{RS}$) What does the right angle symbol indicate? (The line segments are perpendicular.) Using symbols, how would $\overline{AB}$ *is perpendicular to* $\overline{RS}$ be written? ($\overline{AB} \perp \overline{RS}$ or $\overline{RS} \perp \overline{AB}$)

Problem 8

What figures are shown? ($\overleftrightarrow{ED}$ and $\overleftrightarrow{FG}$) What kind of lines are they? (parallel lines) What indicates that the lines are parallel? (the small arrows on each line pointing

in the same direction) **Using symbols, how would $\overleftrightarrow{ED}$ is parallel to $\overleftrightarrow{FG}$ be written?** ($\overleftrightarrow{ED} \parallel \overleftrightarrow{FG}$ or $\overleftrightarrow{FG} \parallel \overleftrightarrow{ED}$)

Problem 9

What is this figure? (a line) **This line can be named in two different ways. What are the two names of this line?** ($\overleftrightarrow{ST}$ and $\overleftrightarrow{TS}$)

Problem 10

What kind of lines are these? (parallel lines) **What tells that the lines are parallel?** (the small arrows on each line pointing in the same direction) **Using symbols, how would $\overleftrightarrow{UV}$ is parallel to $\overleftrightarrow{RS}$ be written?** ($\overleftrightarrow{UV} \parallel \overleftrightarrow{RS}$ or $\overleftrightarrow{RS} \parallel \overleftrightarrow{UV}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 11–20

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 11 and 15 Reminder

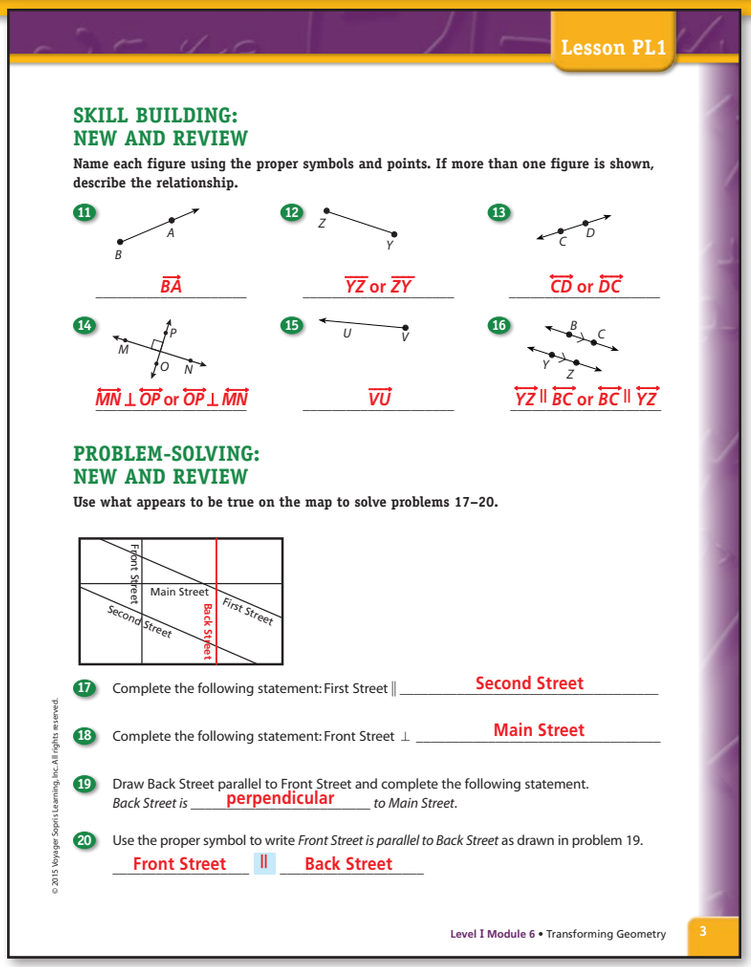
To name a ray, always list the endpoint first.

Problems 14 and 16 Reminder

Students can write $\overleftrightarrow{NM}$ for $\overleftrightarrow{MN}$, $\overleftrightarrow{PO}$ for $\overleftrightarrow{OP}$, $\overleftrightarrow{ZY}$ for $\overleftrightarrow{ZY}$, or $\overleftrightarrow{CB}$ for $\overleftrightarrow{BC}$.

Problem 19 Reminder

Make sure students draw the line parallel to Front Street correctly before answering the question. Review *parallel* and *perpendicular* with students.



Lesson PL1

SKILL BUILDING: NEW AND REVIEW

Name each figure using the proper symbols and points. If more than one figure is shown, describe the relationship.

11.

$\overleftrightarrow{BA}$

12.

$\overleftrightarrow{YZ}$ or $\overleftrightarrow{ZY}$

13.

$\overleftrightarrow{CD}$ or $\overleftrightarrow{DC}$

14.

$\overleftrightarrow{MN} \perp \overleftrightarrow{OP}$ or $\overleftrightarrow{OP} \perp \overleftrightarrow{MN}$

15.

$\overleftrightarrow{VU}$

16.

$\overleftrightarrow{YZ} \parallel \overleftrightarrow{BC}$ or $\overleftrightarrow{BC} \parallel \overleftrightarrow{YZ}$

PROBLEM-SOLVING: NEW AND REVIEW

Use what appears to be true on the map to solve problems 17–20.

17. Complete the following statement: First Street $\parallel$ Second Street

18. Complete the following statement: Front Street $\perp$ Main Street

19. Draw Back Street parallel to Front Street and complete the following statement. *Back Street is* perpendicular *to Main Street.*

20. Use the proper symbol to write *Front Street is parallel to Back Street* as drawn in problem 19. Front Street $\parallel$ Back Street

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Level I Module 6 • Transforming Geometry 3

CHECK UP

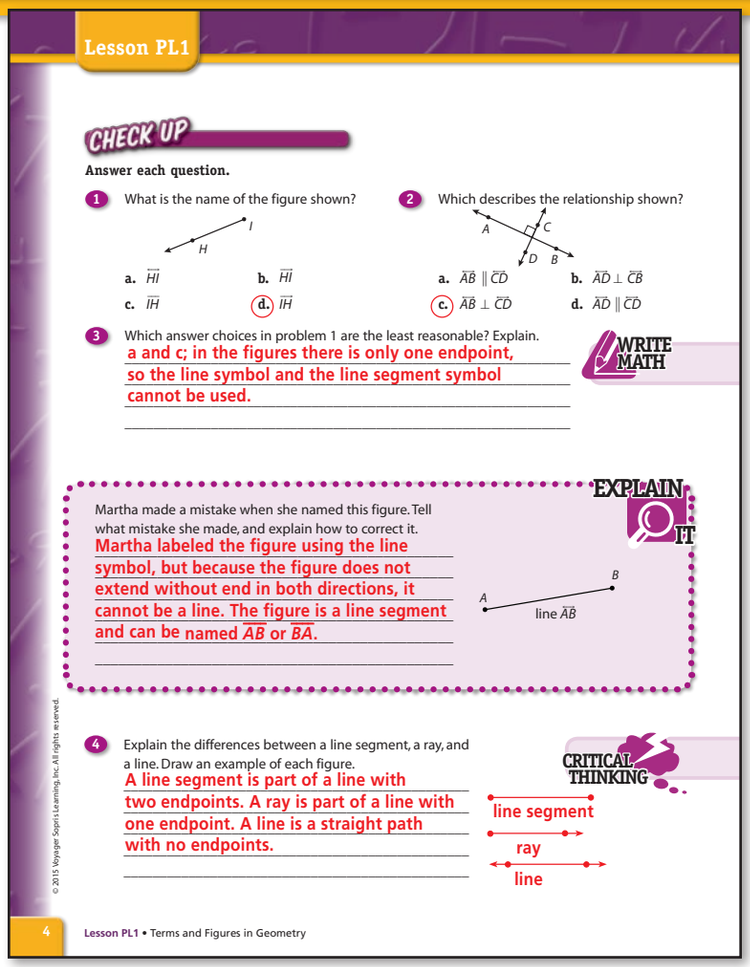
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1c: The student did not use the correct symbol for the figure. Use Additional Resources in I.6.PL1 to reteach how to use symbols to name lines, line segments, or rays.

If student answered 1b: The student did not write the endpoint of the ray first. Remind the student that when naming rays, the endpoint is always written first.



Lesson PL1

CHECK UP

Answer each question.

1 What is the name of the figure shown?

a. $\overleftrightarrow{HI}$ b. $\overrightarrow{HI}$
c. $\overline{IH}$ (d.) $\overrightarrow{IH}$

2 Which describes the relationship shown?

a. $\overline{AB} \parallel \overline{CD}$ b. $\overline{AD} \perp \overline{CB}$
(c.) $\overline{AB} \perp \overline{CD}$ d. $\overline{AD} \parallel \overline{CD}$

3 Which answer choices in problem 1 are the least reasonable? Explain.

WRITE MATH

a and c; in the figures there is only one endpoint, so the line symbol and the line segment symbol cannot be used.

EXPLAIN IT

Martha made a mistake when she named this figure. Tell what mistake she made, and explain how to correct it.

Martha labeled the figure using the line symbol, but because the figure does not extend without end in both directions, it cannot be a line. The figure is a line segment and can be named $\overline{AB}$ or $\overline{BA}$.

4 Explain the differences between a line segment, a ray, and a line. Draw an example of each figure.

CRITICAL THINKING

A line segment is part of a line with two endpoints. A ray is part of a line with one endpoint. A line is a straight path with no endpoints.

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4 Lesson PL1 • Terms and Figures in Geometry

If student answered 2a or 2d: The student used the incorrect symbol for *is perpendicular to*. Use Additional Resources in I.6.PL1 to reteach how to use symbols to name perpendicular lines.

If student answered 2b: The student chose an answer that has lines that are not labeled in the figure. Use Additional Resources in I.6.PL1 to reteach how to name perpendicular lines.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student of how to name a ray using the endpoint and another point on the line.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 6 Lesson PL1

Reteach Teacher Edition Module 6 Lesson PL1

Extra Practice

Student Book page 49

English Language Learners

Use the VmathLive Animated Glossary to review the terms *intersecting lines*, *line segment*, *parallel lines*, *perpendicular lines*, and *ray*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Because there are so many new terms to learn in the lesson, have students create charts like the one in the **HOW TO** box to help organize the definitions, symbols, and interpretations of all the geometric terms covered in the lesson.

Students with Special Needs

It may be difficult for some students to find specific geometric figures when they are embedded within many other figures. Encourage students to use paper or an index card to cover parts of figures to help them focus.

Lesson PL2

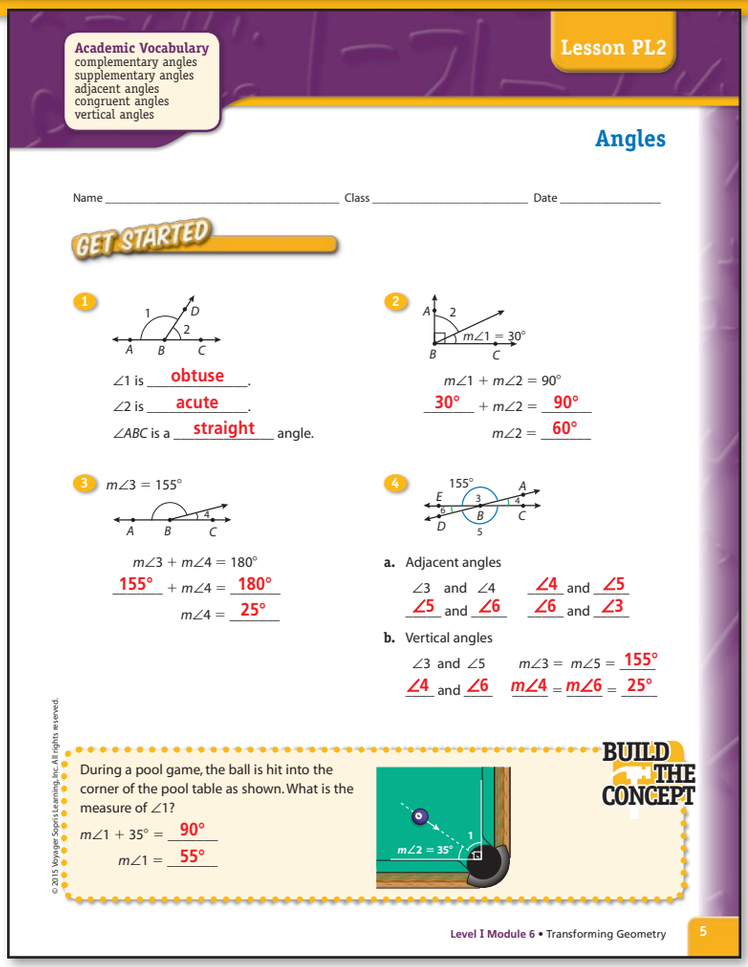
Objective

To identify, name, and classify an angle

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **complementary angles**
  two angles whose measures have a sum of 90°
- **supplementary angles**
  two angles whose measures have a sum of 180°
- **adjacent angles**
  two angles that share a vertex and a common side and do not overlap
- **congruent angles**
  two angles with the same measure
- **vertical angles**
  the opposite congruent angles formed by two intersecting lines


Academic Vocabulary
complementary angles
supplementary angles
adjacent angles
congruent angles
vertical angles

Lesson PL2

Angles

Name _____ Class _____ Date _____

GET STARTED

1

∠1 is obtuse.
∠2 is acute.
∠ABC is a straight angle.

2

$m\angle 1 + m\angle 2 = 90°$
$30° + m\angle 2 = 90°$
$m\angle 2 = 60°$

3 $m\angle 3 = 155°$

$m\angle 3 + m\angle 4 = 180°$
$155° + m\angle 4 = 180°$
$m\angle 4 = 25°$

4

a. Adjacent angles
∠3 and ∠4 ∠4 and ∠5
∠5 and ∠6 ∠6 and ∠3

b. Vertical angles
∠3 and ∠5 $m\angle 3 = m\angle 5 = 155°$
∠4 and ∠6 $m\angle 4 = m\angle 6 = 25°$

BUILD THE CONCEPT

During a pool game, the ball is hit into the corner of the pool table as shown. What is the measure of ∠1?

$m\angle 1 + 35° = 90°$
$m\angle 1 = 55°$

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Level I Module 6 • Transforming Geometry 5

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

In problem 1, three different angles have *B* as a vertex: ∠*ABD*, ∠*DBC*, and ∠*ABC*. To avoid confusion without always having to name angles using three points, arcs and numbers are used. The arc labeled 1 is ∠*ABD*. The arc labeled 2 is ∠*DBC*. An obtuse angle is an angle whose measure is greater than 90° but less than 180°. Look at ∠1. Is ∠1 an obtuse angle? (yes) An angle whose measure is less than 90° is an acute angle. Is ∠2 an acute angle? (yes) A straight angle is an angle that measures 180°. Angle *ABC* is a straight angle.

MODEL NEW SKILLS

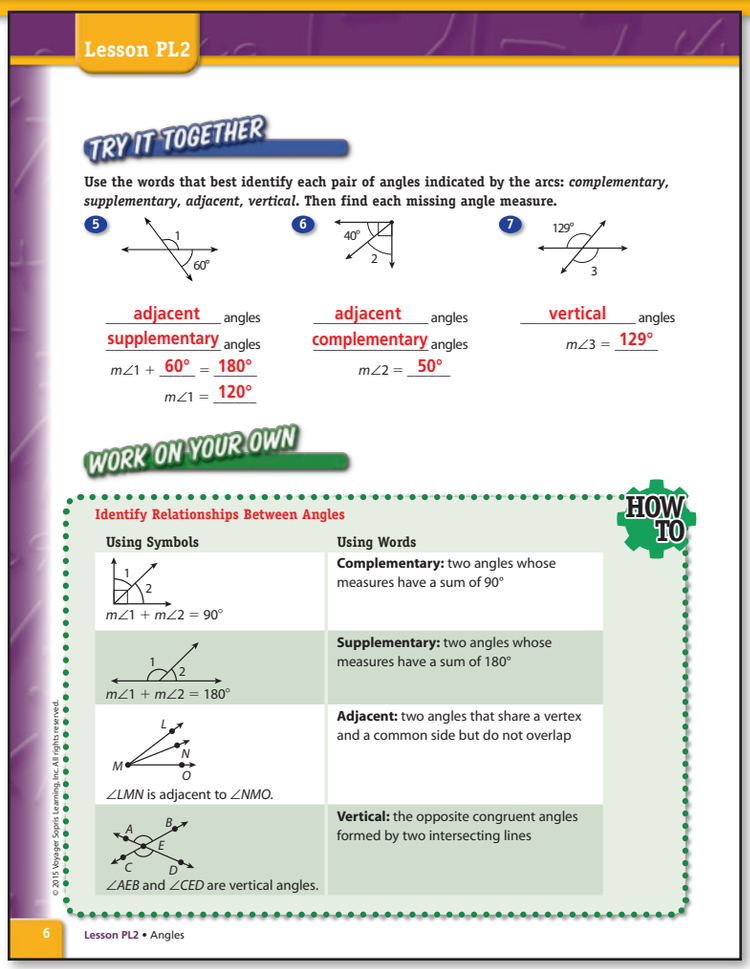
Problem 2

Angle *ABC* is a right angle made up of two angles, ∠1 and ∠2. The lowercase *m* used in the diagram means *the measure of*. The measure of ∠1 is given as 30°. The measure of ∠2 is not given, but together ∠1 and ∠2 form a right angle. This means that the sum of the measures of ∠1 and ∠2 equals 90°. Two angles whose measures have a sum of 90° are complementary angles. Because the sum of the measures of complementary angles is 90°, the missing angle measure can be found using the equation $m\angle 1 + m\angle 2 = 90°$. Substitute 30° for $m\angle 1$ and solve the equation for $m\angle 2$. What number should be subtracted from both sides of the equation? (30°) What is 90° minus 30°? (60°) The measure of ∠2 is 60°.

Problem 3

Angle *ABC* is a straight angle made up of two angles, ∠3 and ∠4. The measure of ∠3 is given as 155°. The measure of ∠4 is not given, but together ∠3 and ∠4 make a straight angle. This means that the sum of the measures of ∠3 and ∠4 equals 180°. Two angles whose measures have a sum of 180° are supplementary angles.

Because the sum of the measures of supplementary angles is 180°, the missing angle measure can be found using the equation $m\angle 3 + m\angle 4 = 180°$. Substitute 155° for $m\angle 3$ and solve the equation for $m\angle 4$. What number should be subtracted from both sides of the equation? (155°) What is 180° minus 155°? (25°) The measure of ∠4 is 25°.



Problem 4

The intersecting lines in this figure create four angles. What are the numbers of the angles? (∠3, ∠4, ∠5, ∠6) **Adjacent angles are two angles that have the same vertex, share a common side, and do not overlap. Angle 3 and ∠4 are adjacent angles. What is the common vertex of ∠3 and ∠4?** (*B*) **What is the common side of ∠3 and ∠4?** ($\overrightarrow{BA}$) **Other pairs of adjacent angles in this figure are ∠4 and ∠5, ∠5 and ∠6, and ∠6 and ∠3.**

Angles that are opposite each other when two lines intersect are vertical angles. Vertical angles are congruent. Angle 3 and ∠5 are vertical, congruent angles, so *m*∠3 equals *m*∠5. The measure of ∠3 is 155° here. What is *m*∠5? (155°) **What other angles are vertical angles?** (∠4 and ∠6) **So, *m*∠4 equals *m*∠6. Use the angle measurement found for the other known angles to help calculate *m*∠4. What is *m*∠4?** (25°) **Why?** ($m\angle 4 = 180° - 155° = 25°$) **What is *m*∠6?** (25°)

BUILD THE CONCEPT

Model how to find the measure of an angle.

During a pool game, the ball is hit into the corner of the pool table as shown. The path of the ball forms ∠1 and ∠2 with the sides of the table. What kind of angle is formed by the sides of the table? (a right angle) **What is the measure of a right angle?** (90°) **What is the sum of the measures of ∠1 and ∠2?** (90°) **What is the relationship between ∠1 and ∠2?** (The angles are complementary.) **What is the measure of ∠2?** (35°) **Find the measure of ∠1 by using the equation $m\angle 1 + 35° = 90°$. What is 90° minus 35°?** (55°) **So the measure of ∠1 is 55°.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

Do ∠1 and the angle marked 60° share a vertex and a common side? (yes) **Do the angles overlap?** (no) **What kind of angles are they?** (adjacent) **Together, what kind of angle do these angles make?** (straight angle) **Are the two angles complementary or supplementary?** (supplementary)

What angle measure is given? (60°) **What equation should be used to find the missing angle measure?** ($m\angle 1 + 60° = 180°$) **Next, solve the equation. What is 180° minus 60°?** (120°) **What is the measure of ∠1?** (120°)

Problem 6

What do these angles share? (a vertex and a side) **What kind of angles are they?** (adjacent) **Together, what kind of angle do these angles make?** (right angle) **Are the two angles complementary or supplementary?** (complementary)

What equation should be used to find the missing angle measure? ($m\angle 2 + 40° = 90°$) **What is the measure of ∠2?** (50°)

Problem 7

Which word or words best identify the angles: complementary, supplementary, adjacent, or vertical? (vertical) **Why?** (They are opposite angles formed by two intersecting lines.)

What is the measure of the missing angle? (129°) **Why?** (Vertical angles are congruent.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using Reasonableness** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 17.

The problem shows a figure with two intersecting lines. The measure of ∠4 is given as 110°. Students are asked to find the measures of ∠1, ∠2, and ∠3.

First, have students look at the figure. They should think about what they know about angles. Vertical angles are congruent, so they have the same measure. Supplementary angles have a total measure of 180°. Students should use these facts to find the measures of the angles.

Angle 4 and ∠2 are vertical angles. So, $m\angle 2$ equals $m\angle 4$, which equals 110°.

Angle 4 and ∠1 are supplementary, so $m\angle 4$ plus $m\angle 1$ equals 180°. The measure of ∠1 equals 70°.

Angle 1 and ∠3 are vertical angles, so $m\angle 3$ equals 70°.

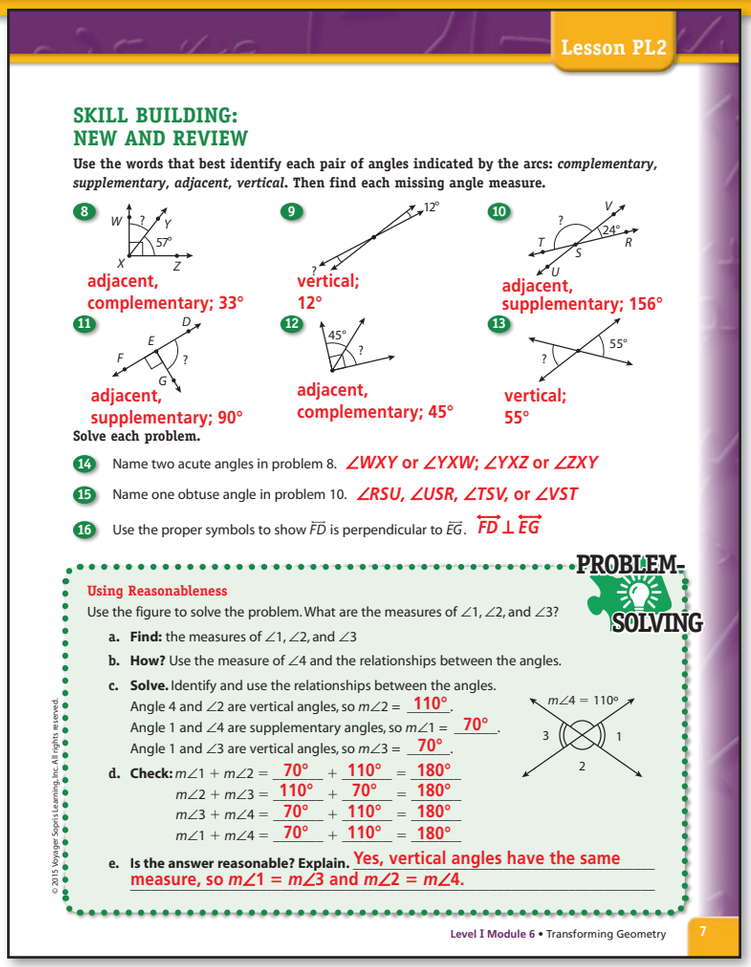
Ask students to explain why the answer is reasonable

Problems 8–20

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 8 and 10–12 Reminder

Remind students that angles can be adjacent and complementary or adjacent and supplementary. These angles should have two words that identify them.



Lesson PL2

SKILL BUILDING: NEW AND REVIEW

Use the words that best identify each pair of angles indicated by the arcs: *complementary, supplementary, adjacent, vertical*. Then find each missing angle measure.

8 adjacent, complementary; 33°

9 vertical; 12°

10 adjacent, supplementary; 156°

11 adjacent, supplementary; 90°

12 adjacent, complementary; 45°

13 vertical; 55°

Solve each problem.

14 Name two acute angles in problem 8. ∠WXY or ∠YXW; ∠YXZ or ∠ZXY

15 Name one obtuse angle in problem 10. ∠RSU, ∠USR, ∠TSV, or ∠VST

16 Use the proper symbols to show $\overline{FD}$ is perpendicular to $\overline{EG}$. $\overline{FD} \perp \overline{EG}$

PROBLEM-SOLVING

Using Reasonableness

Use the figure to solve the problem. What are the measures of ∠1, ∠2, and ∠3?

a. **Find:** the measures of ∠1, ∠2, and ∠3

b. **How?** Use the measure of ∠4 and the relationships between the angles.

c. **Solve.** Identify and use the relationships between the angles.
Angle 4 and ∠2 are vertical angles, so $m\angle 2 =$ 110°.
Angle 1 and ∠4 are supplementary angles, so $m\angle 1 =$ 70°.
Angle 1 and ∠3 are vertical angles, so $m\angle 3 =$ 70°.

$m\angle 4 = 110°$

d. **Check:** $m\angle 1 + m\angle 2 =$ 70° + 110° = 180°
$m\angle 2 + m\angle 3 =$ 110° + 70° = 180°
$m\angle 3 + m\angle 4 =$ 70° + 110° = 180°
$m\angle 1 + m\angle 4 =$ 70° + 110° = 180°

e. **Is the answer reasonable? Explain.** Yes, vertical angles have the same measure, so $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$.

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Level I Module 6 • Transforming Geometry 7

Problems 14 and 15 Reminder

Students need to use the figures in problems 8 and 10 to answer these questions.

Problem 18 Reminder

Be sure students understand how to set up the equation for the complementary angles. They should set up an equation with the sum equal to 90°.

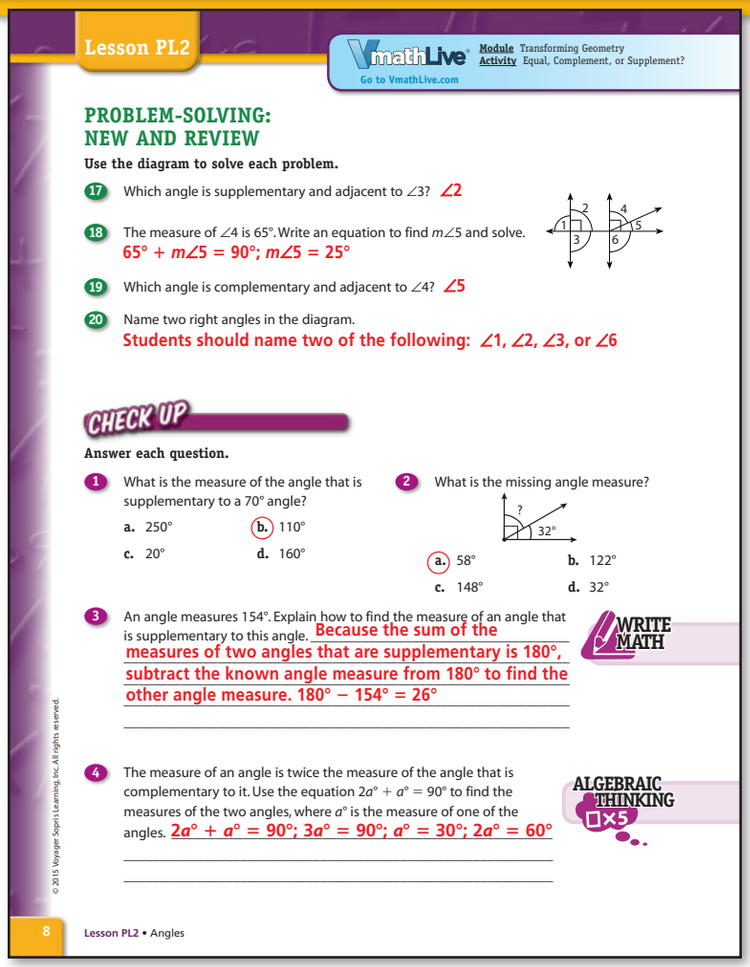
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a, 1d, or 2b: The student added the measure of the angle to 180° or 90°. Use Additional Resources in I.6.PL2 to reteach how to find a missing supplementary or complementary angle measure.



Lesson PL2

VmathLive Module Transforming Geometry Activity Equal, Complement, or Supplement? Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Use the diagram to solve each problem.

17 Which angle is supplementary and adjacent to ∠3? ∠2

18 The measure of ∠4 is 65°. Write an equation to find $m\angle 5$ and solve. $65° + m\angle 5 = 90°$; $m\angle 5 = 25°$

19 Which angle is complementary and adjacent to ∠4? ∠5

20 Name two right angles in the diagram. Students should name two of the following: ∠1, ∠2, ∠3, or ∠6

CHECK UP

Answer each question.

1 What is the measure of the angle that is supplementary to a 70° angle?
a. 250° (b.) 110° c. 20° d. 160°

2 What is the missing angle measure?
(a.) 58° b. 122° c. 148° d. 32°

3 An angle measures 154°. Explain how to find the measure of an angle that is supplementary to this angle. Because the sum of the measures of two angles that are supplementary is 180°, subtract the known angle measure from 180° to find the other angle measure. 180° − 154° = 26°

WRITE MATH

4 The measure of an angle is twice the measure of the angle that is complementary to it. Use the equation $2a° + a° = 90°$ to find the measures of the two angles, where $a°$ is the measure of one of the angles. $2a° + a° = 90°$; $3a° = 90°$; $a° = 30°$; $2a° = 60°$

ALGEBRAIC THINKING □×5

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8 Lesson PL2 • Angles

If student answered 1c or 2c: The student confused supplementary and complementary angles. Remind students that the sum of the measures of supplementary angles is 180° and the sum of the measures of complementary angles is 90°.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student the definition of the term *supplementary angle.*

DIFFERENTIATION

Additional Resources

VmathLive
Module: Transforming Geometry
Activity: Equal, Complement, or Supplement?

Vmath Reteach
Reteach Student Book Module 6 Lesson PL2
Reteach Teacher Edition Module 6 Lesson PL2

Extra Practice
Student Book page 49

English Language Learners

Use the VmathLive Animated Glossary to review the terms *adjacent angles, complementary angles, supplementary angles,* and *vertical angles*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Show students that one angle cannot be adjacent, complementary, supplementary, or vertical by itself. It must have another angle to form a pair. Relate this idea to parallel lines. A line cannot be parallel by itself; it needs another line to be parallel to. Be sure to show examples and nonexamples to clarify the meaning of each term.

Students with Special Needs

Have students form groups of two to four. Give each group five index cards with angles of different measures drawn on them and a protractor. On a sheet of paper, students can draw three columns labeled "card," "measure," and "identification." Underneath the card column, students should number five rows, 1–5. In each group, one player turns over the first index card, another player measures the angle, and another records the angle measure. The group then discusses the identification of the angle (for example, acute, obtuse, or right). Continue until all five cards have been drawn.

Technology

Have students review the Animated Glossary on VmathLive for reinforcement of math vocabulary. Additional digital content is available through this feature in the eBook.

HANDS-ON GUIDED DISCOVERY

Lesson 1

Objective

To use grid paper to determine that similar figures have equal ratios between their corresponding side lengths

CONCEPTUAL DEVELOPMENT

Materials

- Copy Master: Grid Paper
- construction paper
- scissors
- straightedges

Lesson Notes

Before beginning the lesson, be sure students have their Student Books, copies of Copy Master: Grid Paper, construction paper, scissors, and straightedges.

Emphasize that the length of a rectangle is the longer side and the width is the shorter side. Remind students that the order of the numbers in a ratio is important. In these problems, the first number is always a dimension of the original figure, and the second number is always a dimension of the newly created figure.

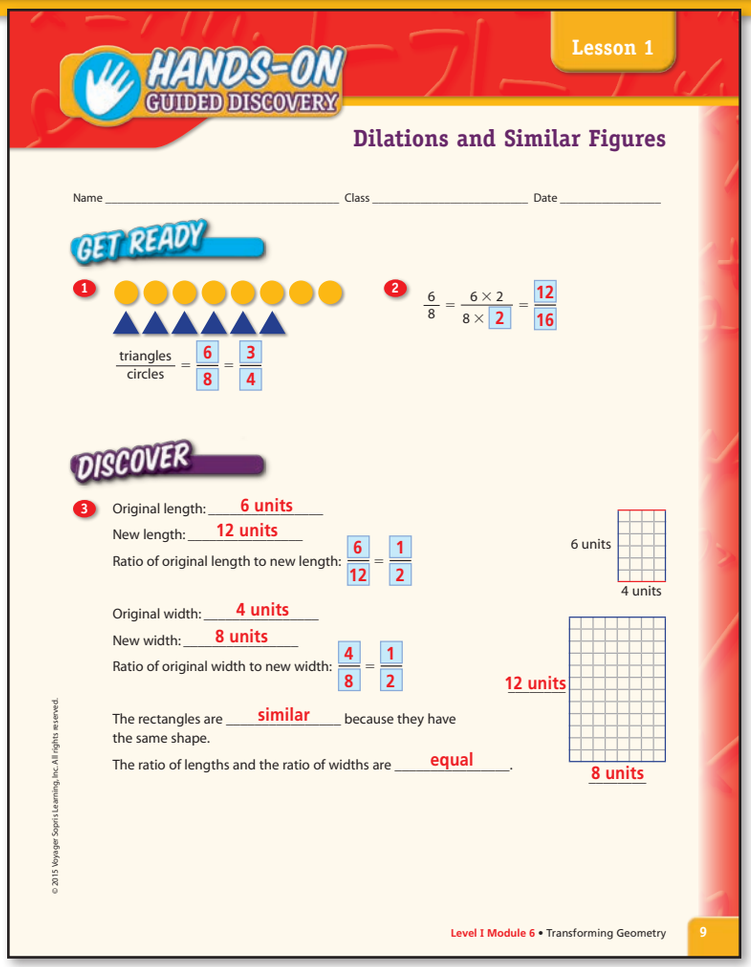
GET READY

Problem 1

How many triangles are in the problem 1 diagram? (6) **How many circles are in the diagram?** (8) **What is the ratio of triangles to circles in the diagram?** ($\frac{6}{8}$) **Is $\frac{6}{8}$ in simplest form?** (no) **What is the greatest common factor of 6 and 8?** (2) **Divide both 6 and 8 by 2. What is this ratio written in simplest form?** ($\frac{3}{4}$)

Problem 2

In problem 2, a ratio that is equivalent to $\frac{6}{8}$ can be found by multiplying the numerator and denominator of $\frac{6}{8}$ by the same number. What number is the numerator multiplied by? (2) **What is 6 times 2?** (12) **What number must the denominator be multiplied by?** (2) **What is 8 times 2?** (16) **What ratio is equivalent to $\frac{6}{8}$?** ($\frac{12}{16}$)



HANDS-ON GUIDED DISCOVERY

Lesson 1

Dilations and Similar Figures

Name _____ Class _____ Date _____

GET READY

1 ●●●●●●●● ▲▲▲▲▲▲

triangles / circles = 6/8 = 3/4

2 6/8 = 6 × 2 / 8 × 2 = 12/16

DISCOVER

3 Original length: 6 units

New length: 12 units

Ratio of original length to new length: 6/12 = 1/2

6 units / 4 units

Original width: 4 units

New width: 8 units

Ratio of original width to new width: 4/8 = 1/2

12 units / 8 units

The rectangles are similar because they have the same shape.

The ratio of lengths and the ratio of widths are equal.

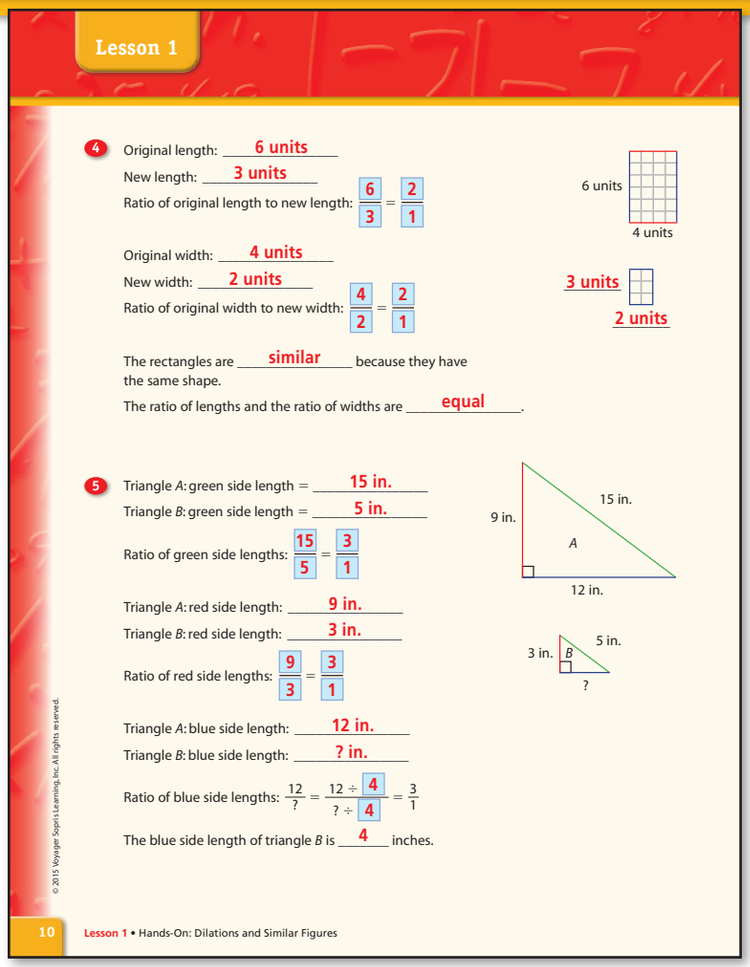
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Level I Module 6 • Transforming Geometry 9

DISCOVER

Problem 3

On grid paper, draw a rectangle in the top left corner that is 6 units long and 4 units wide, as shown in problem 3. Next, cut out a rectangle from the construction paper that is exactly the same size as the one on the grid paper. Use the straightedge to make sure it is the same size. **Note:** Make sure students cut out a rectangle identical to the one drawn on the grid paper. **Using the rectangle cut out of the construction paper and the rectangle drawn on the grid, we will double the dimensions of the rectangle that is 6 units long by 4 units wide. Place the construction paper rectangle directly above the one on the grid paper, lining up the bottom side to double the length. What is the length of the new rectangle?** (12 units) **What is the ratio of the original length to the new length?** ($\frac{6}{12}$) **What is this ratio written in simplest form?** ($\frac{1}{2}$) **Place the construction paper rectangle directly beside the one on the grid paper to double the width. What is the width of the new rectangle?** (8 units) **What is the ratio of the original width to the new width?** ($\frac{4}{8}$) **What is this ratio written in simplest form?** ($\frac{1}{2}$) **The original rectangle and the new rectangle with the dimensions doubled are similar because they have the**



Lesson 1

4 Original length: 6 units
New length: 3 units
Ratio of original length to new length: $\frac{6}{3} = \frac{2}{1}$

6 units
4 units

Original width: 4 units
New width: 2 units
Ratio of original width to new width: $\frac{4}{2} = \frac{2}{1}$

3 units
2 units

The rectangles are similar because they have the same shape.
The ratio of lengths and the ratio of widths are equal.

5 Triangle *A*: green side length = 15 in.
Triangle *B*: green side length = 5 in.
Ratio of green side lengths: $\frac{15}{5} = \frac{3}{1}$

9 in.
15 in.
A
12 in.

Triangle *A*: red side length: 9 in.
Triangle *B*: red side length: 3 in.
Ratio of red side lengths: $\frac{9}{3} = \frac{3}{1}$

3 in.
B
5 in.
?

Triangle *A*: blue side length: 12 in.
Triangle *B*: blue side length: ? in.
Ratio of blue side lengths: $\frac{12}{?} = \frac{12 \div 4}{? \div 4} = \frac{3}{1}$

The blue side length of triangle *B* is 4 inches.

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same shape. How does the ratio of the lengths compare to the ratio of the widths? (The ratios are equal.)

Problem 4

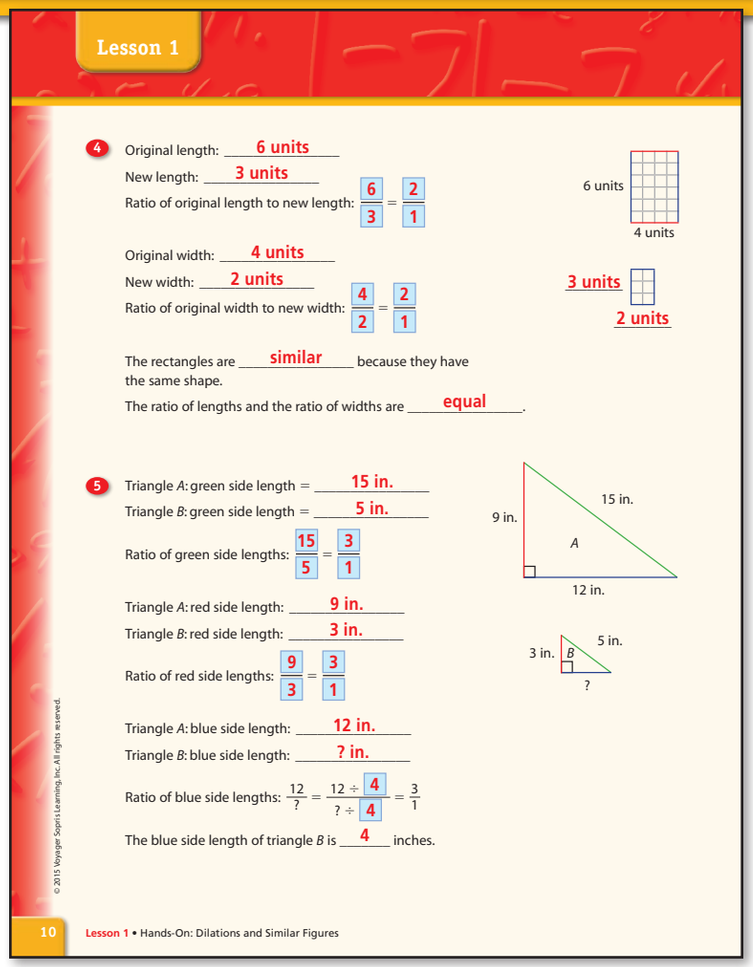
In problem 4, a similar rectangle will be drawn that is smaller than the original rectangle. Take your construction paper rectangle and fold it in half lengthwise. Then fold it in half widthwise. Unfold the construction paper rectangle. Cut out one of the four smaller rectangles. Place this rectangle inside the one drawn on the grid paper so that one pair of corners lines up and the lengths and widths line up. What is the length of the original rectangle? (6 units) **What is the length of the new smaller rectangle?** (3 units) **What is the ratio of the original length to the new length?** ($\frac{6}{3}$) **What is this ratio written in simplest form?** ($\frac{2}{1}$) **What is the width of the original rectangle?** (4 units) **What is the width of the new smaller rectangle?** (2 units) **What is the ratio of the original width to the new width?** ($\frac{4}{2}$) **What is this ratio written in simplest form?** ($\frac{2}{1}$) **The original rectangle and the new rectangle are similar because they have the same shape. How does the ratio of the lengths compare to the ratio of the widths?** (The ratios are equal.)

Problem 5

Triangle *A* and triangle *B* in problem 5 are similar. When figures are similar, the sides of the figures that are in the same relative position are called corresponding sides. So, the green sides are corresponding sides, the red sides are corresponding sides, and the blue sides are corresponding sides.

In triangle *A*, what is the green side length? (15 in.) **In triangle *B*, what is the green side length?** (5 in.) **What is the ratio of the green side length in triangle *A* to the green side length in triangle *B*?** ($\frac{15}{5}$) **What is this ratio written in simplest form?** ($\frac{3}{1}$) **In triangle *A*, what is the red side length?** (9 in.) **In triangle *B*, what is the red side length?** (3 in.) **What is the ratio of the red side length of triangle *A* to the red side length of triangle *B*?** ($\frac{9}{3}$) **What is this ratio written in simplest form?** ($\frac{3}{1}$)

In triangle *A*, what is the blue side length? (12 in.) **In triangle *B*, the length of the blue side is unknown. What should be true about the simplified ratio of the blue side length of triangle *A* to the blue side length of triangle *B*?** (It should be $\frac{3}{1}$.) **So, the ratio of the blue side lengths should be equal to $\frac{3}{1}$. Twelve divided by what number is equal to 3?** (4) **Because the ratios are equivalent, the blue side length of triangle *B* divided by 4 should be equal to 1. What number when divided by 4 is equal to 1?** (4) **So, the blue side length of triangle *B* is 4 inches.**



Lesson 1

4 Original length: 6 units
New length: 3 units
Ratio of original length to new length: $\frac{6}{3} = \frac{2}{1}$

6 units
4 units

Original width: 4 units
New width: 2 units
Ratio of original width to new width: $\frac{4}{2} = \frac{2}{1}$

3 units
2 units

The rectangles are similar because they have the same shape.
The ratio of lengths and the ratio of widths are equal.

5 Triangle *A*: green side length = 15 in.
Triangle *B*: green side length = 5 in.
Ratio of green side lengths: $\frac{15}{5} = \frac{3}{1}$

9 in.
15 in.
A
12 in.

Triangle *A*: red side length: 9 in.
Triangle *B*: red side length: 3 in.
Ratio of red side lengths: $\frac{9}{3} = \frac{3}{1}$

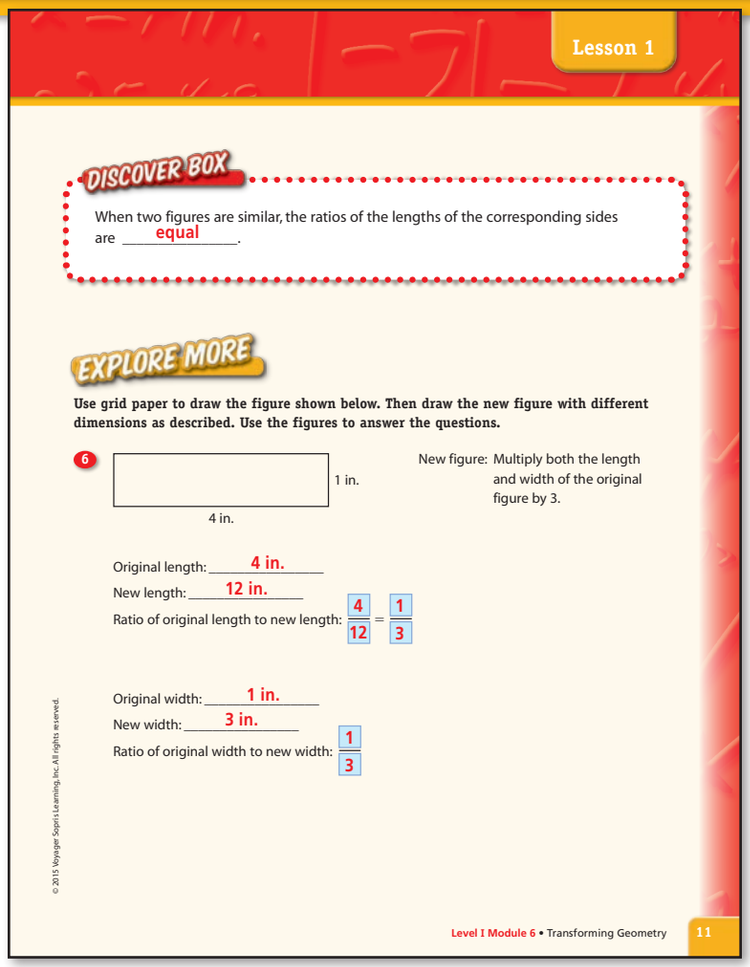
3 in.
5 in.
B
?

Triangle *A*: blue side length: 12 in.
Triangle *B*: blue side length: ? in.
Ratio of blue side lengths: $\frac{12}{?} = \frac{12 \div 4}{? \div 4} = \frac{3}{1}$

The blue side length of triangle *B* is 4 inches.

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10 Lesson 1 • Hands-On: Dilations and Similar Figures



Lesson 1

DISCOVER BOX

When two figures are similar, the ratios of the lengths of the corresponding sides are equal.

EXPLORE MORE

Use grid paper to draw the figure shown below. Then draw the new figure with different dimensions as described. Use the figures to answer the questions.

6

1 in.

4 in.

New figure: Multiply both the length and width of the original figure by 3.

Original length: 4 in.

New length: 12 in.

Ratio of original length to new length: $\frac{4}{12} = \frac{1}{3}$

Original width: 1 in.

New width: 3 in.

Ratio of original width to new width: $\frac{1}{3}$

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DISCOVER BOX

In problem 3, how does the ratio of the lengths compare to the ratio of the widths? (The ratios are equal.) **In problem 4, how does the ratio of the lengths compare to the ratio of the widths?** (The ratios are equal.) **In problem 5, how do the ratio of the red side lengths, the ratio of the blue side lengths, and the ratio of the green side lengths compare?** (The ratios are equal.) **When two figures are similar, what can be concluded about the ratios of the lengths of the corresponding sides?** (They are equal.)

EXPLORE MORE

Problem 6

Have students work independently. In problem 6, students can use the method from problem 3 to find the dimensions of the new rectangle.

Extension: Have students draw a coordinate plane on grid paper. Direct them to graph a rectangle on the coordinate plane and record the coordinates of the four vertices of the rectangle. Then have them draw a dilation by a factor of 2 of the rectangle and record the coordinates of the vertices of the new rectangle.

Lesson Adaptations for Module 6

Lesson 6.2

This lesson focuses on proportions and similar figures. Students may use grid paper to model the problems.

Lesson 2

Objective

To find the ratio of corresponding sides of similar figures and to find an unknown length in similar figures

Preskills	Lesson
Solving Proportions	I.5.PL1

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **similar figures**
  geometric figures that are the same shape but not necessarily the same size

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To solve the proportion in problem 1, write an equation using cross multiplication, then solve the equation. What is 10 times *n*? (10*n*) **What is 25 times 5?** (125) **What is the equation?** (10*n* = 125) **What must be done to solve the equation?** (Divide both sides of the equation by 10.) **What is 125 divided by 10?** (12.5) **What is the solution?** (*n* = 12.5) **How is the proportion read?** (10 is to 25 as 5 is to 12.5)

MODEL NEW SKILLS

Problem 2

Triangles *ABC* and *DEF* are similar triangles. The symbol in the statement below the triangles means *is similar to*. Read the statement as *triangle ABC is similar to triangle DEF*.

Similar figures are figures that have the same shape but not necessarily the same size. The ratios of the lengths of corresponding sides in similar figures are equivalent.

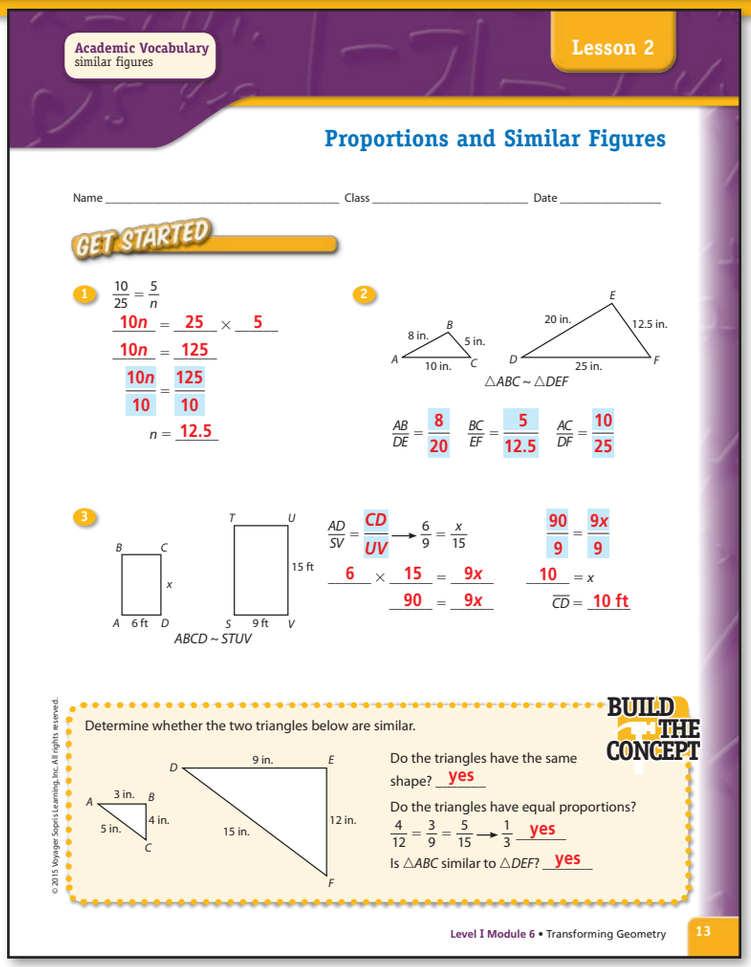
The order of the letters in the names of the similar figures is used to identify corresponding sides and angles. Look at the order of the letters in the similarity statement. For triangle *ABC*, the first two letters are used to identify the line segment *AB*. For the second triangle in the similarity statement, *triangle DEF*, the first two letters are used to identify the line segment *DE*.

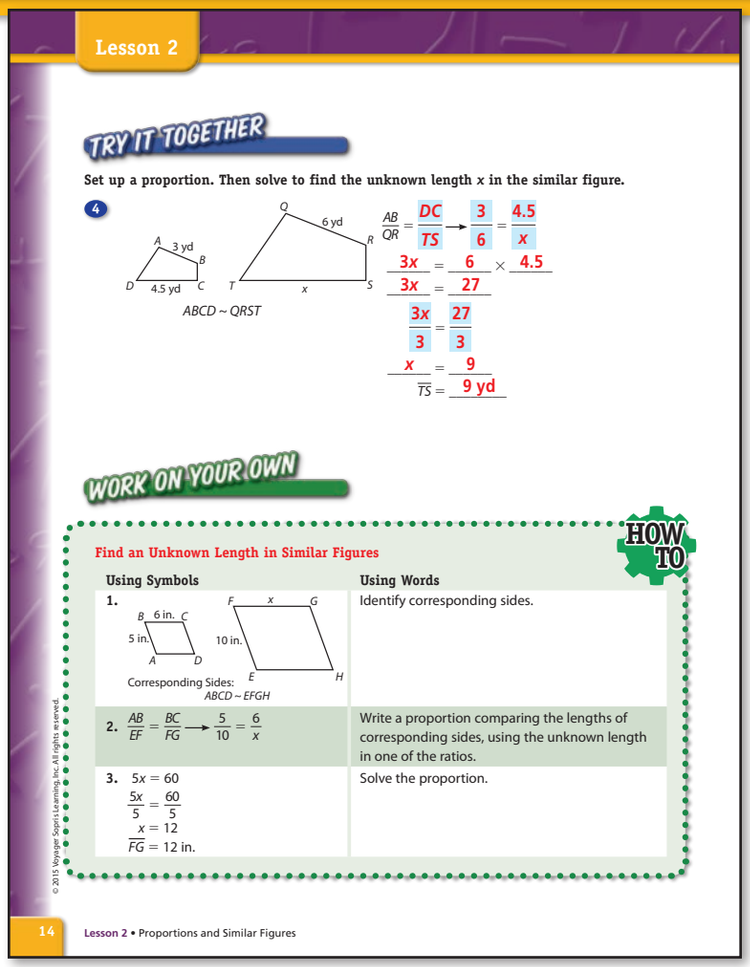
Because the letters identifying these two line segments, $\overline{AB}$ and $\overline{DE}$, appear in the same position in the names of the two similar triangles in the statement, those line segments correspond to each other in the similar triangles.

Look at the drawing. Segment *AB* corresponds to $\overline{DE}$. When writing ratios of the lengths of corresponding sides, it is not necessary to write the units of measure. What is the ratio of the length of $\overline{AB}$ to the length of $\overline{DE}$? $(\frac{8}{20})$ **Segment *BC* corresponds to $\overline{EF}$. What is the ratio of the length of $\overline{BC}$ to the length of $\overline{EF}$?** $(\frac{5}{12.5})$ **Segment *AC* corresponds to $\overline{DF}$. What is the ratio of the length of $\overline{AC}$ to the length of $\overline{DF}$?** $(\frac{10}{25})$ **Because the figures are similar, the ratios of the lengths of corresponding sides of the two triangles are equivalent.**

Problem 3

In this problem, rectangle *ABCD* and rectangle *STUV* are similar figures. The length of side $\overline{CD}$ is unknown and is represented by the variable *x*. Because ratios of





the lengths of corresponding sides in similar figures are equivalent, a proportion can be used to find the unknown side length. Write a proportion comparing the lengths of corresponding sides, using the unknown length in one of the ratios.

One ratio in the proportion compares the lengths of two corresponding sides whose lengths are known. Segment *AD* and $\overline{SV}$ are corresponding sides whose lengths are known. The other ratio in the proportion will compare the length of $\overline{CD}$ and the length of its corresponding side, $\overline{UV}$. Notice that both numerators are measurements from one figure and both denominators are measurements from the other figure.

What is the ratio of the length of $\overline{AD}$ to the length of $\overline{SV}$? ($\frac{6}{9}$) What is the ratio of the length of $\overline{CD}$ to the length of $\overline{UV}$? ($\frac{x}{15}$) Cross multiply to set up an equation. What is 6 times 15? (90) **What is 9 times *x*?** ($9x$) **What is the equation?** ($90 = 9x$) **What must be done to solve the equation?** (Divide both sides of the equation by 9.) **What is the solution?** ($x = 10$) **What is the length of $\overline{CD}$?** (10 ft)

BUILD THE CONCEPT

Model how to find the ratio of corresponding sides of similar triangles.

Similar triangles have the same shape. They may not be the same size, but they do have equal proportions. What does it mean to have equal proportions? (The ratios of corresponding sides are the same.) **In $\triangle ABC$, what is the length of $\overline{AB}$?** (3 in.) **In $\triangle DEF$, what is the length of $\overline{DE}$?** (9 in.) **Write that as a ratio. What is the ratio?** ($\frac{3}{9}$) **Compare that ratio with the ratio of the length of $\overline{BC}$ to the length of $\overline{EF}$. Are the two ratios equivalent?** (yes) **Finally, compare the ratio of the length of $\overline{AC}$ to the length of $\overline{DF}$. What is the ratio?** ($\frac{5}{15}$) **Are all three ratios equal?** (yes) **Why?** (All three ratios written in simplest form equal $\frac{1}{3}$.) **Are the triangles similar?** (yes)

TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 4

Quadrilaterals *ABCD* and *QRST* are similar figures. What is the first step in finding the length of $\overline{TS}$? (Identify corresponding sides.) **The order of the letters in the names of the similar figures is used to identify corresponding sides and angles. What side in quadrilateral *ABCD* corresponds to $\overline{TS}$?** ($\overline{DC}$) **What should be done next to find the unknown length *x*?** (Write a proportion.) **What segments will be used for the corresponding sides whose measures are known?** (*AB* and *QR*) **Use the ratio $\frac{AB}{QR}$ as one of the ratios. What should the other ratio compare?** (the length of corresponding sides including the unknown length) **What ratio involving $\overline{DC}$ is equivalent to $\frac{AB}{QR}$?** ($\frac{DC}{TS}$) **What is the value of the ratio $\frac{AB}{QR}$?** ($\frac{3}{6}$) **What is the ratio of $\frac{DC}{TS}$?** ($\frac{4.5}{x}$) **Solve the proportion. What is the equation after cross multiplying?** ($3x = 27$) **What is the solution?** ($x = 9$) **What is the length of $\overline{TS}$?** (9 yd)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using Reasonableness** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving an equation. The instruction is immediately followed by application of the strategy in problem 11.

Have students read the problem.

Show students that there isn't a statement of similarity beneath the figures. However, similarity is explicitly mentioned in the problem.

Students are to use a proportion to find the missing side length.

Be sure that students correctly solve the proportion:

$$\frac{AD}{EH} = \frac{CD}{GH}$$

$$\overline{CD} = 10 \text{ ft}$$

The answer is reasonable because the ratios of corresponding sides are equal:

$$\frac{AD}{EH} = \frac{CD}{GH} = \frac{2}{1}$$

Problems 5–14

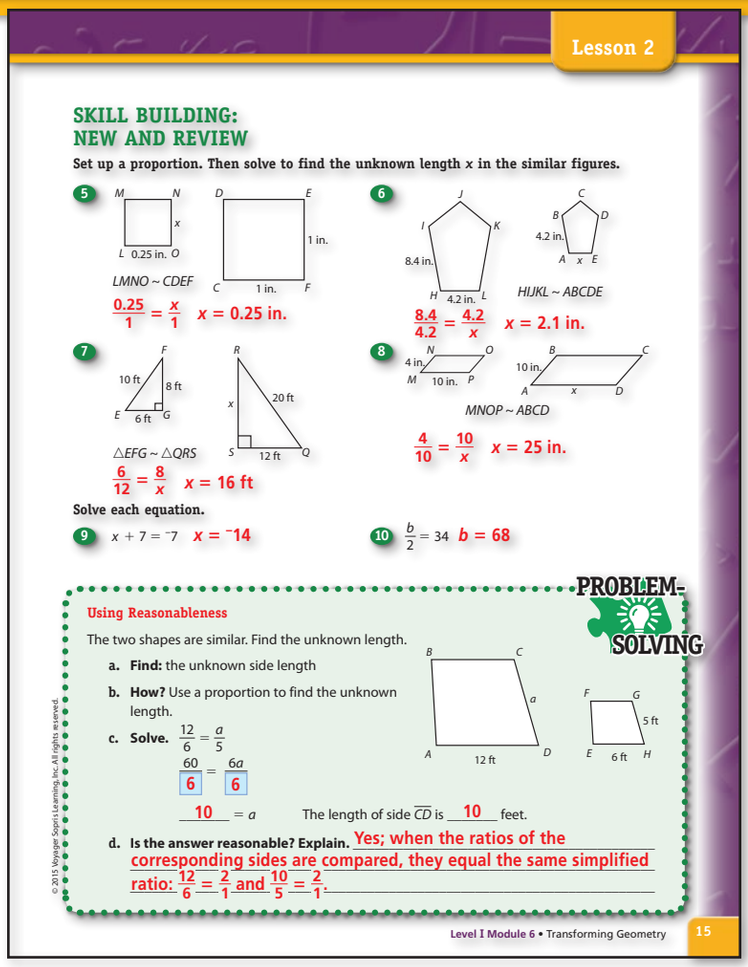
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 7 Reminder

The similar triangles in this problem have different orientations. Be sure students identify corresponding sides correctly before setting up a proportion to solve for x.

Problem 13 Reminder

Students will need to find three ratios, one for each set of corresponding sides.



Lesson 2

SKILL BUILDING: NEW AND REVIEW

Set up a proportion. Then solve to find the unknown length x in the similar figures.

5 $LMNO \sim CDEF$ $\frac{0.25}{1} = \frac{x}{1}$ $x = 0.25$ in.

6 $HIJKL \sim ABCDE$ $\frac{8.4}{4.2} = \frac{4.2}{x}$ $x = 2.1$ in.

7 $\triangle EFG \sim \triangle QRS$ $\frac{6}{12} = \frac{8}{x}$ $x = 16$ ft

8 $MNOP \sim ABCD$ $\frac{4}{10} = \frac{10}{x}$ $x = 25$ in.

Solve each equation.

9 $x + 7 = -7$ $x = -14$

10 $\frac{b}{2} = 34$ $b = 68$

PROBLEM-SOLVING

Using Reasonableness

The two shapes are similar. Find the unknown length.

a. **Find:** the unknown side length

b. **How?** Use a proportion to find the unknown length.

c. **Solve.** $\frac{12}{6} = \frac{a}{5}$

$\frac{60}{6} = \frac{6a}{6}$

10 $= a$ The length of side $\overline{CD}$ is 10 feet.

d. **Is the answer reasonable? Explain.** Yes; when the ratios of the corresponding sides are compared, they equal the same simplified ratio: $\frac{12}{6} = \frac{2}{1}$ and $\frac{10}{5} = \frac{2}{1}$.

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CHECK UP

ASSESS INFORMALLY

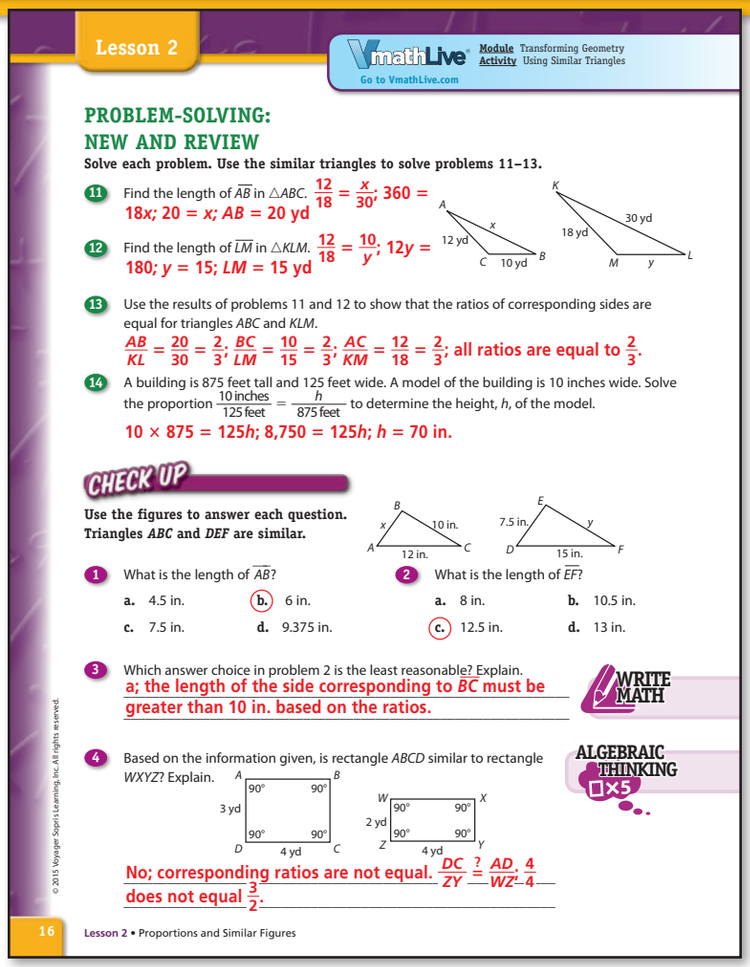
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2a: The student did not set up and solve a proportion correctly to find the unknown length. Use Additional Resources in I.6.2 to reteach how to use proportions to find the unknown length in similar figures.

If student answered 1c or 1d: The student switched the numbers in the second ratio. Remind the student that in the proportion used to solve for the unknown length, both numerators should come from the same figure and both denominators should come from the other figure.

If student answered 2b or 2d: The student added 3 to a given value rather than using a proportion. Use Additional Resources in I.6.2 to reteach how to use proportions to find the unknown length in similar figures.



Lesson 2

VmathLive Module Transforming Geometry Activity Using Similar Triangles
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem. Use the similar triangles to solve problems 11–13.

11 Find the length of $\overline{AB}$ in $\triangle ABC$. $\frac{12}{18} = \frac{x}{30}$; $360 = 18x$; $20 = x$; $AB = 20$ yd

12 Find the length of $\overline{LM}$ in $\triangle KLM$. $\frac{12}{18} = \frac{10}{y}$; $12y = 180$; $y = 15$; $LM = 15$ yd

13 Use the results of problems 11 and 12 to show that the ratios of corresponding sides are equal for triangles *ABC* and *KLM*.
$\frac{AB}{KL} = \frac{20}{30} = \frac{2}{3}$; $\frac{BC}{LM} = \frac{10}{15} = \frac{2}{3}$; $\frac{AC}{KM} = \frac{12}{18} = \frac{2}{3}$; all ratios are equal to $\frac{2}{3}$.

14 A building is 875 feet tall and 125 feet wide. A model of the building is 10 inches wide. Solve the proportion $\frac{10 \text{ inches}}{125 \text{ feet}} = \frac{h}{875 \text{ feet}}$ to determine the height, *h*, of the model.
$10 \times 875 = 125h$; $8{,}750 = 125h$; $h = 70$ in.

CHECK UP

Use the figures to answer each question. Triangles *ABC* and *DEF* are similar.

1 What is the length of $\overline{AB}$?
a. 4.5 in. (b.) 6 in. c. 7.5 in. d. 9.375 in.

2 What is the length of $\overline{EF}$?
a. 8 in. b. 10.5 in. (c.) 12.5 in. d. 13 in.

3 Which answer choice in problem 2 is the least reasonable? Explain.
a; the length of the side corresponding to $\overline{BC}$ must be greater than 10 in. based on the ratios.

WRITE MATH

4 Based on the information given, is rectangle *ABCD* similar to rectangle *WXYZ*? Explain.
No; corresponding ratios are not equal. $\frac{DC}{ZY} \stackrel{?}{=} \frac{AD}{WZ}$; $\frac{4}{4}$ does not equal $\frac{3}{2}$.

ALGEBRAIC THINKING

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WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the corresponding side to $\overline{BC}$ in $\triangle DEF$ must be longer than 10 inches. Remind the student that if one ratio in a proportion compares a lesser number to a greater number, then the other ratio in the proportion must compare a lesser number to a greater number also.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Transforming Geometry
Activity: Using Similar Triangles

Vmath Reteach
Reteach Student Book Module 6 Lesson 2
Reteach Teacher Edition Module 6 Lesson 2

Extra Practice
Student Book page 50

English Language Learners

Use the VmathLive Animated Glossary to review the term *similar figures*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Some students may confuse the meaning of *similar* with the meaning of *same*. While similar figures do look the same and have the same shape, they are not always the same size. Figures that are the same size with the same measurements are not only similar, but they are also identical, or congruent. Have students work in small groups to draw similar shapes and practice identifying shapes that are not similar.

Students with Special Needs

Have the students use grid paper, a ruler, and a protractor to model similar figures. Draw two sets of figures on the board: two rectangles and two right triangles. One rectangle measures 4×2, and the other measures 5×3. The smaller triangle has sides that measure 3, 4, and 5. The larger triangle has sides that measure 6, 8, and 10. Have the students draw the same figures on their grid paper. Next, have them measure the angles of the rectangles using their protractors; then have them measure the angles of the triangles. Using the grids on the paper, have the students write the measurements of the rectangles and triangles and set them up as ratios. Students should be able to tell from their measurements that the rectangles are not similar, but the triangles are similar.

Lesson 3

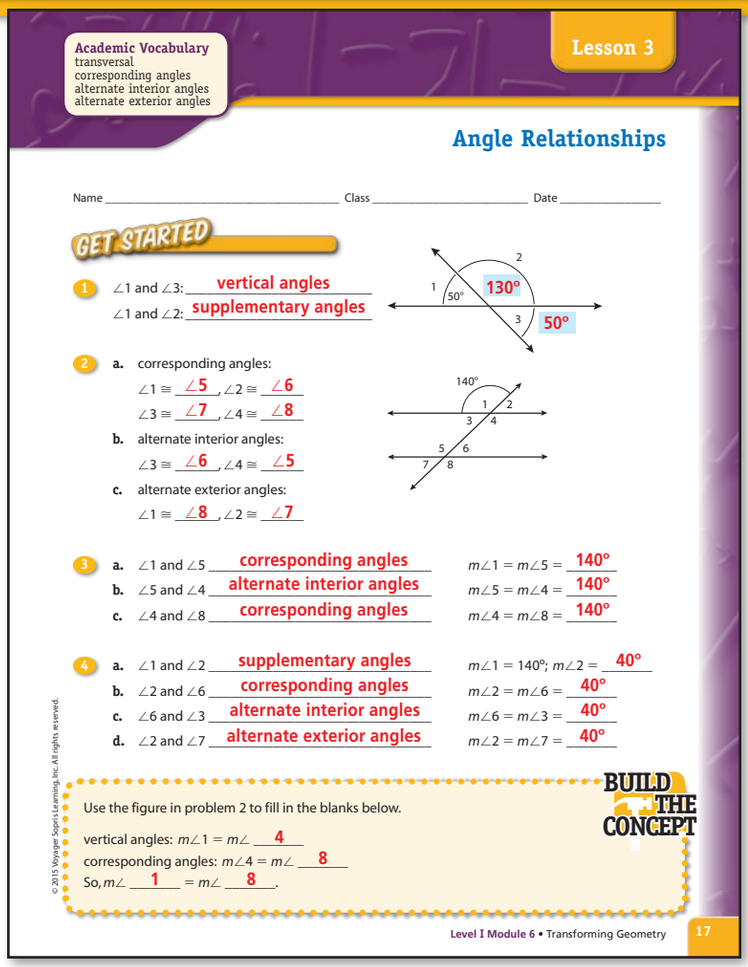
Objective

To use angle relationships to find the measurements of angles

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **transversal**
  a line that intersects two or more lines at different points
- **corresponding angles**
  a pair of angles formed by two lines that are cut by a transversal; one angle is an interior angle and the other an exterior angle on the same side of the transversal at different vertices
- **alternate interior angles**
  a pair of angles formed by two lines that are cut by a transversal; both angles are interior angles on opposite sides of the transversal at different vertices
- **alternate exterior angles**
  a pair of angles formed by two lines that are cut by a transversal; both angles are exterior angles on opposite sides of the transversal at different vertices



Academic Vocabulary
transversal
corresponding angles
alternate interior angles
alternate exterior angles

Lesson 3

Angle Relationships

Name _____ Class _____ Date _____

GET STARTED

1. ∠1 and ∠3: vertical angles
   ∠1 and ∠2: supplementary angles

2. a. corresponding angles:
   ∠1 ≅ ∠5, ∠2 ≅ ∠6
   ∠3 ≅ ∠7, ∠4 ≅ ∠8
   b. alternate interior angles:
   ∠3 ≅ ∠6, ∠4 ≅ ∠5
   c. alternate exterior angles:
   ∠1 ≅ ∠8, ∠2 ≅ ∠7

3. a. ∠1 and ∠5 corresponding angles — $m\angle1 = m\angle5 =$ 140°
   b. ∠5 and ∠4 alternate interior angles — $m\angle5 = m\angle4 =$ 140°
   c. ∠4 and ∠8 corresponding angles — $m\angle4 = m\angle8 =$ 140°

4. a. ∠1 and ∠2 supplementary angles — $m\angle1 = 140°$; $m\angle2 =$ 40°
   b. ∠2 and ∠6 corresponding angles — $m\angle2 = m\angle6 =$ 40°
   c. ∠6 and ∠3 alternate interior angles — $m\angle6 = m\angle3 =$ 40°
   d. ∠2 and ∠7 alternate exterior angles — $m\angle2 = m\angle7 =$ 40°

BUILD THE CONCEPT

Use the figure in problem 2 to fill in the blanks below.

vertical angles: $m\angle1 = m\angle$ 4
corresponding angles: $m\angle4 = m\angle$ 8
So, $m\angle$ 1 $= m\angle$ 8.

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The angles in this figure are formed by two intersecting lines. Angles that are opposite of each other when two lines intersect are vertical angles. Vertical angles are congruent. Angle 1 and ∠3 are vertical angles. What is the measure of ∠1? (50°) **What is the measure of ∠3?** (50°) **The measures of ∠1 and ∠3 are less than 90°. They are acute angles.**

The measure of ∠2 is not given, but ∠1 and ∠2 together form a straight angle. This means that the sum of the measures of ∠1 and ∠2 equals 180°. Two angles whose measures have a sum of 180° are supplementary angles. The missing angle measure can be found by subtracting 50° from 180°. What is 180° minus 50°? (130°) **The measure of ∠2 is 130°. The measure of ∠2 is greater than 90°, so ∠2 is an obtuse angle.**

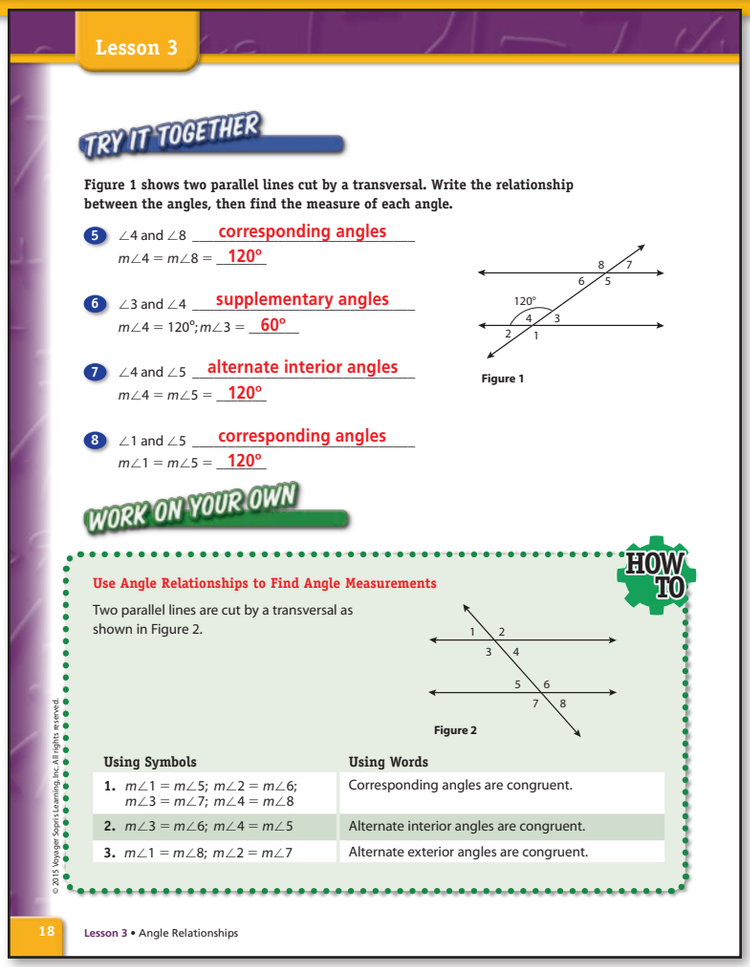
MODEL NEW SKILLS

Problem 2

The figure shows two parallel lines cut by a transversal. When two lines are cut by a transversal, eight angles are formed. In this figure, the exterior angles are above the top line and below the bottom line. Angle 1, ∠2, ∠7, and ∠8 are exterior angles.

The interior angles are between the lines. Angle 3, ∠4, ∠5, and ∠6 are interior angles in this figure. A pair of corresponding angles is made up of an interior angle and an exterior angle that are on the same side of the transversal, in corresponding positions.

Look at ∠1 and ∠5. Angle 1 is an exterior angle and ∠5 is an interior angle. Are both angles on the same side of the transversal? (yes) **Are the angles in corresponding positions?** (yes) **In this figure, there are four pairs of corresponding angles: ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, and ∠4 and ∠8. If the lines cut by the transversal are parallel, corresponding angles are congruent.**



Lesson 3

TRY IT TOGETHER

Figure 1 shows two parallel lines cut by a transversal. Write the relationship between the angles, then find the measure of each angle.

5. ∠4 and ∠8 corresponding angles
$m\angle 4 = m\angle 8 =$ 120°

6. ∠3 and ∠4 supplementary angles
$m\angle 4 = 120°; m\angle 3 =$ 60°

7. ∠4 and ∠5 alternate interior angles
$m\angle 4 = m\angle 5 =$ 120°

8. ∠1 and ∠5 corresponding angles
$m\angle 1 = m\angle 5 =$ 120°

Figure 1

WORK ON YOUR OWN

HOW TO

Use Angle Relationships to Find Angle Measurements

Two parallel lines are cut by a transversal as shown in Figure 2.

Figure 2

Using Symbols	Using Words
1. $m\angle 1 = m\angle 5$; $m\angle 2 = m\angle 6$; $m\angle 3 = m\angle 7$; $m\angle 4 = m\angle 8$	Corresponding angles are congruent.
2. $m\angle 3 = m\angle 6$; $m\angle 4 = m\angle 5$	Alternate interior angles are congruent.
3. $m\angle 1 = m\angle 8$; $m\angle 2 = m\angle 7$	Alternate exterior angles are congruent.

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18 Lesson 3 • Angle Relationships

Which angle is congruent to ∠1? (∠5) **Which angle is congruent to ∠2?** (∠6) **Which angle is congruent to ∠3?** (∠7) **Which angle is congruent to ∠4?** (∠8)

When interior angles are on opposite sides of the transversal at different vertices, they are called alternate interior angles. Angle 3 and ∠6 are alternate interior angles, as are ∠4 and ∠5. Because lines cut by a transversal are parallel, alternate interior angles are congruent. Which angle is congruent to ∠3? (∠6) **Which angle is congruent to ∠4?** (∠5)

When exterior angles are on opposite sides of the transversal at different vertices, they are called alternate exterior angles. Angle 1 and ∠8 are alternate exterior angles, as are ∠2 and ∠7. Because lines cut by a transversal are parallel, alternate exterior angles are congruent. Complete the statements in problem 2c about alternate exterior angles. Which angle is congruent to ∠1? (∠8) **Which angle is congruent to ∠2?** (∠7)

Problem 3

Look at the figure in problem 2. When parallel lines are cut by a transversal, the measures of all the other angles can be found if the measure of one angle is known. In this figure, the measure of ∠1 is given as 140°. Because ∠1 and ∠5 are corresponding angles, they are congruent. This means that if the measure of ∠1 is 140°, then the measure of ∠5 is 140°.

Angle 5 and ∠4 are alternate interior angles, so their angle measures are congruent. If the measure of ∠5 is 140°, then the measure of ∠4 is 140°. Angle 4 and ∠8 are corresponding angles, so if the measure of ∠4 is 140°, then the measure of ∠8 is also 140°.

Problem 4

In problem 3, the measures of ∠1, ∠4, ∠5, and ∠8 of the figure in problem 2 were found. The measures of ∠2, ∠6, ∠3, and ∠7 still need to be found. What is the relationship between ∠1 and ∠2? (They are supplementary.) **If the measure of ∠1 is 140°, how is the measure of ∠2 found?** (Subtract 140° from 180°.) **What is 180° minus 140°?** (40°) **The measure of ∠2 is 40°.**

What is the relationship between ∠2 and ∠6? (They are corresponding angles.) **So, ∠2 is congruent to ∠6. What is the measure of ∠6?** (40°) **What is the relationship between ∠6 and ∠3?** (They are alternate interior angles.) **So, ∠6 and ∠3 are congruent. If the measure of ∠6 is 40°, what is the measure of ∠3?** (40°)

What is the relationship between ∠2 and ∠7? (They are alternate exterior angles.) **So, ∠2 and ∠7 are congruent angles. The measure of ∠2 already has been found. What is the measure of ∠2?** (40°) **What is the measure of ∠7?** (40°)

Notice that the measures of four of the angles are equal to 140°, and the measures of the other four angles are equal to 40°, the supplement of 140°.

BUILD THE CONCEPT

Model how to understand angle relationships.

Look at the figure in problem 2. Vertical angles are congruent. What angle is vertical to ∠1? (∠4) **That means that ∠1 is congruent to ∠4.**

What angle corresponds to ∠4? (∠8) **Therefore, ∠4 is congruent to ∠8. So, because ∠1 is congruent to ∠4 and ∠4 is congruent to ∠8, ∠1 is also congruent to ∠8.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

Look at Figure 1. Are the parallel lines cut by the transversal? (yes) **What angle measure is given?** ($m\angle 4 = 120°$) **What is the relationship between ∠4 and ∠8?** (They are corresponding angles.) **What is true about corresponding angles?** (They are congruent.) **What is the measure of ∠8?** (120°)

Problem 6

What is the relationship between ∠3 and ∠4? (They are supplementary angles.) **How is the measure of ∠3 found?** (Subtract the measurement of ∠4 from 180°.) **What is 180° minus 120°?** (60°) **What is the measure of ∠3?** (60°)

Problem 7

What is the relationship between ∠4 and ∠5? (They are alternate interior angles.) **What is the measure of ∠4?** (120°) **What is the measure of ∠5?** (120°)

Problem 8

What is the measure of ∠1? (120°) **What is the measure of ∠5?** (120°) **Why?** (Corresponding angles are congruent.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

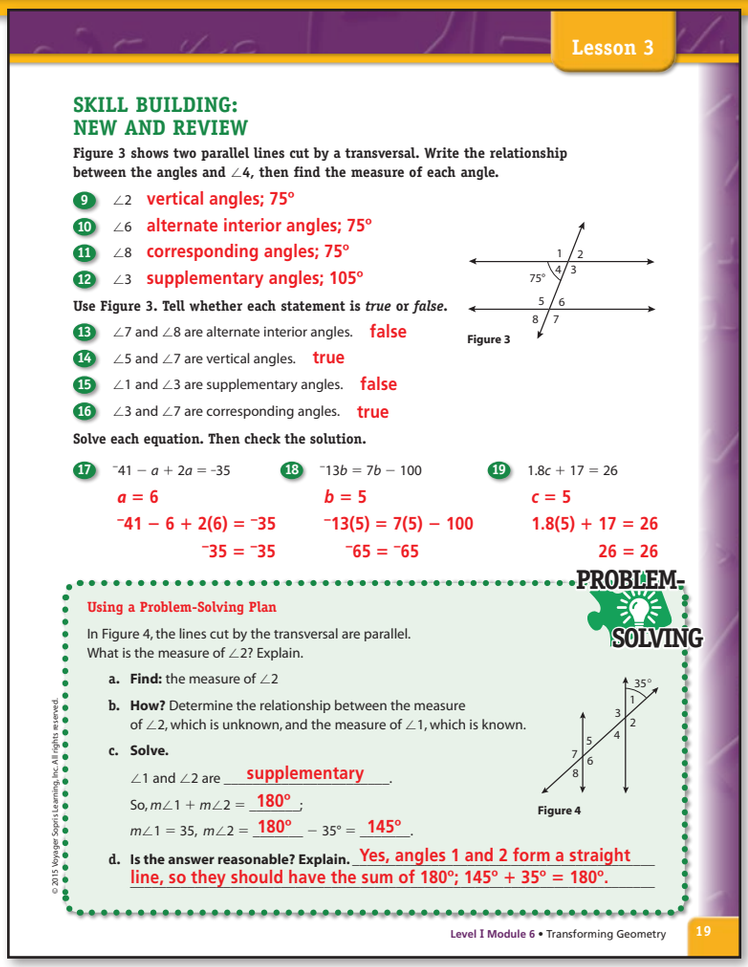
HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using a Problem-Solving Plan** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 20.

Have students read the problem.



Lesson 3

SKILL BUILDING: NEW AND REVIEW

Figure 3 shows two parallel lines cut by a transversal. Write the relationship between the angles and ∠4, then find the measure of each angle.

9. ∠2 vertical angles; 75°
10. ∠6 alternate interior angles; 75°
11. ∠8 corresponding angles; 75°
12. ∠3 supplementary angles; 105°

Figure 3

Use Figure 3. Tell whether each statement is *true* or *false*.

13. ∠7 and ∠8 are alternate interior angles. false
14. ∠5 and ∠7 are vertical angles. true
15. ∠1 and ∠3 are supplementary angles. false
16. ∠3 and ∠7 are corresponding angles. true

Solve each equation. Then check the solution.

17. $^-41 - a + 2a = -35$
$a = 6$
$^-41 - 6 + 2(6) = ^-35$
$^-35 = ^-35$

18. $^-13b = 7b - 100$
$b = 5$
$^-13(5) = 7(5) - 100$
$^-65 = ^-65$

19. $1.8c + 17 = 26$
$c = 5$
$1.8(5) + 17 = 26$
$26 = 26$

PROBLEM-SOLVING

Using a Problem-Solving Plan

In Figure 4, the lines cut by the transversal are parallel. What is the measure of ∠2? Explain.

a. **Find:** the measure of ∠2

b. **How?** Determine the relationship between the measure of ∠2, which is unknown, and the measure of ∠1, which is known.

c. **Solve.**

∠1 and ∠2 are supplementary.

So, $m\angle 1 + m\angle 2 =$ 180°;

$m\angle 1 = 35$, $m\angle 2 =$ 180° − 35° = 145°.

Figure 4

d. **Is the answer reasonable? Explain.** Yes, angles 1 and 2 form a straight line, so they should have the sum of 180°; 145° + 35° = 180°.

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Review the steps of the problem-solving plan:

Step 1 is to determine what needs to be found.

Step 2 is to determine how the problem will be solved.

Step 3 is to solve the problem.

Step 4 is to check the answer for reasonableness.

Show students that ∠1 and ∠2 are supplementary. Be sure that students do not confuse supplementary with complementary.

The sum of supplementary angles is 180°.

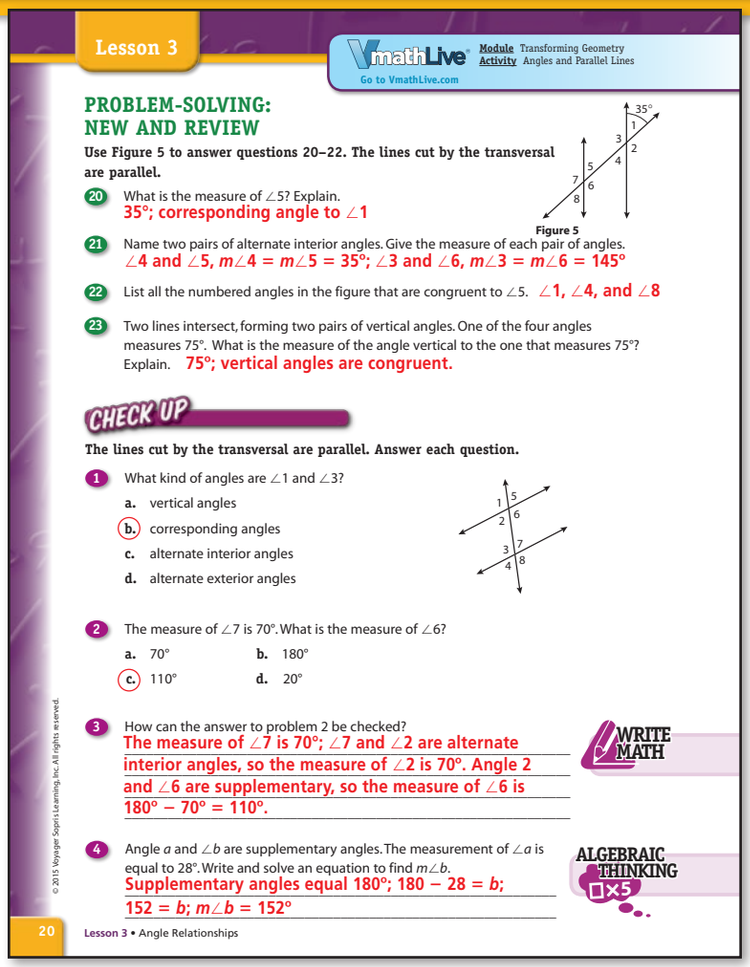
$m\angle 1 = 35°$

$m\angle 2 = 180° - 35° = 145°$

The answer is reasonable because the sum of supplementary angles is 180°; 145° + 35° = 180°.

Problems 9–23

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 3

VmathLive Module Transforming Geometry Activity Angles and Parallel Lines Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Use Figure 5 to answer questions 20–22. The lines cut by the transversal are parallel.

Figure 5

20 What is the measure of ∠5? Explain.
35°; corresponding angle to ∠1

21 Name two pairs of alternate interior angles. Give the measure of each pair of angles.
∠4 and ∠5, $m\angle 4 = m\angle 5 = 35°$; ∠3 and ∠6, $m\angle 3 = m\angle 6 = 145°$

22 List all the numbered angles in the figure that are congruent to ∠5. ∠1, ∠4, and ∠8

23 Two lines intersect, forming two pairs of vertical angles. One of the four angles measures 75°. What is the measure of the angle vertical to the one that measures 75°? Explain. 75°; vertical angles are congruent.

CHECK UP

The lines cut by the transversal are parallel. Answer each question.

1 What kind of angles are ∠1 and ∠3?
a. vertical angles
b. corresponding angles
c. alternate interior angles
d. alternate exterior angles

2 The measure of ∠7 is 70°. What is the measure of ∠6?
a. 70° b. 180°
c. 110° d. 20°

3 How can the answer to problem 2 be checked?
The measure of ∠7 is 70°; ∠7 and ∠2 are alternate interior angles, so the measure of ∠2 is 70°. Angle 2 and ∠6 are supplementary, so the measure of ∠6 is 180° − 70° = 110°.

WRITE MATH

4 Angle a and ∠b are supplementary angles. The measurement of ∠a is equal to 28°. Write and solve an equation to find $m\angle b$.
Supplementary angles equal 180°; 180 − 28 = b; 152 = b; $m\angle b = 152°$

ALGEBRAIC THINKING □×5

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CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly: The student did not correctly identify the relationships between angles. Use Additional Resources in I.6.3 to reteach angle relationships.

If student answered 2a: The student misidentified the two angles as congruent. Use Additional Resources in I.6.3 to reteach angle relationships.

If student answered 2b: The student did not understand the relationships among angles. Use Additional Resources in I.6.3 to reteach angle relationships.

If student answered 2d: The student did not recognize that supplementary angles add up to 180° instead of 90° as in complementary angles. Use Additional Resources in I.6.3 to reteach angle relationships.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that many corresponding and supplementary angles exist when a transversal cuts two parallel lines. Use Additional Resources in I.6.3 to reteach angle relationships.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Transforming Geometry
Activity: Angles and Parallel Lines

Vmath Reteach
Reteach Student Book Module 6 Lesson 3
Reteach Teacher Edition Module 6 Lesson 3

Extra Practice
Student Book page 50

English Language Learners

Use the VmathLive Animated Glossary to review the terms *vertical angles, supplementary angles, intersecting lines, parallel lines, transversal, corresponding angles, alternate interior angles,* and *alternate exterior angles.* Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Have students draw a sketch for each of the vocabulary terms so that they have a visual representation of each term associated with this lesson. Encourage students to constantly use the terms in written and verbal language.

Students with Special Needs

Have students draw a sketch of two parallel lines cut by a transversal. Then have them shade the two sets of four congruent angles with two different colors as a visual reference to the major concept in the lesson.

Lesson 4

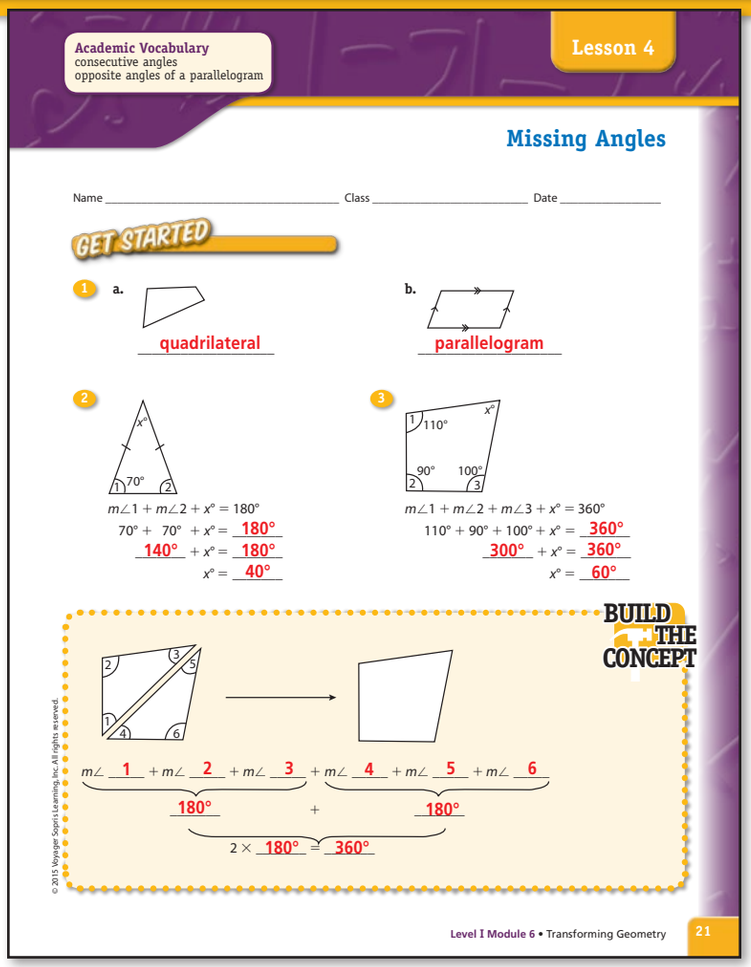
Objective

To use properties of triangles and quadrilaterals to find a missing angle measure

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **consecutive angles**
  angles that share a common side
- **opposite angles of a parallelogram**
  nonconsecutive angles of a parallelogram



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What type of polygon is in problem 1a? (quadrilateral) **A quadrilateral is a polygon with 4 sides. Look at problem 1b. This is a special quadrilateral with two pairs of parallel sides. What is its name?** (parallelogram)

Problem 2

The triangle in problem 2 is an isosceles triangle. Why? (It has 2 congruent sides.) **An isosceles triangle has congruent base angles that are opposite the two congruent sides. Angle 1 is a base angle with a measure of 70°. Angle 2 is the other base angle in this triangle. What is the measure of angle 2?** (70°) **The value of the missing angle measure is indicated by the variable *x*°.**

The sum of the measures of the interior angles in every triangle is 180°. If two angle measures are known, the third angle measure can be found. To find the missing angle measure, use the equation $m\angle 1 + m\angle 2 + x° = 180°$, where *x*° is the missing angle measure, and $m\angle 1$ and $m\angle 2$ are the measures of the known angles.

Substitute the known angle measures into the equation and solve for *x*°. The equation is $70° + 70° + x° = 180°$. What is 70° plus 70°? (140°) **Subtract 140° from both sides of the equation. What is 180° minus 140°?** (40°) **The missing angle measure is 40°.**

MODEL NEW SKILLS

Problem 3

Look at the quadrilateral in problem 3. The measures of three of the four angles are known. What are the known angle measures? (110°, 90°, 100°) **How is the value of the missing angle measure indicated?** (with the variable *x*°)

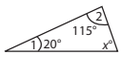
The sum of the measures of the interior angles of any quadrilateral is 360°. So, to find the missing angle measure, use the equation $m\angle 1 + m\angle 2 + m\angle 3 + x° = 360°$, where *x*° is the missing angle measure, and $m\angle 1$, $m\angle 2$, and $m\angle 3$ are the measures of the known angles.

Substitute the known angle measures into the equation and solve for *x*°. The equation is $110° + 90° + 100° + x° = 360°$. What is 110° plus 90° plus 100°? (300°) **Subtract 300° from both sides of the equation. What is 360° minus 300°?** (60°) **The missing angle measure is 60°.**

TRY IT TOGETHER

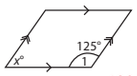
Find the missing angle measure of each figure.

4



$m\angle 1 + m\angle 2 + x° =$ 180°
$20° + 115° + x° =$ 180°
135° $+ x° =$ 180°
$x° =$ 45°

5



$m\angle 1 + x° =$ 180°
125° $+ x° =$ 180°
$x° =$ 55°

WORK ON YOUR OWN

HOW TO

Find Missing Angle Measures in a Triangle







Using Symbols	Using Words
1. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	The sum of the interior angle measures is 180°.
2. $m\angle 4 = m\angle 5$	The base angles in an isosceles triangle are congruent.
3. $m\angle 6 = m\angle 7 = m\angle 8 = 60°$	The angle measures in an equilateral triangle are congruent. Each angle measures 60°.

Find Missing Angle Measures in a Quadrilateral





Using Symbols	Using Words
1. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$	The sum of the interior angle measures is 360°.
2. $m\angle 5 = m\angle 7; m\angle 6 = m\angle 8$	The opposite angles of a parallelogram are congruent.
3. $m\angle 5 + m\angle 6 = 180°$ $m\angle 6 + m\angle 7 = 180°$ $m\angle 7 + m\angle 8 = 180°$ $m\angle 8 + m\angle 5 = 180°$	The sum of the measures of two consecutive angles of a parallelogram is 180°.

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BUILD THE CONCEPT

Model how to use properties of triangles to find the sum of the angles of a quadrilateral.

Look at the two triangles. How is the sum of the angle measures in the triangle on the left written? ($m\angle 1 + m\angle 2 + m\angle 3$) **How is the sum of the angle measures in the triangle on the right written?** ($m\angle 4 + m\angle 5 + m\angle 6$) **The total sum of the angle measures in the two triangles is $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6$. What is the sum of the interior angle measures of any triangle?** (180°) **So, $m\angle 1 + m\angle 2 + m\angle 3$ is 180° and $m\angle 4 + m\angle 5 + m\angle 6$ is 180°.**

The total sum of the measures of the interior angles of both triangles is 2 times 180°, or 360°. If these triangles are put together as shown, what kind of figure is formed? (a quadrilateral) **Notice that when the triangles are put together, the interior angles of both triangles form the interior angles of the quadrilateral. What is the total sum of the measures of the interior angles in the two triangles?** (360°) **So, the sum of the measures of the interior angles of the quadrilateral is 360°.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

What equation is used to find the missing angle measure in a triangle? ($m\angle 1 + m\angle 2 + x° = 180°$) **What is the equation after the known angle measures are substituted?** ($20° + 115° + x° = 180°$) **What is 20° plus 115°?** (135°) **What is 180° minus 135°?** (45°) **What is the missing angle measure?** (45°)

Problem 5

What special type of quadrilateral is the polygon in problem 5? (parallelogram) **Angle 1 and the angle with the missing measure are consecutive angles. In a parallelogram, the sum of the measures of consecutive angles is 180°.**

To find the missing angle measure, use the equation $m\angle 1 + x° = 180°$. What is 180° minus 125°? (55°) **The missing angle measure is 55°.**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Drawing a Picture** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a picture. The instruction is immediately followed by application of the strategy in problem 12.

Have students read the problem.

Have students look at the picture. Have students label the picture with the information in the problem:

$m\angle 1 = 120°$

$m\angle 2 = 40°$

$m\angle 3 = 80°$

$m\angle 4 = ? = x°$

Help students recall that the sum of the angles in a quadrilateral is 360°.

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$$
$$120° + 40° + 80° + x° = 360°$$
$$240° + x° = 360°$$
$$x° = 120°$$

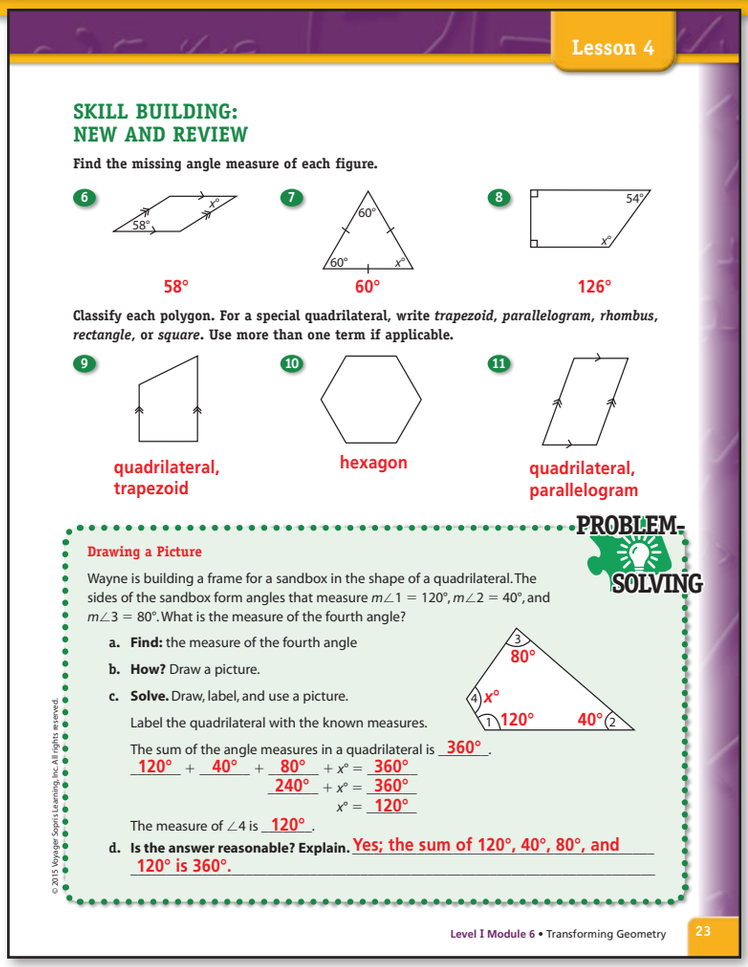
The answer is reasonable because the sum of all the angle measures is 360° and visually, angles 1 and 4 in the drawing appear to have the same measure.

Problems 6–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 12 Reminder

Review types of quadrilaterals with students. Remind them that the sum of the interior angle measures in any quadrilateral is 360°.



Lesson 4

SKILL BUILDING: NEW AND REVIEW

Find the missing angle measure of each figure.

6

58°

7

60°

8

126°

Classify each polygon. For a special quadrilateral, write *trapezoid, parallelogram, rhombus, rectangle,* or *square*. Use more than one term if applicable.

9
quadrilateral, trapezoid

10
hexagon

11
quadrilateral, parallelogram

PROBLEM-SOLVING

Drawing a Picture

Wayne is building a frame for a sandbox in the shape of a quadrilateral. The sides of the sandbox form angles that measure $m\angle 1 = 120°$, $m\angle 2 = 40°$, and $m\angle 3 = 80°$. What is the measure of the fourth angle?

a. **Find:** the measure of the fourth angle

b. **How?** Draw a picture.

c. **Solve.** Draw, label, and use a picture.

Label the quadrilateral with the known measures.

The sum of the angle measures in a quadrilateral is 360°.

120° + 40° + 80° + $x°$ = 360°

240° + $x°$ = 360°

$x°$ = 120°

The measure of ∠4 is 120°.

d. **Is the answer reasonable? Explain.** Yes; the sum of 120°, 40°, 80°, and 120° is 360°.

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CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

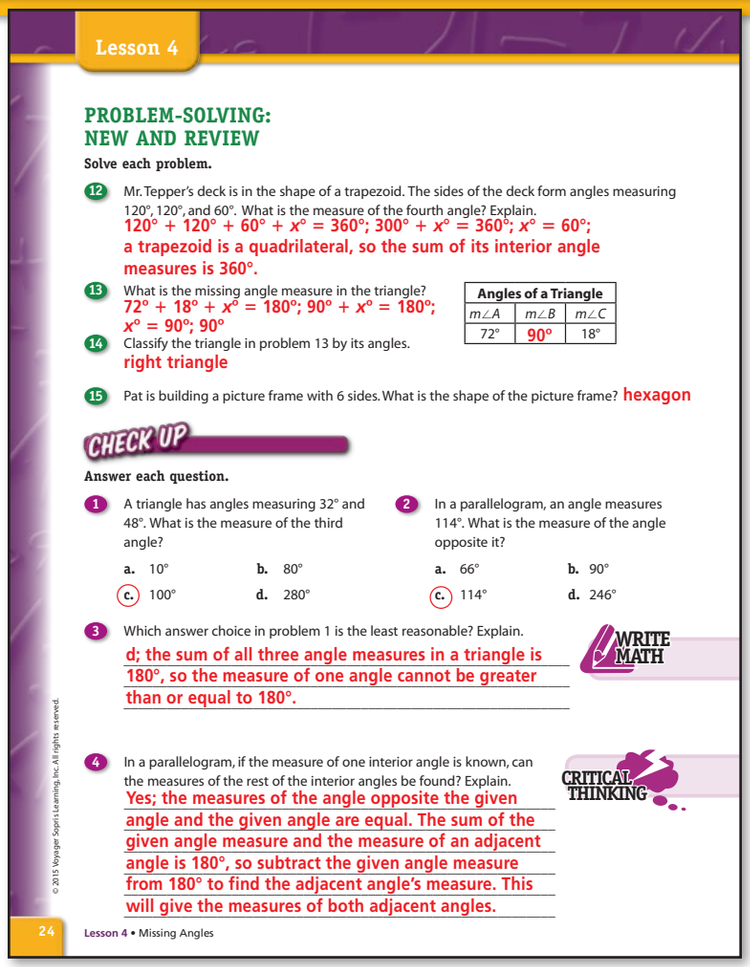
If student answered 1b or 1d: The student did not subtract the sum from 180°, or subtracted from 360° instead of 180°. Use Additional Resources in I.6.4 to reteach how to find the missing angle measure in a triangle.

If student answered problem 2 incorrectly: The student did not apply the property that opposite angles of a parallelogram are congruent. Use Additional Resources in I.6.4 to reteach how to find the missing angle measure in a parallelogram.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the sum of the measures of the interior angles of a triangle is 180°. One angle measure cannot be greater than or equal to 180°.



Lesson 4

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

12. Mr. Tepper's deck is in the shape of a trapezoid. The sides of the deck form angles measuring 120°, 120°, and 60°. What is the measure of the fourth angle? Explain.
$120° + 120° + 60° + x° = 360°$; $300° + x° = 360°$; $x° = 60°$; a trapezoid is a quadrilateral, so the sum of its interior angle measures is 360°.

13. What is the missing angle measure in the triangle?
$72° + 18° + x° = 180°$; $90° + x° = 180°$; $x° = 90°$; 90°

Angles of a Triangle		
$m\angle A$	$m\angle B$	$m\angle C$
72°	90°	18°

14. Classify the triangle in problem 13 by its angles.
right triangle

15. Pat is building a picture frame with 6 sides. What is the shape of the picture frame? hexagon

CHECK UP

Answer each question.

1. A triangle has angles measuring 32° and 48°. What is the measure of the third angle?
   a. 10° b. 80° (c.) 100° d. 280°

2. In a parallelogram, an angle measures 114°. What is the measure of the angle opposite it?
   a. 66° b. 90° (c.) 114° d. 246°

3. Which answer choice in problem 1 is the least reasonable? Explain. WRITE MATH
d; the sum of all three angle measures in a triangle is 180°, so the measure of one angle cannot be greater than or equal to 180°.

4. In a parallelogram, if the measure of one interior angle is known, can the measures of the rest of the interior angles be found? Explain. CRITICAL THINKING
Yes; the measures of the angle opposite the given angle and the given angle are equal. The sum of the given angle measure and the measure of an adjacent angle is 180°, so subtract the given angle measure from 180° to find the adjacent angle's measure. This will give the measures of both adjacent angles.

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24 Lesson 4 • Missing Angles

Technology

Have students use an online search engine to discover other ways to apply finding missing angle measures in triangles and quadrilaterals in real-life situations. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 6 Lesson 4
Reteach Teacher Edition Module 6 Lesson 4

Extra Practice
Student Book page 51

English Language Learners

Use the VmathLive Animated Glossary to review the term *parallelogram*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Make sure students do not confuse the angle *number* with the angle *measurement*. Show that a measurement should have a degree symbol and an angle number should not. When students have found missing angle measures, they can easily check their answers. Encourage them to check that the sum of the angle measures in a triangle is 180°, and the sum of the angle measures in a quadrilateral is 360°.

Students with Special Needs

Encourage students to use diagrams and pictures to their advantage. If they are asked to find missing angle measures, they should label the diagram with all of the known information first. Then they will have all the facts directly in front of them. When students draw the diagrams themselves, encourage them to draw large figures. When figures are small, angle measurements may run into each other.

Gizmos Guided Discovery

Lesson 5

Objective

To determine which congruence properties of triangles will guarantee congruent triangles

WITH CONCEPTUAL DEVELOPMENT TECHNOLOGY

Materials

- Gizmo: Proving Triangles Congruent

Gizmos Log In Instructions

- Log in to vmath.voyagersopris.com using your Username and Password provided in VPORT®.
- Select the Gizmos poster.
- Click on the Proving Triangles Congruent Gizmo link.

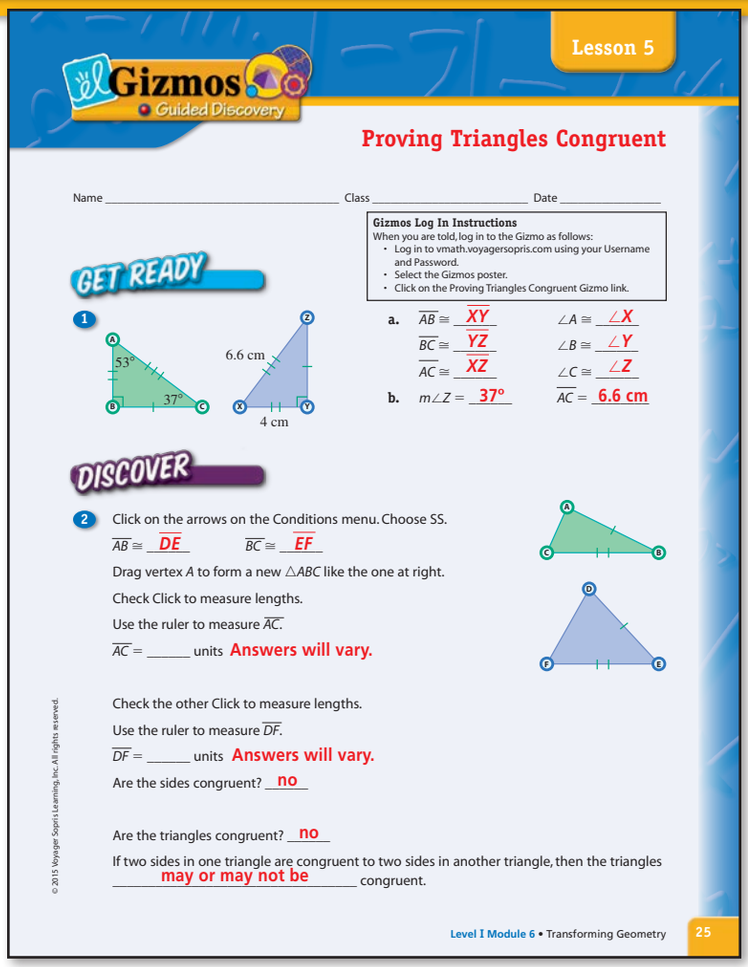
Lesson Notes

Before beginning the lesson, be sure students have their Student Books and are ready to work at the computers.

In this lesson, students will discover the rules for SSS, SAS, and ASA.

Complete problem 1 before students log in to the Proving Triangles Congruent Gizmo.

If students have additional time, have them answer Assessment Questions 1–3 in the Gizmo. They can click on the Check Your Answers button to see how well they did on the assessment.



Gizmos Guided Discovery — Lesson 5

Proving Triangles Congruent

Name _____ Class _____ Date _____

Gizmos Log In Instructions
When you are told, log in to the Gizmo as follows:
- Log in to vmath.voyagersopris.com using your Username and Password.
- Select the Gizmos poster.
- Click on the Proving Triangles Congruent Gizmo link.

GET READY

1. 53°, 37°, 6.6 cm, 4 cm

a. $\overline{AB} \cong \overline{XY}$ $\overline{BC} \cong \overline{YZ}$ $\overline{AC} \cong \overline{XZ}$ $\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\angle C \cong \angle Z$

b. $m\angle Z =$ 37° $\overline{AC} =$ 6.6 cm

DISCOVER

2. Click on the arrows on the Conditions menu. Choose SS.

$\overline{AB} \cong$ DE $\overline{BC} \cong$ EF

Drag vertex A to form a new $\triangle ABC$ like the one at right.

Check Click to measure lengths.

Use the ruler to measure $\overline{AC}$.

$\overline{AC} =$ _____ units Answers will vary.

Check the other Click to measure lengths.

Use the ruler to measure $\overline{DF}$.

$\overline{DF} =$ _____ units Answers will vary.

Are the sides congruent? no

Are the triangles congruent? no

If two sides in one triangle are congruent to two sides in another triangle, then the triangles may or may not be congruent.

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Level I Module 6 • Transforming Geometry 25

GET READY

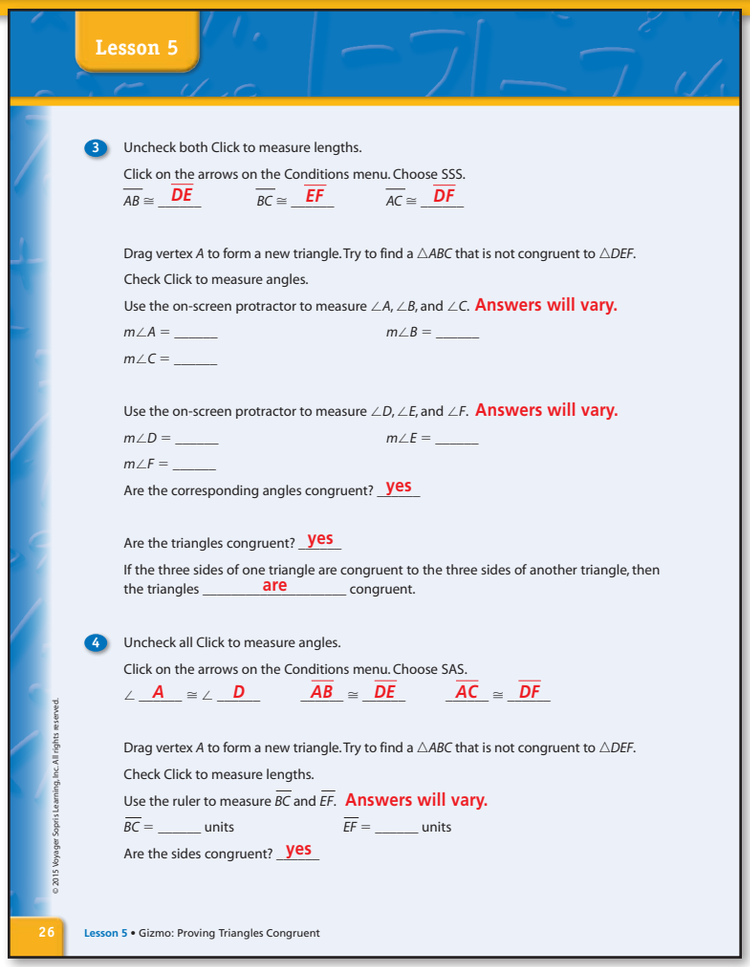
Problem 1

The triangles in problem 1 are congruent. In problem 1a, you will name the corresponding sides and the corresponding angles. Which segment corresponds to $\overline{AB}$? ($\overline{XY}$) **Write this relationship using a congruency symbol. Read the statement as *segment AB is congruent to segment XY*. Which segment corresponds to and is congruent to $\overline{BC}$?** ($\overline{YZ}$) **Which segment corresponds to and is congruent to $\overline{AC}$?** ($\overline{XZ}$) **Which angle corresponds to and is congruent to $\angle A$?** ($\angle X$) **Which angle corresponds to and is congruent to $\angle B$?** ($\angle Y$) **Which angle corresponds to and is congruent to $\angle C$?** ($\angle Z$) **Because congruent triangles have congruent corresponding sides and angles, it is possible to determine the side lengths and angle measures in one figure if the corresponding side lengths and angle measures are known in the congruent figure. To find the measure of $\angle Z$, first find the angle congruent to $\angle Z$. Which angle is congruent to $\angle Z$?** ($\angle C$) **What is the measure of $\angle C$?** (37°) **So, the measure of $\angle Z$ is also 37°. To find the length of $\overline{AC}$, first find the segment that is congruent to $\overline{AC}$. Which segment is congruent to $\overline{AC}$?** ($\overline{XZ}$) **What is the length of $\overline{XZ}$?** (6.6 cm) **So, the length of $\overline{AC}$ is also 6.6 centimeters.**

DISCOVER

Problem 2

Have students log in to the Proving Triangles Congruent Gizmo using the instructions in the box. **Click on the arrows on the Conditions menu. Choose SS. Two pairs of sides are congruent. Which side is congruent to $\overline{AB}$?** ($\overline{DE}$) **How do you know?** ($\overline{AB}$ and $\overline{DE}$ each have one slash mark.) **Which side is congruent to $\overline{BC}$?** ($\overline{EF}$) **How do you know?** ($\overline{BC}$ and $\overline{EF}$ each



Lesson 5

3 Uncheck both Click to measure lengths.

Click on the arrows on the Conditions menu. Choose SSS.

$\overline{AB} \cong$ DE $\quad \overline{BC} \cong$ EF $\quad \overline{AC} \cong$ DF

Drag vertex *A* to form a new triangle. Try to find a $\triangle ABC$ that is not congruent to $\triangle DEF$.

Check Click to measure angles.

Use the on-screen protractor to measure $\angle A$, $\angle B$, and $\angle C$. Answers will vary.

$m\angle A =$ _____ $\quad m\angle B =$ _____

$m\angle C =$ _____

Use the on-screen protractor to measure $\angle D$, $\angle E$, and $\angle F$. Answers will vary.

$m\angle D =$ _____ $\quad m\angle E =$ _____

$m\angle F =$ _____

Are the corresponding angles congruent? yes

Are the triangles congruent? yes

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

4 Uncheck all Click to measure angles.

Click on the arrows on the Conditions menu. Choose SAS.

$\angle$ A $\cong \angle$ D $\quad \overline{AB} \cong \overline{DE} \quad \overline{AC} \cong \overline{DF}$

Drag vertex *A* to form a new triangle. Try to find a $\triangle ABC$ that is not congruent to $\triangle DEF$.

Check Click to measure lengths.

Use the ruler to measure $\overline{BC}$ and $\overline{EF}$. Answers will vary.

$\overline{BC} =$ _____ units $\quad \overline{EF} =$ _____ units

Are the sides congruent? yes

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have two slash marks.) **Drag vertex *A* so that $\triangle ABC$ looks like the one in your book. If $\triangle ABC$ and $\triangle DEF$ are congruent, what must be true about $\overline{AC}$ and $\overline{DF}$?** (They must also be congruent.) **Do these sides look congruent?** (no) **Measure the lengths of the sides by checking Click to measure lengths. Drag one side of the ruler to vertex *A* and the other side of the ruler to vertex *C*. What is the length of $\overline{AC}$?** (Answers will vary.) **Check the other Click to measure lengths and find the length of $\overline{DF}$. What is the length of $\overline{DF}$?** (Answers will vary.) **Are the sides congruent?** (no) **Are the triangles congruent?** (no) **If two sides in one triangle are congruent to two sides in another triangle, can you conclude that the two triangles are congruent?** (no)

Problem 3

Uncheck both Click to measure lengths. Click on the arrows on the Conditions menu. Choose SSS. Three pairs of sides are congruent. Which side is congruent to $\overline{AB}$? ($\overline{DE}$) **Which side is congruent to $\overline{BC}$?** ($\overline{EF}$) **Which side is congruent to $\overline{AC}$?** ($\overline{DF}$) **Drag vertex *A* so that $\triangle ABC$ and $\triangle DEF$ are not congruent. Can you drag vertex *A* so that the triangles are not congruent?** (no) **If the triangles are congruent, what is known about the measures of the corresponding angles?** (They also are congruent.)

Measure the angles by checking Click to measure angles. The middle point of the protractor should lie on the vertex of the angle that is being measured, and the other two points should lie on the other two vertices of the triangle. What is the measure of $\angle A$? (Answers will vary.) **Uncheck and then check Click to measure angles to measure all of the other angles.** Note: Although resetting the protractor is unnecessary, students may inadvertently change their triangle when trying to grab the parts of the protractor. Resetting the protractor will help to avoid this problem. **What is the measure of $\angle B$?** (Answers will vary.) **What is the measure of $\angle C$?** (Answers will vary.) **What is the measure of $\angle D$?** (Answers will vary.) **What is the measure of $\angle E$?** (Answers will vary.) **What is the measure of $\angle F$?** (Answers will vary.)

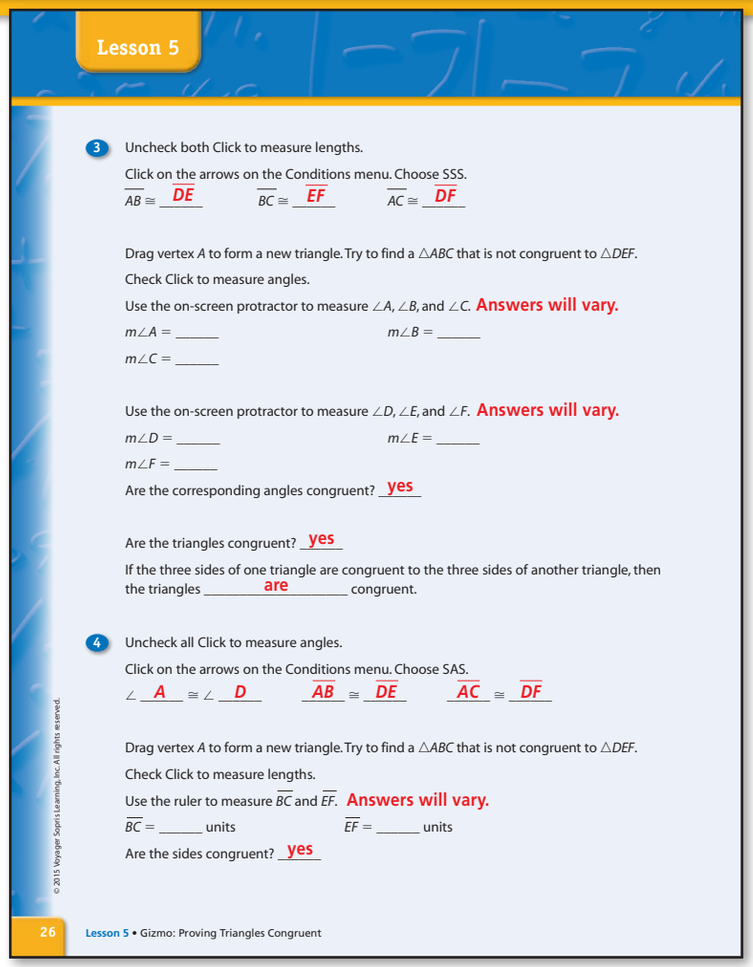
Are the corresponding angles congruent? (yes) **Are the triangles congruent?** (yes) **If the three sides in one triangle are congruent to the three sides in another triangle, can you conclude that the two triangles are congruent?** (yes)

Problem 4

Uncheck all Click to measure angles. Click on the arrows on the Conditions menu. Choose SAS. Two pairs of sides and the pair of angles between the sides are congruent. Which angles are congruent? ($\angle A$ and $\angle D$) **Name one pair of sides that are congruent.** ($\overline{AB}$ and $\overline{DE}$) **Name the other pair of sides that are congruent.** ($\overline{AC}$ and $\overline{DF}$)

Try to drag vertex *A* so that $\triangle ABC$ and $\triangle DEF$ are not congruent. Can you drag vertex *A* so that the triangles are not congruent? (no) **If the triangles are congruent, what do you know about the lengths of $\overline{BC}$ and $\overline{BF}$?** (The lengths are the same.) **Check Click to measure lengths and find the lengths of $\overline{BC}$ and $\overline{BF}$. Are the sides congruent?** (yes) **If the triangles are congruent, what is known about the measures of the corresponding angles?** (They are also congruent.)

Measure $\angle B$, $\angle C$, $\angle E$, and $\angle F$ by checking Click to measure angles and using the on-screen protractor. How do the measures of $\angle B$ and $\angle E$ compare? (They are the same.) **How do the measures of $\angle C$ and $\angle F$ compare?** (They are the same.) **Are all the corresponding sides congruent?** (yes) **Are all the corresponding angles congruent?** (yes) **Are the triangles congruent?** (yes) **If two sides and the angle between them in one triangle are congruent to two sides and the angle between them in another triangle, can you conclude that the two triangles are congruent?** (yes)



Lesson 5

3 Uncheck both Click to measure lengths.

Click on the arrows on the Conditions menu. Choose SSS.

$\overline{AB} \cong$ DE $\quad \overline{BC} \cong$ EF $\quad \overline{AC} \cong$ DF

Drag vertex *A* to form a new triangle. Try to find a $\triangle ABC$ that is not congruent to $\triangle DEF$.

Check Click to measure angles.

Use the on-screen protractor to measure $\angle A$, $\angle B$, and $\angle C$. Answers will vary.

$m\angle A =$ _____ $\quad m\angle B =$ _____

$m\angle C =$ _____

Use the on-screen protractor to measure $\angle D$, $\angle E$, and $\angle F$. Answers will vary.

$m\angle D =$ _____ $\quad m\angle E =$ _____

$m\angle F =$ _____

Are the corresponding angles congruent? yes

Are the triangles congruent? yes

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

4 Uncheck all Click to measure angles.

Click on the arrows on the Conditions menu. Choose SAS.

$\angle$ A $\cong \angle$ D $\quad \overline{AB} \cong \overline{DE} \quad \overline{AC} \cong \overline{DF}$

Drag vertex *A* to form a new triangle. Try to find a $\triangle ABC$ that is not congruent to $\triangle DEF$.

Check Click to measure lengths.

Use the ruler to measure $\overline{BC}$ and $\overline{EF}$. Answers will vary.

$\overline{BC}$ = _____ units $\quad \overline{EF}$ = _____ units

Are the sides congruent? yes

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Uncheck Click to measure lengths.

Check Click to measure angles.

Use the ruler to measure $\angle B$, $\angle C$, $\angle E$, and $\angle F$. **Answers will vary.**

$m\angle B =$ _____ $m\angle C =$ _____

$m\angle E =$ _____ $m\angle F =$ _____

Are the corresponding angles congruent? **yes**

Are the triangles congruent? **yes**

If two sides and the angle between them in one triangle are congruent to two sides and the angle between them in another triangle, then the triangles _____**are**_____ congruent.

DISCOVER BOX

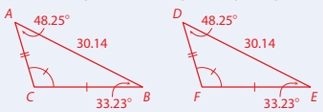
Describe two rules that you can use to determine whether two triangles are congruent.

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. If two sides and the angle between them in one triangle are congruent to two sides and the angle between them in another triangle, then the triangles are congruent.

EXPLORE MORE

Use the Gizmo to answer the question. Draw a diagram to support your answer.

5 Click on the arrows on the Conditions menu. Choose ASA. If two angles and the side between them in one triangle are congruent to two angles and the side between them in another triangle, are the triangles always congruent? **Yes. Sample diagram:**



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DISCOVER BOX

Look at problems 3 and 4. It is not necessary to know that all pairs of corresponding angles and all pairs of corresponding sides are congruent to conclude that two triangles are congruent. In some cases, it is enough to know that some of the pairs of corresponding sides and/or corresponding angles are congruent. Use the results from problems 3 and 4 to write two rules that you could use to determine whether two triangles are congruent. (If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. If two sides and the angle between them in one triangle are congruent to two sides and the angle between them in another triangle, then the triangles are congruent.)

EXPLORE MORE

Problem 5

Have students work individually. In this problem, students will repeat the processes in problems 3 and 4 to decide whether given conditions about congruent angles and sides are enough to determine whether two triangles are congruent.

Extension: Have students or partners draw pairs of triangles, then take turns explaining the rules to determine whether the triangles are congruent.

Lesson 6

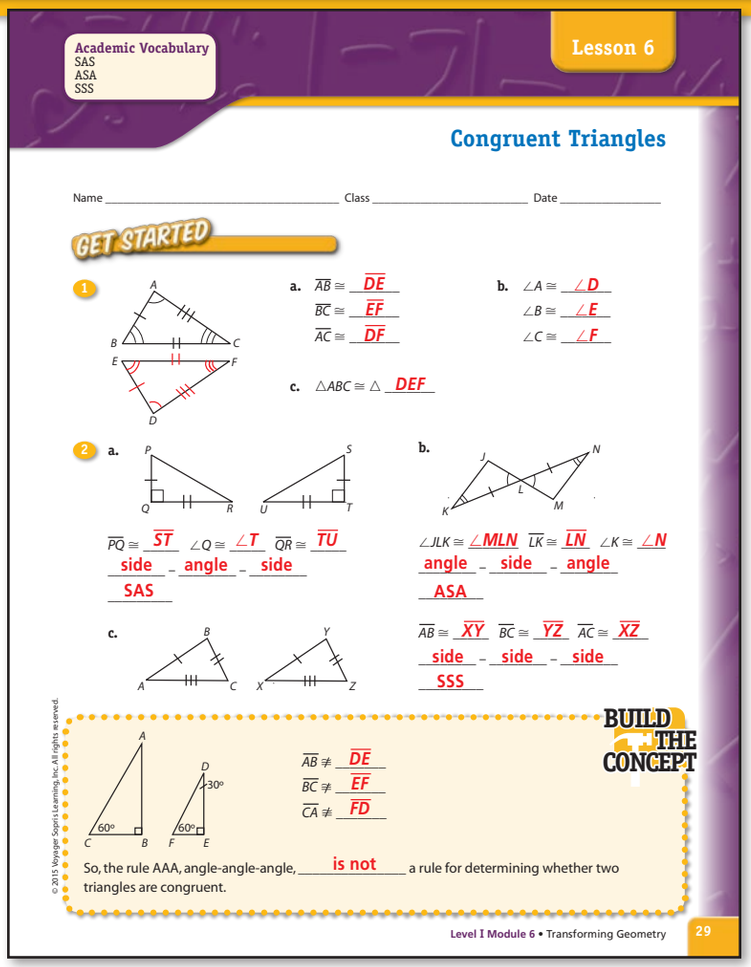
Objective

To apply the rules for congruent triangles to determine if two triangles are congruent

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **SAS**
  SAS stands for side-angle-side. The rule states that if two sides and the angle between the sides of one triangle are congruent to two sides and the angle between the sides of another triangle, the triangles are congruent.
- **ASA**
  ASA stands for angle-side-angle. The rule states that if two angles and the side between the angles of one triangle are congruent to two angles and the side between the angles of another triangle, the triangles are congruent.
- **SSS**
  SSS stands for side-side-side. The rule states that if three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.


Academic Vocabulary
SAS
ASA
SSS

Lesson 6

Congruent Triangles

Name _____ Class _____ Date _____

GET STARTED

1

a. $\overline{AB} \cong$ $\overline{DE}$
$\overline{BC} \cong$ $\overline{EF}$
$\overline{AC} \cong$ $\overline{DF}$

b. $\angle A \cong$ $\angle D$
$\angle B \cong$ $\angle E$
$\angle C \cong$ $\angle F$

c. $\triangle ABC \cong \triangle$ DEF

2 a.

$\overline{PQ} \cong$ $\overline{ST}$ $\angle Q \cong$ $\angle T$ $\overline{QR} \cong$ $\overline{TU}$
side – angle – side
SAS

b.

$\angle JLK \cong \angle MLN$ $\overline{LK} \cong \overline{LN}$ $\angle K \cong \angle N$
angle – side – angle
ASA

c.

$\overline{AB} \cong$ $\overline{XY}$ $\overline{BC} \cong$ $\overline{YZ}$ $\overline{AC} \cong$ $\overline{XZ}$
side – side – side
SSS

BUILD THE CONCEPT

$\overline{AB} \neq$ $\overline{DE}$
$\overline{BC} \neq$ $\overline{EF}$
$\overline{CA} \neq$ $\overline{FD}$

So, the rule AAA, angle-angle-angle, is not a rule for determining whether two triangles are congruent.

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

The triangles in problem 1 are congruent. Which segment corresponds to $\overline{AB}$? ($\overline{DF}$) Read the statement as *segment AB is congruent to segment DE*. Notice that $\overline{AB}$ has one slash mark on it. Draw one slash mark on $\overline{DE}$ to show that the two segments are congruent. Which segment corresponds to and is congruent to $\overline{BC}$? ($\overline{EF}$) Segment BC has two slash marks on it. Draw two slash marks on $\overline{EF}$. Which segment corresponds to and is congruent to $\overline{AC}$? ($\overline{DF}$) How many slash marks should be drawn on $\overline{DF}$? (3) Draw the slash marks on $\overline{DF}$.

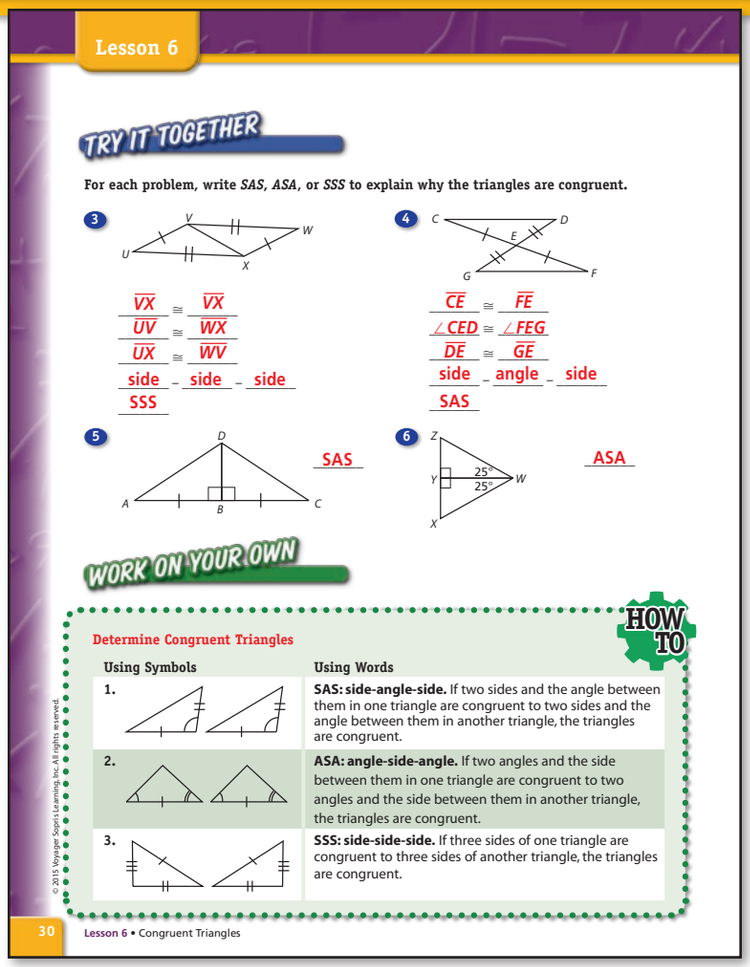
Look at problem 1b. Which angle corresponds and is congruent to $\angle A$? ($\angle D$) Notice that one arc is drawn on $\angle A$. Draw one arc on $\angle D$. Which angle corresponds and is congruent to $\angle B$? ($\angle E$) How many arcs should be drawn on $\angle E$? (2) Which angle corresponds to and is congruent to $\angle C$? ($\angle F$) How many arcs should be drawn on $\angle F$? (3)

Look at problem 1c. To write a statement that shows two figures are congruent, write the vertices of the second figure in the correct order in relation to the order of the vertices in the first figure. What triangle is congruent to $\triangle ABC$? ($\triangle DEF$)

MODEL NEW SKILLS

Problem 2

Special rules can be used to determine whether two triangles are congruent. The first rule states that if two sides and the angle between them in one triangle are congruent to two sides and the angle between them in the other triangle, the triangles are congruent. This rule is called the side-angle-side rule, or SAS.


Lesson 6

TRY IT TOGETHER

For each problem, write *SAS*, *ASA*, or *SSS* to explain why the triangles are congruent.

3

$\overline{VX} \cong \overline{VX}$
$\overline{UV} \cong \overline{WX}$
$\overline{UX} \cong \overline{WV}$
side – side – side
SSS

4

$\overline{CE} \cong \overline{FE}$
$\angle CED \cong \angle FEG$
$\overline{DE} \cong \overline{GE}$
side – angle – side
SAS

5

SAS

6

ASA

WORK ON YOUR OWN

HOW TO

Determine Congruent Triangles

	Using Symbols	Using Words
1.		**SAS: side-angle-side.** If two sides and the angle between them in one triangle are congruent to two sides and the angle between them in another triangle, the triangles are congruent.
2.		**ASA: angle-side-angle.** If two angles and the side between them in one triangle are congruent to two angles and the side between them in another triangle, the triangles are congruent.
3.		**SSS: side-side-side.** If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

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30 Lesson 6 • Congruent Triangles

Look at problem 2a. The slash marks show the sides that are congruent. Which side in $\triangle STU$ is congruent to $\overline{PQ}$? ($\overline{ST}$) **Which side is congruent to $\overline{QR}$?** ($\overline{TU}$) **What kind of angle is between these two sides in each triangle?** (right angle) **Because two sides and the angle between the sides of $\triangle PQR$ are congruent to two sides and the angle between the sides of $\triangle STU$, the SAS rule explains why the triangles are congruent.**

The second rule states that if two angles and the side between the angles of one triangle are congruent to two angles and the side between the angles of the other triangle, then the triangles are congruent. This rule is called the *angle-side-angle rule*, or ASA.

Look at problem 2b. The slash marks show the sides that are congruent. The arcs show the angles that are congruent. Notice that $\angle JLK$ and $\angle MLN$ are vertical angles. What is true about vertical angles? (They are congruent.) **What angle is congruent to $\angle K$?** ($\angle N$) **What side is congruent to $\overline{LK}$?** ($\overline{LN}$) **Because two angles and the side between the angles of $\triangle JLK$ are congruent to two angles and the side between the angles of $\triangle MLN$, the ASA rule explains why the triangles are congruent.**

The third rule states that if three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent. The rule is called the side-side-side rule, or SSS.

Look at problem 2c. What side is congruent to $\overline{AB}$? ($\overline{XY}$) **What side is congruent to $\overline{BC}$?** ($\overline{YZ}$) **What side is congruent to $\overline{AC}$?** ($\overline{XZ}$) **Because all three sides are congruent, the SSS rule explains why these triangles are congruent.**

BUILD THE CONCEPT

Model how to apply the rules for congruent triangles to determine whether two triangles are congruent.

Look at the two triangles, $\triangle ABC$ and $\triangle DEF$. What kind of triangles are they? (right triangles) **If these two triangles were placed on top of each other, what angle would correspond with $\angle A$?** ($\angle D$) **What angle would correspond with $\angle B$?** ($\angle E$) **What angle would correspond with $\angle C$?** ($\angle F$) **How do these angles compare to their corresponding angles?** (They are equal.)

What side corresponds to $\overline{AB}$? ($\overline{DE}$) **Are these two sides equal in length?** (no) **What side corresponds to $\overline{BC}$?** ($\overline{EF}$) **Are these two sides equal in length?** (no) **What side corresponds to $\overline{CA}$?** ($\overline{FD}$) **Are these two sides equal in length?** (no) **Are these triangles congruent?** (no) **So, even though all of the angles have the same measure, the triangles are not congruent because the sides are not equal in length. That is why there is no AAA, angle-angle-angle, rule for proving triangles congruent.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 3

Notice that $\triangle UVX$ and $\triangle WXV$ share a common side, $\overline{VX}$. Because a segment is congruent to itself, it is possible to write a congruency statement that says that $\overline{VX}$ is congruent to itself. What segment is congruent to $\overline{VX}$? ($\overline{VX}$) **This accounts for one pair of congruent sides. What is true about $\overline{UV}$ and $\overline{WX}$?** (They are congruent.) **What is true about $\overline{UX}$ and $\overline{WV}$?** (They are congruent.)

Then three sides of $\triangle UVX$ are congruent to three sides of $\triangle WXV$. Which rule explains why these triangles are congruent? (SSS)

Problem 4

How many pairs of congruent sides are marked on these triangles? (2) **What are the congruent sides?** ($\overline{CE}$ and $\overline{FE}$, and $\overline{DE}$ and $\overline{GE}$) **Are $\angle CED$ and $\angle FEG$ congruent?** (yes) **Why?** (They are vertical angles, and vertical angles are congruent.) **What rule explains why these triangles are congruent?** (SAS)

Problem 5

What common side is shared by $\triangle ABD$ and $\triangle CBD$? ($\overline{BD}$) **What two sides are labeled as congruent in $\triangle ABD$ and $\triangle CBD$?** ($\overline{AB}$ and $\overline{CB}$) **What is true about $\angle ABD$ and $\angle CBD$?** (They are congruent.) **What rule explains why these triangles are congruent?** (SAS)

Problem 6

What rule explains why these triangles are congruent? (ASA)

WORK ON YOUR OWN

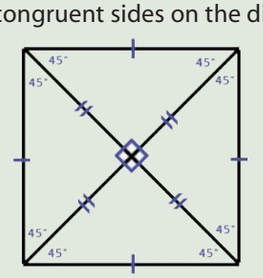
MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Problem-Solving Plan** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 12.

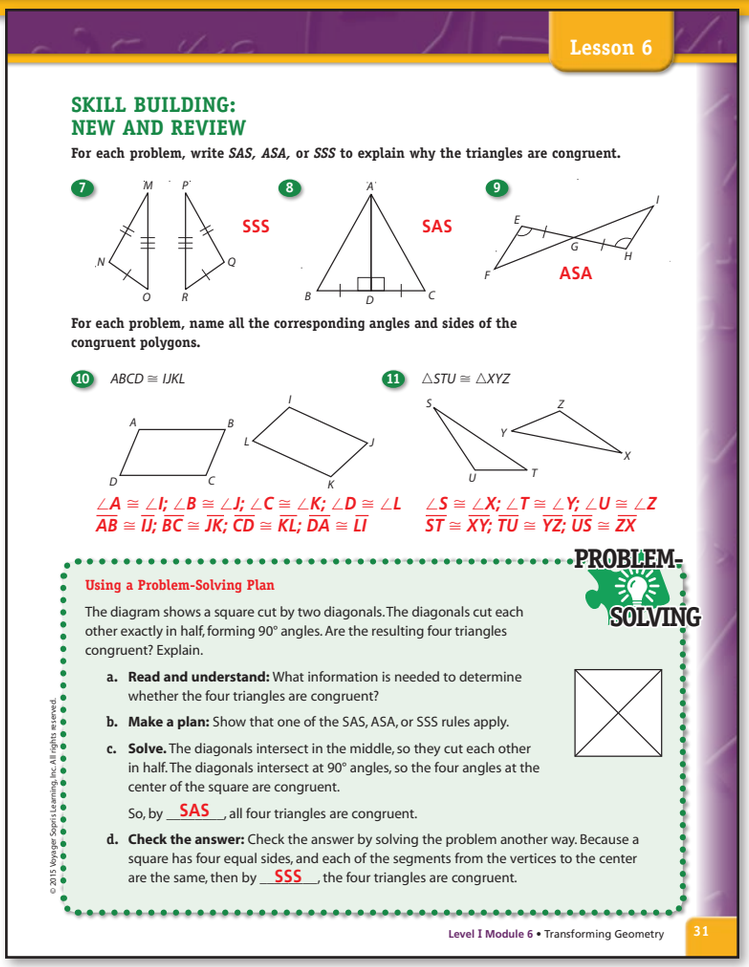
Have students read the problem.

It may be helpful for students to label the angle measures and congruent sides on the diagram.



Students can see that the four triangles are congruent by the SAS rule.

The answer is reasonable because the triangles are also congruent by the SSS rule. This can easily be seen by the diagram.



Lesson 6

SKILL BUILDING: NEW AND REVIEW

For each problem, write *SAS*, *ASA*, or *SSS* to explain why the triangles are congruent.

7 SSS

8 SAS

9 ASA

For each problem, name all the corresponding angles and sides of the congruent polygons.

10 $ABCD \cong IJKL$

$\angle A \cong \angle I; \angle B \cong \angle J; \angle C \cong \angle K; \angle D \cong \angle L$

$\overline{AB} \cong \overline{IJ}; \overline{BC} \cong \overline{JK}; \overline{CD} \cong \overline{KL}; \overline{DA} \cong \overline{LI}$

11 $\triangle STU \cong \triangle XYZ$

$\angle S \cong \angle X; \angle T \cong \angle Y; \angle U \cong \angle Z$

$\overline{ST} \cong \overline{XY}; \overline{TU} \cong \overline{YZ}; \overline{US} \cong \overline{ZX}$

PROBLEM-SOLVING

Using a Problem-Solving Plan

The diagram shows a square cut by two diagonals. The diagonals cut each other exactly in half, forming 90° angles. Are the resulting four triangles congruent? Explain.

a. **Read and understand:** What information is needed to determine whether the four triangles are congruent?

b. **Make a plan:** Show that one of the SAS, ASA, or SSS rules apply.

c. **Solve.** The diagonals intersect in the middle, so they cut each other in half. The diagonals intersect at 90° angles, so the four angles at the center of the square are congruent.

So, by SAS, all four triangles are congruent.

d. **Check the answer:** Check the answer by solving the problem another way. Because a square has four equal sides, and each of the segments from the vertices to the center are the same, then by SSS, the four triangles are congruent.

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Problems 7–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 8 Reminder

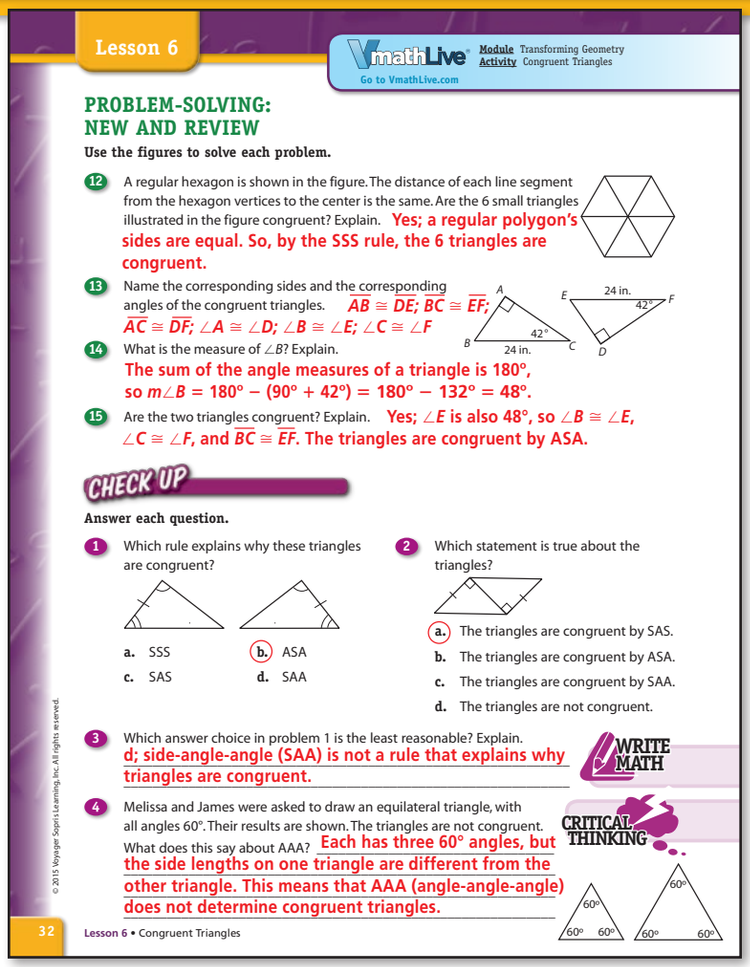
The common line segment between the two triangles is a side of both triangles and accounts for one pair of congruent sides.

Problem 9 Reminder

The vertical angles in the figure are equal.

Problems 14 and 15 Reminder

Students need to use the fact that the sum of the angles in a triangle is 180° to solve problem 14. They will then need to compare the measure of $\angle E$ to the measure of $\angle B$ in order to use ASA in problem 15.



Lesson 6

VmathLive Module Transforming Geometry Activity Congruent Triangles Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Use the figures to solve each problem.

12. A regular hexagon is shown in the figure. The distance of each line segment from the hexagon vertices to the center is the same. Are the 6 small triangles illustrated in the figure congruent? Explain. Yes; a regular polygon's sides are equal. So, by the SSS rule, the 6 triangles are congruent.

13. Name the corresponding sides and the corresponding angles of the congruent triangles. $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$; $\overline{AC} \cong \overline{DF}$; $\angle A \cong \angle D$; $\angle B \cong \angle E$; $\angle C \cong \angle F$

14. What is the measure of $\angle B$? Explain. The sum of the angle measures of a triangle is 180°, so $m\angle B = 180° - (90° + 42°) = 180° - 132° = 48°$.

15. Are the two triangles congruent? Explain. Yes; $\angle E$ is also 48°, so $\angle B \cong \angle E$, $\angle C \cong \angle F$, and $\overline{BC} \cong \overline{EF}$. The triangles are congruent by ASA.

CHECK UP

Answer each question.

1. Which rule explains why these triangles are congruent?

a. SSS (b.) ASA c. SAS d. SAA

2. Which statement is true about the triangles?

(a.) The triangles are congruent by SAS.
b. The triangles are congruent by ASA.
c. The triangles are congruent by SAA.
d. The triangles are not congruent.

3. Which answer choice in problem 1 is the least reasonable? Explain. d; side-angle-angle (SAA) is not a rule that explains why triangles are congruent.

WRITE MATH

4. Melissa and James were asked to draw an equilateral triangle, with all angles 60°. Their results are shown. The triangles are not congruent. What does this say about AAA? Each has three 60° angles, but the side lengths on one triangle are different from the other triangle. This means that AAA (angle-angle-angle) does not determine congruent triangles.

CRITICAL THINKING

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CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problems 1 or 2 incorrectly: The student did not understand the meaning of SAS, ASA, or SSS. Use Additional Resources in I.6.6 to reteach rules for congruent triangles.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that the rules are side-angle-side (SAS), angle-side-angle (ASA), and side-side-side (SSS). Point out that side-angle-angle is not a congruency rule.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Transforming Geometry
Activity: Congruent Triangles

Vmath Reteach
Reteach Student Book Module 6 Lesson 6
Reteach Teacher Edition Module 6 Lesson 6

Extra Practice
Student Book page 51

English Language Learners

Remind students that an acronym is a series of letters, each of which represents a word. For example, PTA stands for Parent Teacher Association, and PC stands for personal computer. Have students brainstorm a few others they know, such as USA, or the school's initials. Explain that some terms used in this lesson are acronyms. Tell students that in all three acronyms in this lesson, the letter *A* stands for *angle* and the letter *S* stands for *side*.

Students with Special Needs

Some students may have difficulty visualizing corresponding sides and corresponding angles of given triangles that are oriented differently. Using an enlarged copy of the lesson, cut out one of each of the paired triangles used to illustrate the lesson examples or problems. Using the cutout as a pattern, trace around one of the triangles on the chalkboard or overhead, oriented as one of the triangles in an example or problem. Reorient the triangle pattern and retrace to show the second triangle. Indicate congruent sides and angles of the triangles. This visual aid will help students get a clearer picture of the corresponding parts.

Lesson 7

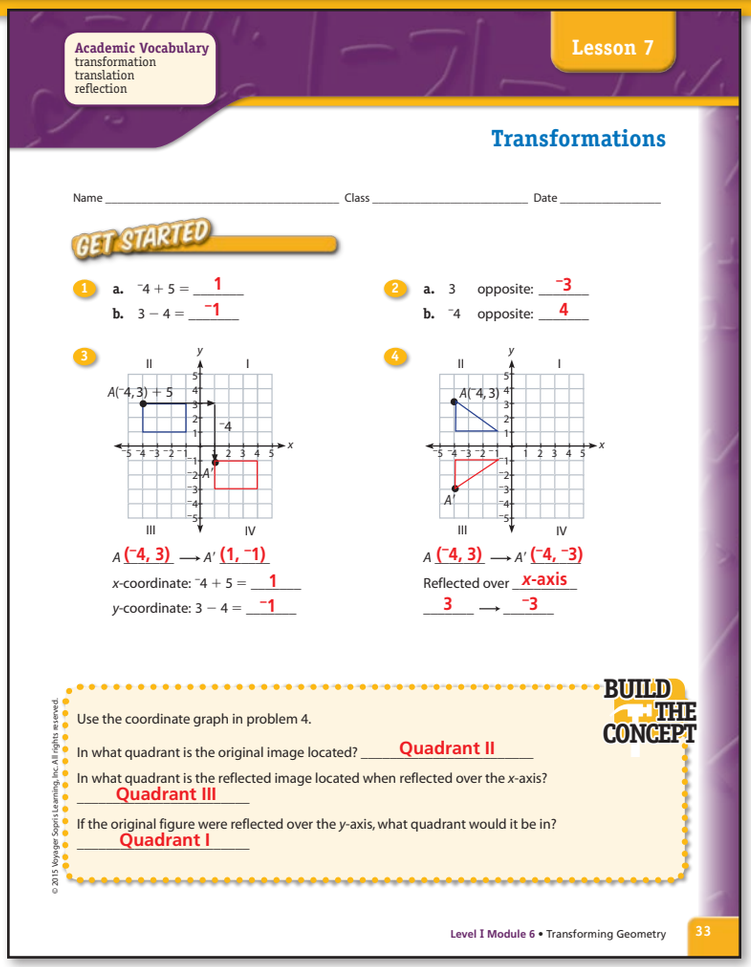
Objective

To identify and perform translations and reflections on the coordinate plane

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **transformation**
  a translation, a reflection, or a rotation
- **translation**
  moving a geometric figure by sliding
- **reflection**
  moving a geometric figure by flipping it over a line



Academic Vocabulary
transformation
translation
reflection

Lesson 7

Transformations

Name _____ Class _____ Date _____

GET STARTED

1 a. $^{-}4 + 5 =$ 1
b. $3 - 4 =$ $^{-}1$

2 a. 3 opposite: $^{-}3$
b. $^{-}4$ opposite: 4

3 A(−4, 3) + 5 → A′ (1, −1)
x-coordinate: $^{-}4 + 5 =$ 1
y-coordinate: $3 - 4 =$ $^{-}1$

4 A (−4, 3) → A′ (−4, −3)
Reflected over x-axis
3 → −3

BUILD THE CONCEPT

Use the coordinate graph in problem 4.

In what quadrant is the original image located? Quadrant II

In what quadrant is the reflected image located when reflected over the *x*-axis? Quadrant III

If the original figure were reflected over the *y*-axis, what quadrant would it be in? Quadrant I

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Level I Module 6 • Transforming Geometry 33

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What is $^{-}4 + 5$? (1) **What is 3 minus 4?** ($^{-}1$)

Problem 2

What is the opposite of 3? ($^{-}3$) **What is the opposite of $^{-}4$?** (4)

MODEL NEW SKILLS

Problem 3

A transformation is the movement of a figure in a plane. In problem 3, the transformation shown is a slide, or translation. The original figure is shown in blue with one vertex labeled *A*. The transformed image is shown in red. Notice that both the original figure and its image are congruent. Only their positions on the coordinate plane are different.

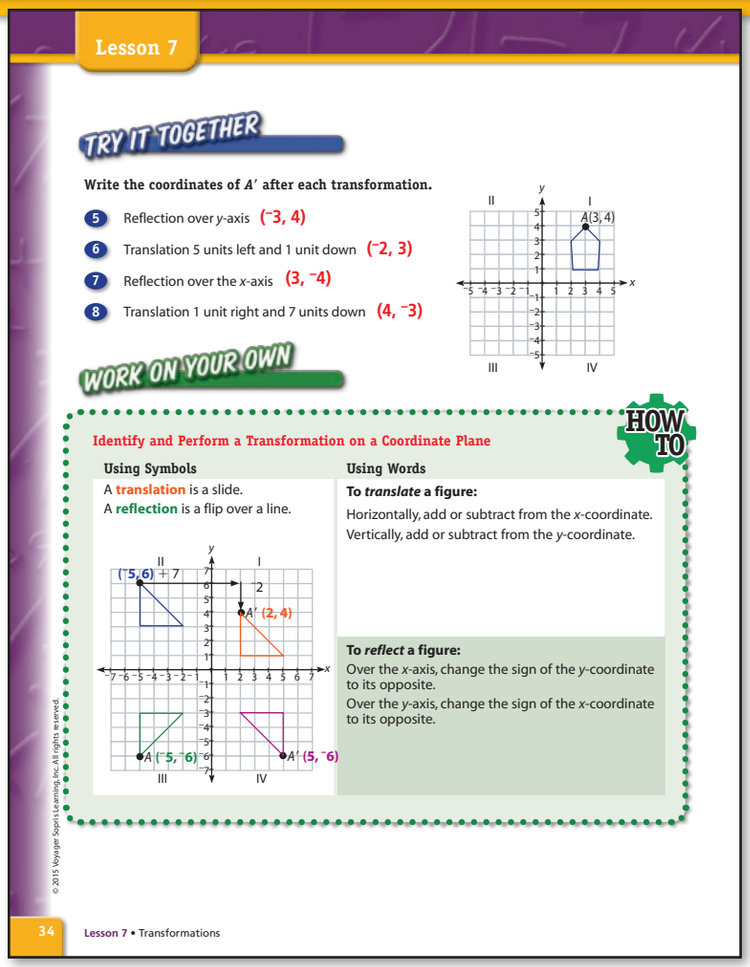
The image of vertex *A* is labeled *A*′, which is read *A prime*. What are the coordinates of *A*? (($^{-}4$, 3)) **It is possible to determine the coordinates of *A*′ when a figure has been translated. For horizontal movement, add to or subtract from the *x*-coordinate. For movement to the right, add; for movement to the left, subtract. For vertical movement, add to or subtract from the *y*-coordinate. For movement up, add; for movement down, subtract.**

The figure in problem 3 has been translated 5 units to the right. Add 5 to the *x*-coordinate. What is $^{-}4$ plus 5? (1) **The *x*-coordinate of *A*′ is 1. The figure has been moved 4 units down. Subtract 4 units from the *y*-coordinate. What is 3 minus 4?** ($^{-}1$) **The *y*-coordinate of *A*′ is $^{-}1$. The coordinates of *A*′ are (1, $^{-}1$).**

Problem 4

In problem 4, a flip over a line, or a reflection, is shown. A reflection is another type of transformation that results in an image congruent to the original figure. The original figure is shown in blue, and its image is shown in red. The figure was reflected over the *x*-axis. Notice that the two triangles are congruent. Only their positions on the coordinate plane are different.

Notice the position of vertex *A* and its image *A*′. What are the coordinates of *A*? (($^{-}4$, 3)) **It is possible to determine the coordinates of *A*′ when a figure has been reflected. For a reflection over the *x*-axis, change the sign of the**


Lesson 7

TRY IT TOGETHER

Write the coordinates of A' after each transformation.

5. Reflection over *y*-axis $(^-3, 4)$
6. Translation 5 units left and 1 unit down $(^-2, 3)$
7. Reflection over the *x*-axis $(3, ^-4)$
8. Translation 1 unit right and 7 units down $(4, ^-3)$

WORK ON YOUR OWN

HOW TO

Identify and Perform a Transformation on a Coordinate Plane

Using Symbols	Using Words
A **translation** is a slide. A **reflection** is a flip over a line.	**To *translate* a figure:** Horizontally, add or subtract from the *x*-coordinate. Vertically, add or subtract from the *y*-coordinate.
	To *reflect* a figure: Over the *x*-axis, change the sign of the *y*-coordinate to its opposite. Over the *y*-axis, change the sign of the *x*-coordinate to its opposite.

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***y*-coordinate to its opposite. For a reflection over the *y*-axis, change the sign of the *x*-coordinate to its opposite. Over which axis was the figure in problem 4 reflected?** (*x*-axis) **What is the *y*-coordinate?** (3) **What is the opposite of 3?** ($^-3$) **The *x*-coordinate stays the same. The coordinates of *A′* are ($^-4$, $^-3$).**

BUILD THE CONCEPT

Model how to identify a reflection on a coordinate plane.

A transformation is movement of a figure within a plane. One type of transformation is called the *reflection*. A reflection occurs over a line. Look at the coordinate plane for problem 4. In what quadrant is the original image? (Quadrant II) **In what quadrant is the reflected image when reflected over the *x*-axis?** (Quadrant III)

Now, imagine the original figure reflected over the *y*-axis. What quadrant would the reflected image be in? (Quadrant I)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What transformation is asked for in this problem? (reflection) **Over which axis is the figure to be reflected?** (*y*-axis) **Which coordinate will change?** (*x*-coordinate) **How will it change?** (It will change to its opposite.)

What is the *x*-coordinate? (3) **What is the opposite of 3?** ($^-3$) **Does the *y*-coordinate change?** (no) **What are the coordinates of *A′*?** ($(^-3, 4)$)

Problem 6

What transformation is asked for in this problem? (translation) **Is the figure to move to the left or right?** (left) **Does this change the *x*-coordinate or *y*-coordinate?** (*x*-coordinate) **How many units to the left is the figure to be moved?** (5) **Is 5 added to or subtracted from the *x*-coordinate?** (subtracted) **What is the *x*-coordinate of *A′*?** ($^-2$)

Is the figure to move up or down? (down) **Will this change the *x*-coordinate or *y*-coordinate?** (*y*-coordinate) **How many units down is the figure to be moved?** (1) **Is 1 added to or subtracted from the *y*-coordinate?** (subtracted) **What is the *y*-coordinate of *A′*?** (3) **What ordered pair gives the coordinates of *A′*?** ($(^-2, 3)$)

Problem 7

Over which axis is the figure to be reflected? (*x*-axis) **Which coordinate will change?** (*y*-coordinate) **How will it change?** (It will change to its opposite.) **What is the opposite of 4?** ($^-4$) **What will be the coordinates of *A′*?** ($(3, ^-4)$)

Problem 8

What are the coordinates of *A′*? ($(4, ^-3)$) **Why?** (The figure is translated 1 unit to the right, so add 1 to the *x*-coordinate, 3, to get 4. The figure is translated down 7 units, so subtract 7 from the *y*-coordinate, 4, to get $^-3$.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Drawing a Picture** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a picture. The instruction is immediately followed by application of the strategy in problem 20.

Have students read the problem.

Be sure that students reflect the images as opposed to translating them.

Be sure that students reflect the images over the correct axis. If students reflect over the x-axis, neither letter will remain the same.

Be sure that students know that the reflections will be in Quadrant II.

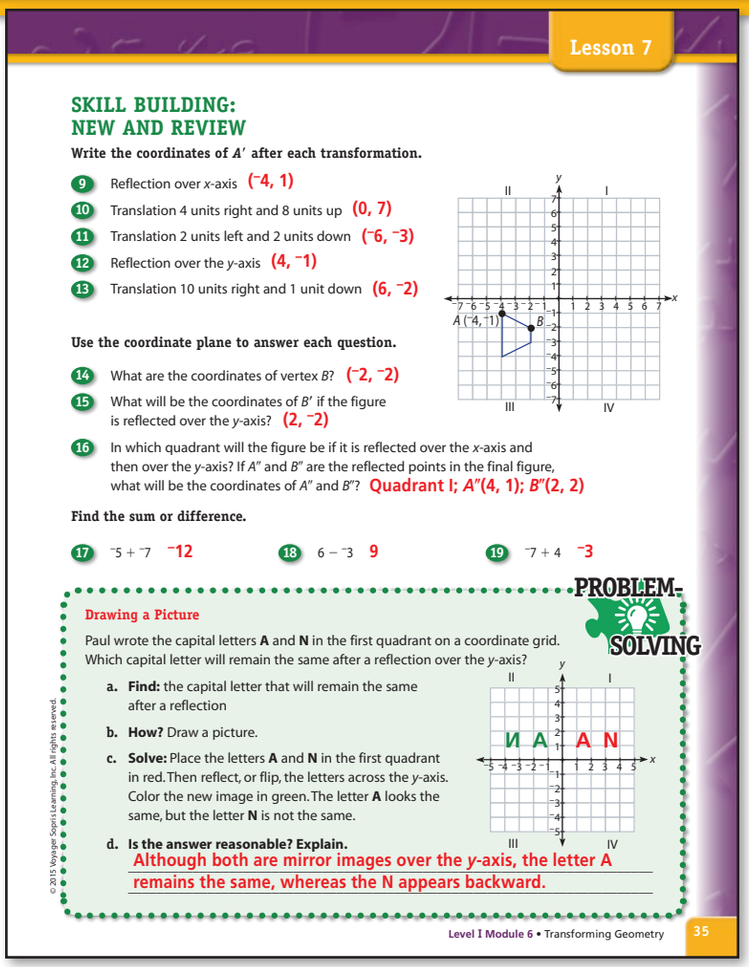
After reflecting, the letter **A** remains the same.

The letter **N** looks like the original, but the slanted line is incorrect.

The answer is reasonable because the letter **A** appears unchanged; the letter **N** appears backward.

Problems 9–22

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.



Lesson 7

SKILL BUILDING: NEW AND REVIEW

Write the coordinates of *A′* after each transformation.

9. Reflection over *x*-axis (−4, 1)
10. Translation 4 units right and 8 units up (0, 7)
11. Translation 2 units left and 2 units down (−6, −3)
12. Reflection over the *y*-axis (4, −1)
13. Translation 10 units right and 1 unit down (6, −2)

Use the coordinate plane to answer each question.

14. What are the coordinates of vertex *B*? (−2, −2)
15. What will be the coordinates of *B′* if the figure is reflected over the *y*-axis? (2, −2)
16. In which quadrant will the figure be if it is reflected over the *x*-axis and then over the *y*-axis? If *A″* and *B″* are the reflected points in the final figure, what will be the coordinates of *A″* and *B″*? Quadrant I; *A″*(4, 1); *B″*(2, 2)

Find the sum or difference.

17. −5 + −7 −12
18. 6 − −3 9
19. −7 + 4 −3

PROBLEM-SOLVING

Drawing a Picture

Paul wrote the capital letters **A** and **N** in the first quadrant on a coordinate grid. Which capital letter will remain the same after a reflection over the *y*-axis?

a. **Find:** the capital letter that will remain the same after a reflection

b. **How?** Draw a picture.

c. **Solve:** Place the letters **A** and **N** in the first quadrant in red. Then reflect, or flip, the letters across the *y*-axis. Color the new image in green. The letter **A** looks the same, but the letter **N** is not the same.

d. **Is the answer reasonable? Explain.** Although both are mirror images over the *y*-axis, the letter A remains the same, whereas the N appears backward.

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Problem 16 Reminder

This problem asks students to perform two reflections of the figure and give the coordinates of *A″* (*A* double prime) and *B″* (*B* double prime). Suggest students apply the rules for reflecting a figure to solve the problem.

Problem 21 Reminder

Students may have to use a coordinate plane to draw the transformation before naming it.

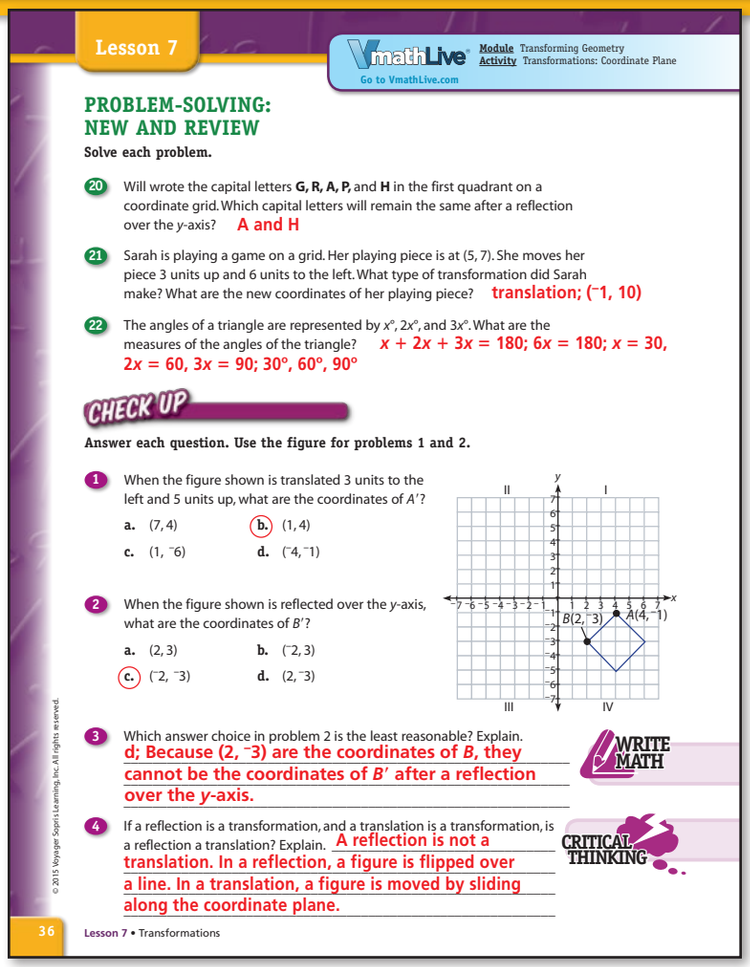
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1c: The student either added to the coordinate instead of subtracting or subtracted from the coordinate instead of adding. Remind the student that when a figure moves left, the number of units is subtracted from the *x*-coordinate. When a figure moves up, the number of units is added to the *y*-coordinate.



Lesson 7

VmathLive Module Transforming Geometry Activity Transformations: Coordinate Plane Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

20 Will wrote the capital letters **G, R, A, P,** and **H** in the first quadrant on a coordinate grid. Which capital letters will remain the same after a reflection over the *y*-axis? A and H

21 Sarah is playing a game on a grid. Her playing piece is at (5, 7). She moves her piece 3 units up and 6 units to the left. What type of transformation did Sarah make? What are the new coordinates of her playing piece? translation; (⁻1, 10)

22 The angles of a triangle are represented by $x°$, $2x°$, and $3x°$. What are the measures of the angles of the triangle? $x + 2x + 3x = 180$; $6x = 180$; $x = 30$, $2x = 60$, $3x = 90$; 30°, 60°, 90°

CHECK UP

Answer each question. Use the figure for problems 1 and 2.

1 When the figure shown is translated 3 units to the left and 5 units up, what are the coordinates of *A*′?

a. (7, 4) (b.) (1, 4)

c. (1, ⁻6) d. (⁻4, ⁻1)

2 When the figure shown is reflected over the *y*-axis, what are the coordinates of *B*′?

a. (2, 3) b. (⁻2, 3)

(c.) (⁻2, ⁻3) d. (2, ⁻3)

3 Which answer choice in problem 2 is the least reasonable? Explain. d; Because (2, ⁻3) are the coordinates of *B*, they cannot be the coordinates of *B*′ after a reflection over the *y*-axis.

WRITE MATH

4 If a reflection is a transformation, and a translation is a transformation, is a reflection a translation? Explain. A reflection is not a translation. In a reflection, a figure is flipped over a line. In a translation, a figure is moved by sliding along the coordinate plane.

CRITICAL THINKING

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36 Lesson 7 • Transformations

If student answered 1d: The student reflected the figure instead of translating it. Use Additional Resources in I.6.7 to reteach transformations.

If student answered problem 2 incorrectly: The student did not change the signs correctly when making a reflection. Use Additional Resources in I.6.7 to reteach transformations.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that when a figure is reflected over the *y*-axis, the sign of the *x*-coordinates change to the opposite. Because (2, ⁻3) are the coordinates of *B*, choice d cannot be the answer.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Transforming Geometry
Activity: Transformations: Coordinate Plane

Vmath Reteach
Reteach Student Book Module 6 Lesson 7
Reteach Teacher Edition Module 6 Lesson 7

Extra Practice
Student Book page 52

English Language Learners

Use the VmathLive Animated Glossary to review the terms *transformation, reflection,* and *translation*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Students may confuse the words *translation* and *transformation*. Explain to students that a translation is one type of transformation. Use a diagram to represent the different types of transformations. Have students say aloud the types of transformations as they are pointed out.

Students with Special Needs

Have students use algebra tiles and grid paper to help them classify types of transformations. Have students mimic the slide or flip with their algebra tiles to determine the type of transformation. For example, place a tile in Quadrant I and place an identical tile in Quadrant II. Ask students to describe what type of moves the tiles represent from Quadrant I to Quadrant II. Repeat with other movements.

Lesson 8

Objective

To identify and perform rotations on the coordinate plane

Preskills	Lesson
Coordinate Planes in Four Quadrants	I.4.PL1

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **rotation about the origin**
  moving a geometric figure by turning it about the origin

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

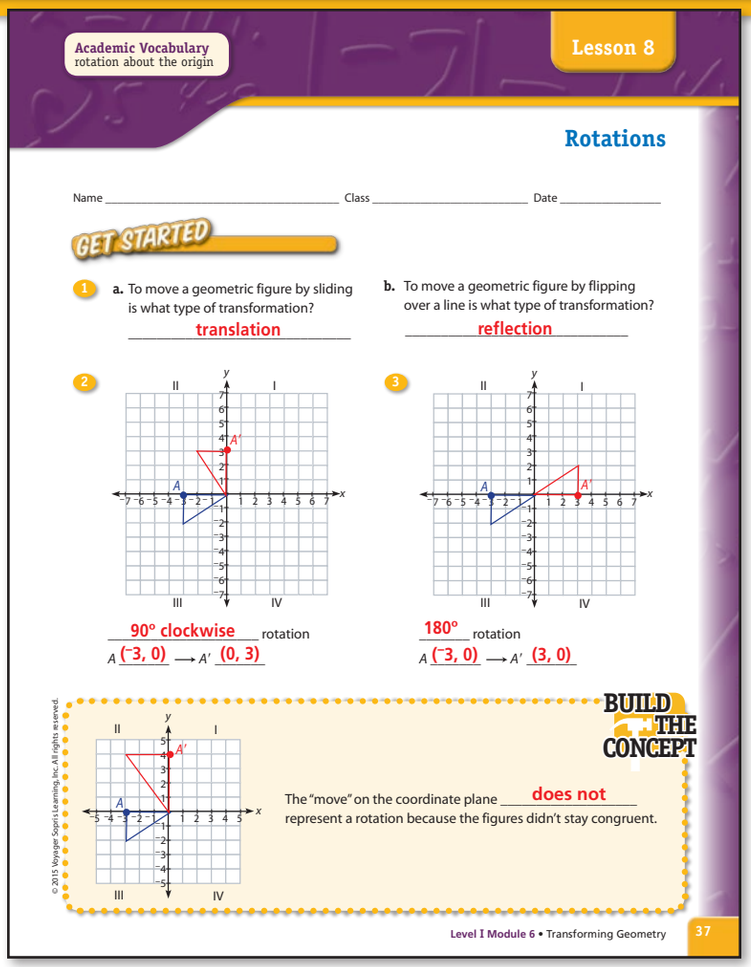
Problem 1

A transformation is a move of a geometric figure without changing the shape of the figure. Look at the question in problem 1a. What type of transformation is a slide? (translation) **What type of transformation is a flip?** (reflection)

MODEL NEW SKILLS

Problem 2

A rotation is another type of transformation that results in an image congruent to the original figure. Some possible ways a figure can be rotated are 90° clockwise, 90° counterclockwise, and 180°. In a clockwise rotation, the figure turns in the direction of the hands of a clock. In a counterclockwise rotation, the figure turns in the opposite direction. In problem 2, the blue figure has been rotated 90° about the origin. The rotated image is shown in red. In which direction was the rotation, clockwise or counterclockwise? (clockwise) **The image was rotated 90° clockwise. The original figure is shown with vertex *A* labeled. In the rotated figure, the image of vertex *A* is labeled *A*′. What are the coordinates of *A*?** ((¯3, 0)) **What are the coordinates of *A*′?** ((0, 3))



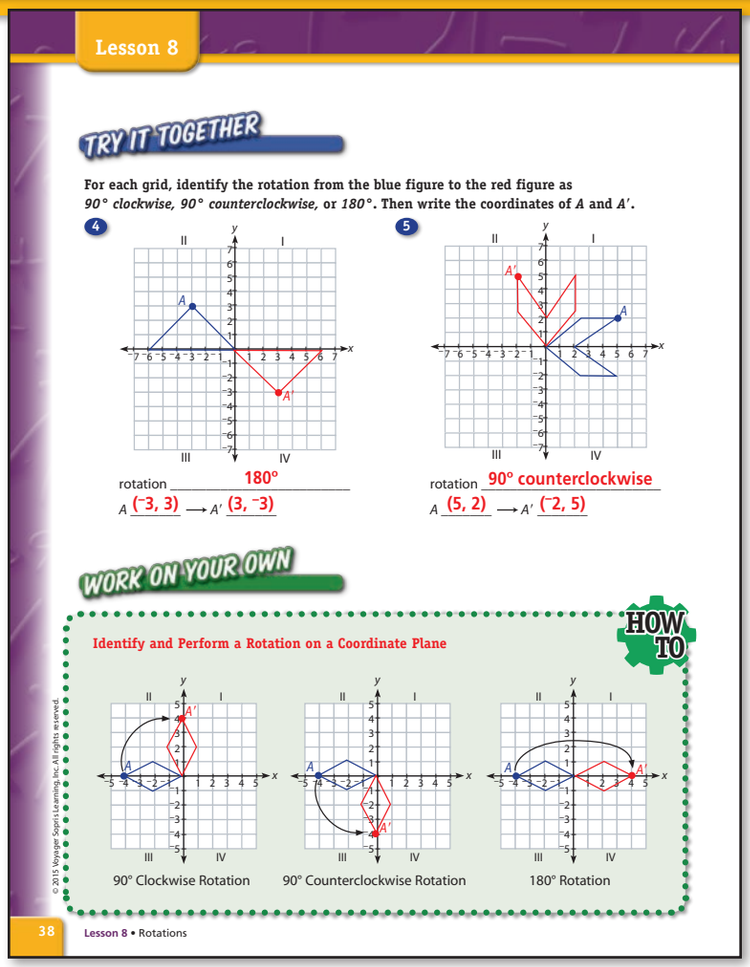
Problem 3

In problem 3, the original figure, shown in blue, has been rotated 180° about the origin. The translated image is red. The original figure is shown with vertex *A* labeled. In the rotated figure, the image of vertex *A* is labeled *A*′. What are the coordinates of *A*? ((¯3, 0)) **What are the coordinates of *A*′?** ((3, 0))

BUILD THE CONCEPT

Model how to identify a rotation on a coordinate plane.

A rotation is a transformation that turns about a fixed point. The point is the center of rotation. Look at the two triangles on the graph. It appears that they are rotating around the origin. Does this model represent a true rotation? (no) **Why not?** (The triangles did not remain congruent.)



TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

Which is the original figure? (blue) **How many degrees about the origin was the blue figure rotated?** (180°) **What are the coordinates for *A*?** ((⁻3, 3)) **What are the coordinates for *A'*?** ((3, ⁻3))

Problem 5

How many degrees about the origin was the figure rotated? (90°) **Was it a clockwise or counterclockwise rotation?** (counterclockwise) **What are the coordinates for *A*?** ((5, 2)) **What are the coordinates for *A'*?** ((⁻2, 5))

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 6–14

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered problem 1 incorrectly:

The student did not understand how to identify a rotation. Use Additional Resources in I.6.8 to reteach how to identify rotating figures in a coordinate plane.

If student answered problem 2 incorrectly:

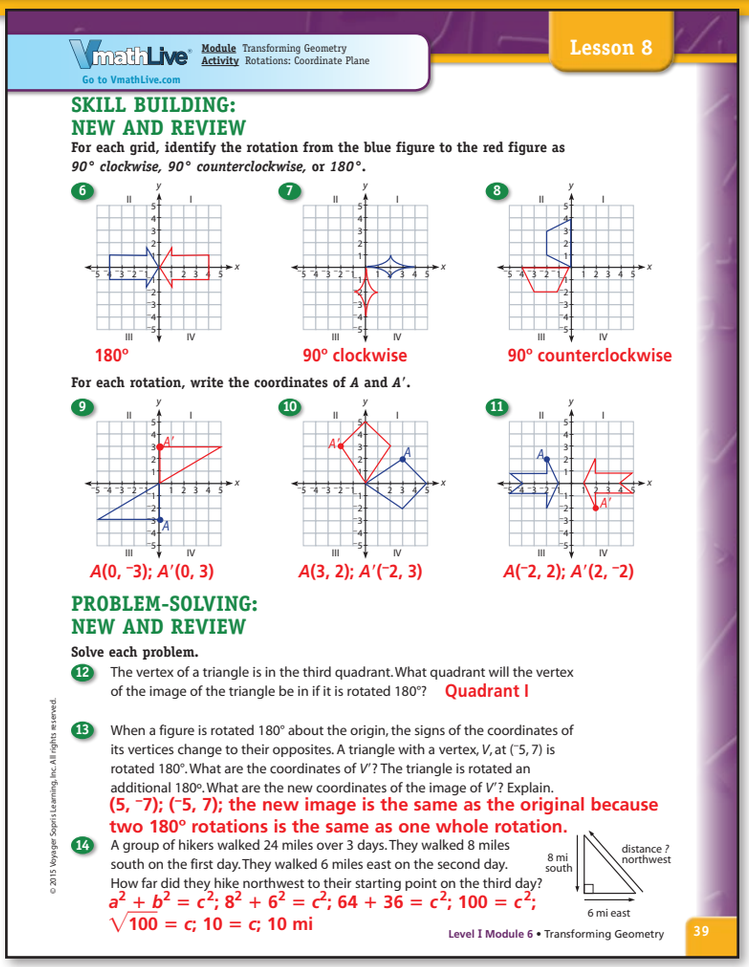
The student did not understand how to write the coordinates of a point. Use Additional Resources in I.6.8 to reteach how to graph in the coordinate plane.



WRITE MATH

If student answered problem 3 incorrectly:

While students work together, remind the student that when the figure was rotated, the point moved. Point *A* is in the first quadrant. Point *A*′ is in the second quadrant.



VmathLive Module Activity Transforming Geometry Rotations: Coordinate Plane
Go to VmathLive.com

Lesson 8

SKILL BUILDING: NEW AND REVIEW

For each grid, identify the rotation from the blue figure to the red figure as *90° clockwise, 90° counterclockwise,* or *180°*.

6 180°

7 90° clockwise

8 90° counterclockwise

For each rotation, write the coordinates of *A* and *A*′.

9 *A*(0, ⁻3); *A*′(0, 3)

10 *A*(3, 2); *A*′(⁻2, 3)

11 *A*(⁻2, 2); *A*′(2, ⁻2)

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

12 The vertex of a triangle is in the third quadrant. What quadrant will the vertex of the image of the triangle be in if it is rotated 180°? Quadrant I

13 When a figure is rotated 180° about the origin, the signs of the coordinates of its vertices change to their opposites. A triangle with a vertex, *V*, at (⁻5, 7) is rotated 180°. What are the coordinates of *V*′? The triangle is rotated an additional 180°. What are the new coordinates of the image of *V*′? Explain.
(5, ⁻7); (⁻5, 7); the new image is the same as the original because two 180° rotations is the same as one whole rotation.

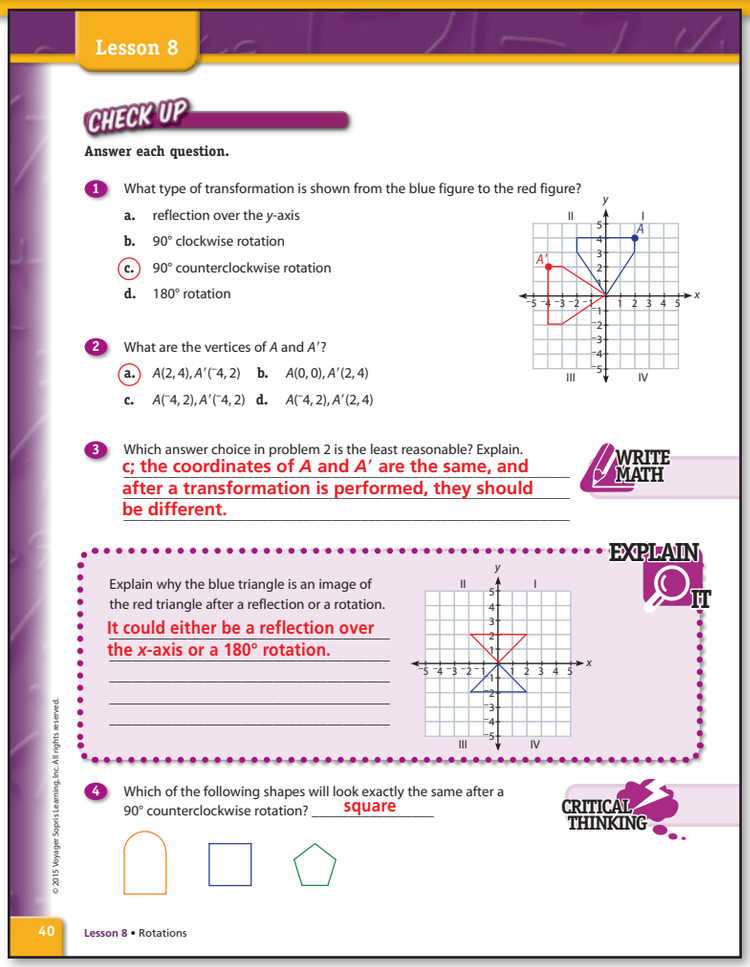
14 A group of hikers walked 24 miles over 3 days. They walked 8 miles south on the first day. They walked 6 miles east on the second day. How far did they hike northwest to their starting point on the third day?
$a^2 + b^2 = c^2$; $8^2 + 6^2 = c^2$; $64 + 36 = c^2$; $100 = c^2$; $\sqrt{100} = c$; $10 = c$; 10 mi

8 mi south; distance ? northwest; 6 mi east

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EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 8

CHECK UP

Answer each question.

1. What type of transformation is shown from the blue figure to the red figure?
   - a. reflection over the *y*-axis
   - b. 90° clockwise rotation
   - (c.) 90° counterclockwise rotation
   - d. 180° rotation

2. What are the vertices of *A* and *A′*?
   - (a.) *A*(2, 4), *A′*(−4, 2)
   - b. *A*(0, 0), *A′*(2, 4)
   - c. *A*(−4, 2), *A′*(−4, 2)
   - d. *A*(−4, 2), *A′*(2, 4)

3. Which answer choice in problem 2 is the least reasonable? Explain.
   c; the coordinates of *A* and *A′* are the same, and after a transformation is performed, they should be different.

WRITE MATH

EXPLAIN IT

Explain why the blue triangle is an image of the red triangle after a reflection or a rotation.
It could either be a reflection over the *x*-axis or a 180° rotation.

4. Which of the following shapes will look exactly the same after a 90° counterclockwise rotation? square

CRITICAL THINKING

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DIFFERENTIATION

Additional Resources

VmathLive
Module: Transforming Geometry
Activity: Rotations: Coordinate Plane

Vmath Reteach
Reteach Student Book Module 6 Lesson 8
Reteach Teacher Edition Module 6 Lesson 8

Extra Practice
Student Book page 52

Technology

Using an online search engine, have students or partners research careers that would need to find transformations or rotations of objects. Encourage them to describe how and why this concept would be needed in specific careers. Additional digital content is available through this feature in the eBook.

English Language Learners

Use the VmathLive Animated Glossary to review the term *rotation*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Be sure students have a good understanding of the term *clockwise*. Figures can rotate about an origin in a *clockwise* or *counterclockwise* direction. Have a clock with a minute hand available for the students or simulate the motion of a clock's hands by drawing a clock face on the board and indicating the different positions of the hands and showing motion with directional arrows. Show students that clockwise means rotation in the direction that the minute hand (or the other clock hand) moves. Explain that *counterclockwise* indicates motion in the opposite direction. Tell students that they can look at an analog clock when unsure to refresh their memory.

Students with Special Needs

For some students, it may be difficult to visualize relationships after movement. To help students understand this concept, give them tracing paper and have them trace the figures. Have students model the rotation by holding a pencil tip on point (0, 0) and turning the tracing. Then have students sketch the rotation on their assignment.

Lesson 9

Objective

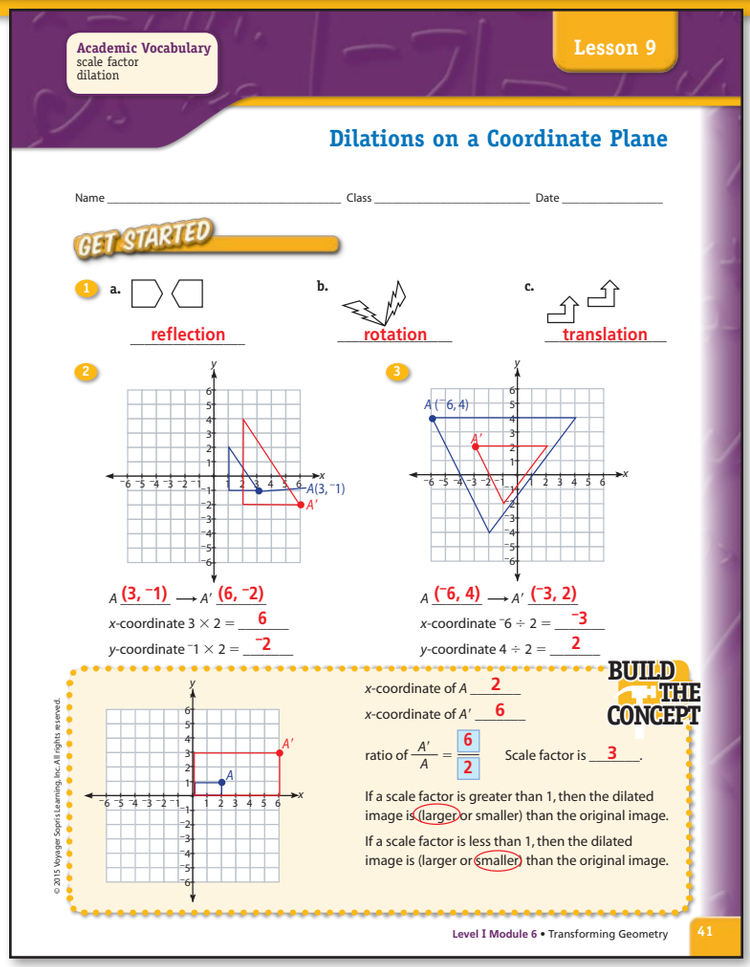
To generate similar figures using dilation and to calculate the scale factor of a dilation

Preskills	Lesson
Transformations	I.6.7

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **scale factor**
  the ratio of the dimensions of an image to the dimensions of an original figure
- **dilation**
  enlarging or reducing a geometric figure to create a similar figure


Academic Vocabulary
scale factor
dilation

Lesson 9

Dilations on a Coordinate Plane

Name _____ Class _____ Date _____

GET STARTED

1 a. reflection b. rotation c. translation

2

A (3, −1) → A′ (6, −2)
x-coordinate 3 × 2 = 6
y-coordinate −1 × 2 = −2

3

A (−6, 4) → A′ (−3, 2)
x-coordinate −6 ÷ 2 = −3
y-coordinate 4 ÷ 2 = 2

BUILD THE CONCEPT

x-coordinate of A 2
x-coordinate of A′ 6
ratio of $\frac{A'}{A} = \frac{6}{2}$ Scale factor is 3.

If a scale factor is greater than 1, then the dilated image is (larger) or smaller) than the original image.

If a scale factor is less than 1, then the dilated image is (larger or smaller) than the original image.

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Level I Module 6 • Transforming Geometry 41

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Look at the figures in problem 1a. Has there been a slide, a flip, or a rotation? (flip) **Slides, flips, and rotations are informal names for transformations. What is the formal name for slide?** (translation) **What is the formal name for flip?** (reflection) **Rotation is used both as the formal and informal term. What type of transformation has occurred with the figure in problem 1a?** (reflection)

Now look at the figures in problem 1b. What type of transformation has occurred with the figure in problem 1b? (rotation) **Now look at the figures in problem 1c. What type of transformation has occurred with the figure in problem 1c?** (translation)

MODEL NEW SKILLS

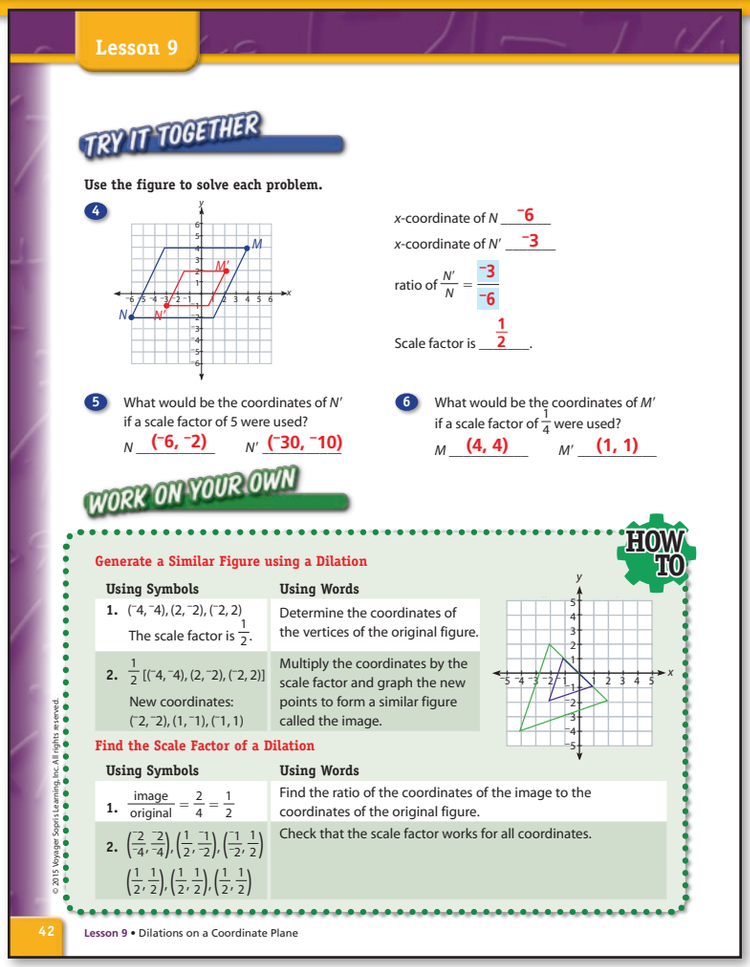
Problem 2

Another type of transformation is a dilation. In a dilation on a coordinate plane, each coordinate of the figure is multiplied by a scale factor. This means that the lengths of the sides of the image will be proportional to the lengths of the sides of the original figure. If the scale factor is greater than 1, the dilation is an enlargement. If the scale factor is between 0 and 1, the dilation is a reduction. The original figure is shown in blue with one vertex labeled *A*. The dilated image is shown in red. Notice that both the original figure and its image are similar. The image of vertex *A* is labeled *A′*, which is read as *A* prime.

Is the red image a reduction or an enlargement of the blue figure? (enlargement) **It is an enlargement because the red figure is larger than the blue figure. A scale factor of 2 was used to dilate the blue figure. Each coordinate of the blue figure was multiplied by the scale factor to generate the red image. What are the coordinates of *A*?** ((3, ⁻1)) **What is 3 times 2?** (6) **The *x*-coordinate of *A′* is 6. What is ⁻1 times 2?** (⁻2) **The *y*-coordinate of *A′* is ⁻2. The coordinates of *A′* are (6, ⁻2).**

Problem 3

The original figure is shown in blue with one vertex labeled *A*. The dilated image is shown in red. The image of vertex *A* is labeled *A′*. Is the red image a reduction or an enlargement of the blue figure? (reduction) **It is a**


Lesson 9

TRY IT TOGETHER

Use the figure to solve each problem.

4

x-coordinate of N ⁻6

x-coordinate of N′ ⁻3

ratio of $\frac{N'}{N} = \frac{-3}{-6}$

Scale factor is $\frac{1}{2}$.

5 What would be the coordinates of N′ if a scale factor of 5 were used?

N (⁻6, ⁻2) N′ (⁻30, ⁻10)

6 What would be the coordinates of M′ if a scale factor of $\frac{1}{4}$ were used?

M (4, 4) M′ (1, 1)

WORK ON YOUR OWN

HOW TO

Generate a Similar Figure using a Dilation

Using Symbols	Using Words
1. (⁻4, ⁻4), (2, ⁻2), (⁻2, 2) The scale factor is $\frac{1}{2}$.	Determine the coordinates of the vertices of the original figure.
2. $\frac{1}{2}$ [(⁻4, ⁻4), (2, ⁻2), (⁻2, 2)] New coordinates: (⁻2, ⁻2), (1, ⁻1), (⁻1, 1)	Multiply the coordinates by the scale factor and graph the new points to form a similar figure called the image.

Find the Scale Factor of a Dilation

Using Symbols	Using Words
1. $\frac{\text{image}}{\text{original}} = \frac{2}{4} = \frac{1}{2}$	Find the ratio of the coordinates of the image to the coordinates of the original figure.
2. $\left(\frac{-2}{-4}, \frac{-2}{-4}\right), \left(\frac{1}{2}, \frac{-1}{-2}\right), \left(\frac{-1}{-2}, \frac{1}{2}\right)$ $\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)$	Check that the scale factor works for all coordinates.

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reduction because the red figure is smaller than the blue figure.

A scale factor of $\frac{1}{2}$ was used to dilate the blue figure. Each coordinate of the blue figure was multiplied by the scale factor to generate the red image.

Remember that multiplying by $\frac{1}{2}$ is the same as dividing by 2. What are the coordinates of *A*? ((⁻6, 4)) **What is ⁻6 divided by 2?** (⁻3) **The *x*-coordinate of *A*′ is ⁻3. What is 4 divided by 2?** (2) **The *y*-coordinate of *A*′ is 2. The coordinates of *A*′ are (⁻3, 2).**

BUILD THE CONCEPT

Model how to calculate the scale factor of a dilation.

The scale factor of the dilation must be determined. The scale factor is found by writing the ratio of the coordinates of *A*′ to *A*. The original figure is shown in blue with one vertex labeled *A*. The transformed image is shown in red. The image of vertex *A* is labeled *A*′. Is the red image a reduction or an enlargement of the blue figure? (enlargement) **It is an enlargement because the red figure is larger than the blue figure. The *x*-coordinates can be used to find the scale factor. What is the *x*-coordinate of *A*?** (2) **What is the *x*-coordinate of *A*′?** (6) **What is the ratio of *A*′ to *A*?** ($\frac{6}{2}$) **The ratio of *A*′ to *A* is 6 divided by 2, which simplifies to 3. The scale factor is 3.**

Notice that if a scale factor is greater than 1, then the dilated image is larger than the original image.

If a scale factor is less than 1, then the dilated image is smaller than the original image.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

Is the red image a reduction or an enlargement of the blue figure? (reduction) **It is a reduction because the red figure is smaller than the blue figure. The *x*-coordinates or the *y*-coordinates can be used to find the scale factor. The *x*-coordinates will be used for this problem.**

What is the *x*-coordinate of *N*? (⁻6) **What is the *x*-coordinate of *N*′?** (⁻3) **What is the ratio of *N*′ to *N*?** ($\frac{-3}{-6}$) **The ratio of *N*′ to *N* is ⁻3 divided by ⁻6, which simplifies to $\frac{1}{2}$. The scale factor is $\frac{1}{2}$.**

Problem 5

What is the scale factor used in this problem? (5) **What are the coordinates of *N*?** ((⁻6, ⁻2)) **What is ⁻6 times 5?** (⁻30) **The *x*-coordinate of *N*′ is ⁻30. What is ⁻2 times 5?** (⁻10) **The *y*-coordinate of *N*′ is ⁻10. What are the coordinates of *N*′?** ((⁻30, ⁻10))

Problem 6

What is the scale factor used in this problem? ($\frac{1}{4}$) **Multiplying by $\frac{1}{4}$ is the same as dividing by what number?** (4) **What are the coordinates of *M*?** ((4, 4)) **What is 4 divided by 4?** (1) **The *x*-coordinate of *M*′ is 1, and the *y*-coordinate of *M*′ is 1. What are the coordinates of *M*′?** ((1, 1))

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Drawing a Picture** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a picture. The instruction is immediately followed by application of the strategy in problem 12.

Have students read the problem.

Be sure that students correctly plot the points for the vertices of the original square and the dilation.

Have students determine the side lengths of both squares:

Side length of original square: 3 units

Side length of dilation: 6 units

Then have students multiply to find the areas of the squares:

Area of original square: $3 \times 3 = 9$ square units

Area of dilation: $6 \times 6 = 36$ square units

The side length of the new square is 2 times the side length of the original square, and the area of the new square is 4 times the area of the original square.

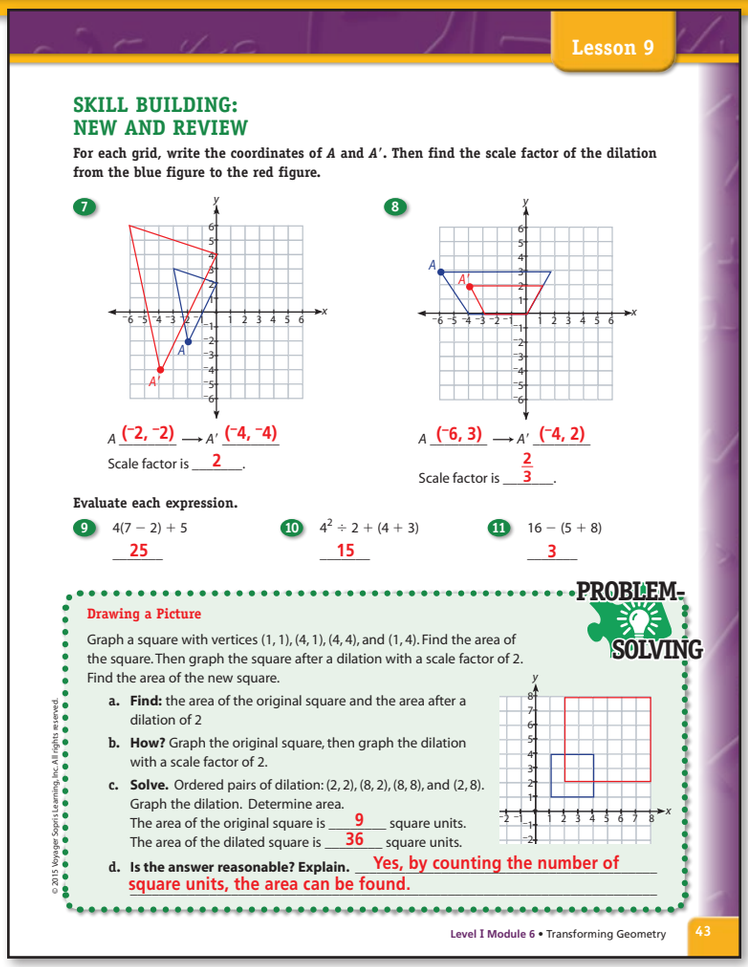
The answer is reasonable because the area can be found by counting square units in the picture.

Problems 7–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 7 and 8 Reminder

Make sure students know that the scale factor is determined by finding the ratio of $\frac{x\text{-coordinate of image}}{x\text{-coordinate of original}}$.



Problem 13 Reminder

Students need to first find the scale factor using the coordinates of F and F' to determine the coordinates of G'.

Problem 15 Reminder

Make sure students know that translating 3 units to the right requires adding 3 to the x-coordinate and translating 2 units down requires subtracting 2 from the y-coordinate.

CHECK UP

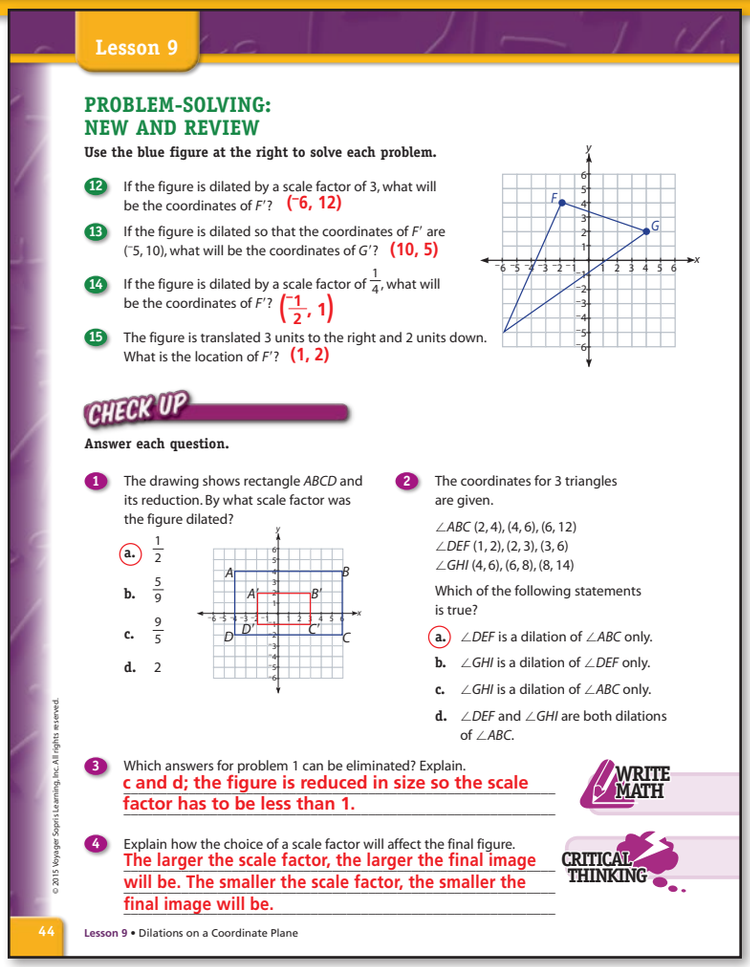
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1b or 1d: The student did not remember that a reduction scale factor must be between 0 and 1. Use Additional Resources in I.6.9 to reteach how to find the scale factor of a dilation.

If student answered 1c: The student used the lengths of the sides for the ratio. Use Additional Resources in I.6.9 to reteach how to find the scale factor of a dilation.



Lesson 9

PROBLEM-SOLVING: NEW AND REVIEW

Use the blue figure at the right to solve each problem.

12. If the figure is dilated by a scale factor of 3, what will be the coordinates of F'? $(^-6, 12)$
13. If the figure is dilated so that the coordinates of F' are $(^-5, 10)$, what will be the coordinates of G'? $(10, 5)$
14. If the figure is dilated by a scale factor of $\frac{1}{4}$, what will be the coordinates of F'? $\left(\frac{^-1}{2}, 1\right)$
15. The figure is translated 3 units to the right and 2 units down. What is the location of F'? $(1, 2)$

CHECK UP

Answer each question.

1. The drawing shows rectangle $ABCD$ and its reduction. By what scale factor was the figure dilated?

   a. $\frac{1}{2}$ b. $\frac{5}{9}$ c. $\frac{9}{5}$ d. 2

2. The coordinates for 3 triangles are given.

   $\angle ABC$ (2, 4), (4, 6), (6, 12)
   $\angle DEF$ (1, 2), (2, 3), (3, 6)
   $\angle GHI$ (4, 6), (6, 8), (8, 14)

   Which of the following statements is true?

   a. $\angle DEF$ is a dilation of $\angle ABC$ only.
   b. $\angle GHI$ is a dilation of $\angle DEF$ only.
   c. $\angle GHI$ is a dilation of $\angle ABC$ only.
   d. $\angle DEF$ and $\angle GHI$ are both dilations of $\angle ABC$.

3. Which answers for problem 1 can be eliminated? Explain.
   c and d; the figure is reduced in size so the scale factor has to be less than 1.

WRITE MATH

4. Explain how the choice of a scale factor will affect the final figure.
   The larger the scale factor, the larger the final image will be. The smaller the scale factor, the smaller the final image will be.

CRITICAL THINKING

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If student answered problem 2 incorrectly: The student did not check each coordinate to determine whether the triangles were dilations. Use Additional Resources in I.6.9 to reteach how to find the scale factor of a dilation.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student the rule about scale factors and resulting size of a dilated or reduced figure: scale factors between 0 and 1 will result in a reduced figure; scale factors greater than 1 will produce an image larger than the original.

Technology



Have students use an online search engine to discover other ways to apply finding scale factors in real-life situations. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 6 Lesson 9
Reteach Teacher Edition Module 6 Lesson 9

Extra Practice
Student Book page 53

English Language Learners

Use the VmathLive Animated Glossary to review the term *scale factor.* Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Give each student two index cards. Have them write *dilation* on one card and *reduction* on the other. Write examples of different scale factors on the board. (For example: $\frac{1}{3}, \frac{4}{1}, \frac{2}{3}, \frac{3}{2}, \frac{2}{5}, \frac{6}{2}$.) Point to each scale factor and have students hold up an index card to indicate whether the scale factor is a reduction or dilation. Have students write the scale factor on the back of each appropriate index card.

Students with Special Needs

If students are incorrectly determining the scale factor, have students write the ratio $\frac{\text{new image}}{\text{original}}$ on an index card to use as a reference. Write the coordinates of two points on the board. Guide students through the process of finding the scale factor. (For example: $A = (1, 3)$; $A' = (4, 12)$.) Students should work in pairs to determine the scale factor, then explain how they formed the ratio.

Lesson 10

Objective

To identify transformations and sequences of transformations that preserve congruence

Preskills	Lesson
Transformations	I.6.7
Rotations	I.6.8
Dilations on a Coordinate Plane	I.6.9



Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What are congruent figures? (figures that have the same shape and same size) **Which two figures are congruent?** (Figure B and Figure D) **Figures B and D are congruent because their corresponding angles and corresponding sides are congruent.**

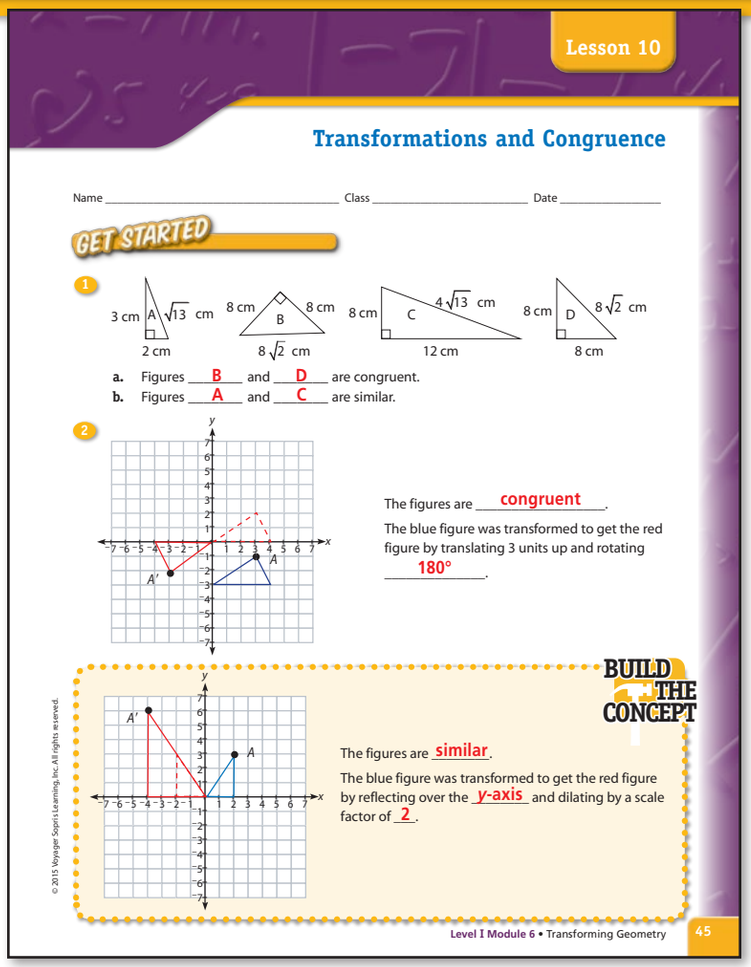
What are similar figures? (figures that have the same shape but not necessarily the same size) **This means that the ratios of the lengths of corresponding sides in similar figures are equivalent. Which figures are similar but not congruent?** (Figure A and Figure C) **What is the ratio of any side length of Figure A to the corresponding side length of Figure C?** ($\frac{1}{4}$)

MODEL NEW SKILLS

Problem 2

Look the blue figure and the red figure. Are the triangles similar, congruent, or neither? (congruent)

Because the figures are congruent, the blue figure was moved to the position of the red figure by using a transformation or a series of transformations. What kinds of transformations are there? (translations, rotations, reflections, and dilations) **Which transformations do not change the shape or size of the figure?** (translations, rotations, and reflections)



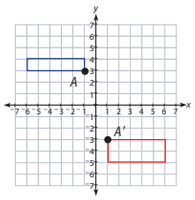
The blue figure is moved to the position of the red figure by using a translation of 3 units upward and a rotation. After the blue figure is moved 3 units upward, what are the coordinates of the vertices? ((0, 0), (3, 2), (4, 0)) **Use dashed lines to draw the triangle after the translation.**

Notice that the orientations of the figures are different. This means that a rotation or reflection was also used. How can the triangle in its new position be rotated to be in the position of the red triangle? (180°)

TRY IT TOGETHER

Determine whether the figures are congruent, similar, or neither. If the figures are congruent or similar, describe the transformations.

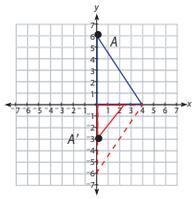
3



neither

Transformations: none

4



similar

Transformations: dilation by scale factor of $\frac{1}{2}$; reflection over *x*-axis

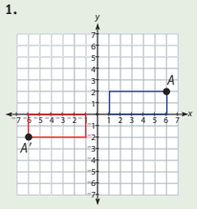
WORK ON YOUR OWN

HOW TO

Determine Whether a Figure Is the Result of Transformations

Using Symbols

1.



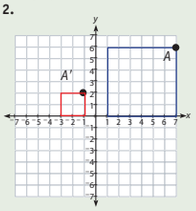
180° counterclockwise rotation, translation 2 units left

Using Words

If the figures are congruent, a translation, rotation, and/or reflection were used in the transformation.

Using Symbols

2.



dilation by a scale factor of $\frac{1}{3}$, translation 4 units left

Using Words

If the figures are similar, a dilation was used in the transformation.

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BUILD THE CONCEPT

Model how to identify a series of transformations by first determining whether the figures are similar or congruent.

Look the blue figure and the red figure. Are the triangles similar, congruent, or neither? (similar)

Because the figures are similar, one of the transformations used to move the blue figure to the red figure must be a dilation. Notice that the orientations of the figures are different. This means that a rotation or reflection was also used.

The blue figure is moved to the position of the red figure by using a reflection and a dilation. Over which axis is the blue figure reflected? (*y*-axis) **Use dashed lines to draw the reflection of the blue triangle over the *y*-axis.**

After the blue figure is reflected over the y-axis, what are the coordinates of the vertices? ((0, 0), (⁻2, 0), (⁻2, 3)) **What are the coordinates of the vertices of the red triangle?** ((0, 0), (⁻4, 0), (⁻4, 6)) **Is the red image a reduction or enlargement of the blue figure?** (enlargement) **So the scale factor must be greater than 1. What are the coordinates of the point on the triangle you drew that corresponds to point *A*′?** (⁻2, 3) **What are the coordinates of *A*′?** (⁻4, 6) **The scale factor is the ratio of ⁻4 to ⁻2 or ⁻6 to ⁻3. What is the scale factor?** (2)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 3

Look the blue figure and the red figure. Are the figures similar, congruent, or neither? (neither) **Because the figures are neither congruent nor similar, a transformation was not used to move the blue figure to the location of the red figure.**

Problem 4

Look the blue figure and the red figure. Are the triangles similar, congruent, or neither? (similar) **What kind of transformation must be used to move the blue figure to the red figure?** (a dilation) **What other kind of transformation is used?** (reflection) **Why?** (The orientations of the blue and red figures are different.)

Over which axis is the blue figure reflected? (*x*-axis) **Use dashed lines to draw the reflection of the blue triangle over the *x*-axis.**

After the blue figure is reflected over the *x*-axis, what are the coordinates of the vertices? ((0, 0), (4, 0), (0, ⁻6)) **What are the coordinates of the vertices of the red triangle?** ((0, 0), (2, 0), (0, ⁻3)) **Is the red image a reduction or enlargement of the blue figure?** (reduction) **So the scale factor must be less than 1. What are the coordinates of the point on the triangle you drew that corresponds to point *A*′?** (0, ⁻6) **What are the coordinates of *A*′?** (0, ⁻3) **The scale factor is the ratio of ⁻3 to ⁻6. What is the scale factor?** ($\frac{1}{2}$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown for each process step.

Problems 5–12

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 10 Reminder

Students should first determine whether the figures are congruent, similar, or neither. From this information, they can determine which kinds of transformations, if any, could have been used to get the position of the second figure from the position of the first figure.

CHECK UP

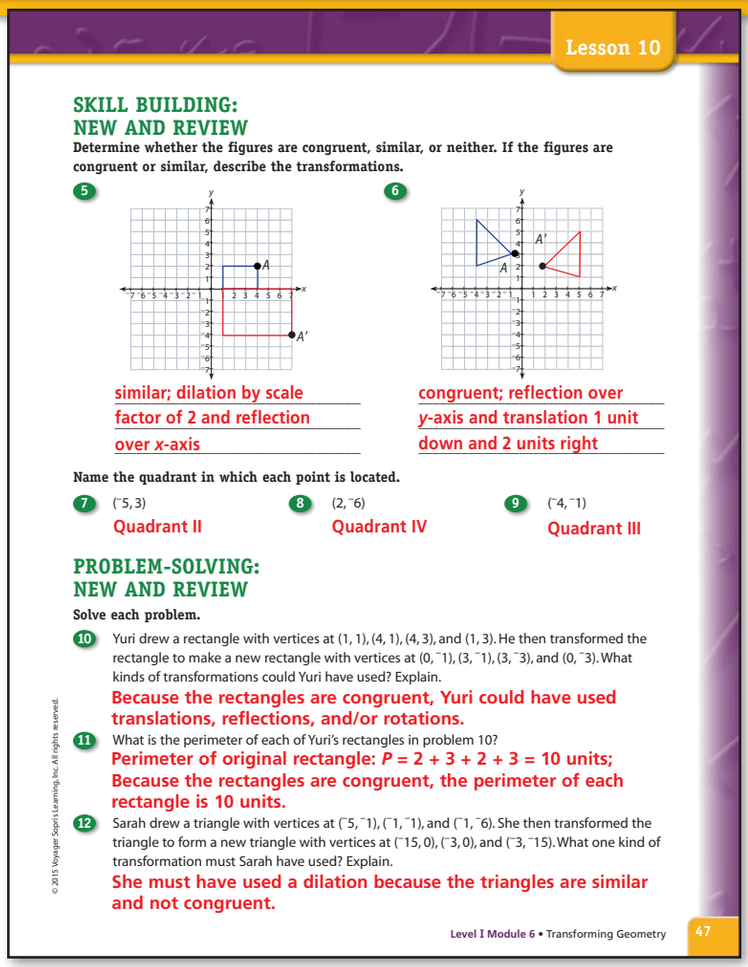
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a, 2a, or 2c: The student identified a triangle that is the result of the incorrect transformation. Use Additional Resources in I.6.10 to review how to use congruence and similarity to identify a transformation.

If student answered 1b, 1d, or 2d: The student identified a triangle that is the result of no transformation of the original figure. Use Additional Resources in I.6.10 to review how to use congruence and similarity to identify a transformation.



Lesson 10

SKILL BUILDING: NEW AND REVIEW

Determine whether the figures are congruent, similar, or neither. If the figures are congruent or similar, describe the transformations.

5

similar; dilation by scale factor of 2 and reflection over *x*-axis

6

congruent; reflection over *y*-axis and translation 1 unit down and 2 units right

Name the quadrant in which each point is located.

7 (⁻5, 3) Quadrant II

8 (2, ⁻6) Quadrant IV

9 (⁻4, ⁻1) Quadrant III

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

10 Yuri drew a rectangle with vertices at (1, 1), (4, 1), (4, 3), and (1, 3). He then transformed the rectangle to make a new rectangle with vertices at (0, ⁻1), (3, ⁻1), (3, ⁻3), and (0, ⁻3). What kinds of transformations could Yuri have used? Explain.

Because the rectangles are congruent, Yuri could have used translations, reflections, and/or rotations.

11 What is the perimeter of each of Yuri's rectangles in problem 10?

Perimeter of original rectangle: $P = 2 + 3 + 2 + 3 = 10$ units; Because the rectangles are congruent, the perimeter of each rectangle is 10 units.

12 Sarah drew a triangle with vertices at (⁻5, ⁻1), (⁻1, ⁻1), and (⁻1, ⁻6). She then transformed the triangle to form a new triangle with vertices at (⁻15, 0), (⁻3, 0), and (⁻3, ⁻15). What one kind of transformation must Sarah have used? Explain.

She must have used a dilation because the triangles are similar and not congruent.

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Level I Module 6 • Transforming Geometry 47

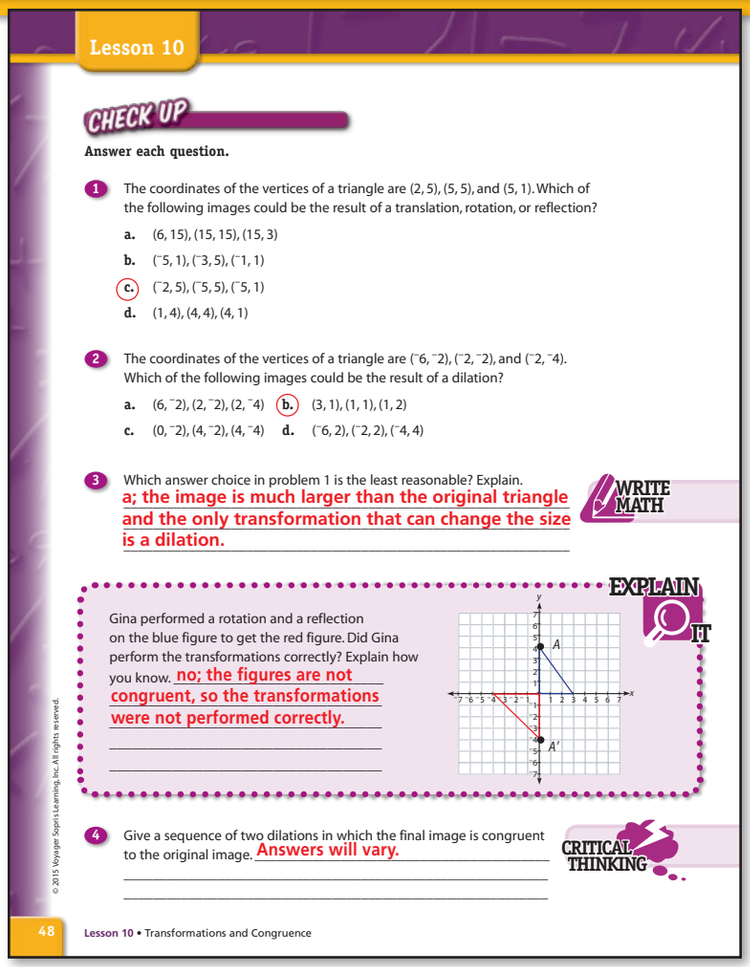


WRITE MATH

If student answered problem 3 incorrectly: While students work together, review the transformations that preserve congruence and the transformation that does not preserve congruence.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.



Lesson 10

CHECK UP

Answer each question.

1. The coordinates of the vertices of a triangle are (2, 5), (5, 5), and (5, 1). Which of the following images could be the result of a translation, rotation, or reflection?
   - **a.** (6, 15), (15, 15), (15, 3)
   - **b.** (⁻5, 1), (⁻3, 5), (⁻1, 1)
   - **c.** (⁻2, 5), (⁻5, 5), (⁻5, 1)
   - **d.** (1, 4), (4, 4), (4, 1)

2. The coordinates of the vertices of a triangle are (⁻6, ⁻2), (⁻2, ⁻2), and (⁻2, ⁻4). Which of the following images could be the result of a dilation?
   - **a.** (6, ⁻2), (2, ⁻2), (2, ⁻4)
   - **b.** (3, 1), (1, 1), (1, 2)
   - **c.** (0, ⁻2), (4, ⁻2), (4, ⁻4)
   - **d.** (⁻6, 2), (⁻2, 2), (⁻4, 4)

3. Which answer choice in problem 1 is the least reasonable? Explain.
   a; the image is much larger than the original triangle and the only transformation that can change the size is a dilation.

WRITE MATH

EXPLAIN IT

Gina performed a rotation and a reflection on the blue figure to get the red figure. Did Gina perform the transformations correctly? Explain how you know. no; the figures are not congruent, so the transformations were not performed correctly.

4. Give a sequence of two dilations in which the final image is congruent to the original image. Answers will vary.

CRITICAL THINKING

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DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 6 Lesson 10

Reteach Teacher Edition Module 6 Lesson 10

Extra Practice

Student Book page 53

English Language Learners

Use the VmathLive Animated Glossary to review the terms *similar, congruent, transformation, reflections, translation, rotation,* and *scale factor.* Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Students may have difficulty remembering the names of each of the four different kinds of transformations. Use a diagram to represent the different types of transformations. Have students say aloud the types of transformations as they are pointed out and tell whether they preserve congruence or not.

Students with Special Needs

When multiple transformations are used to move a figure, students may have difficulty identifying the types of transformations used. Use examples with a right triangle to show students that translations and dilations do not change the orientation of a figure, but rotations and reflections do change the orientation. Point out that dilations are the only transformations that change the size of the figure.

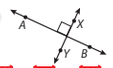
Extra Practice Answer Key

Extra Practice

Name _____ Class _____ Date _____

Lesson PL1 Terms and Figures in Geometry

Name each figure using the proper symbols and points. If more than one figure is shown, describe the relationship.

1. 

$\overleftrightarrow{AB} \perp \overleftrightarrow{XY}$ or $\overleftrightarrow{XY} \perp \overleftrightarrow{AB}$

2. 

$\overleftrightarrow{MN} \parallel \overleftrightarrow{OP}$ or $\overleftrightarrow{OP} \parallel \overleftrightarrow{MN}$

3. 

$\overleftrightarrow{SR}$

4. 

$\overleftrightarrow{UV}$ or $\overleftrightarrow{VU}$

5. 

$\overleftrightarrow{CD}$ or $\overleftrightarrow{DC}$

6. 

$\overleftrightarrow{MN}$

7. 

$\overleftrightarrow{CD}$

8. 

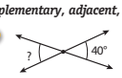
$\overleftrightarrow{ST}$ or $\overleftrightarrow{TS}$

9. 

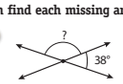
$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

Lesson PL2 Angles

Use the words that best identify each pair of angles indicated by the arcs: *complementary*, *supplementary*, *adjacent*, *vertical*. Then find each missing angle measure.

1. 

vertical; 40°

2. 

adjacent, supplementary; 142°

3. 

adjacent, complementary; 27°

4. 

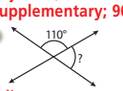
adjacent, supplementary; 90°

5. 

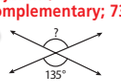
adjacent, complementary; 73°

6. 

vertical; 15°

7. 

adjacent, supplementary; 70°

8. 

vertical; 135°

9. 

adjacent, complementary; 68°

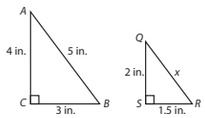
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Extra Practice

Name _____ Class _____ Date _____

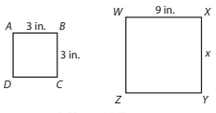
Lesson 2 Proportions and Similar Figures

Set up a proportion. Then solve to find the unknown length *x* in the similar figures.

1. 

$\triangle ABC \sim \triangle QRS$

$\frac{4}{2} = \frac{5}{x}$ $x = 2.5$ in.

2. 

$ABCD \sim WXYZ$

$\frac{3}{9} = \frac{3}{x}$ $x = 9$ in.

Lesson 3 Angle Relationships

Figure 1 shows two parallel lines cut by a transversal. Write the relationship between the angles, then find the measure of each angle.

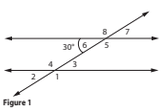


Figure 1

1. ∠2 and ∠6 corresponding angles
$m\angle 2 = m\angle 6 =$ 30°

2. ∠5 and ∠6 supplementary angles
$m\angle 6 = 30°; m\angle 5 =$ 150°

3. ∠6 and ∠7 vertical angles
$m\angle 6 = 30°; m\angle 7 =$ 30°

4. ∠3 and ∠6 alternate interior angles
$m\angle 3 = m\angle 6 =$ 30°

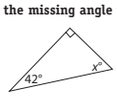
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Extra Practice

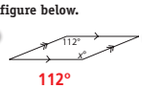
Name _____ Class _____ Date _____

Lesson 4 Missing Angles

Find the missing angle measure of each figure below.

1. 

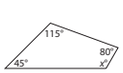
48°

2. 

112°

3. 

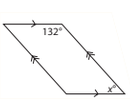
60°

4. 

120°

5. 

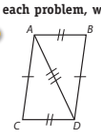
84°

6. 

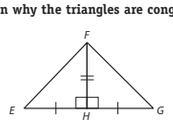
48°

Lesson 6 Congruent Triangles

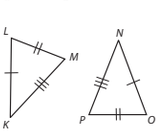
For each problem, write *SAS*, *ASA*, or *SSS* to explain why the triangles are congruent.

1. 

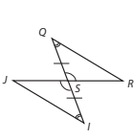
SSS

2. 

SAS

3. 

SSS

4. 

ASA

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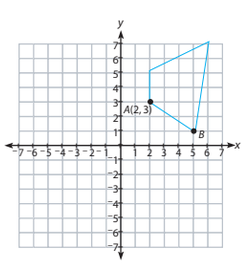
Extra Practice

Name _____ Class _____ Date _____

Lesson 7 Transformations

Write the coordinates of *A′* after each transformation.

1. Reflection over *y*-axis (−2, 3)
2. Translation 6 units left and 1 unit up (−4, 4)
3. Translation 4 units right and 4 units down (6, −1)
4. Reflection over *x*-axis (2, −3)

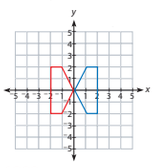


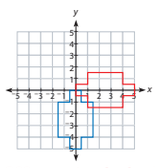
Use the coordinate plane to answer each question.

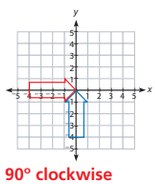
5. What are the coordinates of vertex *B*? (5, 1)
6. What will be the coordinates of *B′* if the figure is reflected over the *y*-axis? (−5, 1)
7. In which quadrant will the figure be if it is reflected over the *y*-axis and then over the *x*-axis? If *A″* and *B″* are the reflected points in the final figure, what will be the coordinates of *A″* and *B″*? Quadrant III; *A″*(−2, −3); *B″*(−5, −1)

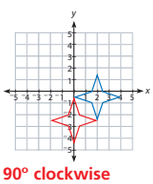
Lesson 8 Rotations

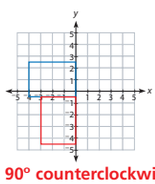
For each grid, identify the rotation from the blue figure to the red figure as *90° clockwise*, *90° counterclockwise*, or *180°*.

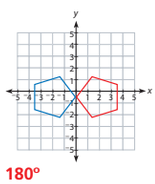
1. 
180°

2. 
90° counterclockwise

3. 
90° clockwise

4. 
90° clockwise

5. 
90° counterclockwise

6. 
180°

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Extra Practice

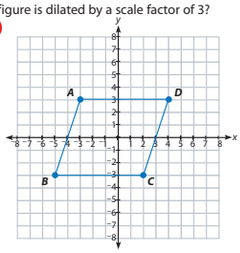
Name _____ Class _____ Date _____

Lesson 9 Dilations on a Coordinate Plane

Answer the question.

1. Figure *WXYZ* has coordinates *W*(1, 3), *X*(⁻5, 2), *Y*(0, ⁻4), and *Z*(⁻2, 4). Using prime notation, what are the coordinates of each vertex of the image if the figure is dilated by a scale factor of 3?
   ***W′*(3, 9), *X′*(⁻15, 6), *Y′*(0, ⁻12), *Z′*(⁻6, 12)**

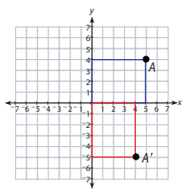
Use the blue figure on the right to solve each problem.



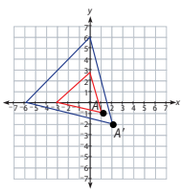
2. If the figure is dilated by a factor of 2, what will be the coordinates of *D′*? **(8, 6)**
3. If the figure is dilated by a factor of 3, what will be the coordinates of *C′*? **(6, ⁻9)**
4. If the figure is dilated by a factor of $\frac{1}{2}$, what will be the coordinates of *A′*? **$(-1\frac{1}{2}, 1\frac{1}{2})$**
5. If the figure is dilated by a factor of 2, what will be the coordinates of *A′* and *B′*?
   ***A′*(⁻6, 6), *B′*(⁻10, ⁻6)**

Lesson 10 Transformations and Congruence

Determine whether the figures are congruent, similar, or neither. If the figures are congruent or similar, describe the transformations.

1. 

   congruent; rotation of 90° clockwise

2. 

   similar; dilation by a scale factor of 2

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Glossary

adjacent angles
two angles that share a vertex and a common side and do not overlap

alternate exterior angles
a pair of angles formed by two lines that are cut by a transversal; both angles are exterior angles on opposite sides of the transversal at different vertices

alternate interior angles
a pair of angles formed by two lines that are cut by a transversal; both angles are interior angles on opposite sides of the transversal at different vertices

ASA
ASA stands for angle-side-angle. The rule states that if two angles and the side between the angles of one triangle are congruent to two angles and the side between the angles of another triangle, the triangles are congruent.

complementary angles
two angles whose measures have a sum of 90°

congruent angles
two angles with the same measure

consecutive angles
angles that share a common side

corresponding angles
a pair of angles formed by two lines that are cut by a transversal; one angle is an interior angle and the other an exterior angle on the same side of the transversal at different vertices

dilation
enlarging or reducing a geometric figure to create a similar figure

intersecting lines
lines that cross at one point

line
a straight path with no endpoints

line segment
part of a line with two endpoints

opposite angles of a parallelogram
nonconsecutive angles of a parallelogram

parallel lines
lines that are in the same plane and never intersect; the symbol || means "is parallel to"

perpendicular lines
lines that intersect to form 90° angles; the symbol ⊥ means "is perpendicular to"

plane
a flat surface with no boundaries

point
a location in space that has no size

ray
part of a line with one endpoint

reflection
moving a geometric figure by flipping it over a line

rotation about the origin
moving a geometric figure by turning it about the origin

SAS
SAS stands for side-angle-side. The rule states that if two sides and the angle between the sides of one triangle are congruent to two sides and the angle between the sides of another triangle, the triangles are congruent.

scale factor
the ratio of the dimensions of an image to the dimensions of an original figure

similar figures
geometric figures that are the same shape but not necessarily the same size

SSS
SSS stands for side-side-side. The rule states that if three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

supplementary angles
two angles whose measures have a sum of 180°

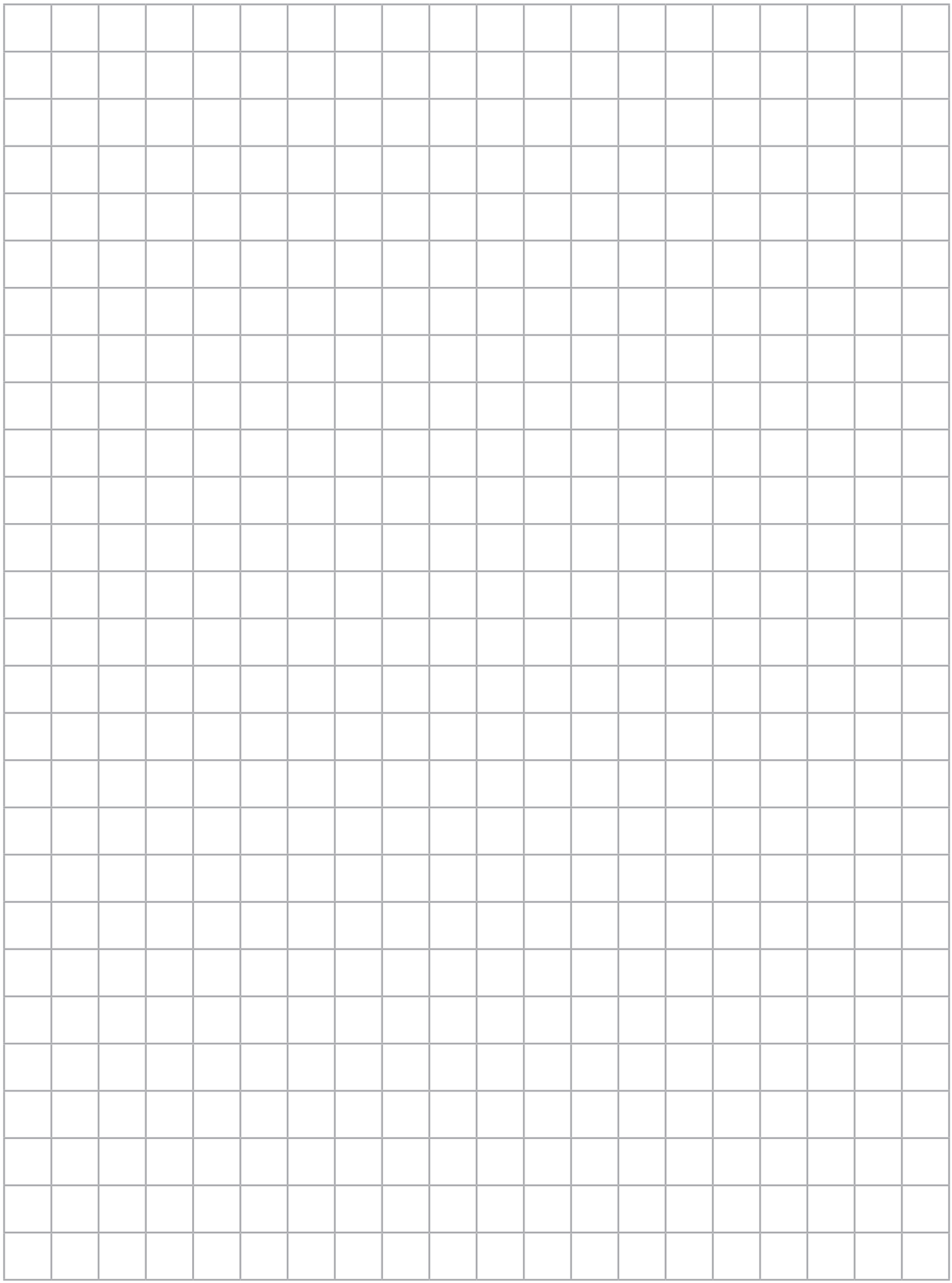
transformation
a translation, a reflection, or a rotation

translation
moving a geometric figure by sliding

transversal
a line that intersects two or more lines at different points

vertical angles
the opposite congruent angles formed by two intersecting lines

Copy Master: Grid Paper



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Geometry

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Lesson PL1

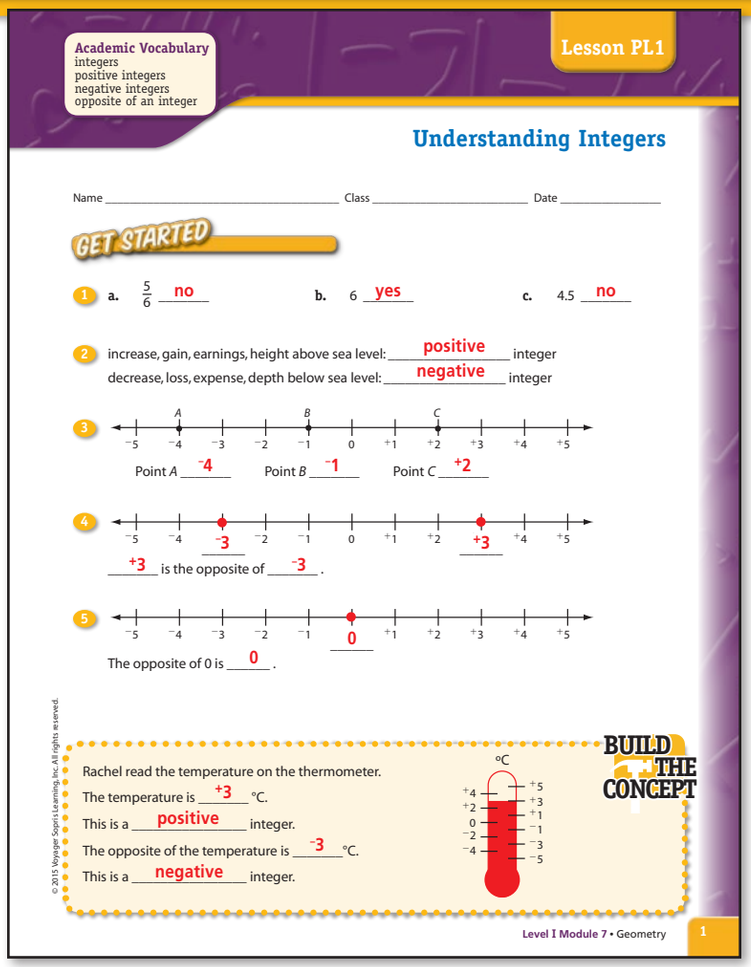
Objective

To match an integer with its point on a number line; to find the opposite of an integer

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **integers**
  the whole numbers and their opposites
- **positive integers**
  integers greater than 0
- **negative integers**
  integers less than 0
- **opposite of an integer**
  the integer that is the same distance from 0 on a number line, but in the opposite direction



Academic Vocabulary
integers
positive integers
negative integers
opposite of an integer

Lesson PL1

Understanding Integers

Name _____ Class _____ Date _____

GET STARTED

1 a. $\frac{5}{6}$ no b. 6 yes c. 4.5 no

2 increase, gain, earnings, height above sea level: positive integer
decrease, loss, expense, depth below sea level: negative integer

3 Point A −4 Point B −1 Point C +2

4 +3 is the opposite of −3.

5 The opposite of 0 is 0.

BUILD THE CONCEPT

Rachel read the temperature on the thermometer.
The temperature is +3 °C.
This is a positive integer.
The opposite of the temperature is −3 °C.
This is a negative integer.

°C

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Level I Module 7 • Geometry 1

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Integers are the whole numbers and their opposites. Positive integers are greater than 0. Negative integers are less than 0. Zero is neither positive nor negative. The opposite of 0 is 0.

What number is shown in problem 1a? ($\frac{5}{6}$) **Is $\frac{5}{6}$ a whole number?** (no) **The fraction $\frac{5}{6}$ is not a whole number, so it is not an integer. What number is shown in problem 1b?** (6) **Is 6 a whole number?** (yes) **Six is an integer because it is a whole number. What number is shown in problem 1c?** (4.5) **Is 4.5 a whole number?** (no) **The decimal 4.5 is not a whole number, so it is not an integer.**

MODEL NEW SKILLS

Problem 2

An integer can be used to represent amounts. To represent an increase, a gain, earnings, or a height above sea level, use a positive integer. To represent a decrease, a loss, an expense, or a depth below sea level, use a negative integer.

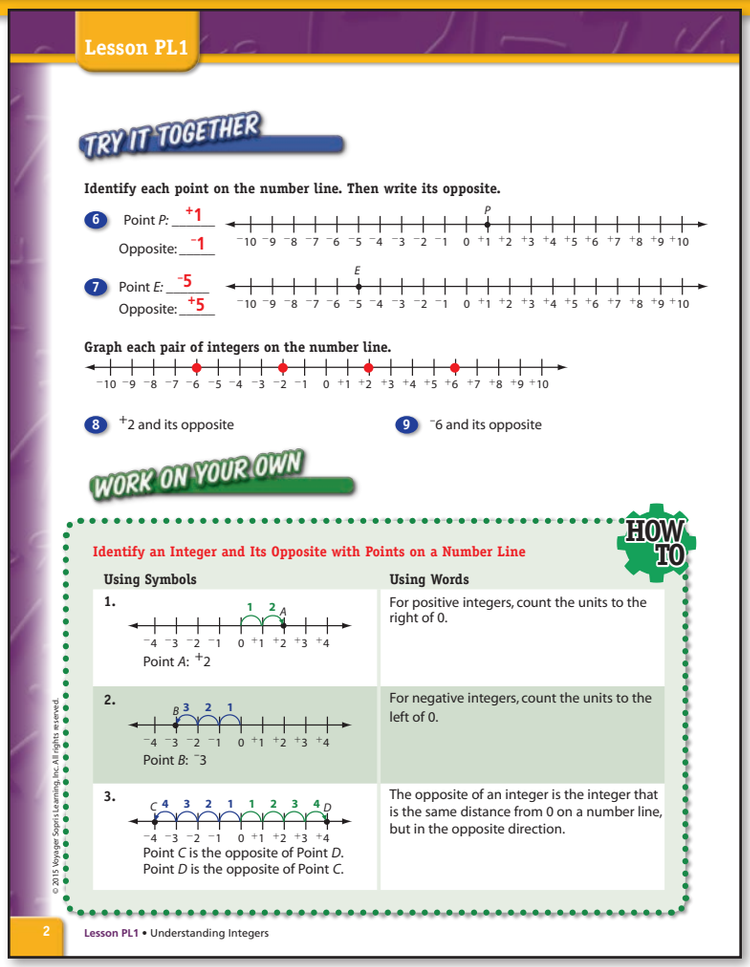
Problem 3

Each tick mark on the number line represents an integer. Integers to the left of 0 are negative. Integers to the right of 0 are positive. In which direction from 0 is point *A*? (left) **Is point *A* a positive or negative integer?** (negative) **How many units to the left of 0 is point *A*?** (4) **Point *A* is graphed at $^{-}4$. Negative numbers are always represented with a negative sign in front of the number.**

To identify point *B*, count from 0 to point *B*. In which direction from 0 is point *B*? (left) **How many units to the left of 0 is point *B*?** (1) **Point *B* is graphed at $^{-}1$. In which direction from 0 is point *C*?** (right) **Is point *C* a positive or negative integer?** (positive) **How many units to the right of 0 is point *C*?** (2) **What integer is graphed at point *C*?** ($^{+}2$)

Problem 4

To graph $^{-}3$ on the number line, count three units to the left of 0. Then draw a point on the number line. Label the point $^{-}3$. The opposite of an integer is the integer that is the same distance from 0 on a number line, but in the opposite direction. The opposite of a negative integer is a positive integer. The opposite of a positive integer is a negative integer. Because 0 is neither positive nor


Lesson PL1

TRY IT TOGETHER

Identify each point on the number line. Then write its opposite.

6 Point P: +1
Opposite: −1

7 Point E: −5
Opposite: +5

Graph each pair of integers on the number line.

8 +2 and its opposite

9 −6 and its opposite

WORK ON YOUR OWN

HOW TO

Identify an Integer and Its Opposite with Points on a Number Line

Using Symbols	Using Words
1. Point A: +2	For positive integers, count the units to the right of 0.
2. Point B: −3	For negative integers, count the units to the left of 0.
3. Point C is the opposite of Point D. Point D is the opposite of Point C.	The opposite of an integer is the integer that is the same distance from 0 on a number line, but in the opposite direction.

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2 Lesson PL1 • Understanding Integers

negative, the opposite of 0 is 0. To graph the opposite of −3, count three units to the right of 0. Draw a point and label it +3. What is the opposite of −3? (+3) **Likewise, −3 is the opposite of +3.**

Problem 5

Is 0 a negative number? (no) **Is 0 a positive number?** (no) **Graph 0 on the number line. What is the opposite of 0?** (0)

BUILD THE CONCEPT

Model how to understand positive and negative integers using a thermometer.

Integers can also be seen on a thermometer. Look at the thermometer. What positive integers are shown? (+1, +2, +3, +4, +5) **What temperatures are associated with these positive integers?** (+1°C, +2°C, +3°C, +4°C, +5°C) **What negative integers are shown?** (−1, −2, −3, −4, −5) **What temperatures are associated with these negative integers?** (−1°C, −2°C, −3°C, −4°C, −5°C) **The temperature shown on a thermometer is marked by where the red fill stops. At what integer does the red fill stop on this thermometer?** (+3) **So, what temperature does the thermometer show?** (+3°C) **Is this a positive or negative integer?** (positive) **How many units above 0 is +3°C?** (3) **In which direction from 0 will the opposite temperature appear?** (below) **How many units below 0?** (3) **Count three units below 0. What temperature is this?** (−3°C) **Is this a positive or negative integer?** (negative)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 6

In which direction is point *P* from 0? (right) **Is point *P* a positive or negative integer?** (positive) **How many units to the right of 0 is point *P*?** (1) **What integer is graphed at point *P*?** (+1) **What is the opposite of +1?** (−1)

Problem 7

Is point *E* graphed at a negative or positive integer? (negative) **Why?** (It is to the left of 0.) **How many units to the left of 0 is point *E*?** (5) **What integer is graphed at point *E*?** (−5) **What is the opposite of −5?** (+5)

Problem 8

How is +2 graphed on the number line? (Count 2 units to the right of 0 and draw a point.) **What is the opposite of +2?** (−2) **How is −2 graphed on the number line?** (Count 2 units to the left of 0 and draw a point there.)

Problem 9

How are −6 and its opposite graphed on the number line? (Draw a point 6 units to the left of 0 and draw a point 6 units to the right of 0.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING

This problem illustrates the **Using a 4-Step Plan** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 19.

Have students read the problem.

Review the four-step plan with students:

Step 1 is to determine what needs to be found.

Step 2 is to determine how the problem will be solved.

Step 3 is to solve the problem.

Step 4 is to check the answer for reasonableness.

Students should then find the opposite of yesterday's temperature, $^{-}2$°F. Have students use the number line.

The opposite of $^{-}2$ is the same distance from 0 as $^{-}2$.

The opposite of $^{-}2$ is $^{+}2$.

The opposite of $^{-}2$°F is $^{+}2$°F.

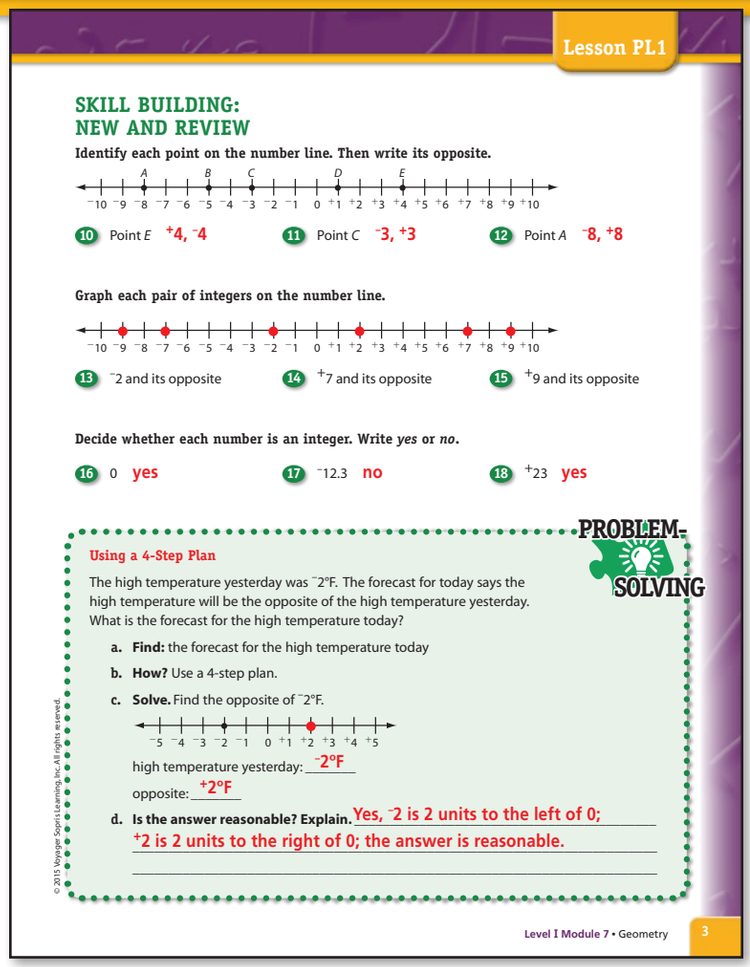
The answer is reasonable because $^{-}2$ is the same distance from 0 as $^{+}2$.

Problems 10–21

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 10–12 Reminder

These problems require identifying the integer graphed on the number line, then writing its opposite.



Lesson PL1

SKILL BUILDING: NEW AND REVIEW

Identify each point on the number line. Then write its opposite.

10 Point E $^{+}4$, $^{-}4$

11 Point C $^{-}3$, $^{+}3$

12 Point A $^{-}8$, $^{+}8$

Graph each pair of integers on the number line.

13 $^{-}2$ and its opposite

14 $^{+}7$ and its opposite

15 $^{+}9$ and its opposite

Decide whether each number is an integer. Write *yes* or *no*.

16 0 yes

17 $^{-}12.3$ no

18 $^{+}23$ yes

PROBLEM-SOLVING

Using a 4-Step Plan

The high temperature yesterday was $^{-}2$°F. The forecast for today says the high temperature will be the opposite of the high temperature yesterday. What is the forecast for the high temperature today?

a. **Find:** the forecast for the high temperature today

b. **How?** Use a 4-step plan.

c. **Solve.** Find the opposite of $^{-}2$°F.

high temperature yesterday: $^{-}2$°F

opposite: $^{+}2$°F

d. **Is the answer reasonable? Explain.** Yes, $^{-}2$ is 2 units to the left of 0; $^{+}2$ is 2 units to the right of 0; the answer is reasonable.

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Level I Module 7 • Geometry 3

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

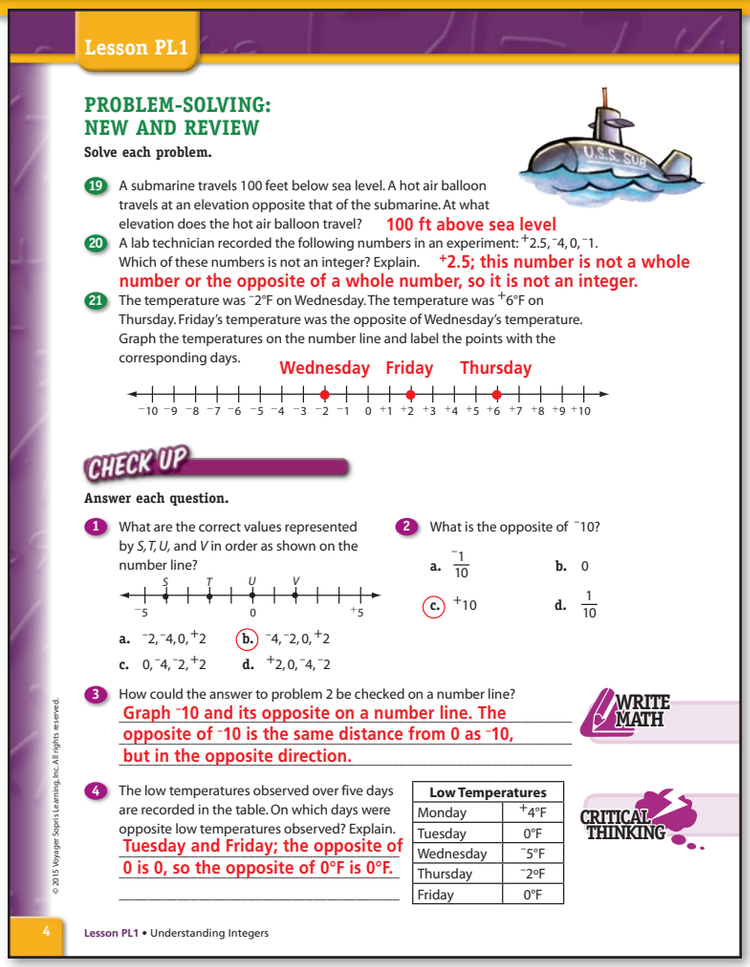
If student answered problem 1 incorrectly: The student did not identify the correct integers on the number line. Use Additional Resources in I.7.PL1 to reteach how to identify integers on a number line.

If student answered problem 2 incorrectly: The student did not apply the definition of opposite integers. Remind the student that the opposite of an integer is the same distance from 0 on the number line, but in the opposite direction.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that opposite integers are the same distance from 0 on a number line, but in opposite directions.



Lesson PL1

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

19 A submarine travels 100 feet below sea level. A hot air balloon travels at an elevation opposite that of the submarine. At what elevation does the hot air balloon travel? 100 ft above sea level

20 A lab technician recorded the following numbers in an experiment: $^{+}2.5, ^{-}4, 0, ^{-}1$. Which of these numbers is not an integer? Explain. $^{+}2.5$; this number is not a whole number or the opposite of a whole number, so it is not an integer.

21 The temperature was $^{-}2$°F on Wednesday. The temperature was $^{+}6$°F on Thursday. Friday's temperature was the opposite of Wednesday's temperature. Graph the temperatures on the number line and label the points with the corresponding days.

Wednesday Friday Thursday

$^{-}10\ ^{-}9\ ^{-}8\ ^{-}7\ ^{-}6\ ^{-}5\ ^{-}4\ ^{-}3\ ^{-}2\ ^{-}1\ 0\ ^{+}1\ ^{+}2\ ^{+}3\ ^{+}4\ ^{+}5\ ^{+}6\ ^{+}7\ ^{+}8\ ^{+}9\ ^{+}10$

CHECK UP

Answer each question.

1 What are the correct values represented by *S*, *T*, *U*, and *V* in order as shown on the number line?

S T U V

$^{-}5$ 0 $^{+}5$

a. $^{-}2, ^{-}4, 0, ^{+}2$ (b.) $^{-}4, ^{-}2, 0, ^{+}2$

c. $0, ^{-}4, ^{-}2, ^{+}2$ d. $^{+}2, 0, ^{-}4, ^{-}2$

2 What is the opposite of $^{-}10$?

a. $\frac{^{-}1}{10}$ b. 0

(c.) $^{+}10$ d. $\frac{1}{10}$

3 How could the answer to problem 2 be checked on a number line?

WRITE MATH

Graph $^{-}10$ and its opposite on a number line. The opposite of $^{-}10$ is the same distance from 0 as $^{-}10$, but in the opposite direction.

4 The low temperatures observed over five days are recorded in the table. On which days were opposite low temperatures observed? Explain.

CRITICAL THINKING

Tuesday and Friday; the opposite of 0 is 0, so the opposite of 0°F is 0°F.

Low Temperatures	
Monday	$^{+}4$°F
Tuesday	0°F
Wednesday	$^{-}5$°F
Thursday	$^{-}2$°F
Friday	0°F

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4 Lesson PL1 • Understanding Integers

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 7 Lesson PL1

Reteach Teacher Edition Module 7 Lesson PL1

Extra Practice

Student Book page 47

English Language Learners

Use the VmathLive Animated Glossary to review the term *integers*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write the symbols + and – on the board. Next to the symbols, write the terms *positive* and *negative*. Read the terms aloud with the students. Work with the students to write various positive and negative integers on the board using these symbols. Read the numbers aloud with the students. Explain to the students that these terms are *opposites*. Show the students other examples of opposites, such as: inside/outside the classroom, lights on/off, smile/frown, open/close a door, girl/boy.

Students with Special Needs

Provide each student with a number line, set of counters, and set of crayons or colored pencils. Show students how to label the number line from $^{-}10$ to $^{+}10$. Ask the students to form a group of six counters. Show students how to place a counter at each integer on the number line from $^{+}1$ to $^{+}6$ and say the number aloud with students. Have students color a dot at the integer $^{+}6$ on their number line using a crayon or colored pencil. Then show students how to place the counters from $^{-}1$ to $^{-}6$ and say the number aloud; color a dot on the number line at $^{-}6$ using the same colored pencil or crayon. Repeat this process for various numbers of counters, using a different colored pencil or crayon for each number. Explain to students that each pair of dots of the same color represents opposite integers.

Lesson PL2

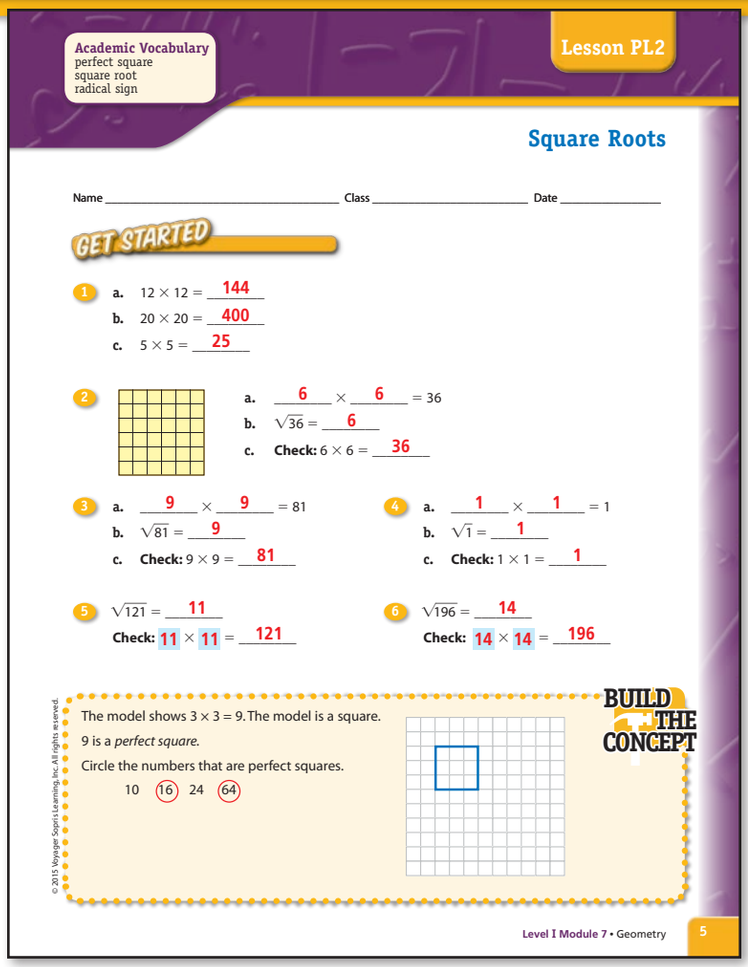
Objective

To find the square root of a perfect square

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **perfect square**
  a whole number that is the product of a number and itself
- **square root**
  a number that when multiplied by itself is equal to a given number
- **radical sign**
  another name for the square root symbol ($\sqrt{\ }$)



Academic Vocabulary
perfect square
square root
radical sign

Lesson PL2

Square Roots

Name _____ Class _____ Date _____

GET STARTED

1 a. $12 \times 12 =$ 144
b. $20 \times 20 =$ 400
c. $5 \times 5 =$ 25

2 a. 6 × 6 = 36
b. $\sqrt{36} =$ 6
c. Check: $6 \times 6 =$ 36

3 a. 9 × 9 = 81
b. $\sqrt{81} =$ 9
c. Check: $9 \times 9 =$ 81

4 a. 1 × 1 = 1
b. $\sqrt{1} =$ 1
c. Check: $1 \times 1 =$ 1

5 $\sqrt{121} =$ 11
Check: $11 \times 11 =$ 121

6 $\sqrt{196} =$ 14
Check: $14 \times 14 =$ 196

BUILD THE CONCEPT

The model shows $3 \times 3 = 9$. The model is a square.
9 is a *perfect square*.
Circle the numbers that are perfect squares.
10 (16) 24 (64)

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Level I Module 7 • Geometry 5

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Multiply 12 and 12. What is 12 times 12? (144) **What is 20 times 20?** (400) **What is 5 times 5?** (25) **Notice that in each problem, a number was multiplied by itself.**

MODEL NEW SKILLS

Problem 2

In this problem, the model is composed of 36 squares. What number times itself equals 36? (6) **Thirty-six is a perfect square because it is the product of a number and itself. Six times 6 equals 36. A number that is multiplied by itself to get a given number is the square root of the given number. For example, the square root of 36 is 6 and is written using a radical sign.**

In problem 2b, read the sentence as *the square root of 36 is 6*. To check, multiply the answer by itself. What is 6 times 6? (36) **The answer is correct because 36 is the number under the radical sign.**

Problem 3

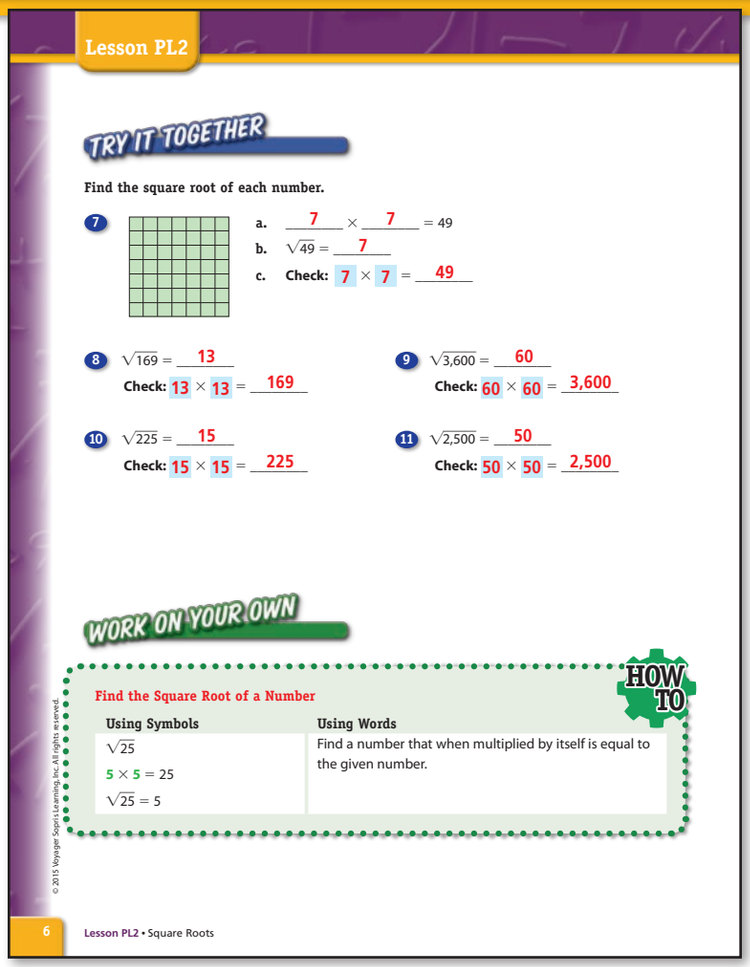
What number times itself equals 81? (9) **Is 81 a perfect square?** (yes) **Why?** (It is the product of a number and itself.) **Because 9 times 9 equals 81, 9 is the square root of 81. To check that the square root of 81 is 9, find the value of 9 multiplied by itself. What is 9 times 9?** (81) **The answer is correct because 81 is the number under the radical sign.**

Problem 4

What number times itself equals 1? (1) **What is the square root of 1?** (1) **To check that the square root of 1 is 1, find the value of 1 multiplied by itself. What is 1 times 1?** (1) **The answer is correct because 1 is the number under the radical sign.**

Problem 5

Sometimes it is necessary to use guess and check to find the square root of a given number. To find the square root of 121, find the number that when multiplied by itself equals 121. Is the square root of 121 greater than 9? (yes) **Why?** (because 9 times 9 equals 81 and 121 is greater than 81) **Is the square root of 121 greater than 10?** (yes) **Why?** (because 10 times 10 equals 100) **What is the number whose square is 121?** (11) **To check the answer, multiply 11 by itself. What is 11 times 11?** (121) **So, the answer is correct.**


Lesson PL2

TRY IT TOGETHER

Find the square root of each number.

7

a. 7 × 7 = 49

b. $\sqrt{49}$ = 7

c. Check: 7 × 7 = 49

8 $\sqrt{169}$ = 13

Check: 13 × 13 = 169

9 $\sqrt{3,600}$ = 60

Check: 60 × 60 = 3,600

10 $\sqrt{225}$ = 15

Check: 15 × 15 = 225

11 $\sqrt{2,500}$ = 50

Check: 50 × 50 = 2,500

WORK ON YOUR OWN

HOW TO

Find the Square Root of a Number

Using Symbols	Using Words
$\sqrt{25}$ 5 × 5 = 25 $\sqrt{25}$ = 5	Find a number that when multiplied by itself is equal to the given number.

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6 Lesson PL2 • Square Roots

Problem 6

Use guess and check to find the square root of 196. The square root of 121 was found in problem 5. Because 196 is greater than 121, the square root of 196 is greater than 11, the square root of 121. What number is multiplied by itself to get 196? (14) **To check the answer, multiply 14 by itself. What is 14 times 14?** (196)

BUILD THE CONCEPT

Model how to identify perfect squares.

A perfect square is the product of a whole number and itself. When the number is thought of as the dimensions of a shape, the shape formed is a square. All four sides are the same length. The area of a square is a square number.

Look at the number 9. Nine is the product of 3 and 3. Look at the shape outlined on the grid. What is the shape's length? (3 units) **What is the shape's width?** (3 units)

The shape outlined is a square with an area of 9 square units. Now, determine which numbers listed are perfect squares. Is there a whole number that when multiplied by itself gives a product of 10? (No) **Ten is not a perfect square.**

Is there a whole number that when multiplied by itself gives a product of 16? (yes) **What number?** (4) **Sixteen is a perfect square. Circle 16. Is 24 a perfect square?** (no) **Is 64 a perfect square?** (yes) **What number when multiplied by itself is 64?** (8) **Circle 64.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 7

How many squares does the model have? (49) **What number times itself equals 49?** (7) **Is 49 a perfect square?** (yes) **Why?** (It is the product of a number and itself.) **What is the square root of 49?** (7) **How can the answer be checked?** (Multiply 7 by itself.) **What is 7 times 7?** (49) **The answer is correct.**

Problem 8

What is the square root of 169? (13) **How can the answer be checked?** (Multiply 13 by itself.) **What is 13 times 13?** (169) **The answer is correct.**

Problem 9

To find the square root of 3,600, think of the square root of 36 first. What is the square root of 36? (6) **Multiply 60 and 60. What is 60 times 60?** (3,600) **What is the square root of 3,600?** (60) **To check, multiply 60 by itself. What is 60 times 60?** (3,600) **The answer is correct.**

Problem 10

What is the square root of 225? (15) **How can the answer be checked?** (Multiply 15 by itself.) **What is 15 times 15?** (225) **The answer is correct.**

Problem 11

To find the square root of 2,500, think of the square root of 25 first. What is the square root of 25? (5) **What is the square root of 2,500?** (50) **To check, multiply 50 and 50. What is 50 times 50?** (2,500) **The answer is correct.**

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the example presented in the **HOW TO** process. As you review, emphasize the words of mathematics by having students read together the words shown at the right.

Problems 12–27

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 24, 25, and 27 Reminder

Students are asked to find the length of the side of a square. To find the square's length, students must find the square root of the area. Remind students to record the answer using the appropriate unit.

Problem 27 Reminder

Students must first find the length of the side of the square, then use that information to find the perimeter of the square.

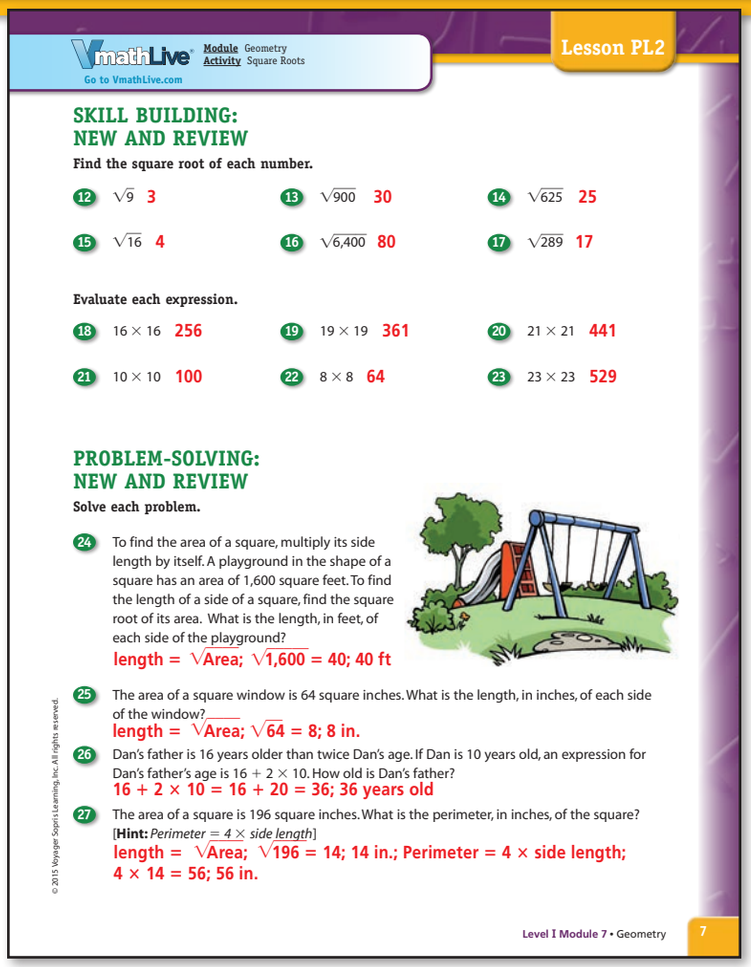
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1c or 2b: The student divided the given number by 4 instead of finding the square root. Remind the student that the square root of a given number is the number that is multiplied by itself to get the given number.



VmathLive Module Activity Geometry Square Roots
Go to VmathLive.com

Lesson PL2

SKILL BUILDING: NEW AND REVIEW

Find the square root of each number.

12. $\sqrt{9}$ 3
13. $\sqrt{900}$ 30
14. $\sqrt{625}$ 25
15. $\sqrt{16}$ 4
16. $\sqrt{6{,}400}$ 80
17. $\sqrt{289}$ 17

Evaluate each expression.

18. 16×16 256
19. 19×19 361
20. 21×21 441
21. 10×10 100
22. 8×8 64
23. 23×23 529

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

24. To find the area of a square, multiply its side length by itself. A playground in the shape of a square has an area of 1,600 square feet. To find the length of a side of a square, find the square root of its area. What is the length, in feet, of each side of the playground?
length = $\sqrt{\text{Area}}$; $\sqrt{1{,}600}$ = 40; 40 ft

25. The area of a square window is 64 square inches. What is the length, in inches, of each side of the window?
length = $\sqrt{\text{Area}}$; $\sqrt{64}$ = 8; 8 in.

26. Dan's father is 16 years older than twice Dan's age. If Dan is 10 years old, an expression for Dan's father's age is $16 + 2 \times 10$. How old is Dan's father?
16 + 2 × 10 = 16 + 20 = 36; 36 years old

27. The area of a square is 196 square inches. What is the perimeter, in inches, of the square? [**Hint:** *Perimeter = 4 × side length*]
length = $\sqrt{\text{Area}}$; $\sqrt{196}$ = 14; 14 in.; Perimeter = 4 × side length; 4 × 14 = 56; 56 in.

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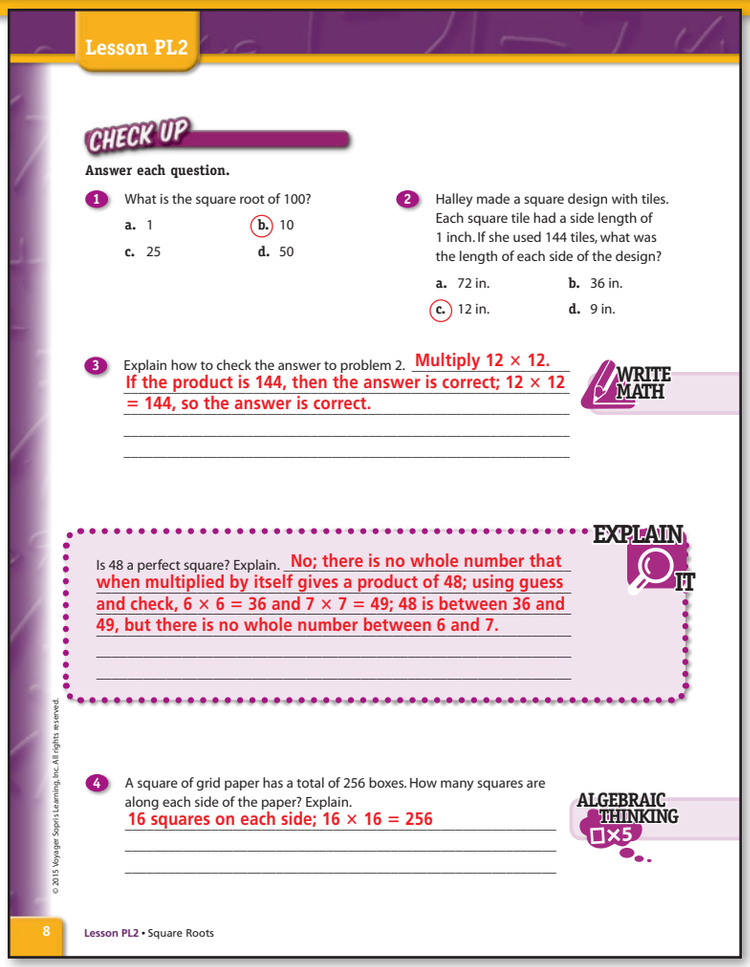
Level I Module 7 • Geometry 7

If student answered 1d or 2a: The student divided the given number by 2 instead of finding the square root. Use Additional Resources in I.7.PL2 to reteach how to find the square root of a perfect square.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the number under the radical sign is the product of a whole number and itself. Review how to find the square root of a perfect square and how to use multiplication to check the answer.



Lesson PL2

CHECK UP

Answer each question.

1. What is the square root of 100?
   a. 1 (b.) 10
   c. 25 d. 50

2. Halley made a square design with tiles. Each square tile had a side length of 1 inch. If she used 144 tiles, what was the length of each side of the design?
   a. 72 in. b. 36 in.
   (c.) 12 in. d. 9 in.

3. Explain how to check the answer to problem 2. Multiply 12 × 12. If the product is 144, then the answer is correct; 12 × 12 = 144, so the answer is correct.

WRITE MATH

EXPLAIN IT

Is 48 a perfect square? Explain. No; there is no whole number that when multiplied by itself gives a product of 48; using guess and check, 6 × 6 = 36 and 7 × 7 = 49; 48 is between 36 and 49, but there is no whole number between 6 and 7.

4. A square of grid paper has a total of 256 boxes. How many squares are along each side of the paper? Explain.
   16 squares on each side; 16 × 16 = 256

ALGEBRAIC THINKING □×5

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8 Lesson PL2 • Square Roots

EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Geometry
Activity: Square Roots

Vmath Reteach
Reteach Student Book Module 7 Lesson PL2
Reteach Teacher Edition Module 7 Lesson PL2

Extra Practice
Student Book page 47

English Language Learners

Use the VmathLive Animated Glossary to review the term *square root*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Write the expression $\sqrt{49}$ 7 on the board. Label the radical sign. Explain that the expression is read as *the square root of 49 is 7*. Point out that 49 is a perfect square. Write other expressions on the board and ask students to read the expression and label the perfect square.

Students with Special Needs

Provide students with grid paper. Have them use the grid paper to find the square root of a number. For example, ask students to draw a square with an area of 25 square units. They should draw a square that measures 5 units on each side. The square root of 25 is 5. Repeat with the numbers 1, 4, 16, and 64.

Lesson 1

Objective

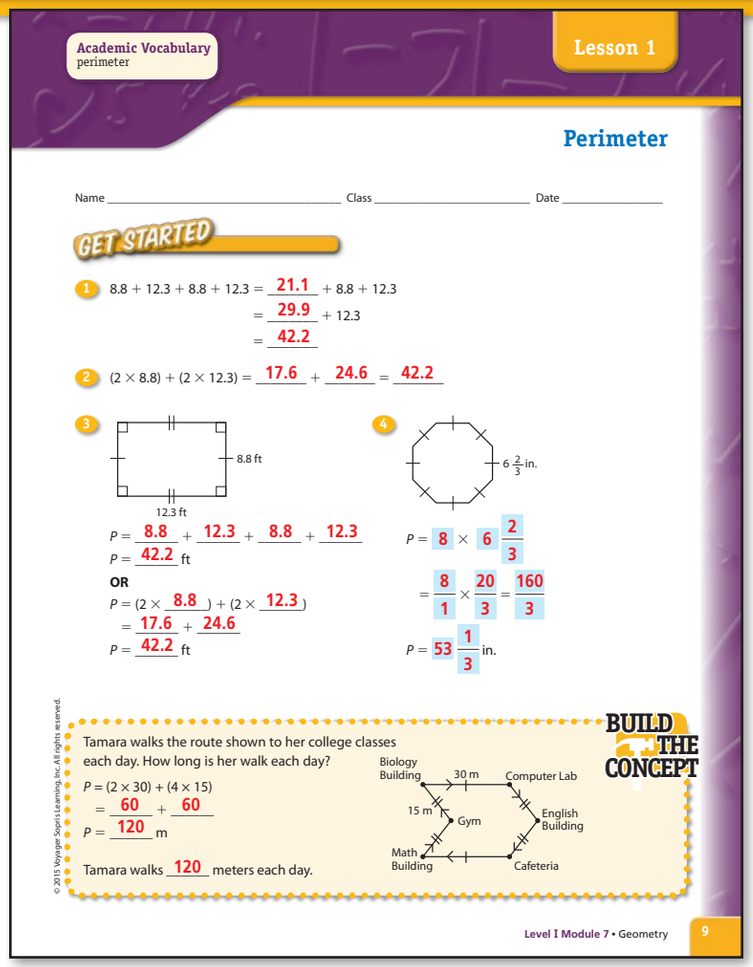
To find the perimeter of a figure

Preskills	Lesson
Math Flash: Adding Positive and Negative Fractions	I.1.13
Evaluating Expressions Using Order of Operations	I.2.2

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **perimeter**
  the distance around the outside of a figure



Academic Vocabulary
perimeter

Lesson 1

Perimeter

Name _____ Class _____ Date _____

GET STARTED

1. 8.8 + 12.3 + 8.8 + 12.3 = 21.1 + 8.8 + 12.3
   = 29.9 + 12.3
   = 42.2

2. (2 × 8.8) + (2 × 12.3) = 17.6 + 24.6 = 42.2

3. 8.8 ft, 12.3 ft
   P = 8.8 + 12.3 + 8.8 + 12.3
   P = 42.2 ft
   OR
   P = (2 × 8.8) + (2 × 12.3)
   = 17.6 + 24.6
   P = 42.2 ft

4. $6\frac{2}{3}$ in.
   $P = 8 \times 6\frac{2}{3}$
   $= \frac{8}{1} \times \frac{20}{3} = \frac{160}{3}$
   $P = 53\frac{1}{3}$ in.

BUILD THE CONCEPT

Tamara walks the route shown to her college classes each day. How long is her walk each day?

P = (2 × 30) + (4 × 15)
= 60 + 60
P = 120 m

Tamara walks 120 meters each day.

Biology Building, 30 m, Computer Lab, 15 m, Gym, English Building, Math Building, Cafeteria

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Level I Module 7 • Geometry 9

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To find the sum of the numbers in problem 1, add the first two numbers. Then add the third number to that sum. Continue until all the numbers have been used.

What is 8.8 plus 12.3? (21.1) **What is 21.1 plus 8.8?** (29.9) **What is 29.9 plus 12.3?** (42.2)

Problem 2

To find the value of the expression in problem 2, perform the operations in parentheses first. What is 2 times 8.8? (17.6) **What is 2 times 12.3?** (24.6) **Now add the products. What is 17.6 plus 24.6?** (42.2)

MODEL NEW SKILLS

Problem 3

Perimeter is the distance around the outside of a figure. Look at the rectangle in this problem. There are two ways to find the perimeter of a rectangle. One way to find the perimeter is to add the lengths of the sides.

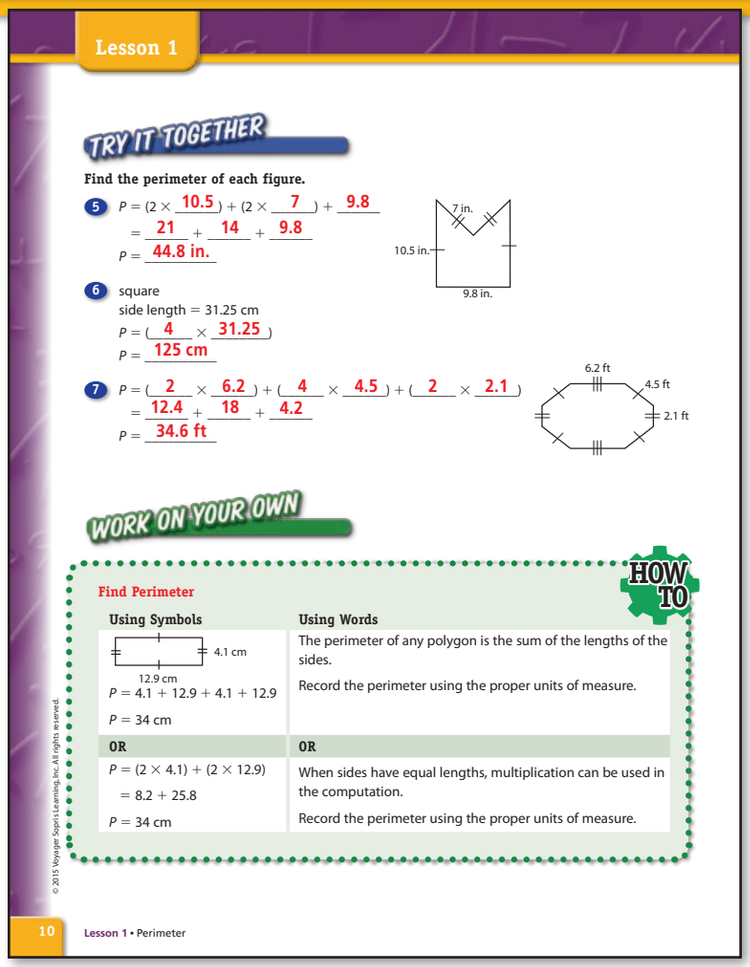
Let *P* represent the perimeter of the rectangle. What are the lengths of the sides? (8.8 ft, 12.3 ft, 8.8 ft, 12.3 ft) **This sum was found in problem 1. What is 8.8 plus 12.3 plus 8.8 plus 12.3?** (42.2) **What is the perimeter of the rectangle?** (42.2 ft)

A second way to find the perimeter of the rectangle is to use multiplication in the computation. There are two sides with a length of 8.8 feet and two sides with a length of 12.3 feet.

Look at problem 2. What is another way to compute the perimeter of this figure? ((2 8.8) + (2 12.3)) **What is 2 times 8.8?** (17.6) **What is 2 times 12.3?** (24.6) **What is the sum of the products?** (42.2) **Record the perimeter using the proper unit of measure. What is the perimeter of the rectangle?** (42.2 ft)

Problem 4

How many sides does this figure have? (8) **Are any sides of equal length?** (yes) **What are they?** (8 sides with lengths of $6\frac{2}{3}$ in.) **Because all eight sides have the same length, multiplication can be used to find the perimeter. What expression can be used to find the perimeter of the figure?** (8 $6\frac{2}{3}$)



Lesson 1

TRY IT TOGETHER

Find the perimeter of each figure.

5 $P = (2 \times 10.5) + (2 \times 7) + 9.8$
$= 21 + 14 + 9.8$
$P = 44.8$ in.

6 square
side length = 31.25 cm
$P = (4 \times 31.25)$
$P = 125$ cm

7 $P = (2 \times 6.2) + (4 \times 4.5) + (2 \times 2.1)$
$= 12.4 + 18 + 4.2$
$P = 34.6$ ft

WORK ON YOUR OWN

HOW TO

Find Perimeter

Using Symbols	Using Words
4.1 cm, 12.9 cm $P = 4.1 + 12.9 + 4.1 + 12.9$ $P = 34$ cm	The perimeter of any polygon is the sum of the lengths of the sides. Record the perimeter using the proper units of measure.
OR	OR
$P = (2 \times 4.1) + (2 \times 12.9)$ $= 8.2 + 25.8$ $P = 34$ cm	When sides have equal lengths, multiplication can be used in the computation. Record the perimeter using the proper units of measure.

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To multiply these numbers, first change each to an improper fraction. What two fractions should be multiplied? ($\frac{8}{1}$ and $\frac{20}{3}$) **What is the product?** ($\frac{160}{3}$) **Is $\frac{160}{3}$ written in simplest form?** (no) **What is $\frac{160}{3}$ written in simplest form?** ($53\frac{1}{3}$) **What is the perimeter of this figure?** ($53\frac{1}{3}$ in.) **Notice that using multiplication in the computation of perimeter when there are sides of equal length can make the computation shorter.**

BUILD THE CONCEPT

Model how to find the perimeter of a figure.

Tamara walks the route shown to her college classes each day. A figure is formed from the path she takes. How many sides does this figure have? (6) **Do any of the sides have equal length?** (yes) **What are they?** (2 sides with lengths of 30 m and 4 sides with lengths of 15 m)

The expression (2 30) (4 15) can be used to find the perimeter of the figure. What is 2 times 30? (60) **What is 4 times 15?** (60) **What is 60 plus 60?** (120) **What is the perimeter of this figure?** (120 m) **So, Tamara walks 120 meters each day to her classes.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

How many sides does this figure have? (5) **Are any sides of equal length?** (yes) **How many of its sides have a length of 10.5 inches?** (2) **How many sides have a length of 7 inches?** (2) **How many sides have a length of 9.8 inches?** (1) **What expression can be used to compute the perimeter of the figure?** ((2 10.5) (2 7) 9.8)

What is 2 times 10.5? (21) **What is 2 times 7?** (14) **What is 21 plus 14 plus 9.8?** (44.8) **What is the perimeter of the figure?** (44.8 in.)

Problem 6

No figure is shown in this problem, but a figure is described. What figure is described? (a square with sides each measuring 31.25 cm) **How many sides have the same length?** (4) **Why?** (A square has 4 sides with equal length.) **How can the perimeter be found using multiplication?** (4 31.25)

What is 4 times 31.25? (125) **What is the perimeter of the figure?** (125 cm)

Problem 7

How many of this figure's sides have a length of 6.2 feet? (2) **How many sides have a length of 4.5 feet?** (4) **How many sides have a length of 2.1 feet?** (2) **What expression can be used to find the perimeter of the figure?** ((2 6.2) (4 4.5) (2 2.1))

What is 2 times 6.2? (12.4) **What is 4 times 4.5?** (18) **What is 2 times 2.1?** (4.2) **What is 12.4 plus 18 plus 4.2?** (34.6) **What is the perimeter of the figure?** (34.6 ft)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 8–20

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 11–13, 17, 18, and 20 Reminder

It may be helpful for students to draw and label each figure before finding the perimeter.

Problem 17 Reminder

To determine the number of inches of wood needed, students need to find the perimeter of the painting. Then they need to compare the answer with 80 inches, the amount of wood Shari has.

Problem 19 Reminder

Remind students that the two parts of the rectangle are congruent. So the area of the yellow triangle is half the area of the rectangle.

CHECK UP

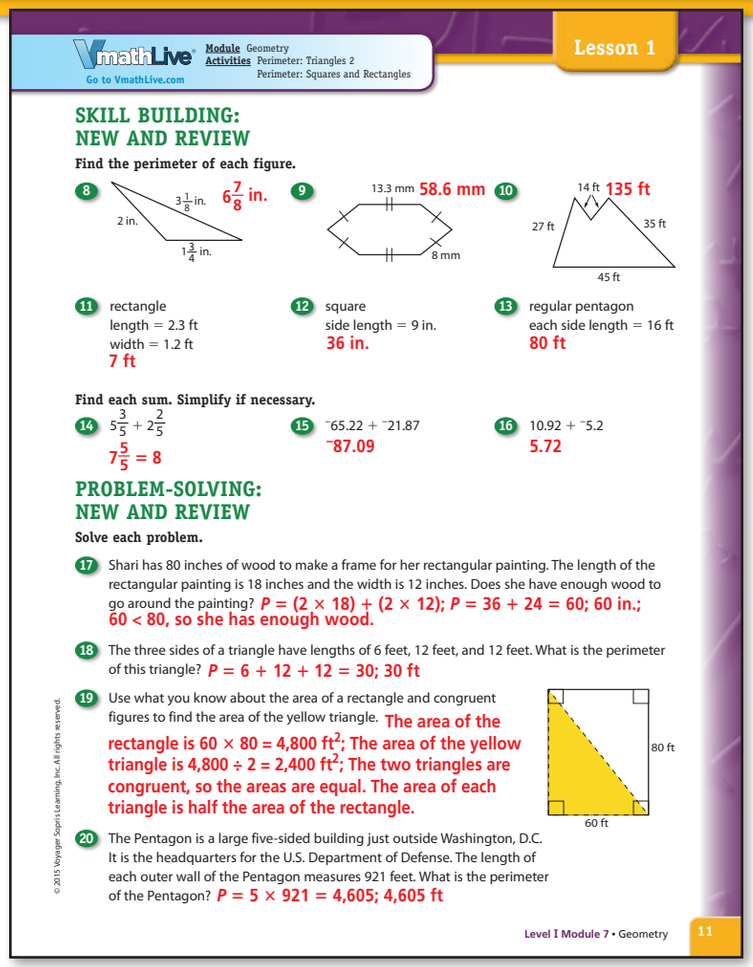
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a, 1d, or 2a: The student did not include all the sides of the figure when finding the perimeter. Use Additional Resources in I.7.1 to reteach how to find the perimeter of a figure.

If student answered 1b: The student multiplied the decimal number incorrectly. Review multiplication of decimal numbers with the student.



VmathLive Go to VmathLive.com
Module Activities: Geometry; Perimeter: Triangles 2; Perimeter: Squares and Rectangles

Lesson 1

SKILL BUILDING: NEW AND REVIEW

Find the perimeter of each figure.

8. $6\frac{7}{8}$ in. (2 in., $3\frac{1}{8}$ in., $1\frac{3}{4}$ in.)
9. 58.6 mm (13.3 mm, 8 mm)
10. 135 ft (14 ft, 27 ft, 35 ft, 45 ft)
11. rectangle; length = 2.3 ft; width = 1.2 ft; 7 ft
12. square; side length = 9 in.; 36 in.
13. regular pentagon; each side length = 16 ft; 80 ft

Find each sum. Simplify if necessary.

14. $5\frac{3}{5} + 2\frac{2}{5} = 7\frac{5}{5} = 8$
15. $^-65.22 + ^-21.87 = ^-87.09$
16. $10.92 + ^-5.2 = 5.72$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

17. Shari has 80 inches of wood to make a frame for her rectangular painting. The length of the rectangular painting is 18 inches and the width is 12 inches. Does she have enough wood to go around the painting? $P = (2 \times 18) + (2 \times 12)$; $P = 36 + 24 = 60$; 60 in.; $60 < 80$, so she has enough wood.
18. The three sides of a triangle have lengths of 6 feet, 12 feet, and 12 feet. What is the perimeter of this triangle? $P = 6 + 12 + 12 = 30$; 30 ft
19. Use what you know about the area of a rectangle and congruent figures to find the area of the yellow triangle. The area of the rectangle is $60 \times 80 = 4{,}800$ ft^2; The area of the yellow triangle is $4{,}800 \div 2 = 2{,}400$ ft^2; The two triangles are congruent, so the areas are equal. The area of each triangle is half the area of the rectangle. (80 ft, 60 ft)
20. The Pentagon is a large five-sided building just outside Washington, D.C. It is the headquarters for the U.S. Department of Defense. The length of each outer wall of the Pentagon measures 921 feet. What is the perimeter of the Pentagon? $P = 5 \times 921 = 4{,}605$; 4,605 ft

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Level I Module 7 • Geometry 11

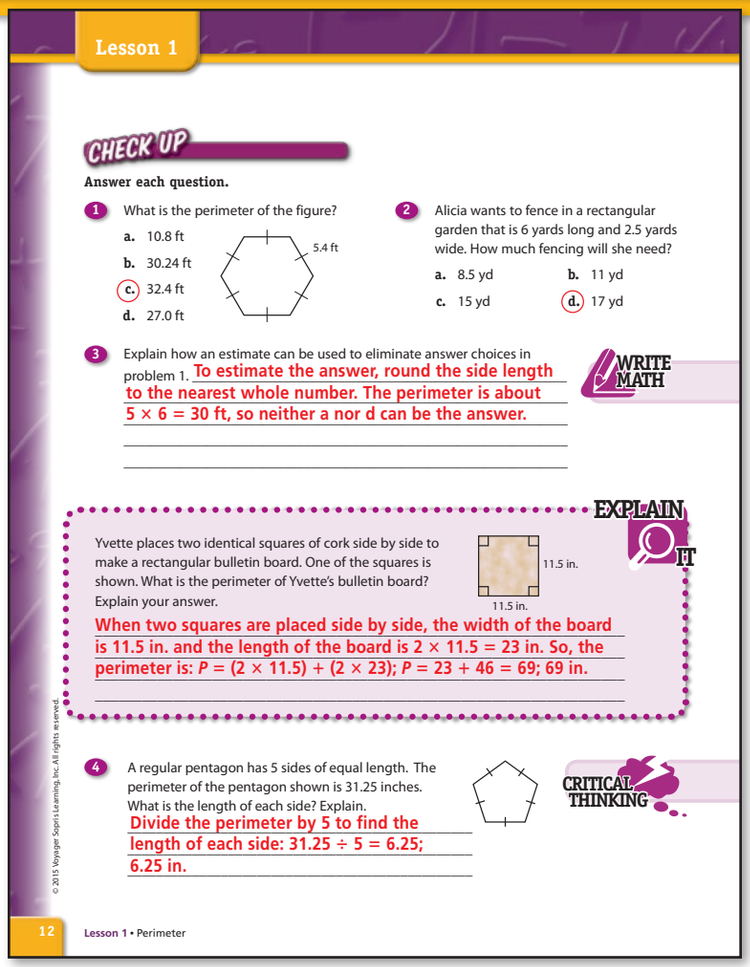
If student answered 2b: The student added the longer side one time in the computation of perimeter. Use Additional Resources in I.7.1 to reteach how to find perimeter.

If student answered 2c: The student multiplied the length by the width to find the perimeter instead of adding the lengths of the four sides. Use Additional Resources in I.7.1 to reteach how to find perimeter.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student how to round a decimal number to the nearest whole number. Encourage the student to use estimation to eliminate answer choices or to check answers for reasonableness.



Lesson 1

CHECK UP

Answer each question.

1. What is the perimeter of the figure?

a. 10.8 ft
b. 30.24 ft
c. 32.4 ft
d. 27.0 ft

2. Alicia wants to fence in a rectangular garden that is 6 yards long and 2.5 yards wide. How much fencing will she need?

a. 8.5 yd
b. 11 yd
c. 15 yd
d. 17 yd

3. Explain how an estimate can be used to eliminate answer choices in problem 1. To estimate the answer, round the side length to the nearest whole number. The perimeter is about 5 × 6 = 30 ft, so neither a nor d can be the answer.

WRITE MATH

EXPLAIN IT

Yvette places two identical squares of cork side by side to make a rectangular bulletin board. One of the squares is shown. What is the perimeter of Yvette's bulletin board? Explain your answer.

When two squares are placed side by side, the width of the board is 11.5 in. and the length of the board is 2 × 11.5 = 23 in. So, the perimeter is: $P = (2 \times 11.5) + (2 \times 23)$; $P = 23 + 46 = 69$; 69 in.

4. A regular pentagon has 5 sides of equal length. The perimeter of the pentagon shown is 31.25 inches. What is the length of each side? Explain.

CRITICAL THINKING

Divide the perimeter by 5 to find the length of each side: 31.25 ÷ 5 = 6.25; 6.25 in.

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EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive

Module: Geometry

Activities: Perimeter: Triangles 2

Perimeter: Squares and Rectangles

Vmath Reteach

Reteach Student Book Module 7 Lesson 1

Reteach Teacher Edition Module 7 Lesson 1

Extra Practice

Student Book page 48

English Language Learners

Use the VmathLive Animated Glossary to review the term *perimeter*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Tell students that the term *shape* used in the dictionary definition is another name for a figure. Encourage students to click on the related term *boundary* in the dictionary to see how the boundary of a shape and its perimeter are related.

Students with Special Needs

Have students label each side of a figure with its length before computing the perimeter. Review with students how to interpret the slash marks on a diagram to identify the length of unmarked sides.

Lesson 2

Objective

To find the area of a triangle and the area of a compound figure formed by triangles or triangles and rectangles

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **height of a triangle**
  the length of the perpendicular line segment drawn from a vertex to the opposite side of the triangle
- **base of a triangle**
  the side of the triangle that is perpendicular to the line segment representing the height

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

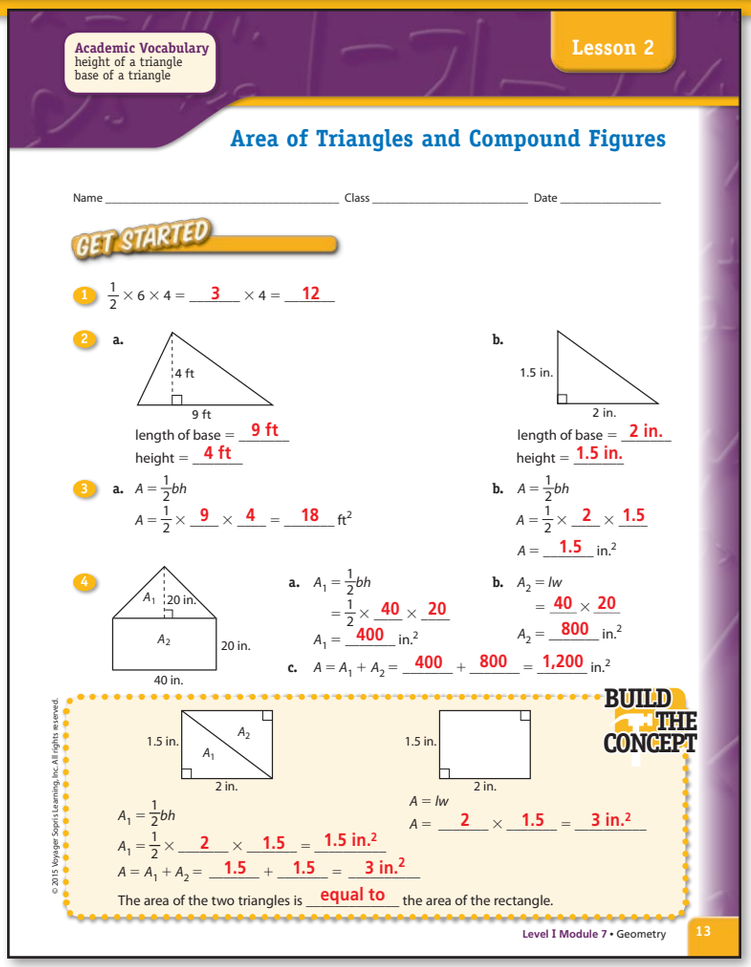
Problem 1

To multiply three or more numbers, multiply two numbers at a time. What is $\frac{1}{2}$ times 6? (3) **What is 3 times 4?** (12) **What is $\frac{1}{2}$ times 6 times 4?** (12)

MODEL NEW SKILLS

Problem 2

From any vertex of a triangle, a perpendicular line segment can be drawn from the vertex to the opposite side or an extension of the opposite side of the triangle. The length of this line segment is called the height of the triangle. It is represented by the letter *h*. The side of the triangle that is perpendicular to the line segment that represents the height is called the base of the triangle. Its length is represented by the letter *b*. In problem 2a, the height of the triangle is shown by the dashed line segment. What is the length of the base of this triangle? (9 ft) **What is the height of this triangle?** (4 ft)



Academic Vocabulary
height of a triangle
base of a triangle

Lesson 2

Area of Triangles and Compound Figures

Name _____ Class _____ Date _____

GET STARTED

1. $\frac{1}{2} \times 6 \times 4 =$ 3 $\times 4 =$ 12

2. a. 4 ft, 9 ft
   length of base = 9 ft
   height = 4 ft

   b. 1.5 in., 2 in.
   length of base = 2 in.
   height = 1.5 in.

3. a. $A = \frac{1}{2}bh$
   $A = \frac{1}{2} \times 9 \times 4 =$ 18 ft^2

   b. $A = \frac{1}{2}bh$
   $A = \frac{1}{2} \times 2 \times 1.5$
   $A =$ 1.5 in.^{2}

4. A_1, 20 in., A_2, 20 in., 40 in.

   a. $A_1 = \frac{1}{2}bh$
   $= \frac{1}{2} \times 40 \times 20$
   $A_1 =$ 400 in.^{2}

   b. $A_2 = lw$
   $= 40 \times 20$
   $A_2 =$ 800 in.^{2}

   c. $A = A_1 + A_2 =$ 400 + 800 = 1,200 in.^{2}

BUILD THE CONCEPT

1.5 in., A_1, A_2, 2 in.

$A_1 = \frac{1}{2}bh$
$A_1 = \frac{1}{2} \times 2 \times 1.5 =$ 1.5 in.^{2}
$A = A_1 + A_2 =$ 1.5 + 1.5 = 3 in.^{2}

1.5 in., 2 in.

$A = lw$
$A =$ 2 $\times$ 1.5 = 3 in.^{2}

The area of the two triangles is equal to the area of the rectangle.

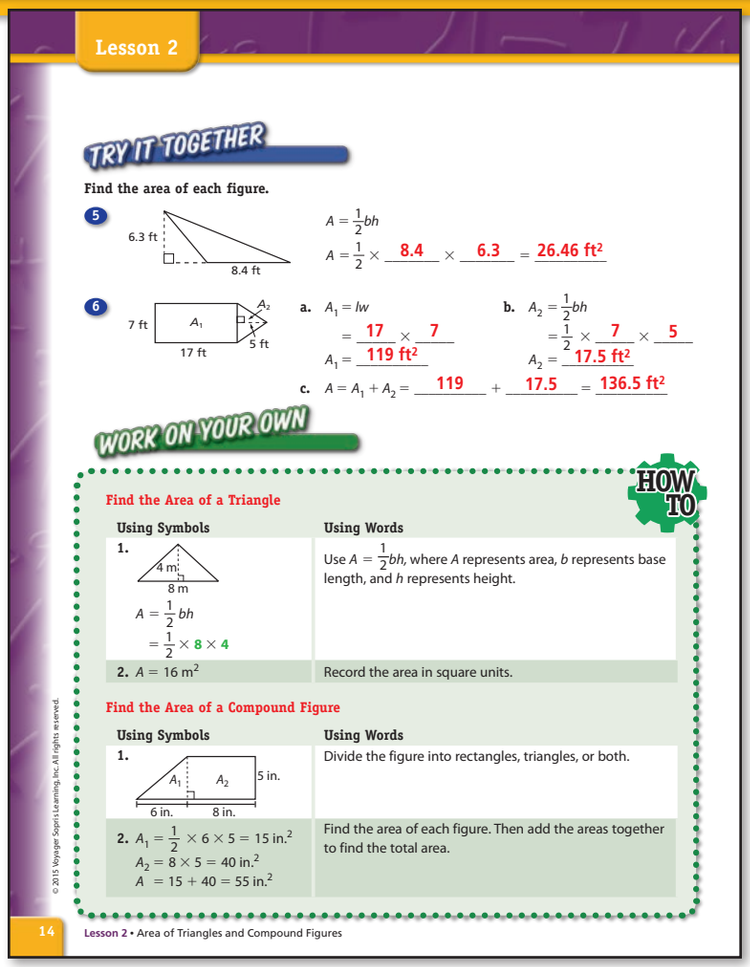
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The triangle in problem 2b is a right triangle. In a right triangle, the height is measured by the length of a leg of the triangle. What is the length of the base of this triangle? (2 in.) **What is the height of this triangle?** (1.5 in.)

Problem 3

The length of the base and the height of a triangle are used to find the area of the triangle. The formula for the area of a triangle is $\frac{1}{2}$ times the length of the base times the height, or $\frac{1}{2}$ *bh*. Use the formula to find the area of the triangle in problem 2a. Substitute the values for *b* and *h* into the formula. In the triangle in problem 2a, what is the length of the base? (9 ft) **What is the height?** (4 ft) **What is $\frac{1}{2}$ times 9 times 4?** (18) **Always record the area of a figure in square units. What is the area of this triangle?** (18 ft^2)

The length of the base and the height of the right triangle in problem 2b have been identified. What is the length of the base of the right triangle in problem 2b? (2 in.) **What is the height?** (1.5 in.) **What is $\frac{1}{2}$ times 2 times 1.5?** (1.5) **What is the area of this triangle?** (1.5 in.^{2})



Problem 4

The figure in problem 4 is made up of a triangle and a rectangle that are joined. The formulas for the area of a triangle and the area of a rectangle can be used to find the area of this figure.

In problem 4a, find the area of the triangle. The triangle's area is labeled as A_1. What is the length of the base of this triangle? (40 in.) **What is the height?** (20 in.) **What is $\frac{1}{2}$ times 40 times 20?** (400) **What is the area of the triangle?** (400 in.^{2})

In problem 4b, find the area of the rectangle. The rectangle's area is labeled as A_2. What is the length of the rectangle? (40 in.) **What is the width of the rectangle?** (20 in.) **What is 40 times 20?** (800) **What is the area of the rectangle?** (800 in.^{2})

In problem 4c, find the total area of the figure by adding the areas A_1 and A_2. What is 400 plus 800? (1,200) **What is the total area?** (1,200 in.^{2})

BUILD THE CONCEPT

Model how to find the area of a triangle and a compound figure formed by triangles.

The rectangle shown at the left is divided into what two shapes? (right triangles) **To find the area of the rectangle, find the area of each triangle and add them together. What are the length of the base and the height of the triangle with area A_1?** (The length of the base is 2 in., and the height is 1.5 in.) **The area of this triangle was found in problem 3b. What is A_1?** (1.5 in.^{2}) **What is A_2?** (1.5 in.^{2}) **Why?** (The area labeled A_2 is the same as A_1 because the triangles are congruent.)

Find the area of the rectangle by adding the areas of the triangles. What is 1.5 plus 1.5? (3) **What is the area of the rectangle?** (3 in.^{2}) **Now look at the rectangle shown at the right. What is the formula for the area of a rectangle?** (*A lw*) **What are the length and width of this rectangle?** (The length is 2 in., and the width is 1.5 in.) **What is 2 times 1.5?** (3) **What is the area of the rectangle?** (3 in.^{2}) **How does the combined area of the two triangles compare to the area of the rectangle?** (They are the same.)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

Notice that the height of this triangle is measured outside the triangle. The height is indicated by the dashed line perpendicular to the extension of the base of the triangle. What is the length of the base of this triangle? (8.4 ft) **What is the height?** (6.3 ft) **What is $\frac{1}{2}$ times 8.4 times 6.3?** (26.46) **What units must be used to record this area?** (square feet) **What is the area of the triangle?** (26.46 ft^2)

Problem 6

What are the dimensions of the rectangle? (17 ft by 7 ft) **What is 17 times 7?** (119) **What is the area of the rectangle?** (119 ft^2) **What is the length of the base of the triangle?** (7 ft) **What is the height of the triangle?** (5 ft) **What is $\frac{1}{2}$ times 7 times 5?** (17.5) **What is the area of the triangle?** (17.5 ft^2) **What is 119 plus 17.5?** (136.5) **What is the area of the entire figure?** (136.5 ft^2)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

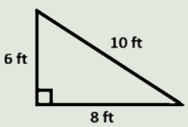
HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Drawing a Picture** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a picture. The instruction is immediately followed by application of the strategy in problem 13.

Have students read the problem.

Have students look at the picture.

Students should label the side lengths with the information given in the problem.



Then students should find the area of the triangle. Show students that the hypotenuse length is superfluous information and is not used to find the area.

$A \quad \frac{1}{2} \quad bh$

$A \quad \frac{1}{2} \quad 8 \quad 6 \quad 24 \text{ ft}^2$

The area is 24 ft^2.

The answer is reasonable because the product of the base and height is 8 6 48; one-half of 48 is 24.

Problems 7–16

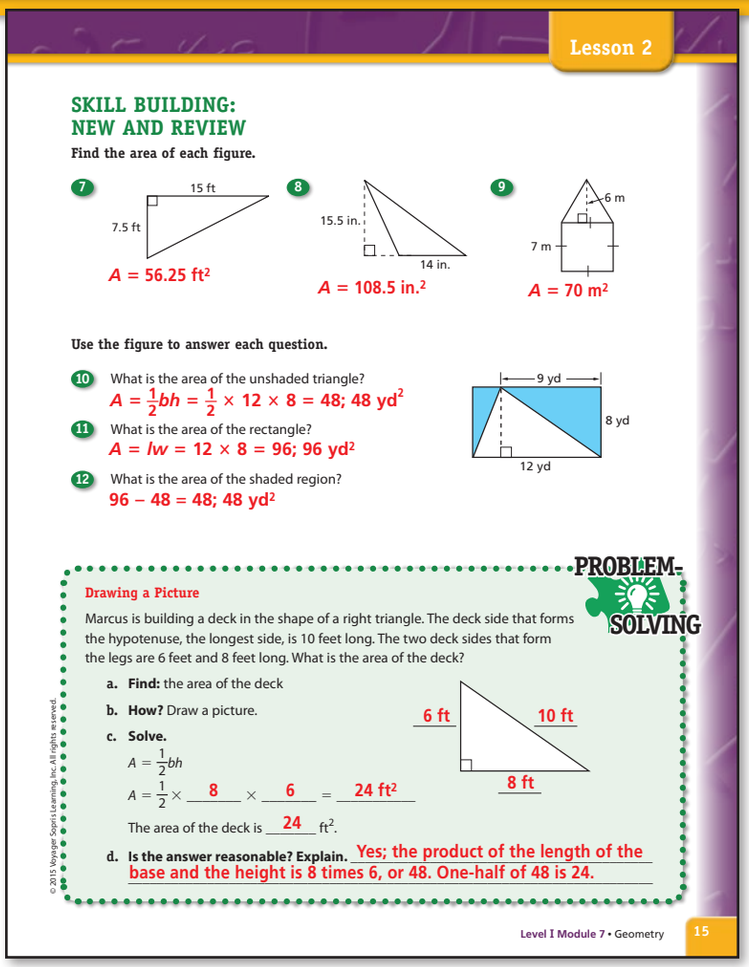
Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 7, 14, and 16 Reminder

Remind the students that in these right triangles, the leg of the triangle is the height.

Problem 12 Reminder

Students will use the results of problems 10 and 11 to find the area of the shaded region, which is the difference between the area of the entire rectangle and the area of the unshaded triangle.



Problem 14 Reminder

Students should notice the orientation of the drawing. Because this figure is a right triangle, the two side lengths given are equal to the length of a base and the height.

Problem 15 Reminder

This problem reviews finding the circumference of a circle. Remind students that the formula for the circumference of a circle is $C \quad \pi d$.

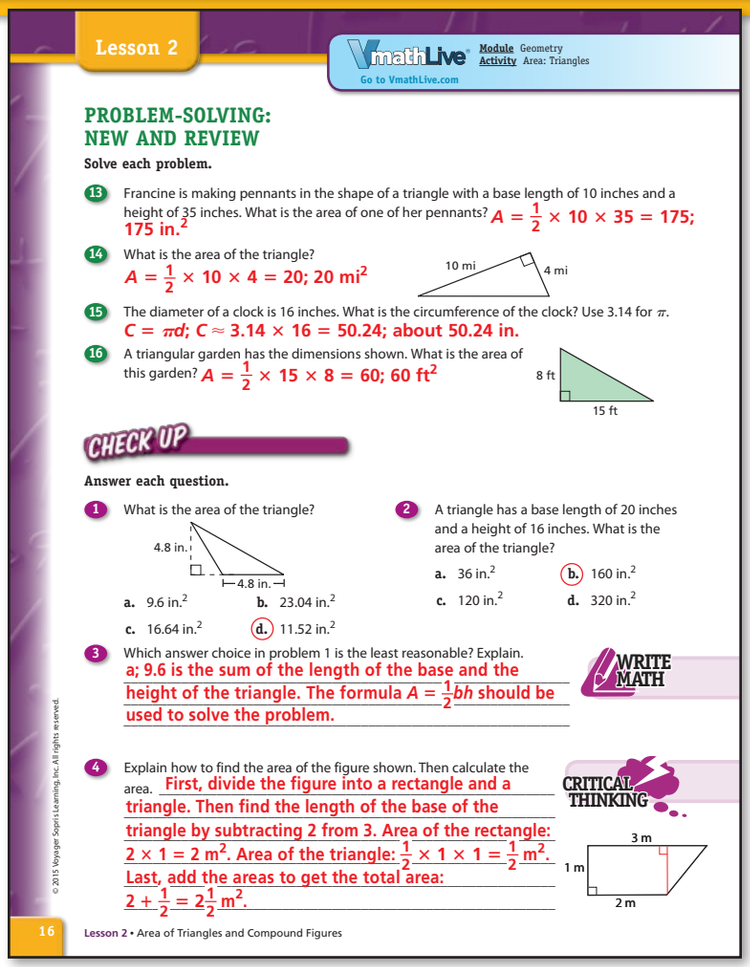
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2a: The student added the length of the base and the height instead of using the formula for area of a triangle. Use Additional Resources in I.7.2 to reteach how to find the area of a triangle.



Lesson 2

VmathLive Module Geometry Activity Area: Triangles
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

13. Francine is making pennants in the shape of a triangle with a base length of 10 inches and a height of 35 inches. What is the area of one of her pennants? $A = \frac{1}{2} \times 10 \times 35 = 175$; 175 in.^{2}

14. What is the area of the triangle?
$A = \frac{1}{2} \times 10 \times 4 = 20$; 20 mi^2

15. The diameter of a clock is 16 inches. What is the circumference of the clock? Use 3.14 for π.
$C = \pi d$; $C \approx 3.14 \times 16 = 50.24$; about 50.24 in.

16. A triangular garden has the dimensions shown. What is the area of this garden? $A = \frac{1}{2} \times 15 \times 8 = 60$; 60 ft^2

CHECK UP

Answer each question.

1. What is the area of the triangle?

a. 9.6 in.^{2} b. 23.04 in.^{2}
c. 16.64 in.^{2} (d.) 11.52 in.^{2}

2. A triangle has a base length of 20 inches and a height of 16 inches. What is the area of the triangle?
a. 36 in.^{2} (b.) 160 in.^{2}
c. 120 in.^{2} d. 320 in.^{2}

3. Which answer choice in problem 1 is the least reasonable? Explain.
a; 9.6 is the sum of the length of the base and the height of the triangle. The formula $A = \frac{1}{2}bh$ should be used to solve the problem.

WRITE MATH

4. Explain how to find the area of the figure shown. Then calculate the area. First, divide the figure into a rectangle and a triangle. Then find the length of the base of the triangle by subtracting 2 from 3. Area of the rectangle: $2 \times 1 = 2$ m^2. Area of the triangle: $\frac{1}{2} \times 1 \times 1 = \frac{1}{2}$ m^2. Last, add the areas to get the total area: $2 + \frac{1}{2} = 2\frac{1}{2}$ m^2.

CRITICAL THINKING

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16 Lesson 2 • Area of Triangles and Compound Figures

If student answered 1b or 2d: The student did not use the factor of $\frac{1}{2}$ in the area formula. Have the student write the formula for area, then substitute the values before multiplying.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that the formula for the area of a triangle involves multiplication, not addition.

DIFFERENTIATION

Additional Resources

VmathLive

VmathLive
Module: Geometry
Activity: Area: Triangles

Vmath Reteach
Reteach Student Book Module 7 Lesson 2
Reteach Teacher Edition Module 7 Lesson 2

Extra Practice
Student Book page 48

English Language Learners

Use the VmathLive Animated Glossary to review the terms *height of a triangle* and *base of a triangle*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Explain to students that the base and the segment that represents the height of a triangle must form a right angle. Draw triangles of various sizes and shapes on the board. Have students use colored chalk to highlight the segment that represents the height and base of each triangle. Point out that the segment that represents the height of a triangle is not necessarily a side of the triangle.

Students with Special Needs

Have students practice finding the product of three numbers. For example, if students are asked to find the product of 3, 4, and 5, first have them find the result of 3 4 5. Then they should find the result of 4 5 3. Continue to change the order of the factors. After this activity, students should realize that changing the order of the factors will not affect the product. Encourage students to use this knowledge to help them with multiplying larger numbers. For example, when multiplying $\frac{1}{2}$ 3 14, show students that it might be easier to first multiply $\frac{1}{2}$ by 14 to get 7, then multiply 7 by 3.

Technology



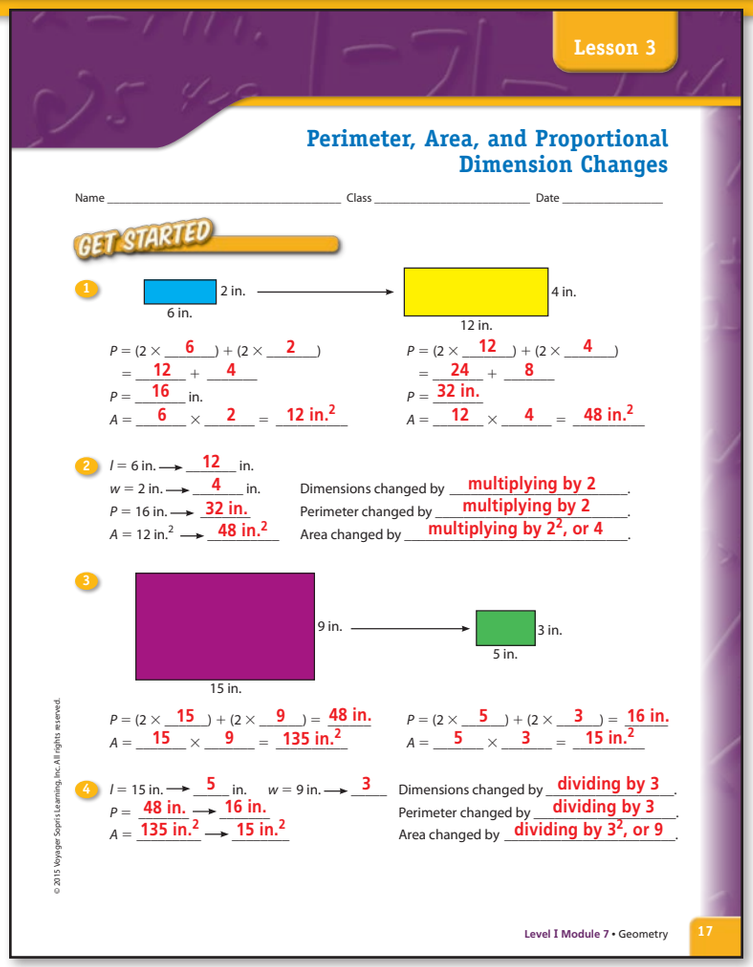
Have students think about math in their world, using an online search engine to find references describing how they could use the area formula to find the area of compound figures. Additional digital content is available through this feature in the eBook.

Lesson 3

Objective

To describe the effects of changes in dimension on perimeter and area

Preskills	Lesson
Exponents	I.2.1
Perimeter	I.7.1
Area of Triangles and Compound Figures	I.7.2



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

Look at the blue rectangle. How is the perimeter of a rectangle found? (by finding the sum of 2 times the length and 2 times the width) **What is the length?** (6 in.) **What is the width?** (2 in.) **What is 2 times 6?** (12) **What is 2 times 2?** (4) **What is 12 plus 4?** (16) **What is the perimeter of the blue rectangle?** (16 in.) **What is the formula for finding the area of a rectangle?** (*A* *lw*) **What is 6 times 2?** (12) **What is the area of the blue rectangle?** (12 in.^{2})

Look at the yellow rectangle. What is the length? (12 in.) **What is the width?** (4 in.) **What is 2 times 12?** (24) **What is 2 times 4?** (8) **What is 24 plus 8?** (32) **What is the perimeter of the yellow rectangle?** (32 in.) **What is 12 times 4?** (48) **What is the area of the yellow rectangle?** (48 in.^{2})

MODEL NEW SKILLS

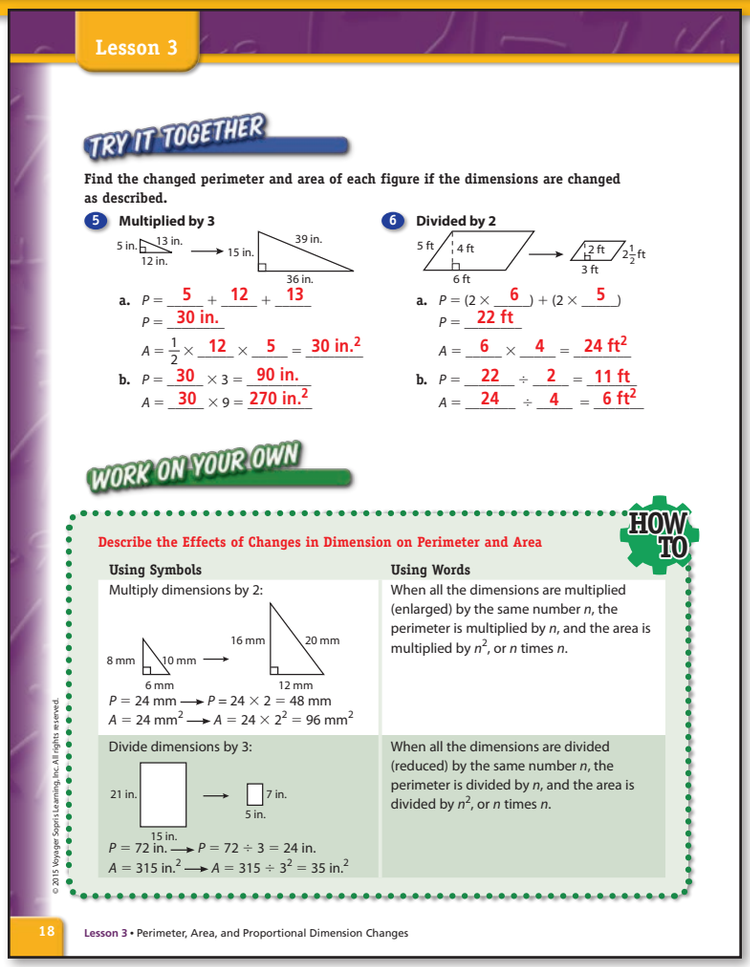
Problem 2

The length of the blue rectangle is 6 inches. What is the length of the yellow rectangle? (12 in.) **How does the length of the yellow rectangle relate to the length of the blue rectangle?** (The length of the yellow rectangle is 2 times the length of the blue rectangle.) **The width of the blue rectangle is 2 inches. What is the width of the yellow rectangle?** (4 in.) **In each case, the dimensions of the yellow rectangle can be found by multiplying the dimensions of the blue rectangle by 2.**

In problem 1, the perimeter of the blue rectangle was found to be 16 inches. What is the perimeter of the yellow rectangle? (32 in.) **The perimeter of the blue rectangle can be multiplied by 2 to get the perimeter of the yellow rectangle. When all the dimensions of a figure are changed by multiplying by 2, the perimeter of the figure is also changed by multiplying by 2.**

The area of the blue rectangle is 12 square inches. What is the area of the yellow rectangle? (48 in.^{2}) **The area of the blue rectangle can be multiplied by 4 to get the area of the yellow rectangle. When all the dimensions of a figure are changed by multiplying by 2, the area of the figure is changed by multiplying by 2^2, or 4.**

Is the yellow rectangle larger or smaller than the blue rectangle? (larger) **The blue rectangle was enlarged to get the yellow rectangle. So when the dimensions of a figure are multiplied by 2, the perimeter is multiplied by 2, and the area is multiplied by 2^2, or 4.**



Lesson 3

TRY IT TOGETHER

Find the changed perimeter and area of each figure if the dimensions are changed as described.

5 Multiplied by 3

5 in., 13 in., 12 in. → 15 in., 39 in., 36 in.

a. $P =$ 5 + 12 + 13
$P =$ 30 in.
$A = \frac{1}{2} \times$ 12 $\times$ 5 $=$ 30 in.^{2}

b. $P =$ 30 $\times 3 =$ 90 in.
$A =$ 30 $\times 9 =$ 270 in.^{2}

6 Divided by 2

5 ft, 4 ft, 6 ft → 2 ft, $2\frac{1}{2}$ ft, 3 ft

a. $P = (2 \times$ 6 $) + (2 \times$ 5 $)$
$P =$ 22 ft
$A =$ 6 $\times$ 4 $=$ 24 ft^2

b. $P =$ 22 $\div$ 2 $=$ 11 ft
$A =$ 24 $\div$ 4 $=$ 6 ft^2

WORK ON YOUR OWN

HOW TO

Describe the Effects of Changes in Dimension on Perimeter and Area

Using Symbols	Using Words
Multiply dimensions by 2: 8 mm, 10 mm, 6 mm → 16 mm, 20 mm, 12 mm $P = 24$ mm $\rightarrow P = 24 \times 2 = 48$ mm $A = 24$ mm^2 $\rightarrow A = 24 \times 2^2 = 96$ mm^2	When all the dimensions are multiplied (enlarged) by the same number n, the perimeter is multiplied by n, and the area is multiplied by n^2, or n times n.
Divide dimensions by 3: 21 in., 15 in. → 7 in., 5 in. $P = 72$ in. $\rightarrow P = 72 \div 3 = 24$ in. $A = 315$ in.^{2} $\rightarrow A = 315 \div 3^2 = 35$ in.^{2}	When all the dimensions are divided (reduced) by the same number n, the perimeter is divided by n, and the area is divided by n^2, or n times n.

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Problem 3

To find the perimeter of the purple rectangle, find the sum of 2 times the length and 2 times the width. What is the length? (15 in.) **What is the width?** (9 in.) **What is 2 times 15?** (30) **What is 2 times 9?** (18) **What is 30 plus 18?** (48) **What is the perimeter of the purple rectangle?** (48 in.) **What is the formula for finding the area of a rectangle?** (A lw) **What is 15 times 9?** (135) **What is the area of the purple rectangle?** (135 in.^{2})

Now look at the green rectangle. What is the perimeter of the green rectangle? (16 in.) **What is the area of the green rectangle?** (15 in.^{2})

Problem 4

The length of the purple rectangle is 15 inches. What is the length of the green rectangle? (5 in.) **The width of the purple rectangle is 9 inches. What is the width of the green rectangle?** (3 in.) **In each case, the dimension of the purple rectangle can be divided by 3 to get the dimension of the green rectangle.**

The perimeter of the purple rectangle is 48 inches. What is the perimeter of the green rectangle? (16 in.) **The perimeter of the purple rectangle can be divided by 3 to get the perimeter of the green rectangle.**

The area of the purple rectangle is 135 square inches. What is the area of the green rectangle? (15 in.^{2}) **The area of the purple rectangle can be divided by 9 to get the area of the green rectangle.**

Is the green rectangle larger or smaller than the purple rectangle? (smaller) **The purple rectangle was reduced to get the green rectangle. When all dimensions of a figure are divided by 3, the perimeter is also divided by 3, and the area of the figure is divided by 3^2, or 9.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

In this problem, consider the effect of multiplying the dimensions of a figure by 3. The triangle on the left is enlarged to create the triangle on the right. What is 5 plus 12 plus 13? (30) **What is the perimeter of the triangle on the left?** (30 in.) **What is the length of the base?** (12 in.) **What is the height?** (5 in.) **What is $\frac{1}{2}$ times 12 times 5?** (30) **What is the area of the triangle on the left?** (30 in.^{2})

Look at problem 5b. What number were the dimensions of the triangle on the left multiplied by to get the dimensions of the triangle on the right? (3)

What number should the perimeter of the triangle on the left be multiplied by to get the perimeter of the triangle on the right? (3) **What is 30 times 3?** (90) **What is the perimeter of the triangle on the right?** (90 in.)

What number should the area of the triangle on the left be multiplied by to find the area of the triangle on the right? (3^2, or 9) **What is 30 times 9?** (270) **What is the area of the triangle on the right?** (270 in.^{2})

Problem 6

In this problem, consider the effect of dividing the dimensions of a parallelogram by 2. First, find the perimeter of the parallelogram on the left. What is 2 times 6? (12) **What is 2 times 5?** (10) **What is 12 plus 10?** (22) **What is the perimeter of the parallelogram**

on the left? (22 ft) **Next, find the area of the parallelogram. What is 6 times 4?** (24) **What is the area of the parallelogram on the left?** (24 ft^2)

Look at problem 6b. The dimensions of the parallelogram on the left were divided by 2 to get the dimensions of the parallelogram on the right.

What number should the perimeter of the parallelogram on the left be divided by to get the perimeter of the parallelogram on the right? (2) **What is 22 divided by 2?** (11) **What is the perimeter of the parallelogram on the right?** (11 ft)

What is 2^2? (4) **What number should the area of the parallelogram on the left be divided by to find the area of the parallelogram on the right?** (4) **What is 24 divided by 4?** (6) **What is the area of the parallelogram on the right?** (6 ft^2)

WORK ON YOUR OWN

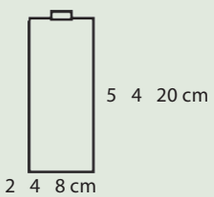
MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Problem-Solving Plan** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 13.

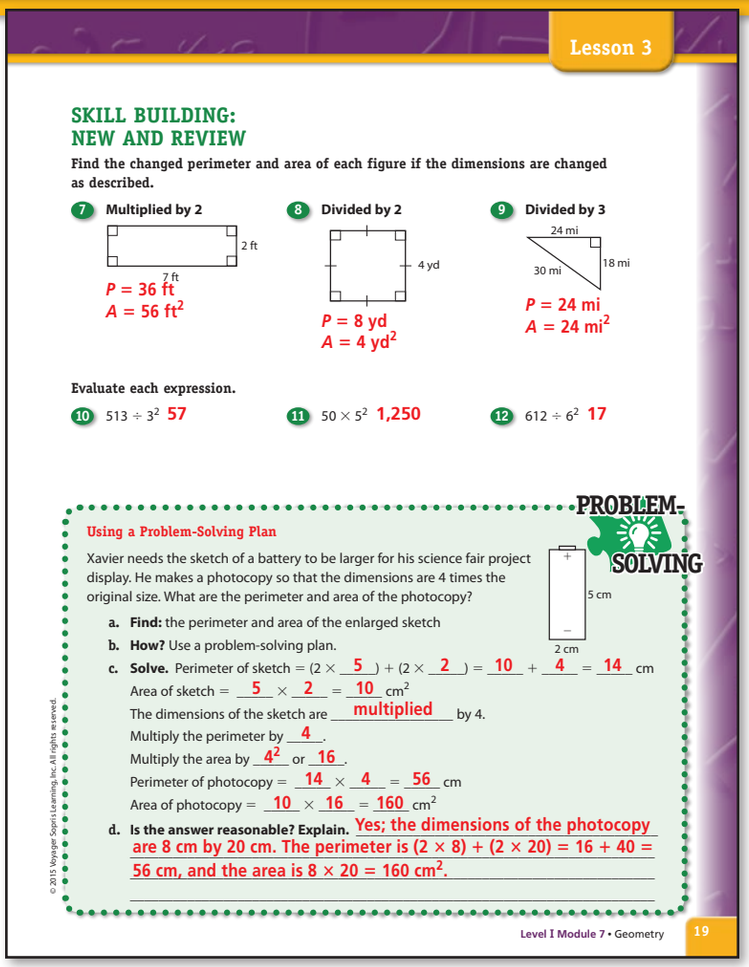
Have students read the problem.

Have students draw a diagram of the photocopy to help them find the perimeter and area.



P (2 8) (2 20) 16 40 56 cm

A 8 20 160 cm^2



Lesson 3

SKILL BUILDING: NEW AND REVIEW

Find the changed perimeter and area of each figure if the dimensions are changed as described.

7 **Multiplied by 2**

2 ft, 7 ft

P = 36 ft
A = 56 ft^2

8 **Divided by 2**

4 yd

P = 8 yd
A = 4 yd^2

9 **Divided by 3**

24 mi, 30 mi, 18 mi

P = 24 mi
A = 24 mi^2

Evaluate each expression.

10 513 ÷ 3^2 57

11 50 × 5^2 1,250

12 612 ÷ 6^2 17

PROBLEM-SOLVING

Using a Problem-Solving Plan

Xavier needs the sketch of a battery to be larger for his science fair project display. He makes a photocopy so that the dimensions are 4 times the original size. What are the perimeter and area of the photocopy?

5 cm, 2 cm

a. **Find:** the perimeter and area of the enlarged sketch

b. **How?** Use a problem-solving plan.

c. **Solve.** Perimeter of sketch = (2 × 5) + (2 × 2) = 10 + 4 = 14 cm
Area of sketch = 5 × 2 = 10 cm^2
The dimensions of the sketch are multiplied by 4.
Multiply the perimeter by 4.
Multiply the area by 4^2 or 16.
Perimeter of photocopy = 14 × 4 = 56 cm
Area of photocopy = 10 × 16 = 160 cm^2

d. **Is the answer reasonable? Explain.** Yes; the dimensions of the photocopy are 8 cm by 20 cm. The perimeter is (2 × 8) + (2 × 20) = 16 + 40 = 56 cm, and the area is 8 × 20 = 160 cm^2.

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The answer is reasonable because the perimeter of the photocopy is 4 times the perimeter of the sketch, *P* (2 5) (2 2) 10 4 14 cm; 4 14 56; and the area of the photocopy is 4^2, or 16, times the area of the sketch, *A* 2 5 10; 16 10 = 160.

Problems 7–15

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 13 Reminder

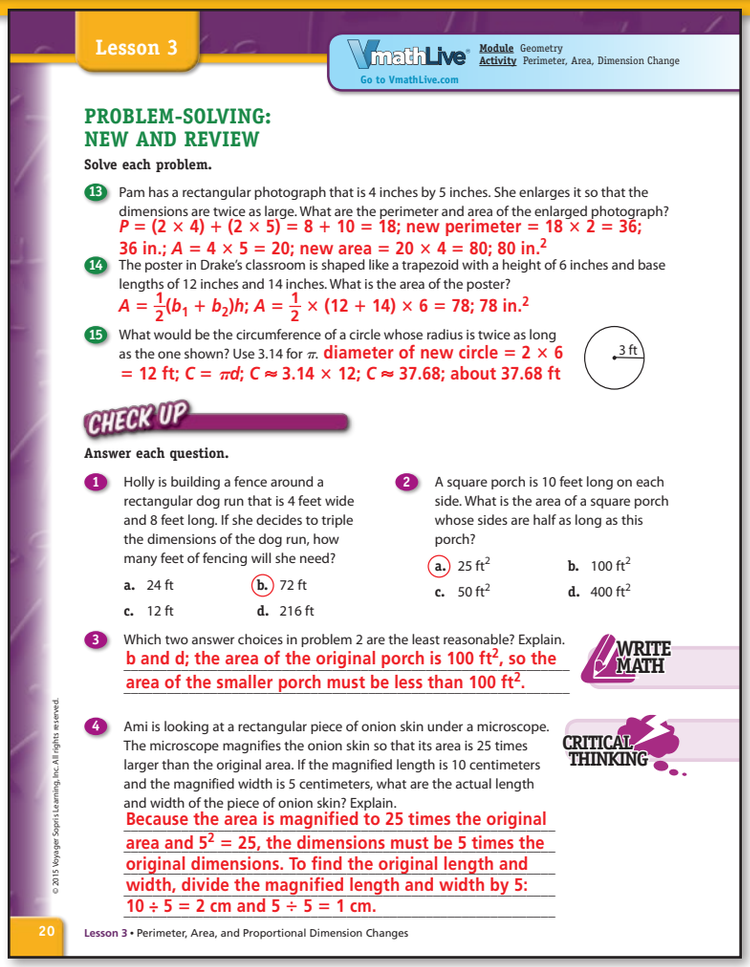
Students will need to find the original perimeter and area first, then apply the dimension changes.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use error analysis to find which skills need review, reteaching, or extra practice.



Lesson 3

VmathLive Module Geometry Activity Perimeter, Area, Dimension Change
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

13 Pam has a rectangular photograph that is 4 inches by 5 inches. She enlarges it so that the dimensions are twice as large. What are the perimeter and area of the enlarged photograph?
$P = (2 \times 4) + (2 \times 5) = 8 + 10 = 18$; new perimeter $= 18 \times 2 = 36$; 36 in.; $A = 4 \times 5 = 20$; new area $= 20 \times 4 = 80$; 80 in.^{2}

14 The poster in Drake's classroom is shaped like a trapezoid with a height of 6 inches and base lengths of 12 inches and 14 inches. What is the area of the poster?
$A = \frac{1}{2}(b_1 + b_2)h$; $A = \frac{1}{2} \times (12 + 14) \times 6 = 78$; 78 in.^{2}

15 What would be the circumference of a circle whose radius is twice as long as the one shown? Use 3.14 for π. diameter of new circle $= 2 \times 6 = 12$ ft; $C = \pi d$; $C \approx 3.14 \times 12$; $C \approx 37.68$; about 37.68 ft

3 ft

CHECK UP

Answer each question.

1 Holly is building a fence around a rectangular dog run that is 4 feet wide and 8 feet long. If she decides to triple the dimensions of the dog run, how many feet of fencing will she need?
a. 24 ft (b.) 72 ft c. 12 ft d. 216 ft

2 A square porch is 10 feet long on each side. What is the area of a square porch whose sides are half as long as this porch?
(a.) 25 ft^2 b. 100 ft^2 c. 50 ft^2 d. 400 ft^2

3 Which two answer choices in problem 2 are the least reasonable? Explain.
WRITE MATH
b and d; the area of the original porch is 100 ft^2, so the area of the smaller porch must be less than 100 ft^2.

4 Ami is looking at a rectangular piece of onion skin under a microscope. The microscope magnifies the onion skin so that its area is 25 times larger than the original area. If the magnified length is 10 centimeters and the magnified width is 5 centimeters, what are the actual length and width of the piece of onion skin? Explain.
CRITICAL THINKING
Because the area is magnified to 25 times the original area and $5^2 = 25$, the dimensions must be 5 times the original dimensions. To find the original length and width, divide the magnified length and width by 5: $10 \div 5 = 2$ cm and $5 \div 5 = 1$ cm.

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20 Lesson 3 • Perimeter, Area, and Proportional Dimension Changes

If student answered 1a or 2b: The student solved the problem using the original dimensions. Use Additional Resources in I.7.3 to reteach the effects of changes in dimensions on perimeter and area.

If student answered 1c : The student added only the lengths of the original two sides. Use Additional Resources in I.7.3 to reteach the effect on perimeter and area when dimensions change.

If student answered 1d or 2c: The student multiplied or divided by the wrong number. Use Additional Resources in I.7.3 to reteach the effects of changes in dimensions on perimeter.

If student answered 2d: The student multiplied when division was needed. Remind the student that if the dimensions of a figure double, multiplication is used. If the dimensions of a figure are halved, division is used.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, have the student sketch the figure whose dimensions are half the original figure, then find the area. Remind the student that if a figure is half as big as another figure, its area will be one-fourth as much as the other figure.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Geometry
Activity: Perimeter, Area, Dimension Change

Vmath Reteach
Reteach Student Book Module 7 Lesson 3
Reteach Teacher Edition Module 7 Lesson 3

Extra Practice
Student Book page 49

English Language Learners

Although there is no new vocabulary defined in this lesson, the term *dimensions* is used often, with the expectation that students understand its meaning. Ask students what is meant when someone asks for the "dimensions of a rug." They will likely know that it has something to do with size. Clarify that it means the length and width (and height for three-dimensional objects).

Students with Special Needs

In this lesson, students need not only to understand the effects of dimension changes on perimeter and area but also to find and verify perimeters and areas of several types of figures. For some students, this may be too much information to recall while solving the problems. Have these students write the formulas for perimeter and area of each figure (square, rectangle, parallelogram, triangle, circle) on index cards. On the front of each card should be a sketch and the name of the figure. On the back should be the formulas for perimeter and area.

HANDS-ON GUIDED DISCOVERY

Lesson 4

Objective

To use grid paper nets to make three-dimensional figures and find the surface area of the figures

CONCEPTUAL DEVELOPMENT

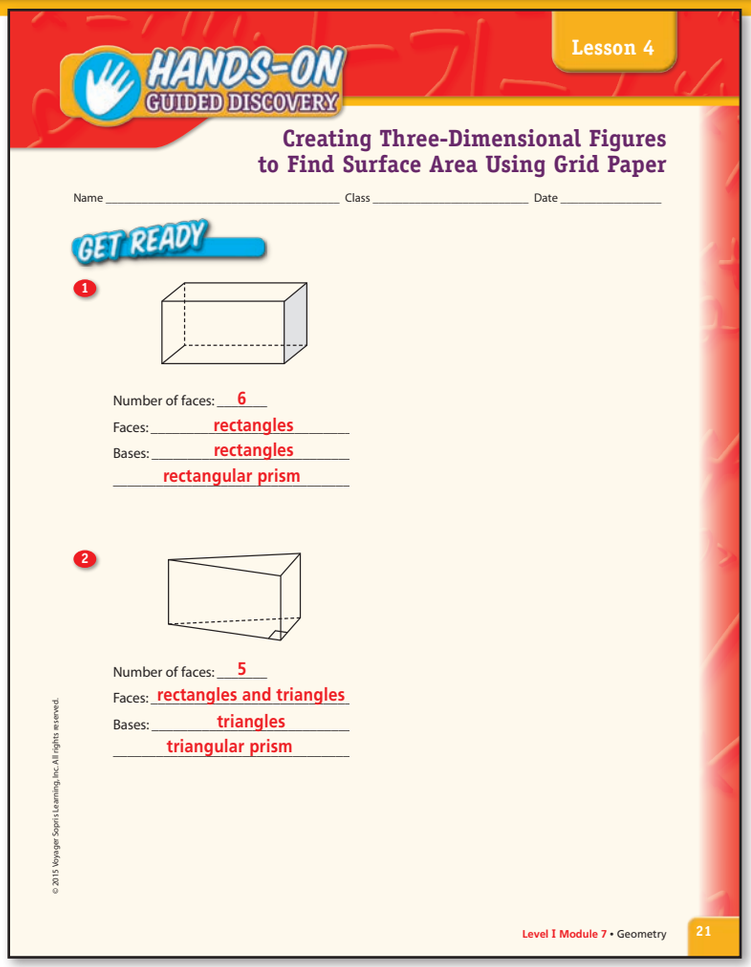
Materials

- Copy Master: Nets
- scissors
- transparent tape

Lesson Notes

Before beginning the lesson, be sure students have their Student Books, copies of Copy Master: Nets, scissors, and transparent tape.

In problem 3, students find the surface area of a rectangular prism. In the Discover Box, students will be guided to notice that the areas of the opposite faces of rectangular prisms are equal. It might be helpful to have students color each pair of opposite faces a different color before students cut out and form the prism from the net for the Discover Box question.



HANDS-ON GUIDED DISCOVERY

Lesson 4

Creating Three-Dimensional Figures to Find Surface Area Using Grid Paper

Name _____ Class _____ Date _____

GET READY

1

Number of faces: 6

Faces: rectangles

Bases: rectangles

rectangular prism

2

Number of faces: 5

Faces: rectangles and triangles

Bases: triangles

triangular prism

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Level I Module 7 • Geometry 21

GET READY

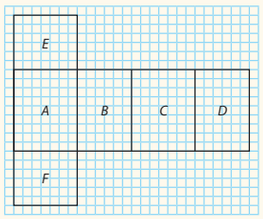
Problem 1

The three-dimensional figure in problem 1 is formed from polygons. How many faces does this figure have? (6) **What is the shape of the faces?** (rectangle) **Two of these faces form the bases of the figure. What is the shape of the bases?** (rectangle) **What kind of three-dimensional figure is shown?** (rectangular prism)

Problem 2

In problem 2, the three-dimensional figure is formed from polygons. How many faces does this figure have? (5) **What are the shapes of the faces?** (rectangles and triangles) **Two of these faces form the bases of the figure. What is the shape of the bases?** (triangle) **What kind of three-dimensional figure is shown?** (triangular prism)

DISCOVER



3 The net forms a rectangular prism.

The faces are rectangles.

Area of face A = 9 × 7 = 63 square units

Area of face B = 9 × 6 = 54 square units

Area of face C = 9 × 7 = 63 square units

Area of face D = 9 × 6 = 54 square units

Area of face E = 6 × 7 = 42 square units

Area of face F = 6 × 7 = 42 square units

Surface Area = 63 + 54 + 63 + 54 + 42 + 42

= 318 square units

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DISCOVER

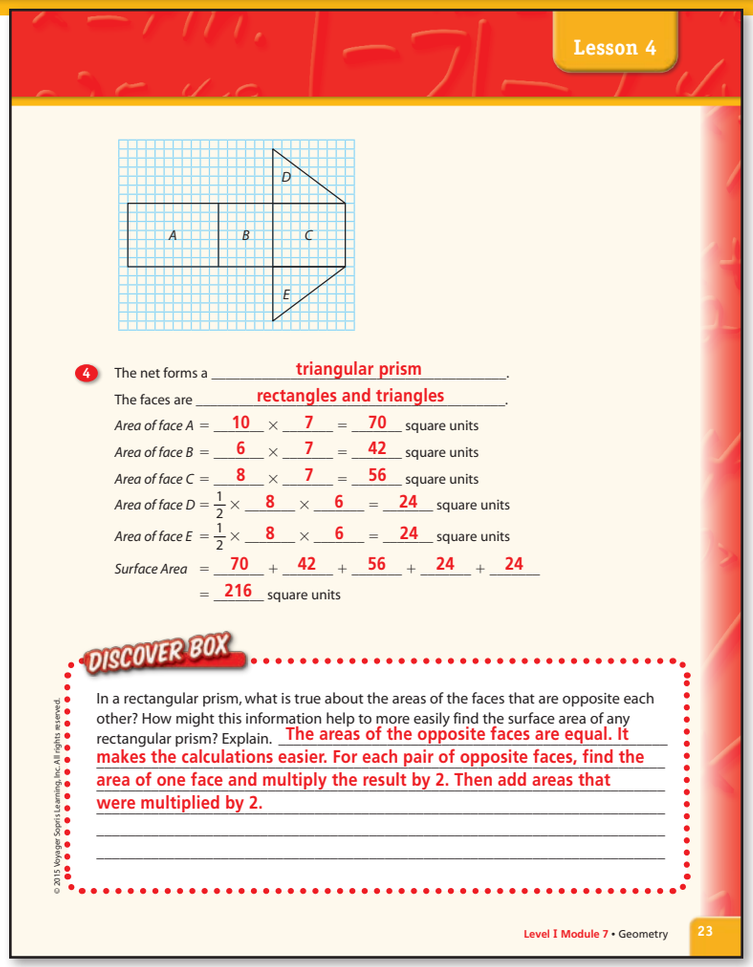
Problem 3

If a three-dimensional figure has at least one flat surface, it can be formed from a pattern called a net. Cut out the net for problem 3 from the nets page. Fold the net on the black lines toward the blank side of the paper, then fold it into a three-dimensional figure. Tape the adjacent faces. What three-dimensional figure is formed by the net? (rectangular prism) **A three-dimensional figure can be described by its surface area, or the area of all its faces. To find the surface area of a three-dimensional figure, add the areas of all its faces.**

In problem 3, each face is labeled with a letter. What is the shape of the faces? (rectangle) **How is the area of a rectangle found?** (by multiplying the length and width) **What are the length and width of face *A*?** (9 units and 7 units) **What is 9 times 7?** (63) **What is the area of face *A*?** (63 square units) **What are the length and width of face *B*?** (9 units and 6 units) **What is 9 times 6?** (54) **What is the area of face *B*?** (54 square units) **What are the length and width of face *C*?** (9 units and 7 units) **What is the area of face *C*?** (63 square units) **What are the length and width of face *D*?** (9 units and 6 units) **What is the area of face *D*?** (54 square units) **What are the length and width of face *E*?** (6 units and 7 units) **What is the area of face *E*?** (42 square units) **What are the length and width of face *F*?** (6 units and 7 units) **What is the area of face *F*?** (42 square units) **The surface area of the prism is the sum of the areas of its faces. What is the sum of 63, 54, 63, 54, 42, and 42?** (318) **So, the surface area of the rectangular prism is 318 square units.**

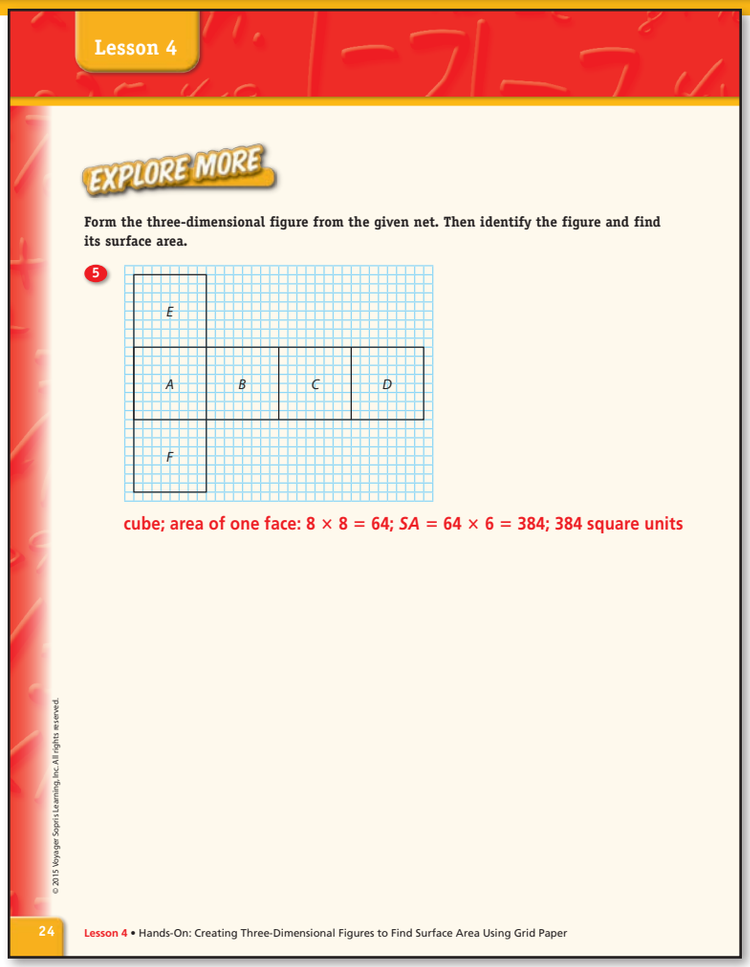
Problem 4

Cut out the net for problem 4 from the nets page. Fold the net on the black lines toward the blank side of the paper, then fold it into a three-dimensional figure. Tape the adjacent faces. What three-dimensional figure is formed by the net? (triangular prism) **Each face is labeled with a letter. What are the shapes of the faces?** (rectangles and triangles) **How is the area of a rectangle found?** (by multiplying its length and width) **What are the length and width of face *A*?** (10 units and 7 units) **What is 10 times 7?** (70) **What is the area of face *A*?** (70 square units) **What are the length and width of face *B*?** (6 units and 7 units) **What is 6 times 7?** (42) **What is the area of face *B*?** (42 square units) **What are the length and width of face *C*?** (8 units and 7 units) **What is the area of face *C*?** (56 square units) **How is the area of a triangle found?** (by multiplying $\frac{1}{2}$, the length of the base, and the height) **What are the length of the base and the height of face *D*?** (8 units and 6 units) **What is $\frac{1}{2}$ times 8 times 6?** (24) **What is the area of face *D*?** (24 square units) **What are the length of the base and the height of face *E*?** (8 units and 6 units) **What is $\frac{1}{2}$ times 8 times 6?** (24) **What is the area of face *E*?** (24 square units) **The surface area of the prism is the sum of the areas of its faces. What is the sum of 70, 42, 56, 24, and 24?** (216) **So, the surface area of the triangular prism is 216 square units.**



DISCOVER BOX

Look at the three-dimensional figure in problem 3. Notice that faces *A* and *C* are opposite each other. How do the lengths and widths of these faces compare? (They are the same.) **How do the areas of these faces compare?** (They are the same.) **Notice that faces *B* and *D* are opposite each other. How do the lengths and widths of these faces compare?** (They are the same.) **How do the areas of these faces compare?** (They are the same.) **Notice that faces *E* and *F* are opposite each other. How do the lengths and widths of these faces compare?** (They are the same.) **How do the areas of these faces compare?** (They are the same.) **What is true about the areas of the faces that are opposite each other in a rectangular prism?** (They are the same.) **How might this information make finding the surface area of a rectangular prism easier?** (The areas of the opposite faces are equal. It makes the calculations easier. For each pair of opposite faces, find the area of one face and multiply the result by 2. Then add areas that were multiplied by 2.)



Lesson 4

EXPLORE MORE

Form the three-dimensional figure from the given net. Then identify the figure and find its surface area.

5

cube; area of one face: 8 × 8 = 64; *SA* = 64 × 6 = 384; 384 square units

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EXPLORE MORE

Problem 5

Students will cut out and form another three-dimensional figure. They should name the figure, then use the method in problems 3 and 4 to find the surface area.

Extension: Have students or partners explore more surface area problems using three-dimensional nets.

Lesson Adaptations for Module 7

Lesson 7.6

Students will find volumes of three-dimensional figures. Constructing three-dimensional figures from grid nets will help students visualize the formula for volume. The grid on the base should help them find area. The grid on the other faces will help give students a sense of the height of the figure.

Lesson 5

Objective

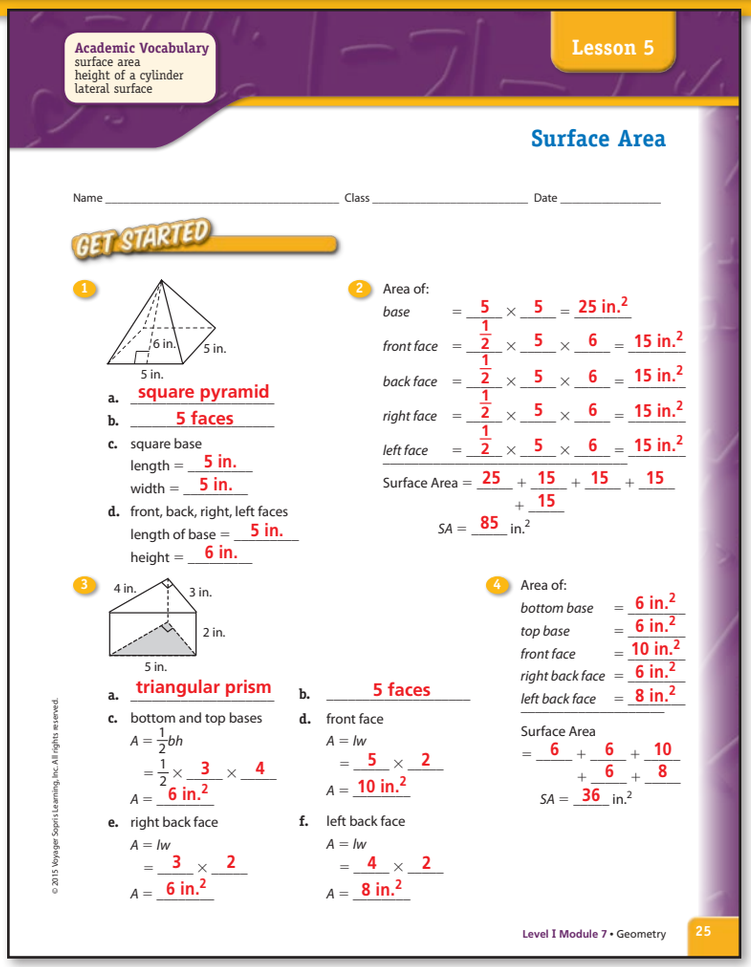
To find the surface area of a rectangular and triangular prism, a cylinder, and a pyramid

Preskills	Lesson
Area of Triangles and Compound Figures	I.7.2

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **surface area**
  the sum of the areas of the faces of a figure
- **height of a cylinder**
  the perpendicular distance between the two bases of a cylinder
- **lateral surface**
  the curved surface of a cylinder



Academic Vocabulary
surface area
height of a cylinder
lateral surface

Lesson 5

Surface Area

Name _____ Class _____ Date _____

GET STARTED

1

a. square pyramid
b. 5 faces
c. square base
length = 5 in.
width = 5 in.
d. front, back, right, left faces
length of base = 5 in.
height = 6 in.

2 Area of:
base = 5 × 5 = 25 in.²
front face = 1/2 × 5 × 6 = 15 in.²
back face = 1/2 × 5 × 6 = 15 in.²
right face = 1/2 × 5 × 6 = 15 in.²
left face = 1/2 × 5 × 6 = 15 in.²
Surface Area = 25 + 15 + 15 + 15 + 15
SA = 85 in.²

3

a. triangular prism
b. 5 faces
c. bottom and top bases
A = 1/2 bh
= 1/2 × 3 × 4
A = 6 in.²
d. front face
A = lw
= 5 × 2
A = 10 in.²
e. right back face
A = lw
= 3 × 2
A = 6 in.²
f. left back face
A = lw
= 4 × 2
A = 8 in.²

4 Area of:
bottom base = 6 in.²
top base = 6 in.²
front face = 10 in.²
right back face = 6 in.²
left back face = 8 in.²
Surface Area = 6 + 6 + 10 + 6 + 6 + 8
SA = 36 in.²

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GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What type of three-dimensional figure is shown in problem 1? (square pyramid) **How many faces does this pyramid have?** (5) **What is the shape of each face?** (4 triangles and 1 square) **What are the dimensions of the base?** (5 in. by 5 in.) **What are the length of the base and height of the front triangular face?** (5 in. and 6 in.)

Does the back face, hidden from view, have the same dimensions? (yes) **What are the dimensions of the right face?** (5 in. and 6 in.) **Does the left face have the same dimensions?** (yes)

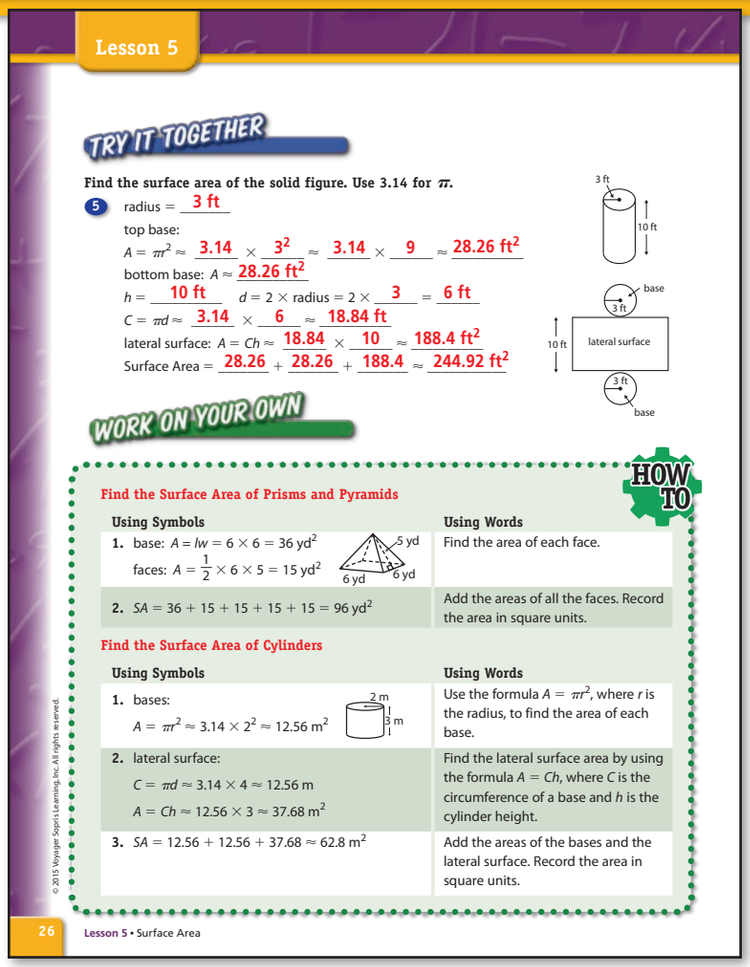
MODEL NEW SKILLS

Problem 2

The surface area of a three-dimensional figure is the sum of the areas of all the faces of the figure. Use the information from problem 1. How many faces does the figure have? (5) **How is the area of a square found?** (by multiplying the length of a side by itself) **Use the dimensions found in problem 1 to calculate the area of each face. What is the area of the base, or 5 times 5?** (25 in.^{2})

The area of a triangle is found by using the formula $A = \frac{1}{2}bh$. What is the area of the front triangular face? (15 in.^{2}) **Is the area of the back face the same?** (yes) **What is the area of the back face?** (15 in.^{2}) **Is the area of the left face the same?** (yes) **What is the area of the left face?** (15 in.^{2}) **Is the area of the right face the same?** (yes) **What is the area of the right face?** (15 in.^{2})

To find the surface area, add the areas of all the faces together. What is the sum of 25, 15, 15, 15, and 15? (85) **What is the surface area, represented by *SA*, of the square pyramid?** (85 in.^{2})


Lesson 5

TRY IT TOGETHER

Find the surface area of the solid figure. Use 3.14 for π.

5 radius = 3 ft

top base:

$A = \pi r^2 \approx$ 3.14 $\times$ 3^2 $\approx$ 3.14 $\times$ 9 $\approx$ 28.26 ft^2

bottom base: $A \approx$ 28.26 ft^2

$h =$ 10 ft $d = 2 \times$ radius $= 2 \times$ 3 $=$ 6 ft

$C = \pi d \approx$ 3.14 $\times$ 6 $\approx$ 18.84 ft

lateral surface: $A = Ch \approx$ 18.84 $\times$ 10 $\approx$ 188.4 ft^2

Surface Area = 28.26 + 28.26 + 188.4 $\approx$ 244.92 ft^2

WORK ON YOUR OWN

HOW TO

Find the Surface Area of Prisms and Pyramids

Using Symbols	Using Words
1. base: $A = lw = 6 \times 6 = 36$ yd^2 faces: $A = \frac{1}{2} \times 6 \times 5 = 15$ yd^2	Find the area of each face.
2. $SA = 36 + 15 + 15 + 15 + 15 = 96$ yd^2	Add the areas of all the faces. Record the area in square units.

Find the Surface Area of Cylinders

Using Symbols	Using Words
1. bases: $A = \pi r^2 \approx 3.14 \times 2^2 \approx 12.56$ m^2	Use the formula $A = \pi r^2$, where r is the radius, to find the area of each base.
2. lateral surface: $C = \pi d \approx 3.14 \times 4 \approx 12.56$ m $A = Ch \approx 12.56 \times 3 \approx 37.68$ m^2	Find the lateral surface area by using the formula $A = Ch$, where C is the circumference of a base and h is the cylinder height.
3. $SA = 12.56 + 12.56 + 37.68 \approx 62.8$ m^2	Add the areas of the bases and the lateral surface. Record the area in square units.

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Problem 3

What type of three-dimensional figure is shown in problem 3? (triangular prism) **How many faces does a triangular prism have?** (5)

What is the shape of the bases of a triangular prism? (triangle) **The shaded triangle is one base of this triangular prism. The formula $A = \frac{1}{2}bh$ can be used to find the area of this triangle. In a right triangle, the length of the base and the height of the triangle are the lengths of the perpendicular sides of the triangle. What are the length of the base and the height of the shaded triangle?** (3 in. and 4 in.) **What is $\frac{1}{2}$ times 3 times 4?** (6) **The area of each triangular base is 6 square inches.**

The formula $A = lw$ can be used to find the area of each rectangular face of the triangular prism. What are the dimensions of the front face? (5 in. and 2 in.) **What is the area of the front face?** (10 in.^{2}) **What are the dimensions of the right back face?** (3 in. and 2 in.) **What is the area of the right back face?** (6 in.^{2}) **What are the dimensions of the left back face?** (4 in. and 2 in.) **What is the area of the left back face?** (8 in.^{2})

Problem 4

The surface area of a prism is the sum of the areas of all its faces. Use the figure in problem 3. How many faces does the triangular prism have? (5) **What is the area of the bottom triangular base?** (6 in.^{2}) **What is the area of the top triangular base?** (6 in.^{2}) **What is the area of the front face?** (10 in.^{2}) **What is the area of the right back face?** (6 in.^{2}) **What is the area of the left back face?** (8 in.^{2})

Now, add all the areas together. What is the sum of 6, 6, 10, 6, and 8? (36) **What is the surface area of the triangular prism?** (36 in.^{2})

TRY IT TOGETHER

Work with students to complete this skill.

SCAFFOLD INSTRUCTION

Problem 5

What figure is shown in problem 5? (cylinder) **How many bases does the cylinder have?** (2) **What is the shape of the bases?** (circle) **What is the formula to find the area of each circular base?** (A r^2) **What is the radius of a base?** (3 ft) **Substitute 3.14 for π in the area formula. What is the area of the top base?** (about 28.26 ft^2) **Is the area of the bottom base the same?** (Yes, the two bases are congruent.) **What is the area of the bottom base?** (about 28.26 ft^2)

Look at the net of the cylinder. What shape is the curved surface in the net? (rectangle) **The area of this rectangle is the area of the curved surface, or the lateral area of the cylinder. The width of the rectangle is given by the height h of the cylinder. What is h?** (10 ft)

The length of the rectangle is given by the circumference of the circular base. What is the formula for the circumference of a circle? (C d) **What is the diameter of the circular base?** (6 ft) **What is the circumference of the base?** (about 18.84 ft)

Both measurements of the rectangle have been found. What is 18.84 times 10? (188.4) **What is the area of the lateral surface?** (about 188.4 ft^2)

Add the areas of the two bases and the area of the lateral surface to find the surface area of the cylinder. What is the sum of 28.26, 28.26, and 188.4? (244.92)

The surface area of the cylinder is about 244.92 ft^2.

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 6–14

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 9–11 Reminder

These problems review identifying three-dimensional figures and counting faces, vertices, and edges. Remind students that only three-dimensional figures that are made up of polygons have faces, vertices, and edges.

Problem 12 Reminder

Students must first find the surface area of the box, then compare the answer with the amount of wrapping paper Jaclyn has.

Problem 13 Reminder

Students should find the area of the pizza by using the formula $A \quad r^2$. Then they should divide the total area by 8 to find the area of each slice of pizza.

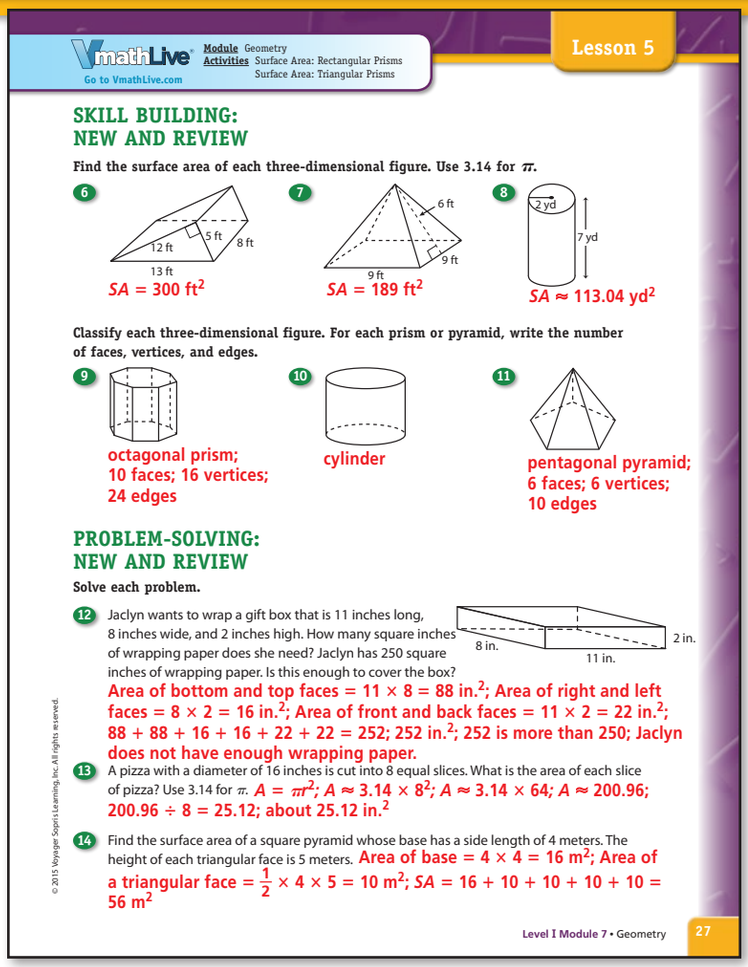
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2a: The student found the area of only one face of the figure. Use Additional Resources I.7.5 to reteach how to find the surface area of prisms and pyramids.



VmathLive Go to VmathLive.com
Module Geometry
Activities Surface Area: Rectangular Prisms; Surface Area: Triangular Prisms

Lesson 5

SKILL BUILDING: NEW AND REVIEW

Find the surface area of each three-dimensional figure. Use 3.14 for π.

6. 12 ft, 5 ft, 8 ft, 13 ft — $SA = 300$ ft^2
7. 6 ft, 9 ft, 9 ft — $SA = 189$ ft^2
8. 2 yd, 7 yd — $SA \approx 113.04$ yd^2

Classify each three-dimensional figure. For each prism or pyramid, write the number of faces, vertices, and edges.

9. octagonal prism; 10 faces; 16 vertices; 24 edges
10. cylinder
11. pentagonal pyramid; 6 faces; 6 vertices; 10 edges

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

12. Jaclyn wants to wrap a gift box that is 11 inches long, 8 inches wide, and 2 inches high. How many square inches of wrapping paper does she need? Jaclyn has 250 square inches of wrapping paper. Is this enough to cover the box? 8 in., 11 in., 2 in. Area of bottom and top faces = 11 × 8 = 88 in.^{2}; Area of right and left faces = 8 × 2 = 16 in.^{2}; Area of front and back faces = 11 × 2 = 22 in.^{2}; 88 + 88 + 16 + 16 + 22 + 22 = 252; 252 in.^{2}; 252 is more than 250; Jaclyn does not have enough wrapping paper.
13. A pizza with a diameter of 16 inches is cut into 8 equal slices. What is the area of each slice of pizza? Use 3.14 for π. $A = \pi r^2$; $A \approx 3.14 \times 8^2$; $A \approx 3.14 \times 64$; $A \approx 200.96$; $200.96 \div 8 = 25.12$; about 25.12 in.^{2}
14. Find the surface area of a square pyramid whose base has a side length of 4 meters. The height of each triangular face is 5 meters. Area of base = 4 × 4 = 16 m^2; Area of a triangular face = $\frac{1}{2} \times 4 \times 5 = 10$ m^2; $SA = 16 + 10 + 10 + 10 + 10 = 56$ m^2

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Level I Module 7 • Geometry 27

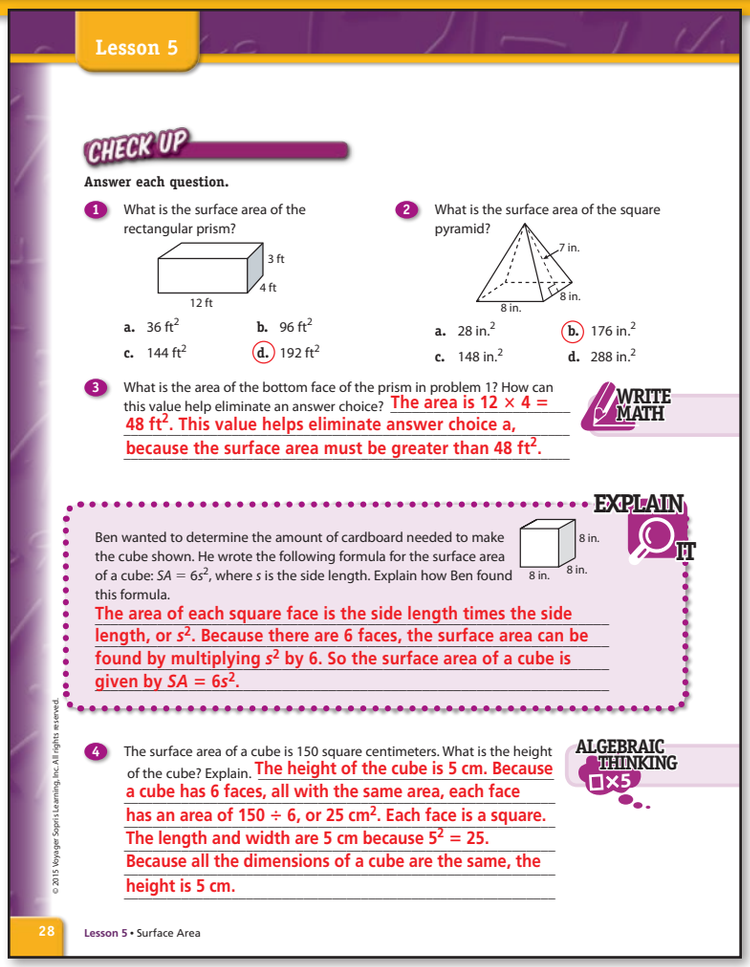
If student answered 1b: The student added the areas of only three faces of the prism when finding the surface area of the prism. Review the number of faces on the prism and use Additional Resources I.7.5 to reteach how to find the surface area of prisms.

If student answered 1c or 2c: The student omitted one of the faces when finding the surface area of the prism or pyramid. Use Additional Resources I.7.5 to reteach how to find the surface area of prisms and pyramids.

If student answered 2d: The student used the formula $A \quad bh$ for the area of each triangular face instead of using $A \quad \frac{1}{2}bh$. Review the formula for the area of a triangle.

WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student that answer choice a is too small because it is less than the area of the bottom face of the figure. Remind the student that the surface area of a prism is the sum of the areas of all the faces.



Lesson 5

CHECK UP

Answer each question.

1 What is the surface area of the rectangular prism?

a. 36 ft^2 b. 96 ft^2
c. 144 ft^2 d. 192 ft^2

2 What is the surface area of the square pyramid?

a. 28 in.^{2} b. 176 in.^{2}
c. 148 in.^{2} d. 288 in.^{2}

3 What is the area of the bottom face of the prism in problem 1? How can this value help eliminate an answer choice? The area is 12 × 4 = 48 ft^2. This value helps eliminate answer choice a, because the surface area must be greater than 48 ft^2.

WRITE MATH

EXPLAIN IT

Ben wanted to determine the amount of cardboard needed to make the cube shown. He wrote the following formula for the surface area of a cube: $SA = 6s^2$, where s is the side length. Explain how Ben found this formula.

The area of each square face is the side length times the side length, or s^2. Because there are 6 faces, the surface area can be found by multiplying s^2 by 6. So the surface area of a cube is given by $SA = 6s^2$.

4 The surface area of a cube is 150 square centimeters. What is the height of the cube? Explain. The height of the cube is 5 cm. Because a cube has 6 faces, all with the same area, each face has an area of 150 ÷ 6, or 25 cm^2. Each face is a square. The length and width are 5 cm because $5^2 = 25$. Because all the dimensions of a cube are the same, the height is 5 cm.

ALGEBRAIC THINKING □×5

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EXPLAIN IT As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

VmathLive

Module: Geometry

Activities: Surface Area: Rectangular Prisms
Surface Area: Triangular Prisms

Vmath Reteach

Reteach Student Book Module 7 Lesson 5

Reteach Teacher Edition Module 7 Lesson 5

Extra Practice

Student Book page 49

English Language Learners

Use the VmathLive Animated Glossary to review the terms *surface area*, *surface*, and *height*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Explain that *lateral surface* is a special name given to the part of a cylinder without the bases. Help students visualize that the "unrolled" lateral surface of a cylinder is actually a rectangle. Do this by bringing in several paper towel or toilet paper tubes and having the students make a vertical cut and unroll the cardboard until it is flat.

Students with Special Needs

Provide small groups of students with a pair of scissors and a photocopy of the net. Allow them to cut out the net.

Use the net of the cylinder to help students understand why the lateral surface area can be found using the formula for area of a rectangle, and why the rectangle's length is equal to the circumference of the circular base. Being able to actually roll the lateral surface around the circular base and unroll it into a rectangle will give some students the hands-on manipulation they need to see these relationships.

Lesson 6

Objective

To find the volume of a cylinder, a sphere, and a cone

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What figure is shown in problem 1? (circle) **The area of a circle is found by multiplying and r^2. What is the value that we substitute for ?** (3.14) **What does r represent in the formula?** (radius) **Now look at the circle. What is the radius of this circle?** (3 m) **What is 3^2?** (9) **What is 3.14 times 9?** (28.26) **What is the area of this circle?** (about 28.26 m^2)

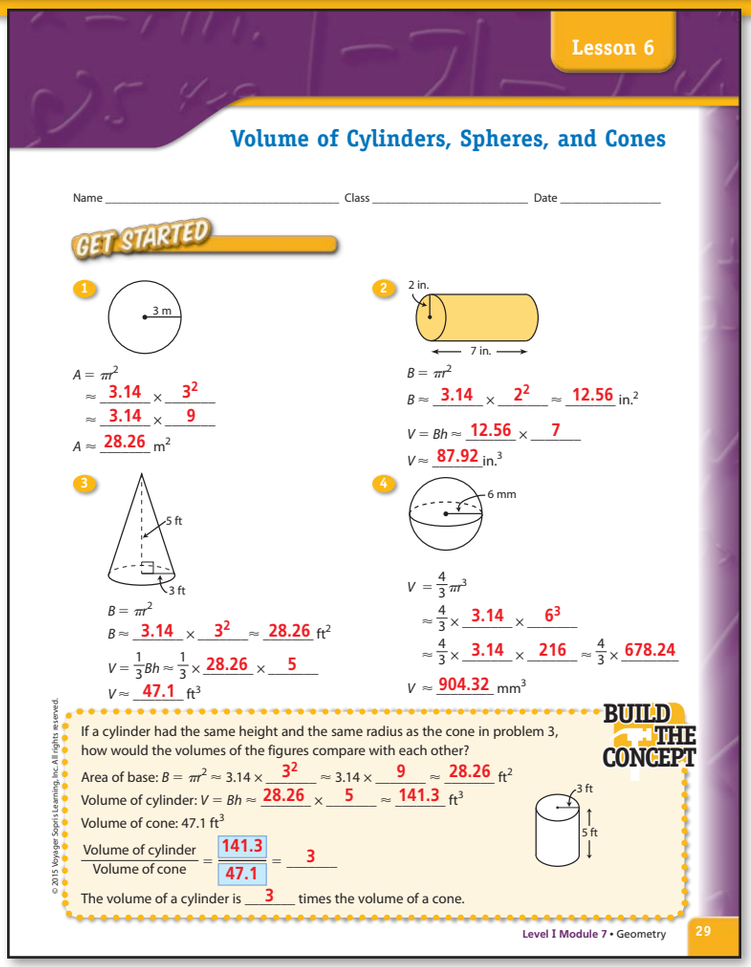
MODEL NEW SKILLS

Problem 2

What figure is shown in problem 2? (cylinder) **It has a base, which is a circle, and it has height. The volume of a cylinder is found by multiplying the area of the base and the height. The formula is V Bh, where B represents the area of the base and h represents the height of the cylinder.**

What type of figure is the base of the cylinder? (circle) **What formula is used to find the area of a circle?** (A r^2) **What is the radius for the base of the cylinder?** (2 in.) **What is 2^2?** (4) **Use 3.14 for . What is the area of the base of the cylinder?** (about 12.56 $in.^2$)

What is the height of the cylinder? (7 in.) **The volume of the cylinder is found by multiplying the area of the base and the height. What is 12.56 times 7?** (87.92) **What is the volume of the cylinder?** (about 87.92 $in.^3$) **Remember to use cubic units for the units of measure when finding volume.**



Lesson 6

Volume of Cylinders, Spheres, and Cones

Name _____ Class _____ Date _____

GET STARTED

1 3 m

$A = \pi r^2$
$\approx 3.14 \times 3^2$
$\approx 3.14 \times 9$
$A \approx 28.26\ m^2$

2 2 in., 7 in.

$B = \pi r^2$
$B \approx 3.14 \times 2^2 \approx 12.56\ in.^2$
$V = Bh \approx 12.56 \times 7$
$V \approx 87.92\ in.^3$

3 5 ft, 3 ft

$B = \pi r^2$
$B \approx 3.14 \times 3^2 \approx 28.26\ ft^2$
$V = \frac{1}{3}Bh \approx \frac{1}{3} \times 28.26 \times 5$
$V \approx 47.1\ ft^3$

4 6 mm

$V = \frac{4}{3}\pi r^3$
$\approx \frac{4}{3} \times 3.14 \times 6^3$
$\approx \frac{4}{3} \times 3.14 \times 216 \approx \frac{4}{3} \times 678.24$
$V \approx 904.32\ mm^3$

BUILD THE CONCEPT

If a cylinder had the same height and the same radius as the cone in problem 3, how would the volumes of the figures compare with each other?

Area of base: $B = \pi r^2 \approx 3.14 \times 3^2 \approx 3.14 \times 9 \approx 28.26\ ft^2$

Volume of cylinder: $V = Bh \approx 28.26 \times 5 \approx 141.3\ ft^3$

Volume of cone: $47.1\ ft^3$

$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{141.3}{47.1} = 3$

The volume of a cylinder is 3 times the volume of a cone.

3 ft, 5 ft

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Problem 3

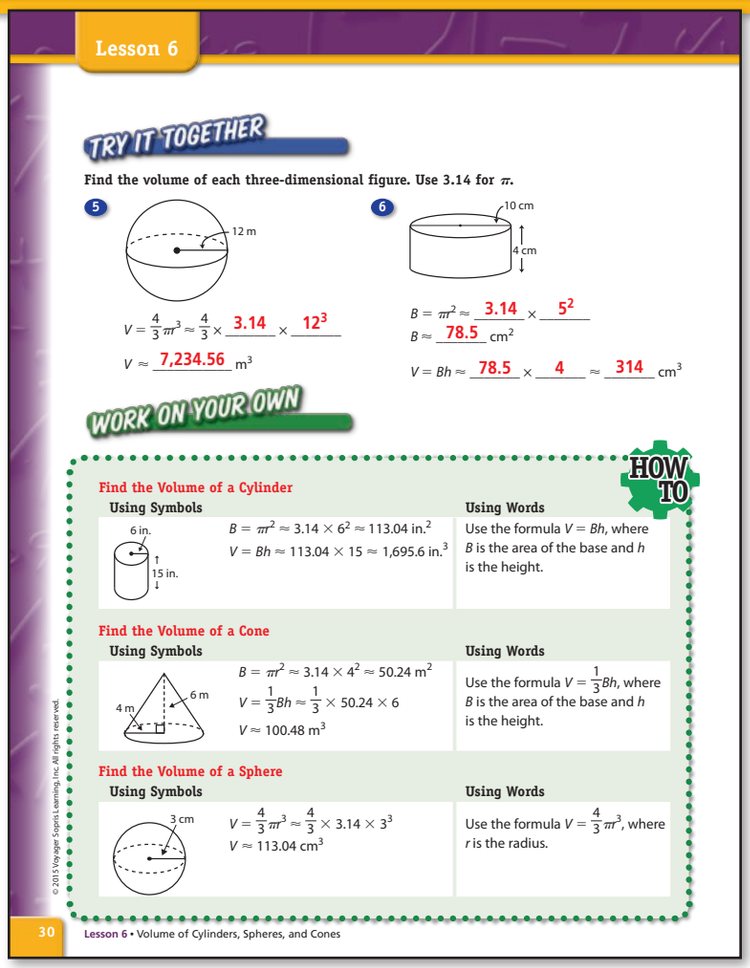
What figure is shown in problem 3? (cone) **It has a base, which is a circle, and it has height. The volume of the cone is found by multiplying $\frac{1}{3}$, the area of the base, and the height. The formula is V $\frac{1}{3}Bh$, where B represents the area of the base and h represents the height of the cone.**

What shape is the base of the cone? (circle) **What formula is used to find the area of a circle?** (A r^2) **What is the radius of the base?** (3 ft) **What is 3^2?** (9) **Use 3.14 for . What is 3.14 times 9?** (28.26)

What is the area of the base? (about 28.26 ft^2) **What is the height of the cone?** (5 ft) **The volume is found by multiplying $\frac{1}{3}$, the area of the base, and the height. What is $\frac{1}{3}$ times 28.26 times 5?** (47.1) **What is the volume of the cone?** (about 47.1 ft^3)

Problem 4

What figure is shown in problem 4? (sphere) **The volume of a sphere is found by multiplying $\frac{4}{3}$, , and the cube of the radius of the sphere. The formula is V $\frac{4}{3}r^3$, where r represents the radius of the sphere.**



Lesson 6

TRY IT TOGETHER

Find the volume of each three-dimensional figure. Use 3.14 for π.

5

$V = \frac{4}{3}\pi r^3 \approx \frac{4}{3} \times$ 3.14 $\times$ 12^3

$V \approx$ 7,234.56 m^3

6

$B = \pi r^2 \approx$ 3.14 $\times$ 5^2

$B \approx$ 78.5 cm^2

$V = Bh \approx$ 78.5 $\times$ 4 $\approx$ 314 cm^3

WORK ON YOUR OWN

HOW TO

Find the Volume of a Cylinder

Using Symbols	Using Words
$B = \pi r^2 \approx 3.14 \times 6^2 \approx 113.04$ in.^{2} $V = Bh \approx 113.04 \times 15 \approx 1{,}695.6$ in.^{3}	Use the formula $V = Bh$, where B is the area of the base and h is the height.

Find the Volume of a Cone

Using Symbols	Using Words
$B = \pi r^2 \approx 3.14 \times 4^2 \approx 50.24$ m^2 $V = \frac{1}{3}Bh \approx \frac{1}{3} \times 50.24 \times 6$ $V \approx 100.48$ m^3	Use the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.

Find the Volume of a Sphere

Using Symbols	Using Words
$V = \frac{4}{3}\pi r^3 \approx \frac{4}{3} \times 3.14 \times 3^3$ $V \approx 113.04$ cm^3	Use the formula $V = \frac{4}{3}\pi r^3$, where r is the radius.

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Look for the radius in the sphere. The radius must start at the center of the sphere and extend to the edge of the sphere. Remember that it can extend to any edge. What is the radius of the sphere? (6 mm)

What is 6^3? (216) **Use 3.14 for . What is 3.14 times 216?** (678.24) **What is $\frac{4}{3}$ times 678.24?** (904.32) **What is the volume of the sphere?** (about 904.32 mm^3)

BUILD THE CONCEPT

Model how to compare the volumes of a cylinder and a cone.

Look at problem 3. What is the height of the cone? (5 ft) **What is the radius of the cone?** (3 ft)

Suppose that a cylinder has this same height and radius. To find the volume of the cylinder, first find the area of its base. How is the area of the base found? (by multiplying and the square of the radius) **What is the square of the radius?** (9 ft^2) **What is 3.14 times 9?** (28.26) **What is the area of the base of the cylinder?** (about 28.26 ft^2)

Now multiply the area of the base by the height. What is 28.26 times 5? (141.3) **What is the volume of the cylinder?** (about 141.3 ft^3) **What is the volume of the cone in problem 3?** (about 47.1 ft^3) **To compare the volumes, divide the volume of the cylinder by the volume of the cone. What is 141.3 divided by 47.1?** (3) **So the cylinder's volume is 3 times the cone's volume.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What figure is shown in problem 5? (sphere) **What is the formula for the volume of the sphere?** (V $\frac{4}{3}$ r^3)

What measure is needed to find the volume of a sphere? (radius of the sphere) **What is the radius of the sphere?** (12 m)

What is 12^3? (1,728) **Multiply $\frac{4}{3}$, 3.14, and 1,728. What is the volume of the sphere?** (about 7,234.56 m^3)

Problem 6

What figure is shown in problem 6? (cylinder) **What is the formula for the volume of the cylinder?** (V Bh)

What is the shape of the base of the cylinder? (circle) **Is the diameter or the radius shown?** (diameter) **What is the radius?** (5 cm) **What is 5^2?** (25) **What is the area of the base of the cylinder?** (about 78.5 cm^2)

What is the height of the cylinder? (4 cm) **What is 78.5 times 4?** (314) **What is the volume of the cylinder?** (about 314 cm^3)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 7–16

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 7 Reminder

Explain that the base of the cylinder is not always oriented at the top or bottom. The base is always a circle.

Problem 10 Reminder

Remind students that the radius of the base of a cone is needed to find the volume of the cone. The diameter of the base is given.

Problem 11 Reminder

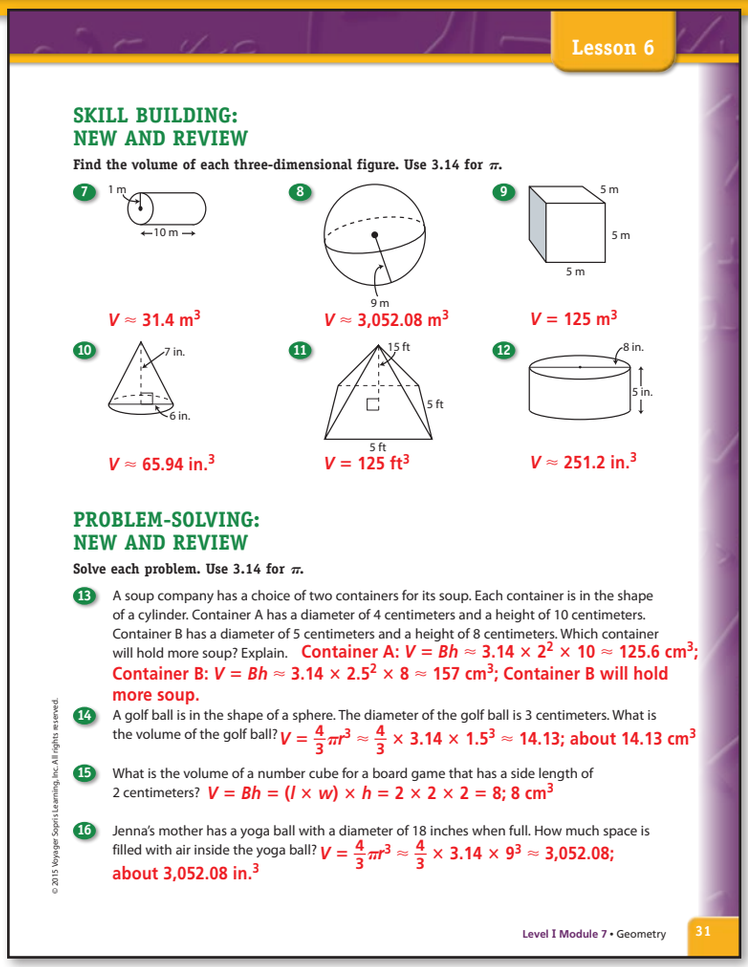
Point out to students that although the formula for the volume of a pyramid is the same as the formula for the volume of a cone, finding the area of the base of each figure requires a different formula. The area of the base of a square pyramid is equal to the side length squared.

Problems 13 Reminder

Remind students that they need to first find the volumes of Container A and Container B, then compare their volumes.

Problems 14 Reminder

Remind students that the radius of the sphere is needed to find the volume of the sphere. The diameter of the sphere is given.



Lesson 6

SKILL BUILDING: NEW AND REVIEW

Find the volume of each three-dimensional figure. Use 3.14 for π.

7. 1 m, 10 m — $V \approx 31.4\ m^3$

8. 9 m — $V \approx 3{,}052.08\ m^3$

9. 5 m, 5 m, 5 m — $V = 125\ m^3$

10. 7 in., 6 in. — $V \approx 65.94\ in.^3$

11. 15 ft, 5 ft, 5 ft — $V = 125\ ft^3$

12. 8 in., 5 in. — $V \approx 251.2\ in.^3$

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem. Use 3.14 for π.

13. A soup company has a choice of two containers for its soup. Each container is in the shape of a cylinder. Container A has a diameter of 4 centimeters and a height of 10 centimeters. Container B has a diameter of 5 centimeters and a height of 8 centimeters. Which container will hold more soup? Explain. Container A: $V = Bh \approx 3.14 \times 2^2 \times 10 \approx 125.6\ cm^3$; Container B: $V = Bh \approx 3.14 \times 2.5^2 \times 8 \approx 157\ cm^3$; Container B will hold more soup.

14. A golf ball is in the shape of a sphere. The diameter of the golf ball is 3 centimeters. What is the volume of the golf ball? $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3} \times 3.14 \times 1.5^3 \approx 14.13$; about $14.13\ cm^3$

15. What is the volume of a number cube for a board game that has a side length of 2 centimeters? $V = Bh = (l \times w) \times h = 2 \times 2 \times 2 = 8$; $8\ cm^3$

16. Jenna's mother has a yoga ball with a diameter of 18 inches when full. How much space is filled with air inside the yoga ball? $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3} \times 3.14 \times 9^3 \approx 3{,}052.08$; about $3{,}052.08\ in.^3$

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Level I Module 7 • Geometry 31

CHECK UP

ASSESS INFORMALLY

Error Analysis

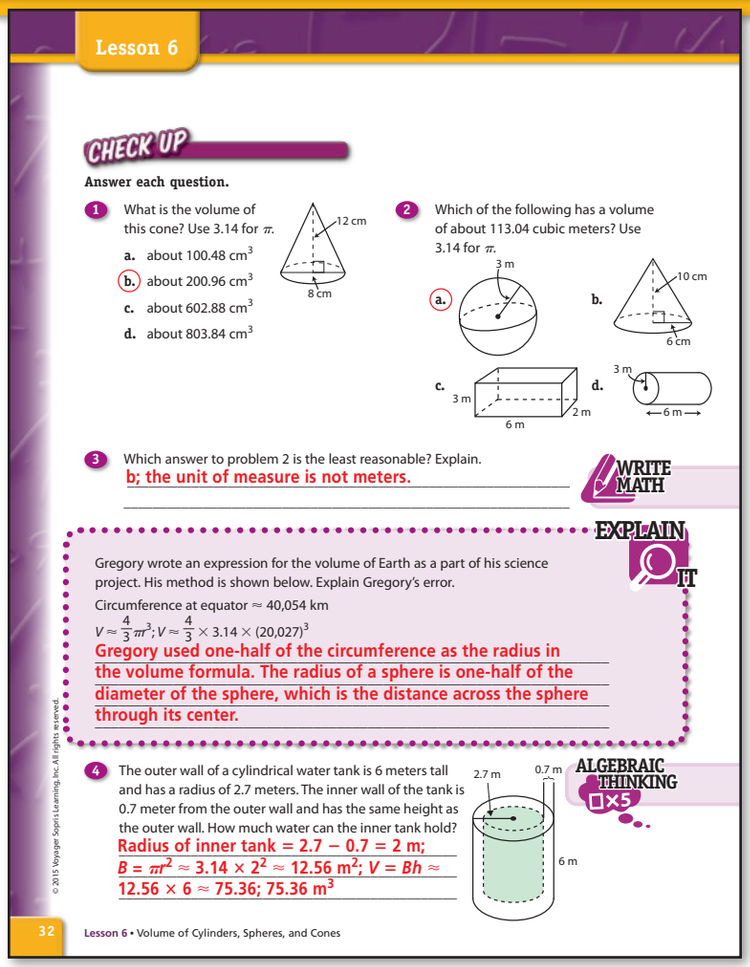
Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student divided the diameter and the height by 2. Remind the student that the radius is found by dividing only the diameter by 2.

If student answered 1c: The student forgot to multiply by $\frac{1}{3}$. Use Additional Resources in I.7.6 to review the formula for the volume of a cone.

If student answered 1d: The student forgot to divide the diameter by 2 to get the radius. Remind the student that the formula requires the radius.

If student answered problem 2 incorrectly: The student did not correctly find the volumes of the objects. Use Additional Resources in I.7.6 to reteach how to find the volumes of a cylinder, sphere, cone, and rectangular prism.



Lesson 6

CHECK UP

Answer each question.

1 What is the volume of this cone? Use 3.14 for π.

12 cm, 8 cm

a. about 100.48 cm^3
b. about 200.96 cm^3
c. about 602.88 cm^3
d. about 803.84 cm^3

2 Which of the following has a volume of about 113.04 cubic meters? Use 3.14 for π.

a. 3 m
b. 10 cm, 6 cm
c. 3 m, 6 m, 2 m
d. 3 m, 6 m

3 Which answer to problem 2 is the least reasonable? Explain.

WRITE MATH

b; the unit of measure is not meters.

EXPLAIN IT

Gregory wrote an expression for the volume of Earth as a part of his science project. His method is shown below. Explain Gregory's error.

Circumference at equator ≈ 40,054 km

$V \approx \frac{4}{3}\pi r^3$; $V \approx \frac{4}{3} \times 3.14 \times (20{,}027)^3$

Gregory used one-half of the circumference as the radius in the volume formula. The radius of a sphere is one-half of the diameter of the sphere, which is the distance across the sphere through its center.

4 The outer wall of a cylindrical water tank is 6 meters tall and has a radius of 2.7 meters. The inner wall of the tank is 0.7 meter from the outer wall and has the same height as the outer wall. How much water can the inner tank hold?

ALGEBRAIC THINKING □×5

2.7 m, 0.7 m, 6 m

Radius of inner tank = 2.7 − 0.7 = 2 m;
$B = \pi r^2 \approx 3.14 \times 2^2 \approx 12.56\ m^2$; $V = Bh \approx$
$12.56 \times 6 \approx 75.36$; $75.36\ m^3$

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32 Lesson 6 • Volume of Cylinders, Spheres, and Cones



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review with the student how to correctly identify the units of measure of the figure. Use Additional Resources in I.7.6 to reteach how to find the volume of a cylinder, sphere, cone, or rectangular prism.

EXPLAIN IT

As you review the Explain It problem together, understand that the correct answer is only part of the solution. Appropriate math vocabulary, a logical method for solving the problem, and the justification of a reasonable answer complete the student response. Explain It problem answers can be used as part of a daily math journal.

DIFFERENTIATION

Additional Resources

Vmath Reteach

Reteach Student Book Module 7 Lesson 6

Reteach Teacher Edition Module 7 Lesson 6

Extra Practice

Student Book page 50

English Language Learners

Use the VmathLive Animated Glossary to review the terms *cylinder*, *cone*, and *sphere*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Show students at least one real-life example of each of the three-dimensional figures in this lesson. Some common items are a metal food can or tennis ball can for a *cylinder*, an ice-cream cone for a *cone*, and a ball or a globe for a *sphere*. Elicit from students the proper geometric names for each of these three-dimensional shapes.

Students with Special Needs

To help students make the connection between the volume formulas they have previously learned and the volume formulas in this lesson, have students create volume reference cards. Review the formula for the volume of a prism, $V = Bh$. Have students write the formula on one side of the card and the description, volume of a prism, on the other side. Point out that the formula for the volume of a cylinder is the same as the formula for the volume of a prism. Have students add the description, volume of a cylinder, to the reference card.

Have them add the formulas for the different bases to the reference card: prism, $B = l \times w$ and cylinder, $B = r^2$.

Have students create a similar card for the volume of a pyramid and the volume of a cone, which have the same volume formula, $V = \frac{1}{3}Bh$.

Technology



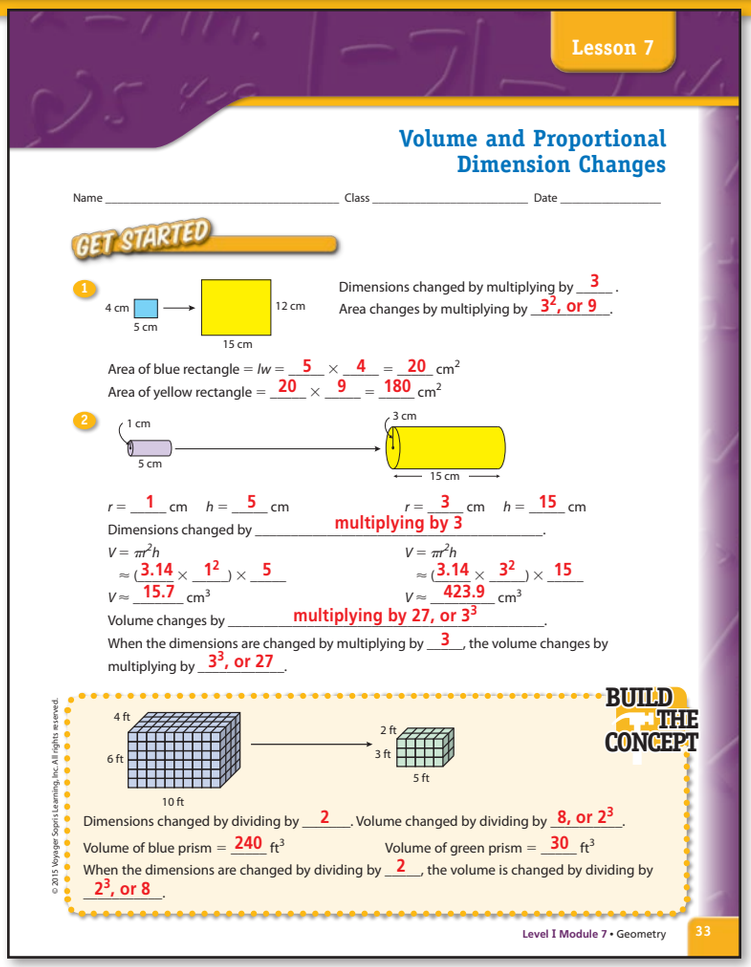
Have students use an online search engine to discover other ways to apply finding volume of three-dimensional objects in real-life situations. Additional digital content is available through this feature in the eBook.

Lesson 7

Objective

To describe the effects of changes in dimension on volume

Preskills	Lesson
Volume of Cylinders, Spheres, and Cones	I.7.6



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What is the length of the blue rectangle? (5 cm) **What is the width?** (4 cm) **What is the length of the yellow rectangle?** (15 cm) **What is the width?** (12 cm) **The yellow rectangle is an enlargement of the blue rectangle.**

First, find the area of the blue rectangle. How is the area of a rectangle found? (Multiply the length by the width.) **What is 5 times 4?** (20) **What is the area of the blue rectangle?** (20 cm^2)

Notice the dimension changes on the rectangles. Each dimension was changed by multiplying by 3. Because the dimensions were changed by multiplying by 3, the area of the yellow rectangle can be found by multiplying the area of the blue rectangle by 3^2, or 9.

What is 20 times 9? (180) **What is the area of the yellow rectangle?** (180 cm^2)

MODEL NEW SKILLS

Problem 2

What figures are shown in problem 2? (cylinders) **The radius of the purple cylinder is 1 centimeter. What is the radius of the yellow cylinder?** (3 cm) **What are the heights of the purple and yellow cylinders?** (5 cm and 15 cm)

The dimensions of the purple cylinder were multiplied by 3 to get the dimensions of the yellow cylinder.

What is the formula for finding the volume of a cylinder? (V Bh) **To find the area of the base, B, use the formula A r^2. What is the radius of the purple cylinder?** (1 cm) **Multiply 3.14, 1, and 5. What is the product?** (15.7) **What is the volume of the purple cylinder?** (about 15.7 cm^3)

Now, find the volume of the yellow cylinder. What is 3.14 times 3^2 times 15? (423.9) **What is the volume of the yellow cylinder?** (about 423.9 cm^3)

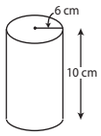
Consider the effect of multiplying the dimensions of the figure by 3. What is 3^3? (27) **The volume of the yellow cylinder can be found by multiplying the volume of the purple cylinder by 27.**

When the dimensions of a figure are changed by multiplying by 3, the volume changes by multiplying by what number? (3^3, or 27)

TRY IT TOGETHER

Find the changed volume of each figure if the dimensions are changed as described. Use 3.14 for π.

3 **Divided by 2**



$V = Bh$

$V = \pi r^2 h$

$\approx$ 3.14 $\times$ 6^2 $\times$ 10

$V \approx$ 1,130.4 cm^3

The dimensions are divided by 2, so the volume is divided by 2^3 or 8.

$V \approx$ 1,130.4 $\div$ 8 $=$ 141.3 cm^3

4 All the dimensions of a rectangular prism with a volume of 60 cubic inches are enlarged by multiplying by 3.

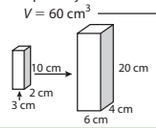
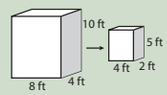
The dimensions are multiplied by 3, so the volume is multiplied by 3^3 or 27.

$V =$ 60 $\times$ 27 $=$ 1,620 $in.^3$

WORK ON YOUR OWN

HOW TO

Describe the Effects of Proportional Dimension Changes on Volume

Using Symbols	Using Words
Multiplied by 2: $V = 60\ cm^3 \rightarrow V = 60 \times 2^3 = 60 \times 8$, $V = 480\ cm^3$ 	When all the dimensions are multiplied (enlarged) by the same number n, the volume is multiplied by n^3.
Divided by 2: $V = 320\ ft^3 \rightarrow V = 320 \div 2^3 = 320 \div 8$, $V = 40\ ft^3$ 	When all the dimensions are divided (reduced) by the same number n, the volume is divided by n^3.

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BUILD THE CONCEPT

Model how to describe the effect of proprotional dimension changes on volume of a rectangular prism.

Two rectangular prisms are shown. Compare the dimensions of the prisms. The length of the blue prism is 10 feet. What is the length of the green prism? (5 ft) **The width of the blue prism is 4 feet. What is the width of the green prism?** (2 ft) **The height of the blue prism is 6 feet. What is the height of the green prism?** (3 ft) **The dimensions of the blue prism were divided by 2 to get the dimensions of the green prism.**

Each prism is made of cubes that are 1 foot long, 1 foot wide, and 1 foot tall. To find the volume of the blue prism, count the number of cubes in the prism. How many cubes are used to form the blue prism? (240) **What is the volume of the blue prism?** (240 ft^3) **Now, find the volume of the green prism. How many cubes are used to form the green prism?** (30) **What is the volume of the green prism?** (30 ft^3)

Compare the volume of the blue prism with the volume of the green prism. The volume of the blue prism could be divided by 8, or 2^3, to find the volume of the green prism. When the dimensions are divided by 2, the volume is divided by what number? (2^3, or 8)

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 3

What is the formula for the volume of a cylinder? (V Bh) **What is the formula for the area of the base of the cylinder?** (A r^2)

What is the radius? (6 cm) **What is 3.14 times 6^2 times 10?** (1,130.4) **What is the volume of the cylinder?** (about 1,130.4 cm^3)

Consider the effect of dividing all the dimensions of the figure by 2. When the dimensions of a figure are divided by the same number, the volume is divided by the cube of that number. What number is each dimension of the cylinder being divided by? (2) **So what number should the volume be divided by?** (2^3) **What is 2^3?** (8)

What is 1,130.4 divided by 8? (141.3) **What is the volume of the reduced cylinder?** (about 141.3 cm^3)

Problem 4

What is the volume of the rectangular prism in problem 4? (60 $in.^3$) **How are the dimensions of the prism to be changed?** (by multiplying each dimension by 3) **How will the volume be changed?** (by multiplying by 3^3, or 27) **What numbers should be multiplied to find the changed volume?** (60 and 27) **What is 60 times 27?** (1,620) **What is the volume of the enlarged rectangular prism?** (1,620 $in.^3$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using Reasonableness** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem. The instruction is immediately followed by application of the strategy in problem 9.

Have students read the problem.

Show students that this is a multi-step problem. Students should first find the volume of the smaller box by dividing the dimensions of the original box by 3. Then students should compare the volume with 40 cm^3.

Original dimensions:

r 6 cm

h 9 cm

Original volume:

V Bh $r^2h \approx 3.14$ 6^2 9 1,017.36 cm^3

New dimensions:

r 6 3 2 cm

h 9 3 3 cm

New volume:

V Bh $r^2h \approx 3.14$ 2^2 3 37.68 cm^3

37.68 $cm^3 < 40$ cm^3

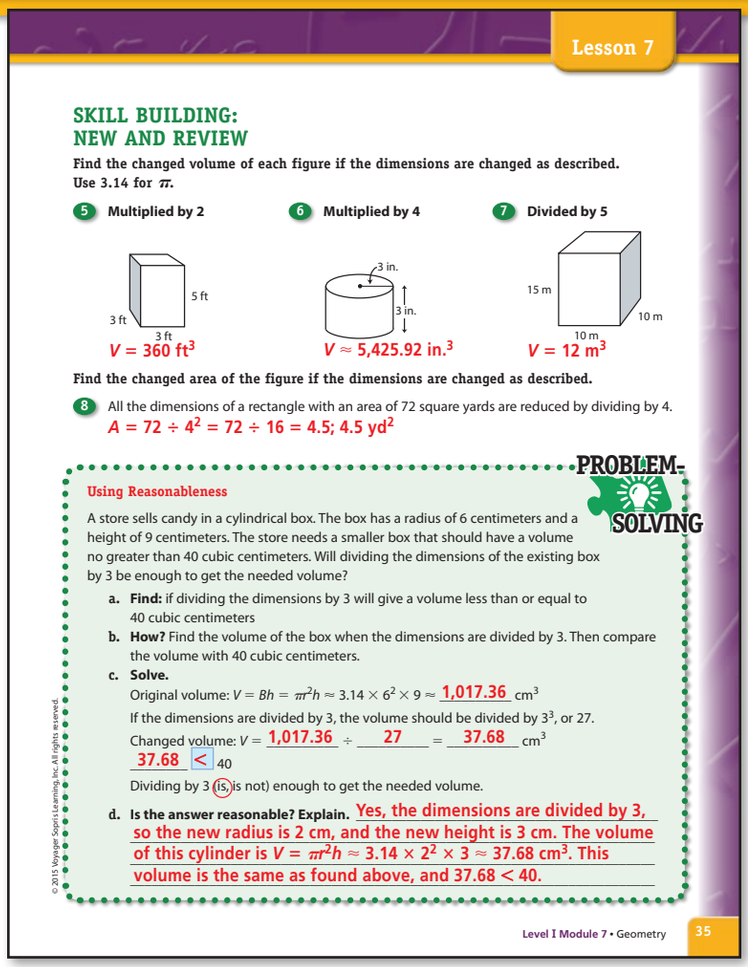
The answer is reasonable because dividing the dimensions by 3 means the volume should be divided by 3^3, or 27; 1,017.36 27 37.68.

Problems 5–12

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problems 5–7 Reminder

Have students multiply or divide the volume by the cube of the given number to find the changed volume.



Lesson 7

SKILL BUILDING: NEW AND REVIEW

Find the changed volume of each figure if the dimensions are changed as described. Use 3.14 for π.

5 Multiplied by 2

3 ft, 5 ft, 3 ft

$V = 360$ ft^3

6 Multiplied by 4

3 in., 3 in.

$V \approx 5{,}425.92$ $in.^3$

7 Divided by 5

15 m, 10 m, 10 m

$V = 12$ m^3

Find the changed area of the figure if the dimensions are changed as described.

8 All the dimensions of a rectangle with an area of 72 square yards are reduced by dividing by 4.

$A = 72 \div 4^2 = 72 \div 16 = 4.5$; 4.5 yd^2

PROBLEM-SOLVING

Using Reasonableness

A store sells candy in a cylindrical box. The box has a radius of 6 centimeters and a height of 9 centimeters. The store needs a smaller box that should have a volume no greater than 40 cubic centimeters. Will dividing the dimensions of the existing box by 3 be enough to get the needed volume?

a. **Find:** if dividing the dimensions by 3 will give a volume less than or equal to 40 cubic centimeters

b. **How?** Find the volume of the box when the dimensions are divided by 3. Then compare the volume with 40 cubic centimeters.

c. **Solve.**

Original volume: $V = Bh = \pi r^2 h \approx 3.14 \times 6^2 \times 9 \approx$ 1,017.36 cm^3

If the dimensions are divided by 3, the volume should be divided by 3^3, or 27.

Changed volume: $V =$ 1,017.36 $\div$ 27 $=$ 37.68 cm^3

37.68 < 40

Dividing by 3 (is) is not) enough to get the needed volume.

d. **Is the answer reasonable? Explain.** Yes, the dimensions are divided by 3, so the new radius is 2 cm, and the new height is 3 cm. The volume of this cylinder is $V = \pi r^2 h \approx 3.14 \times 2^2 \times 3 \approx 37.68$ cm^3. This volume is the same as found above, and 37.68 < 40.

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Problem 8 Reminder

Make sure students know that when dimensions of a two-dimensional figure have been changed proportionally, the square of the given number must be used to find the new area.

Problem 9 Reminder

Students must first find the volume of the given box, then multiply the volume by 2^3, or 8, to get the new volume.

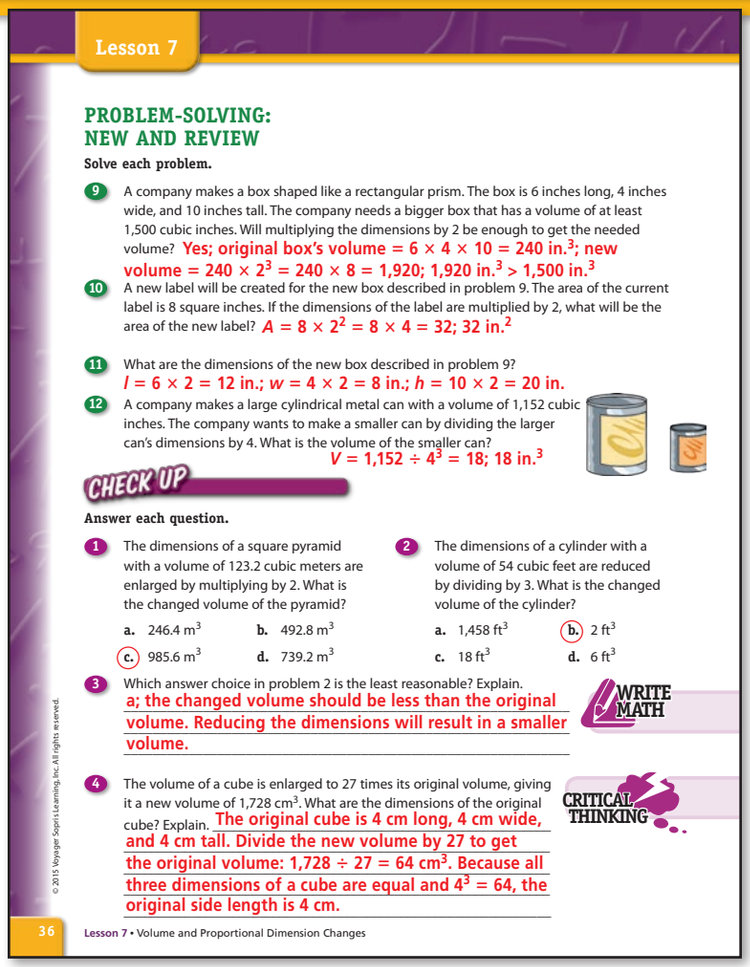
CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student multiplied the old volume by the given number. Use Additional Resources in I.7.7 to reteach how to find the volume when dimensions are changed proportionally.



Lesson 7

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

9 A company makes a box shaped like a rectangular prism. The box is 6 inches long, 4 inches wide, and 10 inches tall. The company needs a bigger box that has a volume of at least 1,500 cubic inches. Will multiplying the dimensions by 2 be enough to get the needed volume? Yes; original box's volume = 6 × 4 × 10 = 240 in.^{3}; new volume = 240 × 2^3 = 240 × 8 = 1,920; 1,920 in.^{3} > 1,500 in.^{3}

10 A new label will be created for the new box described in problem 9. The area of the current label is 8 square inches. If the dimensions of the label are multiplied by 2, what will be the area of the new label? $A = 8 \times 2^2 = 8 \times 4 = 32$; 32 in.^{2}

11 What are the dimensions of the new box described in problem 9?
$l = 6 \times 2 = 12$ in.; $w = 4 \times 2 = 8$ in.; $h = 10 \times 2 = 20$ in.

12 A company makes a large cylindrical metal can with a volume of 1,152 cubic inches. The company wants to make a smaller can by dividing the larger can's dimensions by 4. What is the volume of the smaller can?
$V = 1{,}152 \div 4^3 = 18$; 18 in.^{3}

CHECK UP

Answer each question.

1 The dimensions of a square pyramid with a volume of 123.2 cubic meters are enlarged by multiplying by 2. What is the changed volume of the pyramid?

a. 246.4 m^3 b. 492.8 m^3 (c.) 985.6 m^3 d. 739.2 m^3

2 The dimensions of a cylinder with a volume of 54 cubic feet are reduced by dividing by 3. What is the changed volume of the cylinder?

a. 1,458 ft^3 (b.) 2 ft^3 c. 18 ft^3 d. 6 ft^3

3 Which answer choice in problem 2 is the least reasonable? Explain.
a; the changed volume should be less than the original volume. Reducing the dimensions will result in a smaller volume.

WRITE MATH

4 The volume of a cube is enlarged to 27 times its original volume, giving it a new volume of 1,728 cm^3. What are the dimensions of the original cube? Explain. The original cube is 4 cm long, 4 cm wide, and 4 cm tall. Divide the new volume by 27 to get the original volume: 1,728 ÷ 27 = 64 cm^3. Because all three dimensions of a cube are equal and $4^3 = 64$, the original side length is 4 cm.

CRITICAL THINKING

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If student answered 1b: The student multiplied the original volume by the square of the given number. Use Additional Resources in I.7.7 to reteach how to find the volume when dimensions are changed proportionally.

If student answered 1d: The student multiplied the original volume by 3 times the given number instead of the cube of the number. Use Additional Resources in I.7.7 to reteach how to find the volume when dimensions are changed proportionally.

If student answered problem 2 incorrectly: The student did not divide by the correct number. Use Additional Resources in I.7.7 to reteach how to find the volume when dimensions are changed proportionally.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, remind the student that when a figure is reduced, the volume of the reduced figure will be less than the volume of the original figure. Use Additional Resources in I.7.7 to reteach how to find the volume when dimensions are changed proportionally.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 7 Lesson 7
Reteach Teacher Edition Module 7 Lesson 7

Extra Practice
Student Book page 50

English Language Learners

Although there are no new math terms for this lesson, it is important that students understand the terms *enlarge* and *reduce*. Ask students to name some other pairs of opposites, perhaps math related, like add/subtract, more/less, greater than/less than, multiplication/division, and so on. Explain that the terms *enlarge* and *reduce* are also opposites, and elicit informal definitions from students, such as "to make bigger" and "to make smaller."

Students with Special Needs

Allow students to use calculators for the many computations in this lesson. Review some of the important functions needed for these calculations. For example, show students that there are two ways to do calculations with exponents. Depending on the type of calculator, 4^3 could be keyed in as 4^3 or as 4 4 4. Show students how to do fraction multiplication as well. For example, $\frac{4}{3}$ 81 could be keyed in as (4 3) 81 or as 81 4 ANS. 3.

HANDS-ON GUIDED DISCOVERY

Lesson 8

Objective

To use grid paper to show how to square the sides of a right triangle to demonstrate the Pythagorean Theorem

CONCEPTUAL DEVELOPMENT

Materials

- Copy Master: Grid Paper
- scissors
- straightedges
- transparent tape

Lesson Notes

Before beginning the lesson, be sure students have their Student Books, copies of Copy Master: Grid Paper, scissors, straightedges, and transparent tape.

Remind students that in a right triangle, the hypotenuse is the side opposite the right angle and the legs are the other two sides.

The concepts in this lesson also can be presented using geoboards, tangrams, and snap cubes if these manipulatives are available.

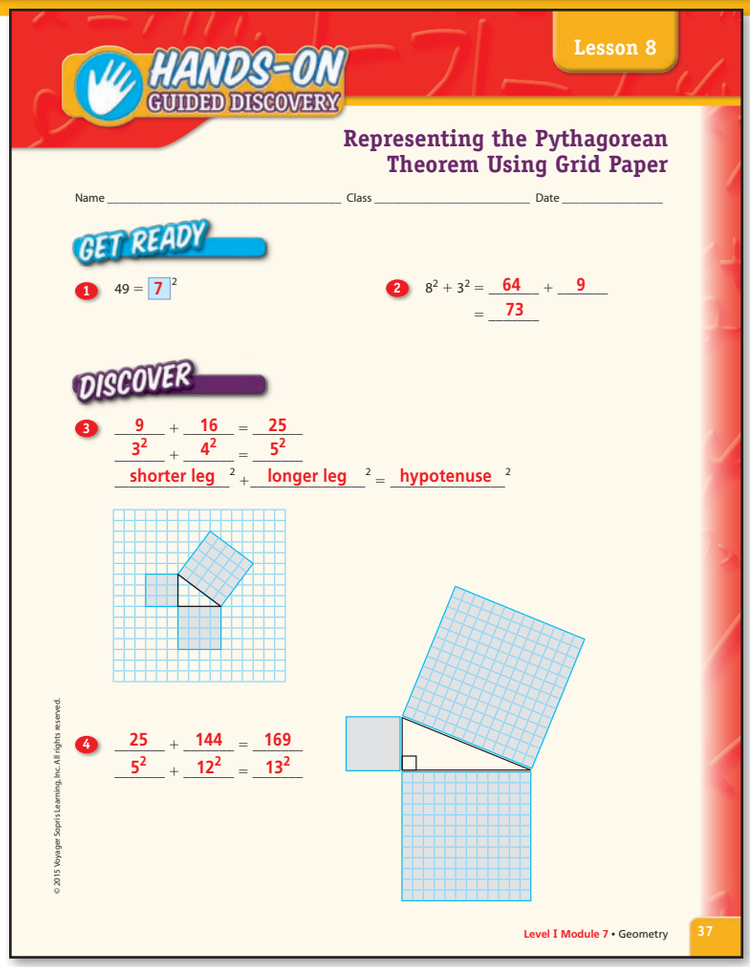
GET READY

Problem 1

Forty-nine is a perfect square in problem 1. What number when multiplied by itself, or squared, is 49? (7)

Problem 2

The order of operations is used to evaluate numerical expressions. For problem 2, what is the first step in evaluating 8^2 3^2? (Evaluate the exponential expressions.) **What is 8^2?** (64) **What is 3^2?** (9) **What is the sum of 64 and 9?** (73)



HANDS-ON GUIDED DISCOVERY

Lesson 8

Representing the Pythagorean Theorem Using Grid Paper

Name _____ Class _____ Date _____

GET READY

1. $49 = 7^2$

2. $8^2 + 3^2 = 64 + 9 = 73$

DISCOVER

3. $9 + 16 = 25$; $3^2 + 4^2 = 5^2$; shorter leg ^{2} + longer leg ^{2} = hypotenuse ^{2}

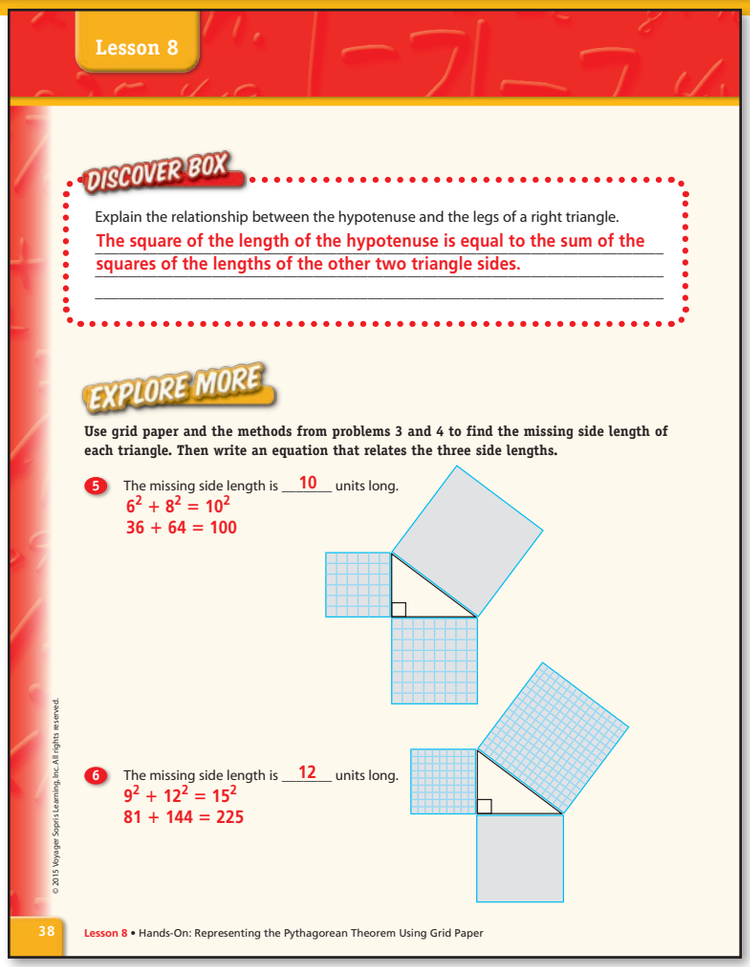
4. $25 + 144 = 169$; $5^2 + 12^2 = 13^2$

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DISCOVER

Problem 3

On grid paper, draw a right triangle that has one leg 3 units long and the other leg 4 units long as shown in the figure for problem 3. Leave some space around the triangle. On the leg that is 3 units, draw a square so that the leg of the triangle and a side of the square are the same size. How many square units are inside this square? (9) **On the leg that is 4 units, draw a square so that the leg of the triangle and a side of the square are the same size. How many square units are inside this square?** (16) **Cut out each square. Then cut the squares into strips that are 1 unit wide. Try arranging these strips into a square along the remaining side of the triangle as shown in the figure.** Students will have to cut one of the strips into single squares to make the pieces fit. **Do they fit into a square?** (yes) **So, the two smaller squares can be arranged into a larger square. Tape the strips to the grid paper. How many square units make up this square?** (25) **What equation relates the numbers 9, 16, and 25?** (9 16 25) **Are these numbers perfect squares?** (yes) **What is 9 written as an exponential expression?** (3^2) **What is 16 written as an exponential expression?** (4^2) **What is 25 written as an exponential**



Lesson 8

DISCOVER BOX

Explain the relationship between the hypotenuse and the legs of a right triangle.

The square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two triangle sides.

EXPLORE MORE

Use grid paper and the methods from problems 3 and 4 to find the missing side length of each triangle. Then write an equation that relates the three side lengths.

5 The missing side length is 10 units long.

$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

6 The missing side length is 12 units long.

$9^2 + 12^2 = 15^2$

$81 + 144 = 225$

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expression? (5^2) **In a right triangle, the side opposite the right angle is called the hypotenuse, and the other sides are called the legs of the triangle. What measurement is 3 units on the triangle?** (the length of the shorter leg) **What measurement is 4 units on the triangle?** (the length of the longer leg) **So, the shorter leg is 3 units long, and the longer leg is 4 units long. What measurement is 5 units on the triangle?** (the length of the hypotenuse) **How do you know the length?** (A square with a side length of 5 units lines up with the hypotenuse on the grid paper.)

Problem 4

In problem 4, a right triangle is shown. Use the information shown to find the side lengths of the triangle. How many square units are covered by the square along the longer leg of the right triangle? (144) **How many square units are covered by the square along the hypotenuse of the right triangle?** (169) **Draw and cut out each of these squares on grid paper. How can this information be used to find the number of grid squares that should cover the square along the shorter leg of the right triangle?** (Subtract or remove the number of square units in the smaller square from the number of square units in the larger square. Do this by placing the smaller square on top of the larger square so that a pair of corners match. Cut out the pieces that are not covered and form the square units that are left into a square.) **Why?** (The sum of the number of square units along the legs equals the number of grid squares in the square along the hypotenuse.) **Cut what remains of the larger square into strips to form the square units into a square. How many square units should cover the square?** (25) **So, 25 plus 144 equals 169. What is 25 written as an exponential expression?** (5^2) **What is 144 written as an exponential expression?** (12^2) **What is 169 written as an exponential expression?** (13^2) **What measurement is 13 units on the triangle?** (the length of the hypotenuse) **What measurement is 12 units on the triangle?** (the length of the longer leg) **What measurement is 5 units on the triangle?** (the length of the shorter leg) **So, the shorter leg is 5 units long.**

DISCOVER BOX

Use the information from problems 3 and 4 to explain the relationship between the hypotenuse and the legs of a right triangle. (The square of the length of the hypotenuse is always equal to the sum of the squares of the lengths of the other two sides.)

EXPLORE MORE

Problem 5

Have students work independently. Students should draw the triangle on grid paper, then draw and cut squares along the legs of the triangle. Students then should cut the squares into strips as they did in problem 3 and arrange them into a square along the hypotenuse.

Problem 6

Have students work independently. Students should cut squares like those shown. Students then should remove the number of square units in the smaller square from the larger square as in problem 4 and form the remaining square units into a square along the longer leg.

Extension: Have students or partners explore whether this method works on triangles other than right triangles.

Lesson 9

Objective

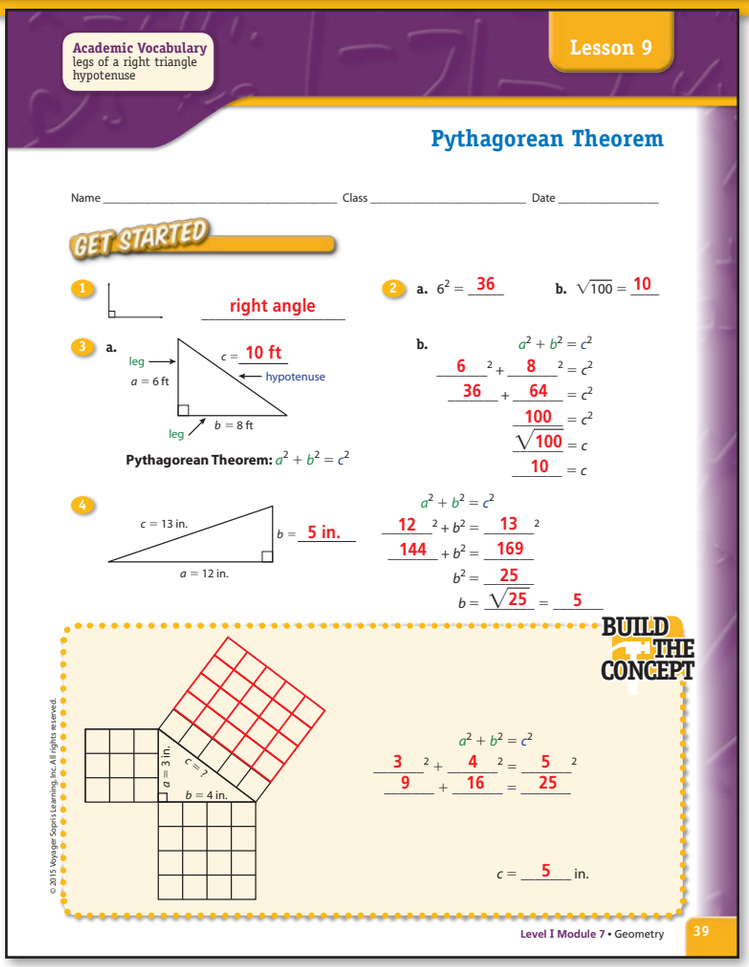
To use the Pythagorean Theorem to find the lengths of sides of right triangles

Preskills	Lesson
Exponents	I.2.1
Square Roots	I.7.PL2

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **legs of a right triangle**
  the two sides of the triangle that form the right angle
- **hypotenuse**
  the side opposite the right angle of the triangle and the longest side in a right triangle



GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

What kind of angle is shown? (right angle) **A right angle measures exactly 90° and is indicated by a right angle box.**

Problem 2

How is the expression in problem 2a read? (6 to the second power, or 6 squared) **What does 6 squared mean?** (6 6) **What is 6 times 6?** (36) **Then 6^2 is equal to 36.**

Look at problem 2b. A number whose square is equal to a given number is called the square root of that number. How is the expression in problem 2b read? (the square root of 100) **What is the square root of 100?** (10) **Why?** (10 times 10 is 100)

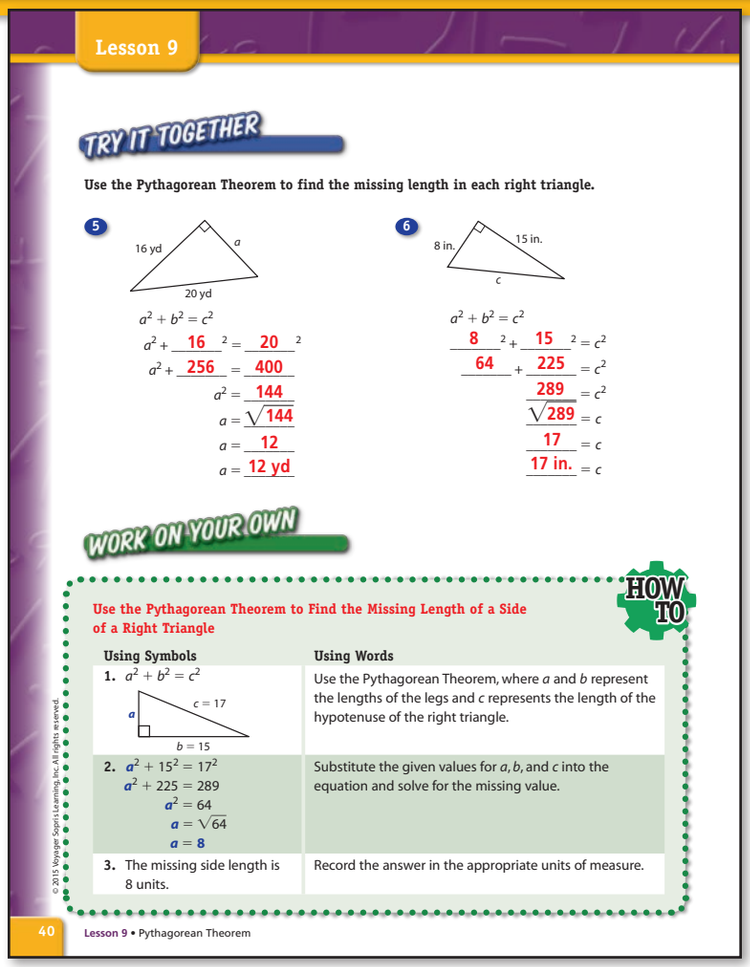
MODEL NEW SKILLS

Problem 3

What does the small square box in the angle of this triangle indicate? (a right angle) **Triangles that have a right angle are called right triangles. The two sides of the triangle that form the right angle are called the legs of the right triangle. It is customary to use the letters *a* and *b* to represent the lengths of the legs of a right triangle. What is the length of *a* in this triangle?** (6 ft) **What is the length of *b* in this triangle?** (8 ft)

The side of a right triangle opposite the right angle is called the hypotenuse. The hypotenuse is always the longest side of a right triangle. It is customary to label the length of the hypotenuse with the letter *c*.

The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of a right triangle. The relationship is stated in the equation a^2 b^2 c^2. This equation is called the Pythagorean Theorem, and it is true for all right triangles where *a* and *b* are the lengths of the triangle's legs and *c* is the length of its hypotenuse. To find the length of the hypotenuse in this triangle, first substitute the given values for *a* and *b* into the equation. What is 6^2? (36) **What is 8^2?** (64) **Then solve the equation. What is 36 plus 64?** (100) **So, c^2 is equal to 100. To find *c*, find the square root of**



Lesson 9

TRY IT TOGETHER

Use the Pythagorean Theorem to find the missing length in each right triangle.

5

$a^2 + b^2 = c^2$

$a^2 + \underline{16}^2 = \underline{20}^2$

$a^2 + \underline{256} = \underline{400}$

$a^2 = \underline{144}$

$a = \sqrt{\underline{144}}$

$a = \underline{12}$

$a = \underline{12 \text{ yd}}$

6

$a^2 + b^2 = c^2$

$\underline{8}^2 + \underline{15}^2 = c^2$

$\underline{64} + \underline{225} = c^2$

$\underline{289} = c^2$

$\sqrt{\underline{289}} = c$

$\underline{17} = c$

$\underline{17 \text{ in.}} = c$

WORK ON YOUR OWN

HOW TO

Use the Pythagorean Theorem to Find the Missing Length of a Side of a Right Triangle

Using Symbols	Using Words
1. $a^2 + b^2 = c^2$	Use the Pythagorean Theorem, where a and b represent the lengths of the legs and c represents the length of the hypotenuse of the right triangle.
2. $a^2 + 15^2 = 17^2$ $a^2 + 225 = 289$ $a^2 = 64$ $a = \sqrt{64}$ $a = 8$	Substitute the given values for a, b, and c into the equation and solve for the missing value.
3. The missing side length is 8 units.	Record the answer in the appropriate units of measure.

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c^2, or 100. What is the square root of 100? (10) What is the length of the hypotenuse of this right triangle? (10 ft) Remember to record the answer using the appropriate unit of measure.

Problem 4

What kind of triangle is shown in this problem? (right triangle) Notice that the length of one leg and the length of the hypotenuse of the right triangle are given. The length of the other leg is missing. Use the Pythagorean Theorem to find the missing length. Substitute the given values for *a* and *c* into the equation.

What is *a*? (12 in.) What is *c*? (13 in.) Now solve the equation. What is 12^2? (144) What is 13^2? (169) Subtract 144 from both sides of the equation. What is 169 minus 144? (25) What is the resulting equation? (b^2 25)

To find *b*, find the square root of 25. What is the square root of 25? (5) What is the missing length? (5 in.) Remember to record the answer using the appropriate unit of measure.

BUILD THE CONCEPT

Model how to find the missing length of the side of a triangle using the Pythagorean Theorem.

Look at the right triangle. What are the lengths of the legs? (3 units and 4 units) Notice that there are blocks that make the square of each side length on each leg. How many total boxes are there on the side that has length 3? (9) How many are on the side with length 4? (16)

Now look at the hypotenuse. What is the length of it? (5 units) Notice that there are already 5 blocks started. Finish the square. How many blocks should be drawn on each side to make a square? (5) How many total blocks are there? (25)

Now use the Pythagorean Theorem to find the length to see if the diagram matches the formula. What is 3^2? (9) What is 4^2? (16) What is 5^2? (25) Does 9 plus 16 equal 25? (yes) So the model of the Pythagorean Theorem is the same as using the formula.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 5

What kind of triangle is shown in problem 5? (right triangle) Is the length of the hypotenuse or the length of a leg missing? (a leg) Remember the length of the legs are represented by *a* and *b* in the equation; the length of the hypotenuse is represented by *c*. What value is missing? (*a*) What values are given? (*b* and *c*) What is *b*? (16 yd) What is *c*? (20 yd)

Substitute the given values into the equation. What equation needs to be solved? (a^2 16^2 20^2) What is 16^2? (256) What is 20^2? (400) What is the next step to solve the equation? (Subtract 256 from both sides of the equation.) What is 400 minus 256? (144) To find *a*, find the square root of 144. What is the square root of 144? (12) What is the missing length? (12 yd)

Problem 6

Is the triangle a right triangle? (yes) Is the measure of the hypotenuse or the measure of the leg missing? (the hypotenuse) Why? (The hypotenuse is the longest side opposite the right angle.)

What is the equation that needs to be solved? (8^2 15^2 c^2) What is 8^2? (64) What is 15^2? (225) What is 225 plus 64? (289) What is the square root

of 289? (17) **What is the solution of the equation?** (17) **What is the length of the hypotenuse?** (17 in.)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

Before students begin independent work, review the **HOW TO** process example.

As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Formula** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a formula. The instruction is immediately followed by application of the strategy in problem 17.

Have students read the problem.

Have students describe the diagram in words:

> The picture shows a ramp. The ramp forms a right triangle. The hypotenuse of the triangle is 13 feet long. One of the legs is 12 feet long. The other leg's length is unknown.

Have students solve for the missing side length using the Pythagorean Theorem.

a^2 b^2 c^2

a unknown side length

b 12 ft

c 13 ft

a 5 ft

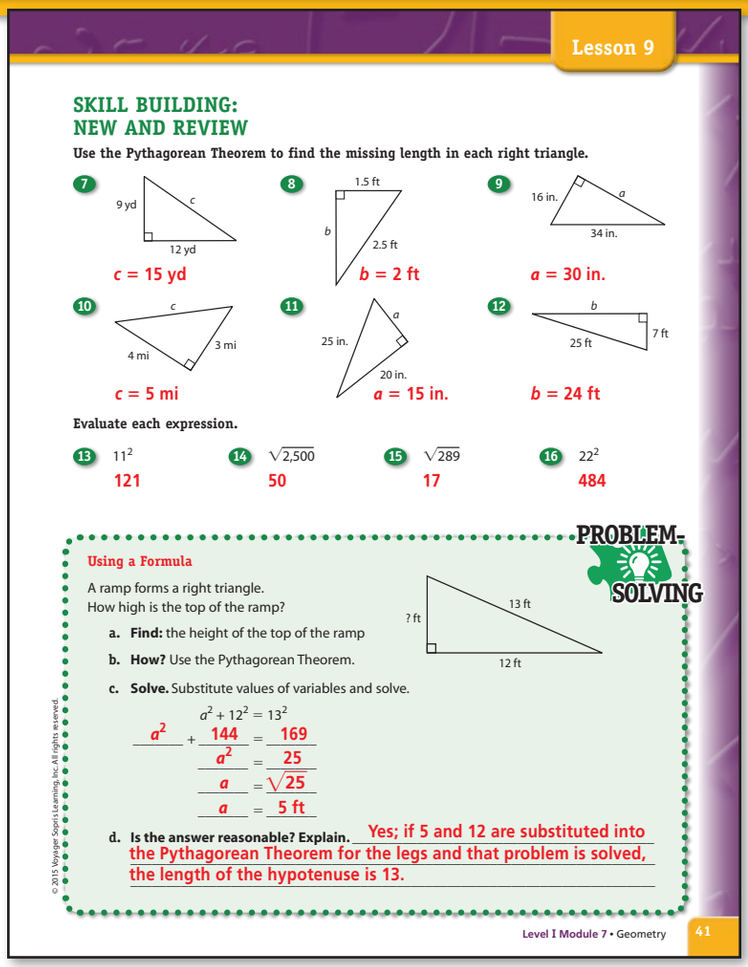
The answer is reasonable because if 5 and 12 are substituted into the Pythagorean Theorem, the result is 13.

Problems 7–20

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

Problem 8 **Reminder**

Be sure students place the decimal point correctly when multiplying and adding decimals.



Lesson 9

SKILL BUILDING: NEW AND REVIEW

Use the Pythagorean Theorem to find the missing length in each right triangle.

7. 9 yd, c, 12 yd — c = 15 yd
8. 1.5 ft, b, 2.5 ft — b = 2 ft
9. 16 in., a, 34 in. — a = 30 in.
10. c, 4 mi, 3 mi — c = 5 mi
11. a, 25 in., 20 in. — a = 15 in.
12. b, 25 ft, 7 ft — b = 24 ft

Evaluate each expression.

13. 11^2 — 121
14. $\sqrt{2{,}500}$ — 50
15. $\sqrt{289}$ — 17
16. 22^2 — 484

Using a Formula PROBLEM-SOLVING

A ramp forms a right triangle. How high is the top of the ramp?

a. **Find:** the height of the top of the ramp

b. **How?** Use the Pythagorean Theorem.

c. **Solve.** Substitute values of variables and solve.

$a^2 + 12^2 = 13^2$

$a^2 + 144 = 169$

$a^2 = 25$

$a = \sqrt{25}$

a = 5 ft

d. **Is the answer reasonable? Explain.** Yes; if 5 and 12 are substituted into the Pythagorean Theorem for the legs and that problem is solved, the length of the hypotenuse is 13.

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Problem 18 **Reminder**

Students may need to draw a diagram of the situation described in the problem to determine which side lengths are given and which needs to be found.

CHECK UP

ASSESS INFORMALLY

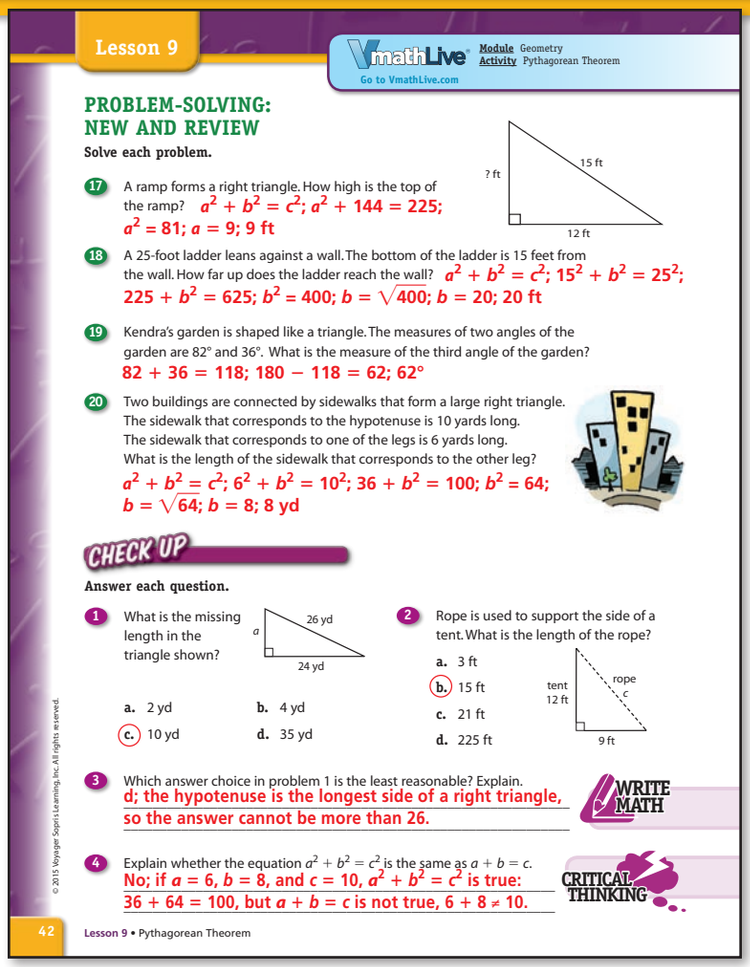
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 2c: The student did not square the values when using the Pythagorean Theorem. Use Additional Resources in I.7.9 to reteach the Pythagorean Theorem.

If student answered 1b: The student did not use the Pythagorean Theorem correctly. Use Additional Resources in I.7.9 to reteach the Pythagorean Theorem.

If student answered 1d or 2a: The student solved for the wrong value in the equation. Remind the student that the side opposite the right angle is the hypotenuse and the other two sides are legs.



Lesson 9

VmathLive Module Geometry Activity Pythagorean Theorem
Go to VmathLive.com

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

17 A ramp forms a right triangle. How high is the top of the ramp? $a^2 + b^2 = c^2$; $a^2 + 144 = 225$; $a^2 = 81$; $a = 9$; 9 ft

18 A 25-foot ladder leans against a wall. The bottom of the ladder is 15 feet from the wall. How far up does the ladder reach the wall? $a^2 + b^2 = c^2$; $15^2 + b^2 = 25^2$; $225 + b^2 = 625$; $b^2 = 400$; $b = \sqrt{400}$; $b = 20$; 20 ft

19 Kendra's garden is shaped like a triangle. The measures of two angles of the garden are 82° and 36°. What is the measure of the third angle of the garden? $82 + 36 = 118$; $180 - 118 = 62$; 62°

20 Two buildings are connected by sidewalks that form a large right triangle. The sidewalk that corresponds to the hypotenuse is 10 yards long. The sidewalk that corresponds to one of the legs is 6 yards long. What is the length of the sidewalk that corresponds to the other leg? $a^2 + b^2 = c^2$; $6^2 + b^2 = 10^2$; $36 + b^2 = 100$; $b^2 = 64$; $b = \sqrt{64}$; $b = 8$; 8 yd

CHECK UP

Answer each question.

1 What is the missing length in the triangle shown?

a. 2 yd
b. 4 yd
c. 10 yd
d. 35 yd

2 Rope is used to support the side of a tent. What is the length of the rope?

a. 3 ft
b. 15 ft
c. 21 ft
d. 225 ft

3 Which answer choice in problem 1 is the least reasonable? Explain.
d; the hypotenuse is the longest side of a right triangle, so the answer cannot be more than 26.

WRITE MATH

4 Explain whether the equation $a^2 + b^2 = c^2$ is the same as $a + b = c$.
No; if $a = 6$, $b = 8$, and $c = 10$, $a^2 + b^2 = c^2$ is true: $36 + 64 = 100$, but $a + b = c$ is not true, $6 + 8 \neq 10$.

CRITICAL THINKING

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If student answered 2d: The student did not take the square root of the sum when using the Pythagorean Theorem. Use Additional Resources in I.7.9 to reteach the Pythagorean Theorem.



WRITE MATH

If student answered problem 3 incorrectly: While students work together, review the parts of a right triangle with the student. Because the hypotenuse is given as 26 yards, the correct answer must be less than 26.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Geometry
Activity: Pythagorean Theorem

Vmath Reteach
Reteach Student Book Module 7 Lesson 9
Reteach Teacher Edition Module 7 Lesson 9

Extra Practice
Student Book page 51

English Language Learners

Use the VmathLive Animated Glossary to review the term *hypotenuse*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Draw different right triangles on the board in different orientations. Have students come to the board and label the legs and hypotenuse of the different triangles. Then have students label the sides *a*, *b*, and *c*.

Students with Special Needs

Emphasize to students that the Pythagorean Theorem can only be used for right triangles. Have students use rulers to draw a large right triangle on grid paper. Students should use the ruler to measure two side lengths in centimeters. Make sure they are using whole number lengths. Have students use the Pythagorean Theorem and a calculator to determine the third side length. Students can work with a partner. Then students should measure the third side to check their answers.

Lesson 10

Objective

To use the Pythagorean Theorem to find the distance between two points in the coordinate plane

Preskills	Lesson
Pythagorean Theorem	I.7.9

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

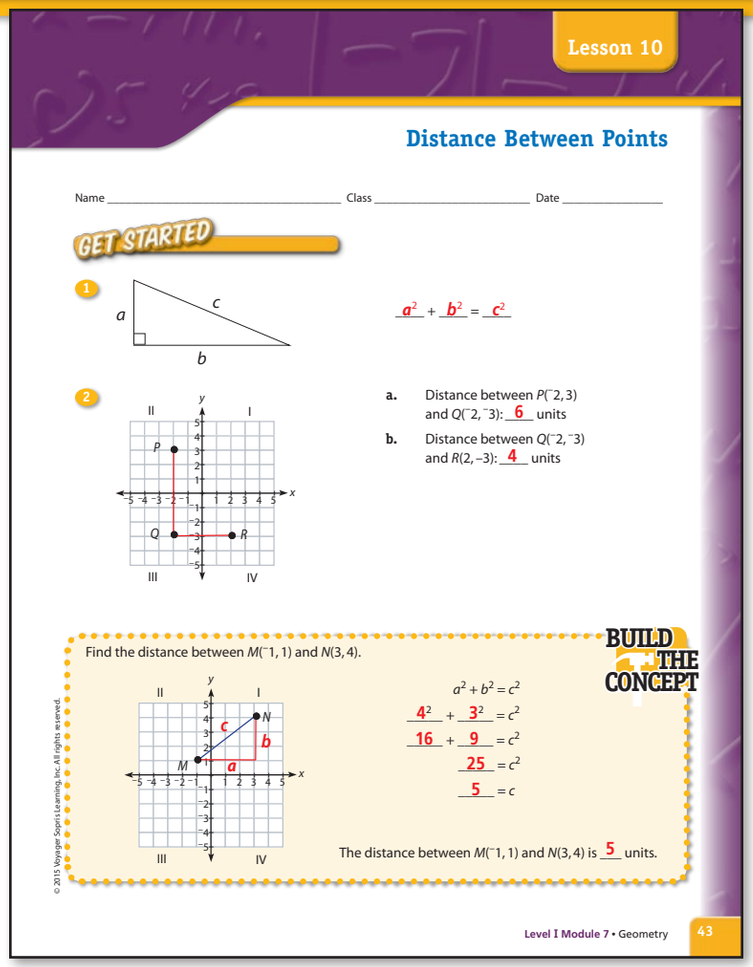
Look at the right triangle. Which side of the triangle is the hypotenuse? (the side opposite the right angle) **What letter represents the length of the hypotenuse?** (*c*) **What letters represent the lengths of the legs?** (*a* and *b*) **The Pythagorean Theorem describes the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle. The theorem says that the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. How do you write this equation using the letters *a*, *b*, and *c*?** ($a^2 + b^2 = c^2$)

MODEL NEW SKILLS

Problem 2

Look at points *P* and *Q* on the coordinate plane. Draw a straight line segment between point *P* and point *Q*. This line segment represents the distance between the points. Remember that the side length of a grid square is 1 unit. How many units long is the line segment? (6) **So, the distance between point *P* and point *Q* is 6 units.**

To find the distance between points *Q* and *R*, draw a straight line segment between the points. How many units long is the line segment? (4) **What is the distance between point *Q* and point *R*?** (4 units)



Lesson 10

Distance Between Points

Name _____ Class _____ Date _____

GET STARTED

1

$_a^2_ + _b^2_ = _c^2_$

2

a. Distance between $P(^-2, 3)$ and $Q(^-2, ^-3)$: 6 units

b. Distance between $Q(^-2, ^-3)$ and $R(2, -3)$: 4 units

BUILD THE CONCEPT

Find the distance between $M(^-1, 1)$ and $N(3, 4)$.

$a^2 + b^2 = c^2$

$_4^2_ + _3^2_ = c^2$

$_16_ + _9_ = c^2$

$_25_ = c^2$

$_5_ = c$

The distance between $M(^-1, 1)$ and $N(3, 4)$ is 5 units.

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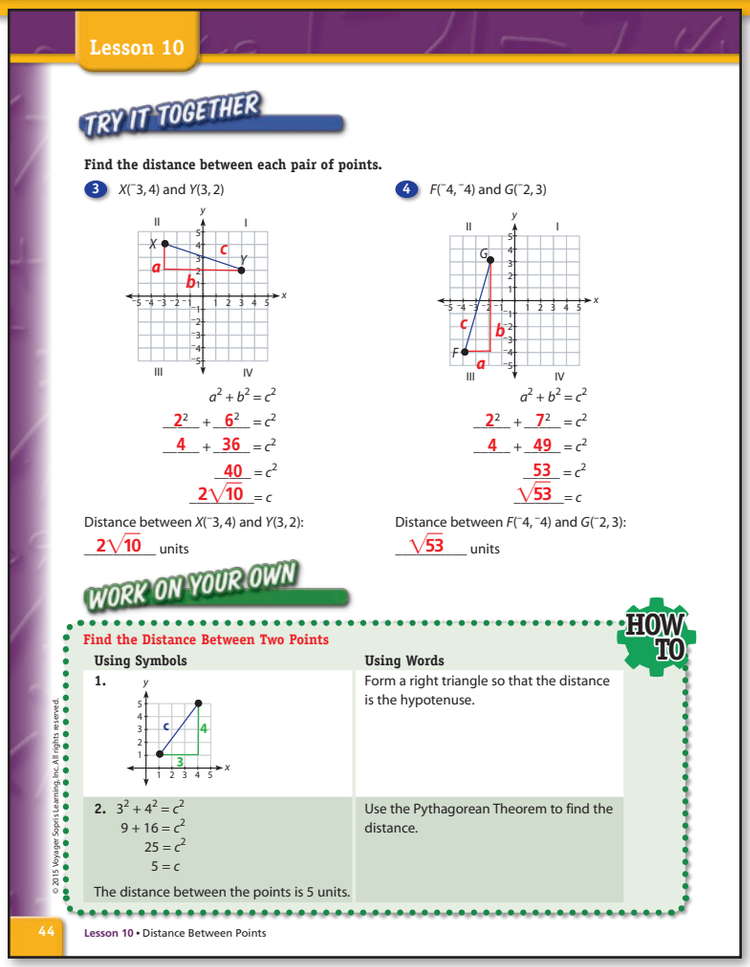
BUILD THE CONCEPT

Model how to use the Pythagorean Theorem to find the distance between two points in the plane.

Look at the line segment that represents the distance between points *M* and *N*. Because the line segment is not vertical or horizontal, its length cannot be determined directly. If this segment were the side length of a right triangle, the Pythagorean Theorem could be used to find the length of the segment.

Make the segment between point *M* and point *N* the hypotenuse of a right triangle. First, draw a horizontal line segment from $M(^-1, 1)$ to (3, 1). Draw a vertical line segment from $N(3, 4)$ to (3, 1). Do the three segments form a right triangle? (yes)

Now use the Pythagorean Theorem. What does *c* represent in the Pythagorean Theorem? (the length of the hypotenuse) **Write *c* on the plane to label the length of the hypotenuse. What do *a* and *b* represent in the Pythagorean Theorem?** (the lengths of the legs) **Label the horizontal leg as *a* and the vertical leg as *b* on the plane. What is *a*?** (4 units) **Substitute 4 for *a* in the equation $a^2 + b^2 = c^2$. What is *b*?** (3 units) **Substitute 3 for *b* in the equation. What is the equation?** ($4^2 + 3^2 = c^2$)



Lesson 10

TRY IT TOGETHER

Find the distance between each pair of points.

3 $X(^{-}3, 4)$ and $Y(3, 2)$

4 $F(^{-}4, ^{-}4)$ and $G(^{-}2, 3)$

$a^2 + b^2 = c^2$
$2^2 + 6^2 = c^2$
$4 + 36 = c^2$
$40 = c^2$
$2\sqrt{10} = c$

Distance between $X(^{-}3, 4)$ and $Y(3, 2)$:
$2\sqrt{10}$ units

$a^2 + b^2 = c^2$
$2^2 + 7^2 = c^2$
$4 + 49 = c^2$
$53 = c^2$
$\sqrt{53} = c$

Distance between $F(^{-}4, ^{-}4)$ and $G(^{-}2, 3)$:
$\sqrt{53}$ units

WORK ON YOUR OWN

HOW TO

Find the Distance Between Two Points

Using Symbols	Using Words
1.	Form a right triangle so that the distance is the hypotenuse.
2. $3^2 + 4^2 = c^2$ $9 + 16 = c^2$ $25 = c^2$ $5 = c$ The distance between the points is 5 units.	Use the Pythagorean Theorem to find the distance.

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Now use the Pythagorean Theorem. What does *c* represent in the Pythagorean Theorem? (the length of the hypotenuse) **Write *c* on the plane to label the length of the hypotenuse. What do *a* and *b* represent in the Pythagorean Theorem?** (the lengths of the legs) **Label the vertical leg as *a* and the horizontal leg as *b* on the plane. What is *a*?** (2 units) **Substitute 2 for *a* in the equation $a^2 + b^2 = c^2$. What is *b*?** (6 units) **Substitute 6 for *b* in the equation. What is the equation?** ($2^2 + 6^2 = c^2$)

Now solve the equation. What is 2^2? (4) **What is 6^2?** (36) **What is 4 plus 36?** (40) **Is 40 the length of the hypotenuse?** (no) **What needs to be done to find the length?** (Find the square root of 40.) **Because 40 is not a perfect square, simplify the square root of 40. What is the simplified form of the square root of 40?** ($2\sqrt{10}$) **What is the length of the hypotenuse?** ($2\sqrt{10}$ units) **What does this length represent?** (the distance between points *X* and *Y*) **So, the distance between point *X* and point *Y* is $2\sqrt{10}$ units.**

Problem 4

What is the first step in finding the distance between points *F* and *G*? (Make the segment between points *F* and *G* the hypotenuse of a right triangle.) **Draw the legs of the right triangle. What are the lengths of the legs?** (2 units and 7 units) **What is the equation after 2 and 7 are substituted for *a* and *b* in the equation $a^2 + b^2 = c^2$?** ($2^2 + 7^2 = c^2$)

Now solve the equation. What is the equation after the exponential expressions are evaluated? ($4 + 49 = c^2$) **What is 4 plus 49?** (53) **What is the distance between points *F* and *G*?** ($\sqrt{53}$ units)

Now solve the equation. What is 4^2? (16) **What is 3^2?** (9) **What is 16 plus 9?** (25) **Is 25 the length of the hypotenuse?** (no) **What needs to be done to find the length?** (Find the square root of 25.) **What is the square root of 25?** (5) **What is the length of the hypotenuse?** (5 units) **What does this length represent?** (the distance between points *M* and *N*) **So, the distance between point *M* and point *N* is 5 units.**

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 3

To find the distance between points *X* and *Y*, make the segment between point *X* and point *Y* the hypotenuse of a right triangle. First, draw a vertical line segment from *X*$(^{-}3, 4)$ to $(^{-}3, 2)$. Draw a horizontal line segment from $(^{-}3, 2)$ to *Y*$(3, 2)$. Do the three segments form a right triangle? (yes)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO

Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

Problems 5–12

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

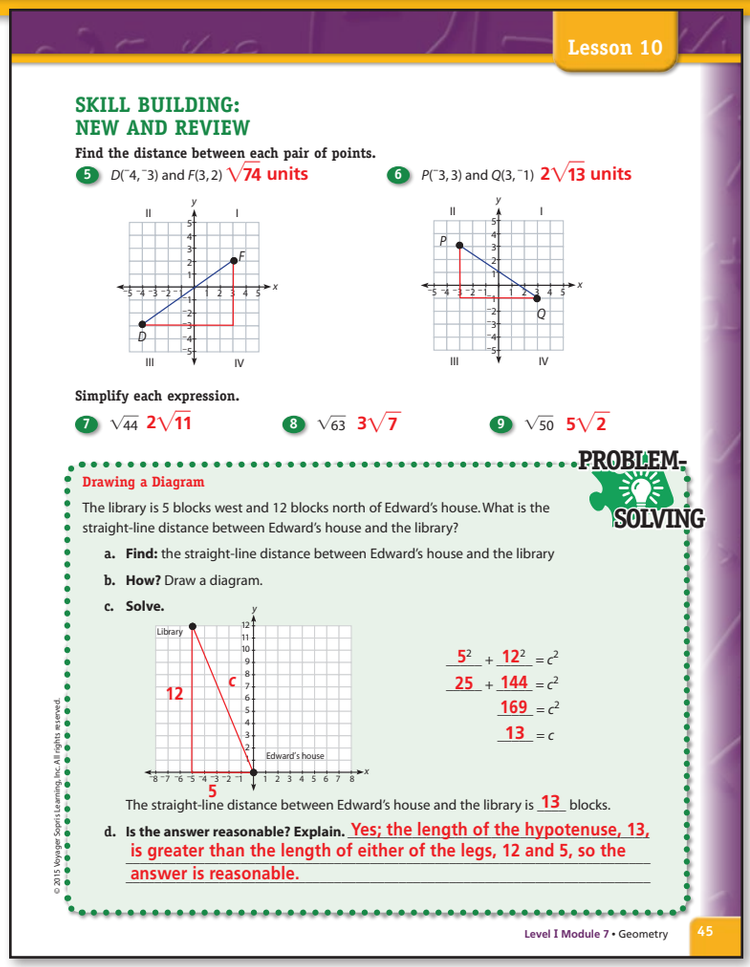
PROBLEM-SOLVING

This problem illustrates the **Drawing a Diagram** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving an unknown distance. The instruction is immediately followed by application of the strategy in problem 10.

This problem describes the location of one building in terms of its directional distance from another building. Students are asked to find the straight-line distance between the buildings.

Because the location of the library is described in terms of its directional distance from Edward's house, the location of Edward's house is graphed at the origin. Have students draw the point that shows the location of the library. Remind students that north is typically oriented up, south is down, east is right, and west is left. Ask them to name the coordinates of the point for the library. Then have students draw a straight line between the points. Now have students draw the right triangle and use the Pythagorean Theorem.

Finally, have students complete the statement. To check their answer, ask them to think about how the length of the hypotenuse should compare to the length of either leg.



Lesson 10

SKILL BUILDING: NEW AND REVIEW

Find the distance between each pair of points.

5 $D(^-4, ^-3)$ and $F(3, 2)$ $\sqrt{74}$ units

6 $P(^-3, 3)$ and $Q(3, ^-1)$ $2\sqrt{13}$ units

Simplify each expression.

7 $\sqrt{44}$ $2\sqrt{11}$

8 $\sqrt{63}$ $3\sqrt{7}$

9 $\sqrt{50}$ $5\sqrt{2}$

PROBLEM-SOLVING

Drawing a Diagram

The library is 5 blocks west and 12 blocks north of Edward's house. What is the straight-line distance between Edward's house and the library?

a. **Find:** the straight-line distance between Edward's house and the library

b. **How?** Draw a diagram.

c. **Solve.**

Library, 12, c, 5, Edward's house

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$169 = c^2$

$13 = c$

The straight-line distance between Edward's house and the library is 13 blocks.

d. **Is the answer reasonable? Explain.** Yes; the length of the hypotenuse, 13, is greater than the length of either of the legs, 12 and 5, so the answer is reasonable.

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Level I Module 7 • Geometry 45

CHECK UP

ASSESS INFORMALLY

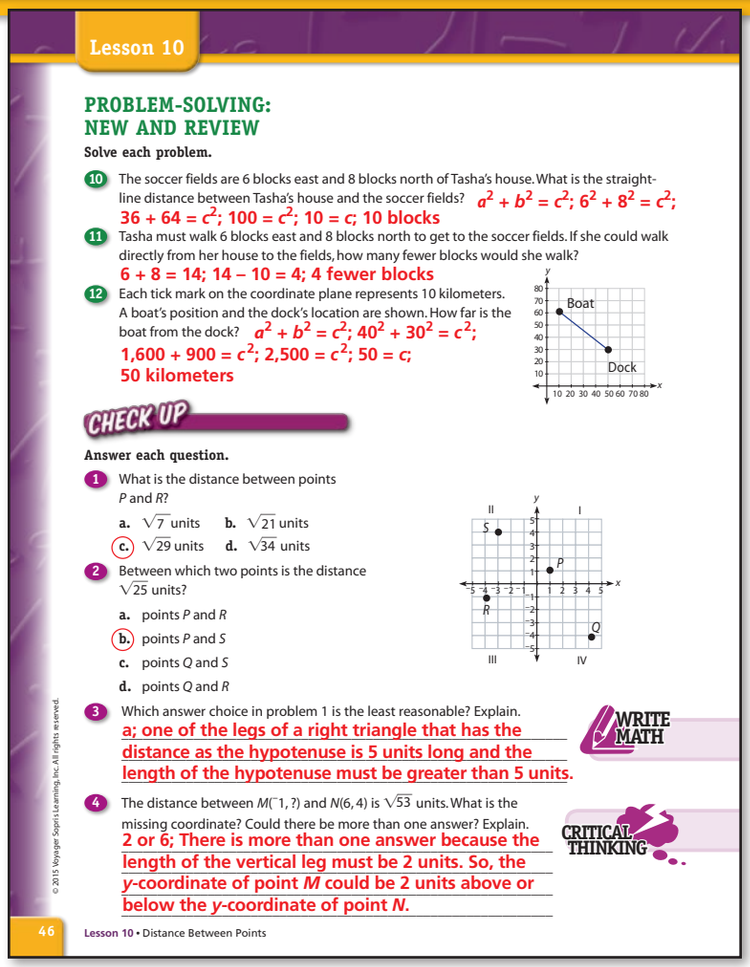
Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a or 1b: The student did not use the Pythagorean Theorem correctly. Use Additional Resources in I.7.9 to review the Pythagorean Theorem.

If student answered 1d: The student found the distance between the wrong points. Use Additional Resources in I.7.10 to review how to draw a right triangle to find the distance between two points in the plane.

If student answered problem 2 incorrectly: The student did not identify the points correctly or made an error when using the Pythagorean Theorem. Use Additional Resources in I.7.10 to review how to draw a right triangle to find the distance between two points in the plane.



Lesson 10

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

10. The soccer fields are 6 blocks east and 8 blocks north of Tasha's house. What is the straight-line distance between Tasha's house and the soccer fields? $a^2 + b^2 = c^2$; $6^2 + 8^2 = c^2$; $36 + 64 = c^2$; $100 = c^2$; $10 = c$; 10 blocks

11. Tasha must walk 6 blocks east and 8 blocks north to get to the soccer fields. If she could walk directly from her house to the fields, how many fewer blocks would she walk?
6 + 8 = 14; 14 − 10 = 4; 4 fewer blocks

12. Each tick mark on the coordinate plane represents 10 kilometers. A boat's position and the dock's location are shown. How far is the boat from the dock? $a^2 + b^2 = c^2$; $40^2 + 30^2 = c^2$; $1{,}600 + 900 = c^2$; $2{,}500 = c^2$; $50 = c$; 50 kilometers

Boat

Dock

CHECK UP

Answer each question.

1. What is the distance between points P and R?
   a. $\sqrt{7}$ units b. $\sqrt{21}$ units
   (c.) $\sqrt{29}$ units d. $\sqrt{34}$ units

2. Between which two points is the distance $\sqrt{25}$ units?
   a. points P and R
   (b.) points P and S
   c. points Q and S
   d. points Q and R

3. Which answer choice in problem 1 is the least reasonable? Explain.
a; one of the legs of a right triangle that has the distance as the hypotenuse is 5 units long and the length of the hypotenuse must be greater than 5 units.

4. The distance between $M(-1, ?)$ and $N(6, 4)$ is $\sqrt{53}$ units. What is the missing coordinate? Could there be more than one answer? Explain.
2 or 6; There is more than one answer because the length of the vertical leg must be 2 units. So, the y-coordinate of point M could be 2 units above or below the y-coordinate of point N.

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46 Lesson 10 • Distance Between Points



If student answered problem 3 incorrectly: While students work together, review the relationships between the side lengths of right triangles. Use Additional Resources in I.7.9 to review right triangles and the Pythagorean Theorem.

Technology

Have students use an online search engine to discover other ways to apply the Pythagorean Theorem in real-life situations. Additional digital content is available through this feature in the eBook.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 7 Lesson 10
Reteach Teacher Edition Module 7 Lesson 10

Extra Practice
Student Book page 51

English Language Learners

Use the VmathLive Animated Glossary to review the terms *legs, hypotenuse,* and *right triangle*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Practice drawing two points in the plane and constructing the right triangle needed to find the distance between the points. Have students label the lengths of the legs as a and b and the hypotenuse as c. Remind students that c represents the length of the hypotenuse *and* the distance between the points.

Students with Special Needs

Remind students that the triangles they draw must be right triangles in order to use the Pythagorean Theorem. If the distance between two points is represented by a nonvertical or nonhorizontal line segment, they will always draw the legs so that one leg is horizontal and the other leg is vertical. If students are having difficulty with finding the length of each leg, have them draw a point at the intersection of the legs.

Extra Practice Answer Key

Extra Practice

Name _____ Class _____ Date _____

Lesson PL1 Understanding Integers

Identify each point on the number line. Then write its opposite.



1. Point *D* +3, −3
2. Point *A* −6, +6
3. Point *E* +5, −5

Identify each point on the number line.

4. Point *B* −3
5. Point *C* 0

Graph each pair of integers on the number line above.

6. −8 and its opposite
7. −4 and its opposite
8. −1 and its opposite
9. +2 and its opposite
10. −10 and its opposite
11. +6 and its opposite

Lesson PL2 Square Roots

Find the square root of each number.

1. $\sqrt{4}$ 2
2. $\sqrt{441}$ 21
3. $\sqrt{25}$ 5
4. $\sqrt{64}$ 8
5. $\sqrt{100}$ 10
6. $\sqrt{324}$ 18
7. $\sqrt{144}$ 12
8. $\sqrt{256}$ 16
9. $\sqrt{81}$ 9
10. $\sqrt{225}$ 15
11. $\sqrt{121}$ 11
12. $\sqrt{400}$ 20

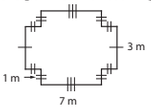
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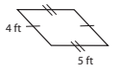
Extra Practice

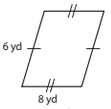
Name _____ Class _____ Date _____

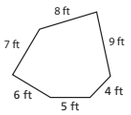
Lesson 1 Perimeter

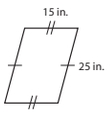
Find the perimeter of each figure.

1. 

P = 28 m
2. 

P = 18 ft
3. 

P = 28 yd
4. 

P = 39 ft
5. 

P = 80 in.
6. 

P = 45 ft

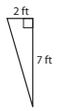
Lesson 2 Area of Triangles and Compound Figures

Find the area of each figure.

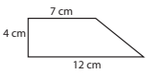
1. 

A = 21 ft²
2. 

A = 4 in.²
3. 

A = 27 mi²
4. 

A = 7 ft²
5. 

A = 10.5 in.²
6. 

A = 38 cm²

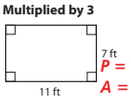
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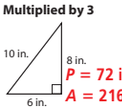
Extra Practice

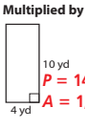
Name _____ Class _____ Date _____

Lesson 3 Perimeter, Area, and Proportional Dimension Changes

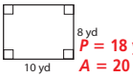
Find the changed perimeter and area of each figure if the dimensions are changed as described.

1. **Multiplied by 3**


P = 108 ft
A = 693 ft²
2. **Multiplied by 3**


P = 72 in.
A = 216 in.²
3. **Multiplied by 5**


P = 140 yd
A = 1,000 yd²
4. **Divided by 4**

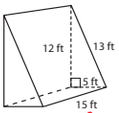

P = 7.5 ft
A = 1.875 ft²
5. **Divided by 2**


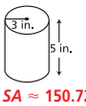
P = 18 yd
A = 20 yd²
6. **Divided by 3**

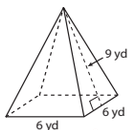

P = 8 mi
A = 4 mi²

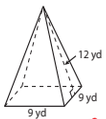
Lesson 5 Surface Area

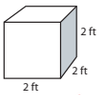
Find the surface area of each three-dimensional figure. Use 3.14 for π.

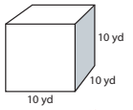
1. 

SA = 510 ft²
2. 

SA ≈ 150.72 in.²
3. 

SA = 144 yd²
4. 

SA = 297 yd²
5. 

SA = 24 ft²
6. 

SA = 600 yd²

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Extra Practice

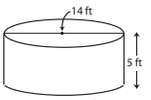
Name _____ Class _____ Date _____

Lesson 6 Volume of Cylinders, Spheres, and Cones

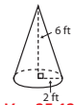
Find the volume of each three-dimensional figure. Use 3.14 for π.

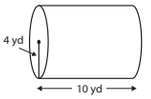
1. 

V ≈ 150.72 m³
2. 

V ≈ 113.04 mm³
3. 

V ≈ 769.3 ft³
4. 

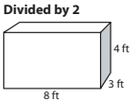
V ≈ 904.32 in.³
5. 

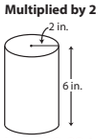
V ≈ 25.12 ft³
6. 

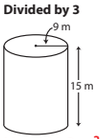
V ≈ 502.4 yd³

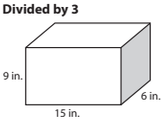
Lesson 7 Volume and Proportional Dimension Changes

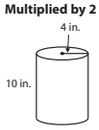
Find the changed volume of each figure if the dimensions are changed as described. Use 3.14 for π.

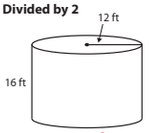
1. **Divided by 2**


V = 12 ft³
2. **Multiplied by 2**


V ≈ 602.88 in.³
3. **Divided by 3**


V ≈ 141.3 m³
4. **Divided by 3**


V = 30 in.³
5. **Multiplied by 2**


V ≈ 4,019.2 in.³
6. **Divided by 2**


V ≈ 904.32 ft³

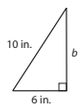
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Extra Practice

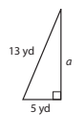
Name _____ Class _____ Date _____

Lesson 9 Pythagorean Theorem

Use the Pythagorean Theorem to find the missing length in each right triangle.

1. 

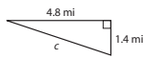
$b = 8$ in.

2. 

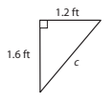
$a = 12$ yd

3. 

$a = 9$ mi

4. 

$c = 5$ mi.

5. 

$c = 2$ ft

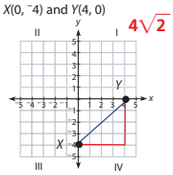
6. 

$a = 14$ ft

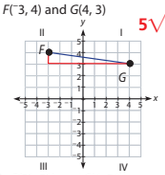
Lesson 10 Distance Between Points

Find the distance between each pair of points.

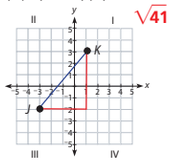
1. $X(0, -4)$ and $Y(4, 0)$ — $4\sqrt{2}$ units



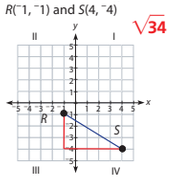
2. $F(-3, 4)$ and $G(4, 3)$ — $5\sqrt{2}$ units



3. $J(-3, -2)$ and $K(1, 3)$ — $\sqrt{41}$ units



4. $R(-1, -1)$ and $S(4, -4)$ — $\sqrt{34}$ units



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Glossary

base of a triangle
the side of the triangle that is perpendicular to the line segment representing the height

height of a cylinder
the perpendicular distance between the two bases of a cylinder

height of a triangle
the length of the perpendicular line segment drawn from a vertex to the opposite side of the triangle

hypotenuse
the side opposite the right angle of the triangle and the longest side in a right triangle

integers
the whole numbers and their opposites

lateral surface
the curved surface of a cylinder

legs of a right triangle
the two sides of the triangle that form the right angle

negative integers
integers less than 0

opposite of an integer
the integer that is the same distance from 0 on a number line, but in the opposite direction

perfect square
a whole number that is the product of a number and itself

perimeter
the distance around the outside of a figure

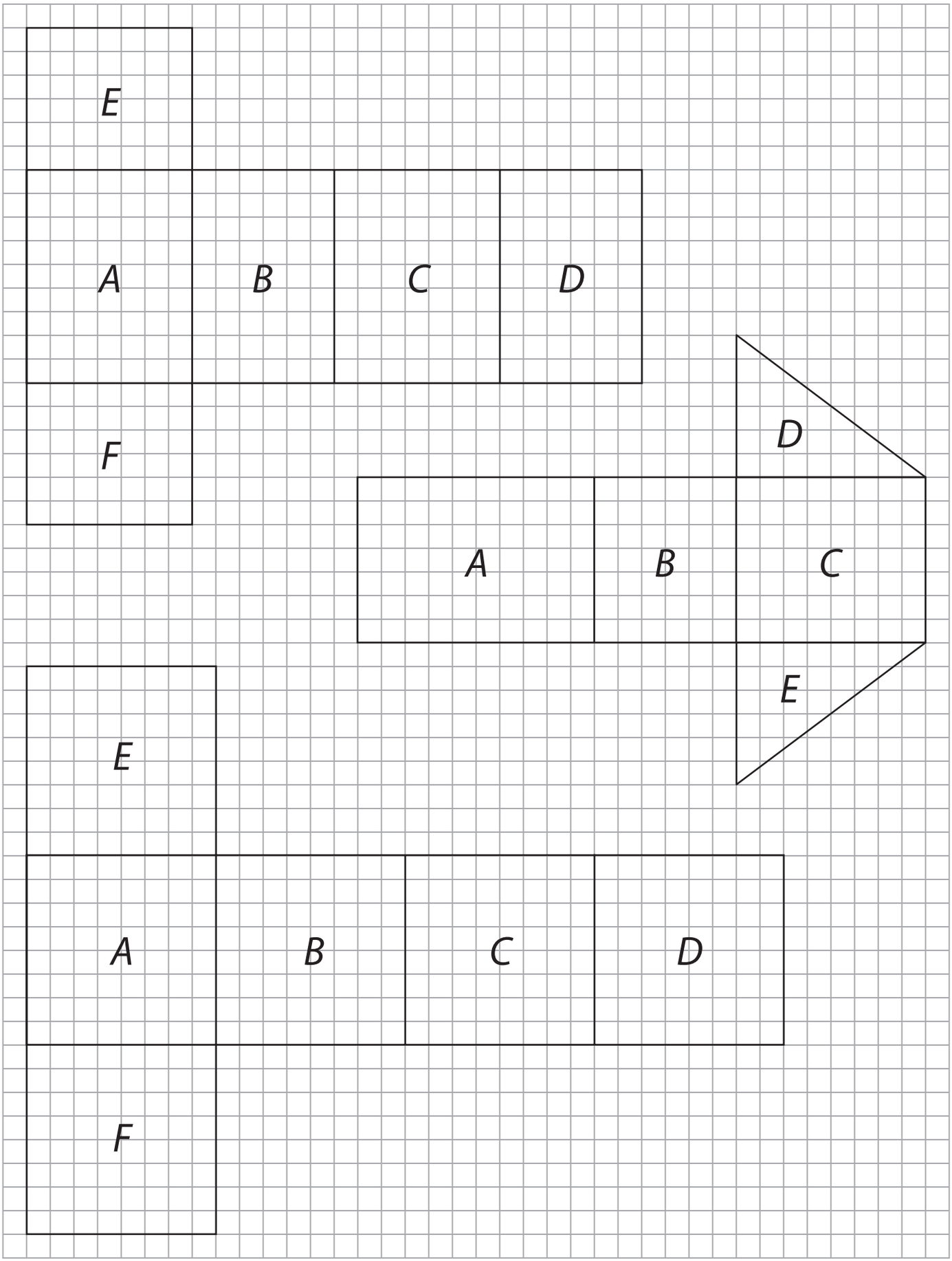
positive integers
integers greater than 0

radical sign
another name for the square root symbol $\sqrt{}$

square root
a number that when multiplied by itself is equal to a given number

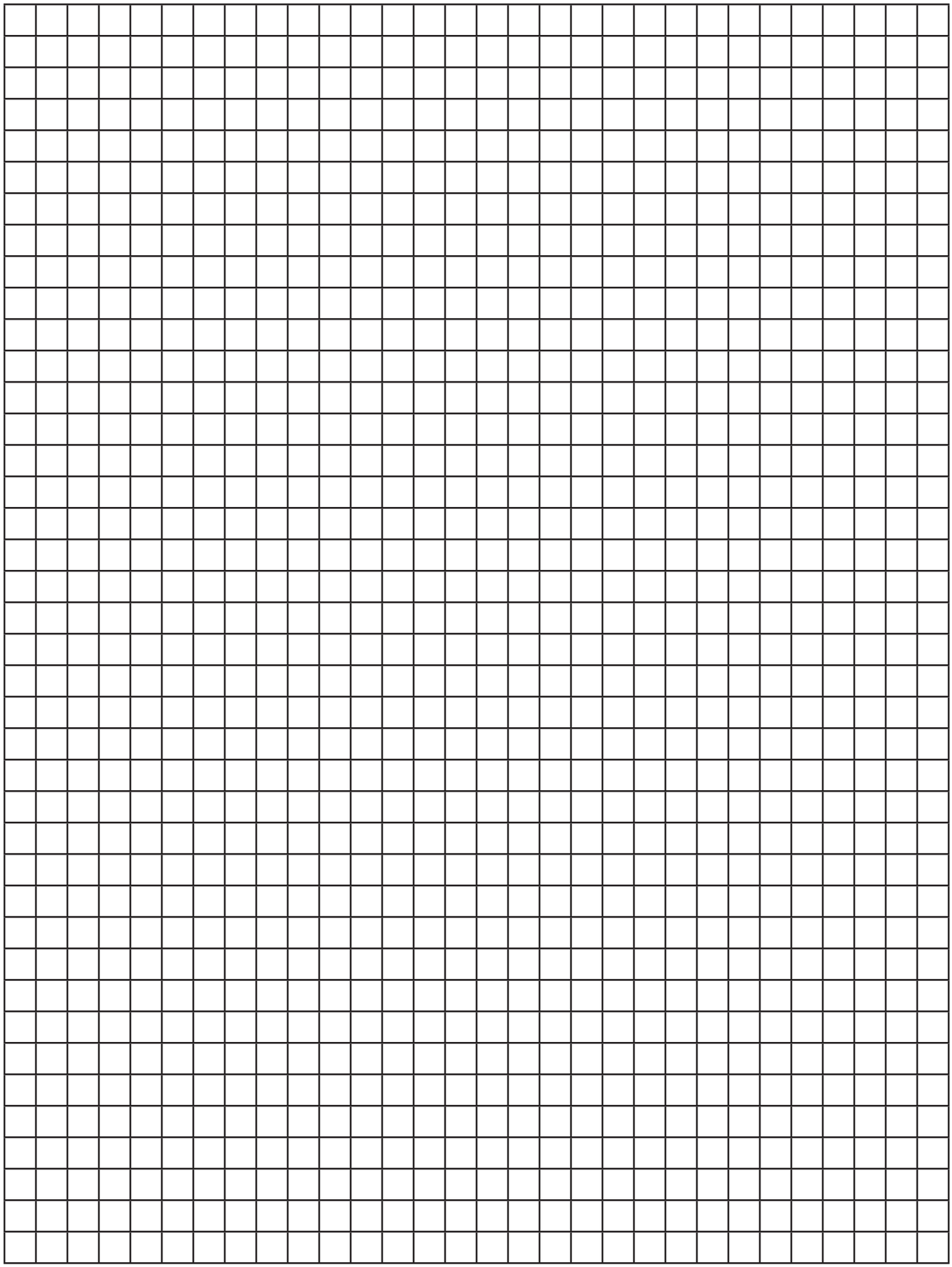
surface area
the sum of the areas of the faces of a figure

Copy Master: Nets



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Copy Master: Grid Paper



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Additional Materials

Name _____ Class _____ Date _____

Addition Facts to 18

1 + 1 = _____	1 + 2 = _____	1 + 3 = _____
2 + 1 = _____	2 + 2 = _____	2 + 3 = _____
3 + 1 = _____	3 + 2 = _____	3 + 3 = _____
4 + 1 = _____	4 + 2 = _____	4 + 3 = _____
5 + 1 = _____	5 + 2 = _____	5 + 3 = _____
6 + 1 = _____	6 + 2 = _____	6 + 3 = _____
7 + 1 = _____	7 + 2 = _____	7 + 3 = _____
8 + 1 = _____	8 + 2 = _____	8 + 3 = _____
9 + 1 = _____	9 + 2 = _____	9 + 3 = _____

1 + 4 = _____	1 + 5 = _____	1 + 6 = _____
2 + 4 = _____	2 + 5 = _____	2 + 6 = _____
3 + 4 = _____	3 + 5 = _____	3 + 6 = _____
4 + 4 = _____	4 + 5 = _____	4 + 6 = _____
5 + 4 = _____	5 + 5 = _____	5 + 6 = _____
6 + 4 = _____	6 + 5 = _____	6 + 6 = _____
7 + 4 = _____	7 + 5 = _____	7 + 6 = _____
8 + 4 = _____	8 + 5 = _____	8 + 6 = _____
9 + 4 = _____	9 + 5 = _____	9 + 6 = _____

1 + 7 = _____	1 + 8 = _____	1 + 9 = _____
2 + 7 = _____	2 + 8 = _____	2 + 9 = _____
3 + 7 = _____	3 + 8 = _____	3 + 9 = _____
4 + 7 = _____	4 + 8 = _____	4 + 9 = _____
5 + 7 = _____	5 + 8 = _____	5 + 9 = _____
6 + 7 = _____	6 + 8 = _____	6 + 9 = _____
7 + 7 = _____	7 + 8 = _____	7 + 9 = _____
8 + 7 = _____	8 + 8 = _____	8 + 9 = _____
9 + 7 = _____	9 + 8 = _____	9 + 9 = _____

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Name _____ Class _____ Date _____

Subtraction Facts to 18

18 − 9 = _____	14 − 9 = _____	13 − 4 = _____
17 − 9 = _____	14 − 8 = _____	12 − 9 = _____
17 − 8 = _____	14 − 7 = _____	12 − 8 = _____
16 − 9 = _____	14 − 6 = _____	12 − 7 = _____
16 − 8 = _____	14 − 5 = _____	12 − 6 = _____
16 − 7 = _____	13 − 9 = _____	12 − 5 = _____
15 − 9 = _____	13 − 8 = _____	12 − 4 = _____
15 − 8 = _____	13 − 7 = _____	12 − 3 = _____
15 − 7 = _____	13 − 6 = _____	11 − 9 = _____
15 − 6 = _____	13 − 5 = _____	11 − 8 = _____
11 − 7 = _____	10 − 5 = _____	9 − 4 = _____
11 − 6 = _____	10 − 4 = _____	9 − 3 = _____
11 − 5 = _____	10 − 3 = _____	9 − 2 = _____
11 − 4 = _____	10 − 2 = _____	9 − 1 = _____
11 − 3 = _____	10 − 1 = _____	8 − 8 = _____
11 − 2 = _____	9 − 9 = _____	8 − 7 = _____
10 − 9 = _____	9 − 8 = _____	8 − 6 = _____
10 − 8 = _____	9 − 7 = _____	8 − 5 = _____
10 − 7 = _____	9 − 6 = _____	8 − 4 = _____
10 − 6 = _____	9 − 5 = _____	8 − 3 = _____
8 − 2 = _____	6 − 5 = _____	4 − 4 = _____
8 − 1 = _____	6 − 4 = _____	4 − 3 = _____
7 − 7 = _____	6 − 3 = _____	4 − 2 = _____
7 − 6 = _____	6 − 2 = _____	4 − 1 = _____
7 − 5 = _____	6 − 1 = _____	3 − 3 = _____
7 − 4 = _____	5 − 5 = _____	3 − 2 = _____
7 − 3 = _____	5 − 4 = _____	3 − 1 = _____
7 − 2 = _____	5 − 3 = _____	2 − 2 = _____
7 − 1 = _____	5 − 2 = _____	2 − 1 = _____
6 − 6 = _____	5 − 1 = _____	1 − 1 = _____

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Name _____ Class _____ Date _____

Multiplication Facts

$1 \times 1 =$ _____	$1 \times 2 =$ _____	$1 \times 3 =$ _____	$1 \times 4 =$ _____
$2 \times 1 =$ _____	$2 \times 2 =$ _____	$2 \times 3 =$ _____	$2 \times 4 =$ _____
$3 \times 1 =$ _____	$3 \times 2 =$ _____	$3 \times 3 =$ _____	$3 \times 4 =$ _____
$4 \times 1 =$ _____	$4 \times 2 =$ _____	$4 \times 3 =$ _____	$4 \times 4 =$ _____
$5 \times 1 =$ _____	$5 \times 2 =$ _____	$5 \times 3 =$ _____	$5 \times 4 =$ _____
$6 \times 1 =$ _____	$6 \times 2 =$ _____	$6 \times 3 =$ _____	$6 \times 4 =$ _____
$7 \times 1 =$ _____	$7 \times 2 =$ _____	$7 \times 3 =$ _____	$7 \times 4 =$ _____
$8 \times 1 =$ _____	$8 \times 2 =$ _____	$8 \times 3 =$ _____	$8 \times 4 =$ _____
$9 \times 1 =$ _____	$9 \times 2 =$ _____	$9 \times 3 =$ _____	$9 \times 4 =$ _____

$1 \times 5 =$ _____	$1 \times 6 =$ _____	$1 \times 7 =$ _____	$1 \times 8 =$ _____
$2 \times 5 =$ _____	$2 \times 6 =$ _____	$2 \times 7 =$ _____	$2 \times 8 =$ _____
$3 \times 5 =$ _____	$3 \times 6 =$ _____	$3 \times 7 =$ _____	$3 \times 8 =$ _____
$4 \times 5 =$ _____	$4 \times 6 =$ _____	$4 \times 7 =$ _____	$4 \times 8 =$ _____
$5 \times 5 =$ _____	$5 \times 6 =$ _____	$5 \times 7 =$ _____	$5 \times 8 =$ _____
$6 \times 5 =$ _____	$6 \times 6 =$ _____	$6 \times 7 =$ _____	$6 \times 8 =$ _____
$7 \times 5 =$ _____	$7 \times 6 =$ _____	$7 \times 7 =$ _____	$7 \times 8 =$ _____
$8 \times 5 =$ _____	$8 \times 6 =$ _____	$8 \times 7 =$ _____	$8 \times 8 =$ _____
$9 \times 5 =$ _____	$9 \times 6 =$ _____	$9 \times 7 =$ _____	$9 \times 8 =$ _____

$1 \times 9 =$ _____	$1 \times 10 =$ _____
$2 \times 9 =$ _____	$2 \times 10 =$ _____
$3 \times 9 =$ _____	$3 \times 10 =$ _____
$4 \times 9 =$ _____	$4 \times 10 =$ _____
$5 \times 9 =$ _____	$5 \times 10 =$ _____
$6 \times 9 =$ _____	$6 \times 10 =$ _____
$7 \times 9 =$ _____	$7 \times 10 =$ _____
$8 \times 9 =$ _____	$8 \times 10 =$ _____
$9 \times 9 =$ _____	$9 \times 10 =$ _____

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Name _____ Class _____ Date _____

Division Facts

1 ÷ 1 = _____	2 ÷ 2 = _____	3 ÷ 3 = _____	4 ÷ 4 = _____
2 ÷ 1 = _____	4 ÷ 2 = _____	6 ÷ 3 = _____	8 ÷ 4 = _____
3 ÷ 1 = _____	6 ÷ 2 = _____	9 ÷ 3 = _____	12 ÷ 4 = _____
4 ÷ 1 = _____	8 ÷ 2 = _____	12 ÷ 3 = _____	16 ÷ 4 = _____
5 ÷ 1 = _____	10 ÷ 2 = _____	15 ÷ 3 = _____	20 ÷ 4 = _____
6 ÷ 1 = _____	12 ÷ 2 = _____	18 ÷ 3 = _____	24 ÷ 4 = _____
7 ÷ 1 = _____	14 ÷ 2 = _____	21 ÷ 3 = _____	28 ÷ 4 = _____
8 ÷ 1 = _____	16 ÷ 2 = _____	24 ÷ 3 = _____	32 ÷ 4 = _____
9 ÷ 1 = _____	18 ÷ 2 = _____	27 ÷ 3 = _____	36 ÷ 4 = _____
10 ÷ 1 = _____	20 ÷ 2 = _____	30 ÷ 3 = _____	40 ÷ 4 = _____

5 ÷ 5 = _____	6 ÷ 6 = _____	7 ÷ 7 = _____	8 ÷ 8 = _____
10 ÷ 5 = _____	12 ÷ 6 = _____	14 ÷ 7 = _____	16 ÷ 8 = _____
15 ÷ 5 = _____	18 ÷ 6 = _____	21 ÷ 7 = _____	24 ÷ 8 = _____
20 ÷ 5 = _____	24 ÷ 6 = _____	28 ÷ 7 = _____	32 ÷ 8 = _____
25 ÷ 5 = _____	30 ÷ 6 = _____	35 ÷ 7 = _____	40 ÷ 8 = _____
30 ÷ 5 = _____	36 ÷ 6 = _____	42 ÷ 7 = _____	48 ÷ 8 = _____
35 ÷ 5 = _____	42 ÷ 6 = _____	49 ÷ 7 = _____	56 ÷ 8 = _____
40 ÷ 5 = _____	48 ÷ 6 = _____	56 ÷ 7 = _____	64 ÷ 8 = _____
45 ÷ 5 = _____	54 ÷ 6 = _____	63 ÷ 7 = _____	72 ÷ 8 = _____
50 ÷ 5 = _____	60 ÷ 6 = _____	70 ÷ 7 = _____	80 ÷ 8 = _____

9 ÷ 9 = _____	10 ÷ 10 = _____
18 ÷ 9 = _____	20 ÷ 10 = _____
27 ÷ 9 = _____	30 ÷ 10 = _____
36 ÷ 9 = _____	40 ÷ 10 = _____
45 ÷ 9 = _____	50 ÷ 10 = _____
54 ÷ 9 = _____	60 ÷ 10 = _____
63 ÷ 9 = _____	70 ÷ 10 = _____
72 ÷ 9 = _____	80 ÷ 10 = _____
81 ÷ 9 = _____	90 ÷ 10 = _____
90 ÷ 9 = _____	100 ÷ 10 = _____

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Name _____ Class _____ Date _____

Addition Grid

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

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Name _____ Class _____ Date _____

Multiplication Grid

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

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Name _____ Class _____ Date _____

Decimal Place Value Chart

Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

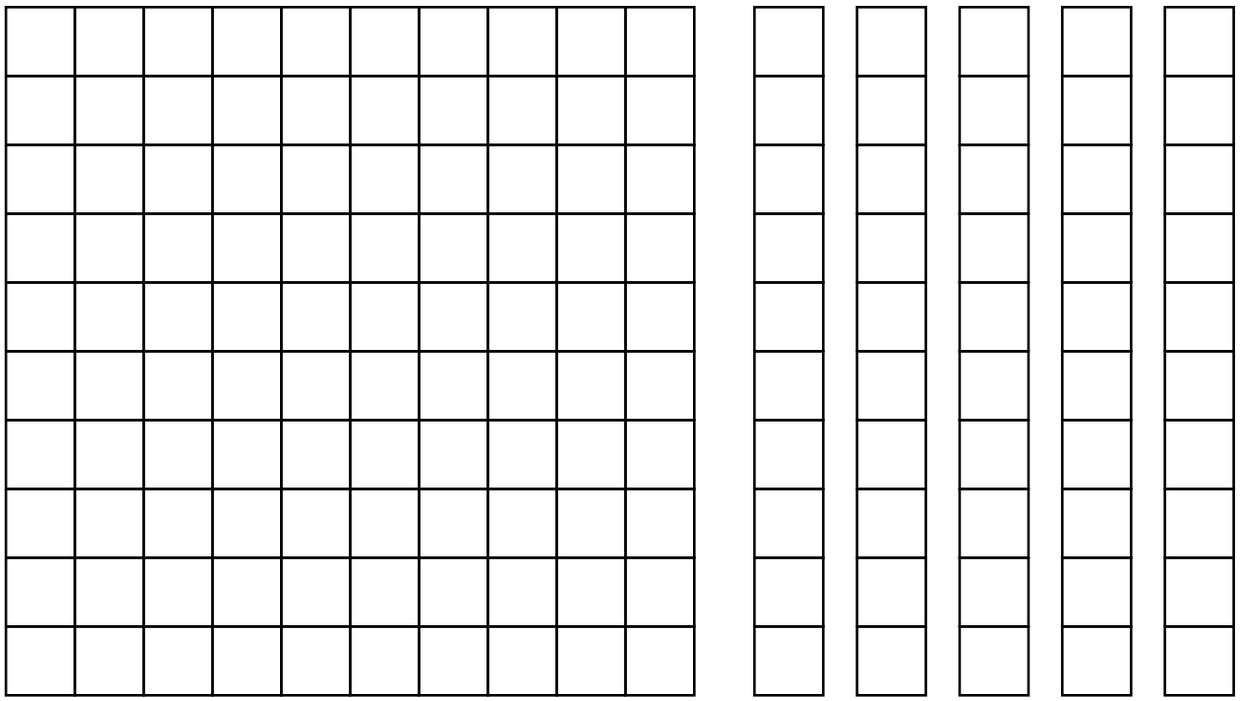
Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

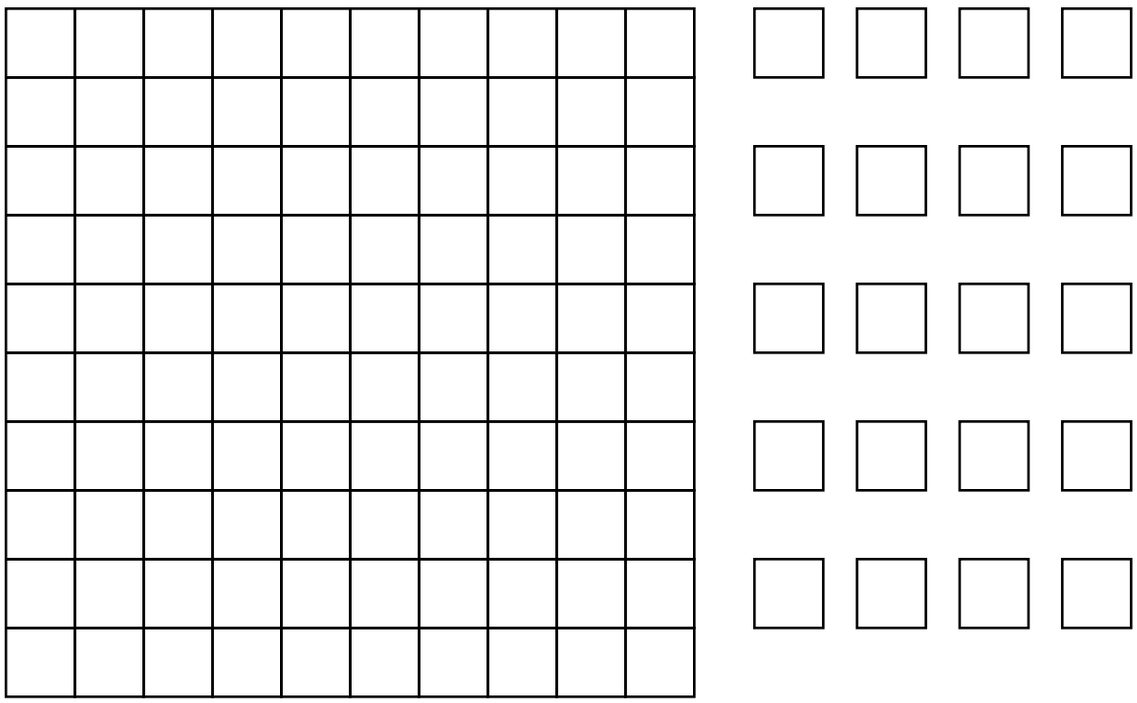
Whole Number Places				.	Decimal Places	
Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths
				.		
				.		

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Name _____ Class _____ Date _____

Base-10 Pieces

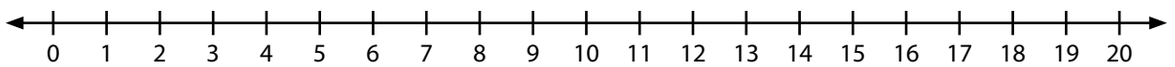


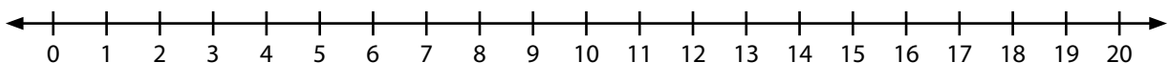


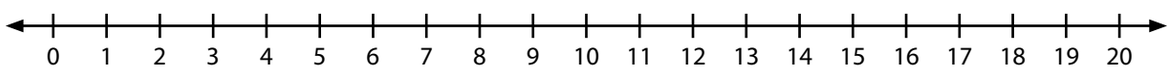
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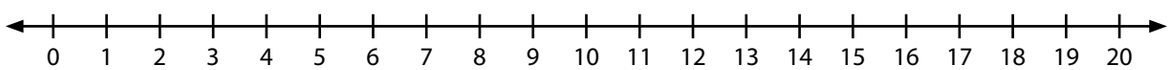
Name _____ Class _____ Date _____

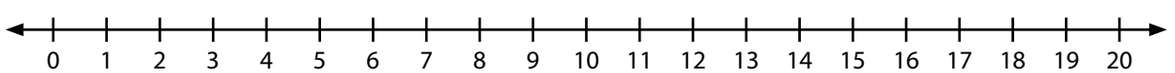
Number Lines

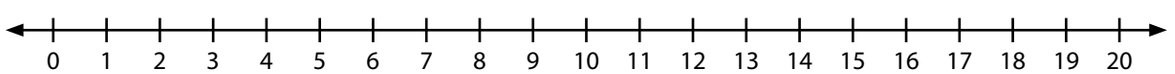


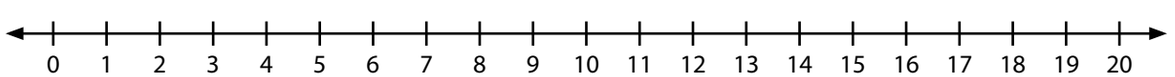


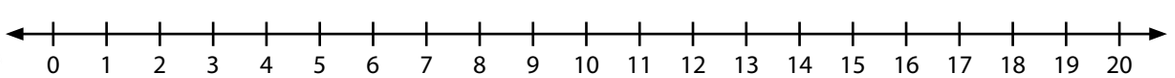












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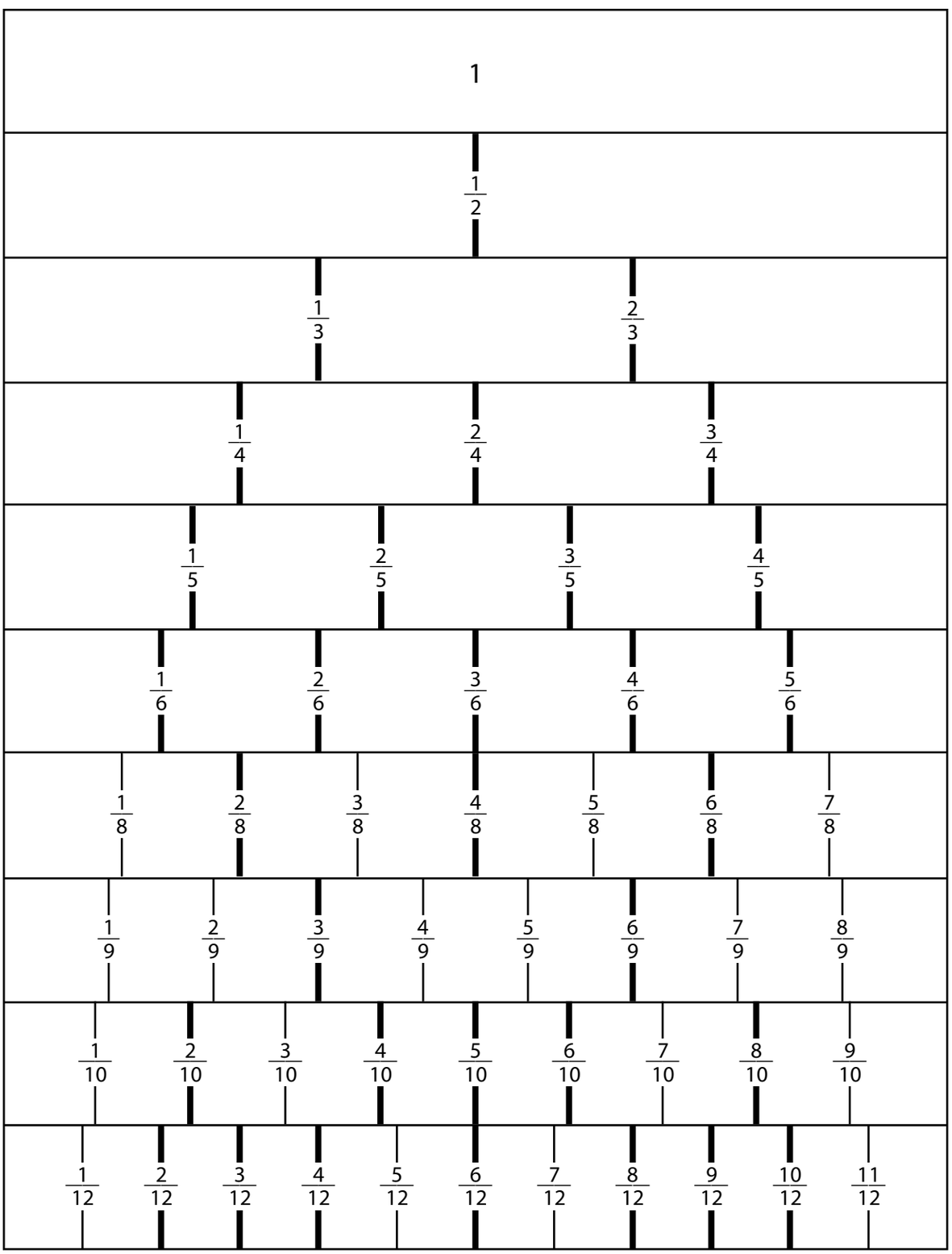
Fraction Strips

1											
$\frac{1}{2}$						$\frac{1}{2}$					
$\frac{1}{3}$				$\frac{1}{3}$				$\frac{1}{3}$			
$\frac{1}{4}$			$\frac{1}{4}$			$\frac{1}{4}$			$\frac{1}{4}$		
$\frac{1}{5}$		$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$
$\frac{1}{6}$			$\frac{1}{6}$			$\frac{1}{6}$			$\frac{1}{6}$		
$\frac{1}{8}$				$\frac{1}{8}$				$\frac{1}{8}$			
$\frac{1}{9}$				$\frac{1}{9}$				$\frac{1}{9}$			
$\frac{1}{10}$				$\frac{1}{10}$				$\frac{1}{10}$			
$\frac{1}{12}$				$\frac{1}{12}$				$\frac{1}{12}$			

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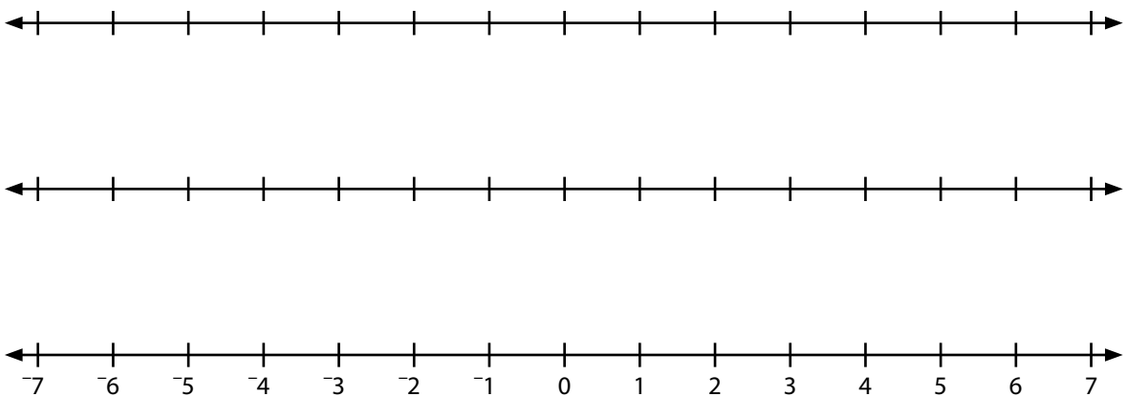
Equivalent Fractions

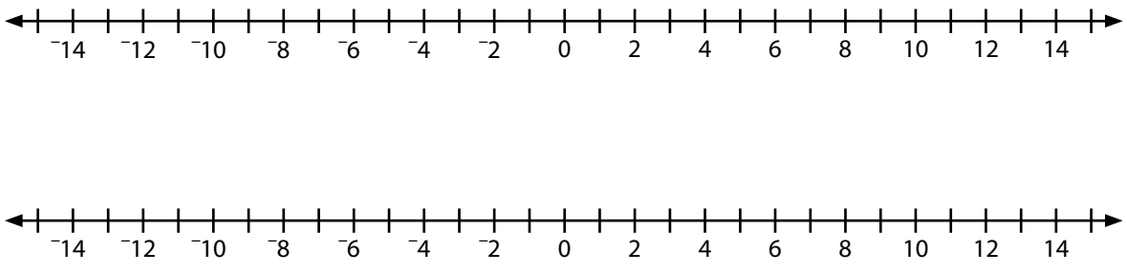


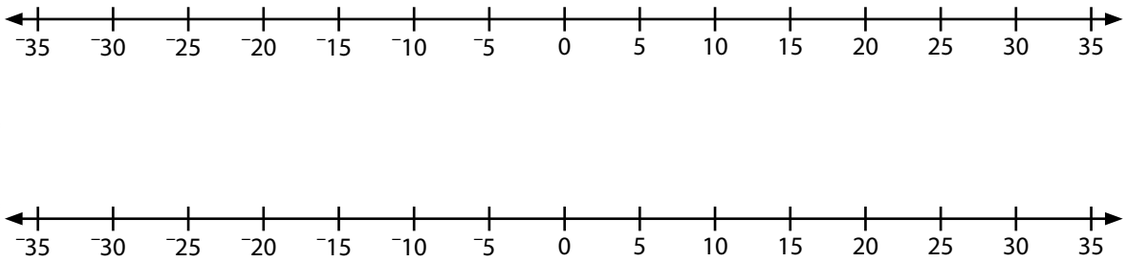
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Name _____ Class _____ Date _____

Integer Number Lines



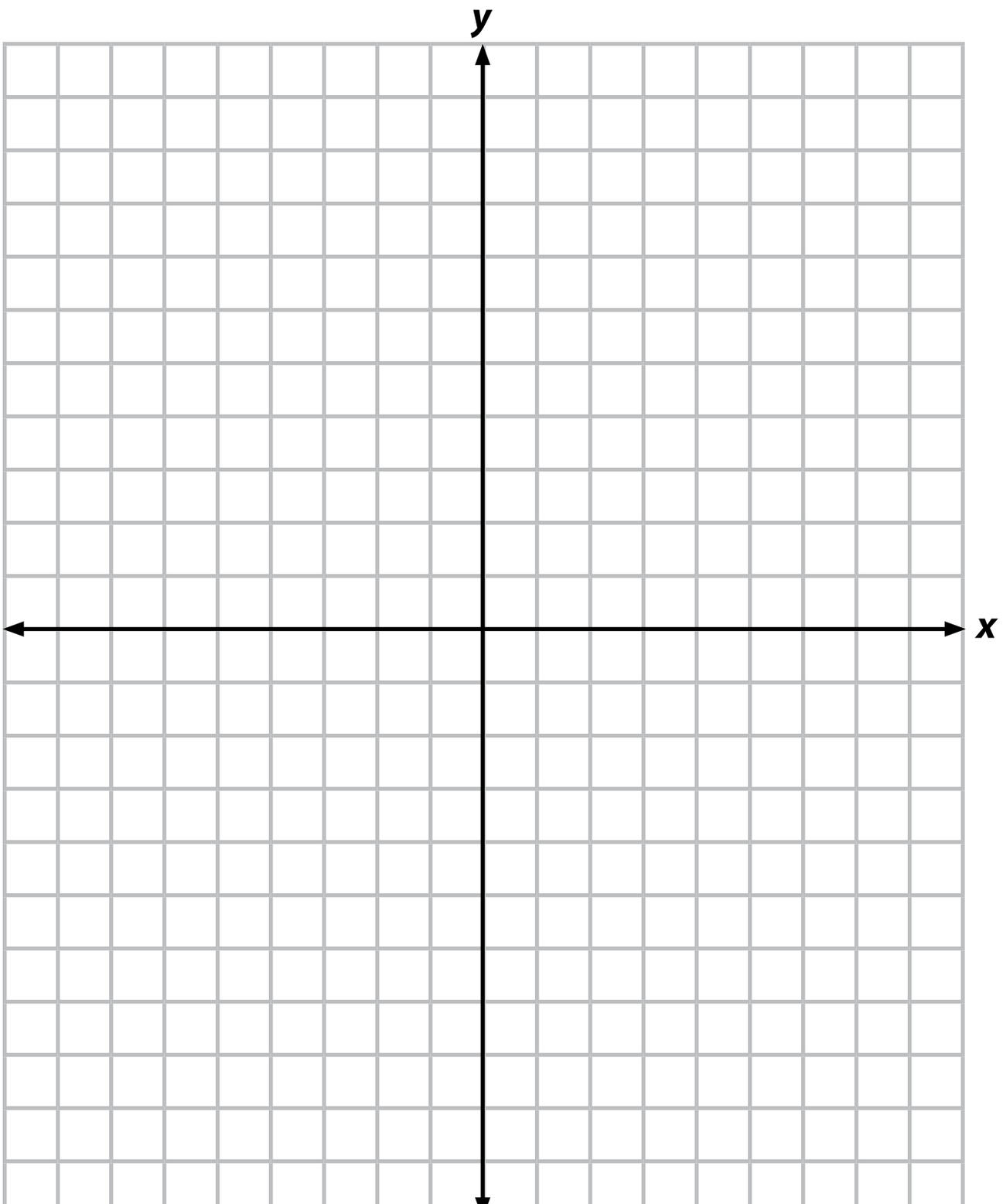




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Name _____ Class _____ Date _____

Coordinate Graph



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Glossary

Glossary

adjacent angles
two angles that share a vertex and a common side and do not overlap

algebraic expression
an expression that can contain one or more numerals, one or more variables, and one or more operations

alternate exterior angles
a pair of angles formed by two lines that are cut by a transversal; both angles are exterior angles on opposite sides of the transversal at different vertices

alternate interior angles
a pair of angles formed by two lines that are cut by a transversal; both angles are interior angles on opposite sides of the transversal at different vertices

ASA
ASA stands for angle-side-angle. The rule states that if two angles and the side between the angles of one triangle are congruent to two angles and the side between the angles of another triangle, the triangles are congruent.

Associative Property of Multiplication
three factors can be grouped in different ways and the product will be the same

base
the number used as a factor in repeated multiplication

base of a triangle
the side of the triangle that is perpendicular to the line segment representing the height

Commutative Property of Multiplication
two factors can be multiplied in any order and the product will be the same

complementary angles
two angles whose measures have a sum of 90°

congruent angles
two angles with the same measure

consecutive angles
angles that share a common side

coordinate plane
a graph formed by two number lines intersecting at a right angle

corresponding angles
a pair of angles formed by two lines that are cut by a transversal; one angle is an interior angle and the other an exterior angle on the same side of the transversal at different vertices

data point
the representation of related values on a graph

data set
a collection of related values

dilation
enlarging or reducing a geometric figure to create a similar figure

Distributive Property
the property of distributing one operation over another, and the answer is the same

Examples: $a(b + c) = a \times b + a \times c$ and $a(b - c) = a \times b - a \times c$

domain
the set of all the first elements of the ordered pairs of a relation

equivalent ratios
ratios that can be written as equivalent fractions

exponent
a number that indicates how many times a base is used as a factor in repeated multiplication

function
a pairing of input and output values in which each input value is paired with exactly one output value

function rule
an equation that shows how an input value, x, and an output value, y, are related

height of a cylinder
the perpendicular distance between the two bases of a cylinder

height of a triangle
the length of the perpendicular line segment drawn from a vertex to the opposite side of the triangle

Glossary

hypotenuse
the side opposite the right angle of the triangle and the longest side in a right triangle

inequality
a number sentence that contains the symbol > (greater than) or < (less than)

input-output table
a table that shows how an input value, x, is paired with an output value, y

integers
the whole numbers and their opposites

intersecting lines
lines that cross at one point

lateral surface
the curved surface of a cylinder

legs of a right triangle
the two sides of the triangle that form the right angle

line
a straight path with no endpoints

line of best fit
the line that comes closest to all points on a scatter plot

line segment
part of a line with two endpoints

linear equation
an equation whose graph is a straight line

linear function
a function whose graph is a line

mean absolute deviation
the average distance between each data value and the mean

multiple
the product of the whole number and another whole number

Example: $4 \times 10 = 40$; 40 is the fourth multiple of 10.

negative correlation
the trend of values decreasing as related values increase

negative integers
integers less than 0

nonlinear function
a function whose graph is not a line

opposite angles of a parallelogram
nonconsecutive angles of a parallelogram

opposite of an integer
the integer that is the same distance from 0 on a number line, but in the opposite direction

ordered pair
a pair of numbers used to locate a point on a coordinate plane

origin
the point where the x-axis and y-axis intersect

parallel lines
lines that are in the same plane and never intersect; the symbol || means "is parallel to"

perfect square
a number that is the product of an integer with itself

perimeter
the distance around the outside of a figure

perpendicular lines
lines that intersect to form 90° angles; the symbol $\perp$ means "is perpendicular to"

plane
a flat surface with no boundaries

point
a location in space that has no size

point-slope form
the equation of a straight line in the form $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) are the coordinates of a given point on the line

positive correlation
the trend of values increasing as related values increase

positive integers
integers greater than 0

proportion
an equation that shows two ratios are equivalent

Glossary

proportional
a linear relationship that can be represented in the form $y = mx$

quadrants
the four sections formed by the intersection of the x-axis and y-axis; labeled with Roman numerals

radical sign
another name for the square root symbol

range
the set of all the second elements of the ordered pairs of a relation

ratio
a comparison of two quantities by division

rational number
a number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not 0

ray
part of a line with one endpoint

real number
a number that represents a quantity along a continuous line

reciprocal
for any nonzero number x, the reciprocal is $\frac{1}{x}$; the number 0 has no reciprocal; the reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$

reflection
moving a geometric figure by flipping over a line

relation
a pairing of numbers in one set, the domain, with numbers in another set, the range

rise
the vertical change between any two points on a line

rotation about the origin
moving a geometric figure by turning it about the origin

run
the horizontal change between any two points on a line

SAS
SAS stands for side-angle-side. The rule states that if two sides and the angle between the sides of one triangle are congruent to two sides and the angle between the sides of another triangle, the triangles are congruent.

scale factor
the ratio of the dimensions of an image to the dimensions of an original figure

scatter plot
a graph made by plotting points on a coordinate plane to show the relationship between two variables in a data set

scientific notation
a number expressed as the product of two factors; the first factor is a number greater than or equal to 1 and less than 10, and the second is a power of 10

similar figures
geometric figures that are the same shape but not necessarily the same size

slope
the ratio of the vertical change, rise, to the horizontal change, run, between any two points on a line

slope-intercept form
the equation of a straight line in the form $y = mx + b$, where m is the slope of the line and b is its y-intercept

solution of a system of equations
an ordered pair that is a solution of every equation in the system

square root
a number that when multiplied by itself is equal to a given number

SSS
SSS stands for side-side-side. The rule states that if three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

standard form
the equation of a straight line in the form $ax + by = c$, where $a \neq 0$ and $b \neq 0$

supplementary angles
two angles whose measures have a sum of 180°

surface area
the sum of the areas of the faces of a figure

system of linear equations
two or more linear equations considered together

transformation
a translation, a reflection, or a rotation

translation
moving a geometric figure by sliding

transversal
a line that intersects two or more lines at different points

variable
a letter or symbol that represents a number

vertical angles
the opposite congruent angles formed by two intersecting lines

***x*-axis**
the horizontal number line of a coordinate plane

***x*-intercept**
the point where a graph crosses the *x*-axis

***y*-axis**
the vertical number line of a coordinate plane

***y*-intercept**
the point where a graph crosses the *y*-axis

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